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Multi-Ball Collisions

Terry L. Smith, *University of North Florida*
Jay S Huebner

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Jay S. Huebner and Terry L. Smith

University of North Florida, Jacksonville, FL 32216-6699

Professor Egler's suggestion[1] to use the "double-ball" demonstration as a model for the supernova core bounce is interesting and useful. However, a demonstration risking student injury from the softball possibly rebounding from the basketball into the class should be published with more of a warning. Softballs are hard and heavy, typically with a mass of 190 g,[2] and should be replaced in these demonstrations with tennis balls. Even soft foam "Nerf" balls will perform well enough to make the physical action clear. Or, alternatively, the balls could be confined inside transparent plastic or screen tubes, or perhaps tethered. Furthermore, higher velocities are achieved using more balls and multiple collisions, which although harder to control, give a more dramatic display and a better approximation of collisions between multiple layers of a collapsing star.

The reason for the increased velocities of the top ball can easily be explained by switching between various moving (Galilean) frames of reference. It is assumed here that the collisions are perfectly elastic, occur between the balls sequentially from the bottom up, and the mass of each ball is negligible compared to the ball below it. Also, friction with any air is ignored. The figures consider the case where three balls are dropped in a vertical stack and reach a velocity, V, in the Earth's frame just before the first collision. Ball 1 rebounds from the Earth after that collision with velocity V, and encounters Ball 2 still coming down. From Ball 1's frame, Ball 2 has a velocity of 2 V before the second collision (between Balls 1 and 2) and so it rebounds with a velocity of 2 V relative to Ball 1. From Earth's frame, Ball 2 then has an upward velocity of 3 V (see the figures).

Although this double-ball collision is interesting and gives students the general idea, the discussion shouldn't stop here. Tripling the velocity is hardly adequate to explain the relativistic velocities achieved in a supernova core bounce, and just a little more discussion, as follows, will yield much more student interest and understanding.

The figures show that Ball 3 has a downward velocity of 4 V from Ball 2's frame just after the second collision. So it will rebound from Ball 2 with 4 V, giving Ball 3 an upward velocity (in the Earth's frame) of 7 V. The velocity of the n^{th} ball, V_n, can be seen from this to be $V_n = 2\,V_{n-1} + V$, which gives 15V for a fourth ball, 31 V for a fifth ball, etc.

A general formula for the velocity achieved by the top ball on a stack of n balls is desired. The velocities given above can be seen to involve a sum on n powers of 2, as: $V_1 = (2^0)\ V = V$, $V_2 = (2^1 + 2^0)\ V = 3\ V$, $V_3 = (2^2 + 2^1 + 2^0)\ V = 7\ V$, etc. These values are generated by the sum,

$$V_n = V \sum_{k=0}^{n-1} 2^k \qquad (1)$$

and are also given by $V_n = V\,(2^n - 1)$. This last equation is a convenient general formula.

Students respond to this demonstration because it produces a surprising result: the top ball rebounds to an unexpected height. Now we understand why higher velocities are achieved. But, just how high will the top ball go? Analysis of this question is simplified by making what should be called "the flat Earth assumption," namely that the gravitational force on a ball of mass M is constant (Mg), independent of its distance above the Earth. This produces the well-known but approximate result for the ball's potential energy, Mgh, where h is the ball's height above the Earth's surface. A rebounding ball converts its kinetic energy into potential energy. The maximum height reached is obtained by

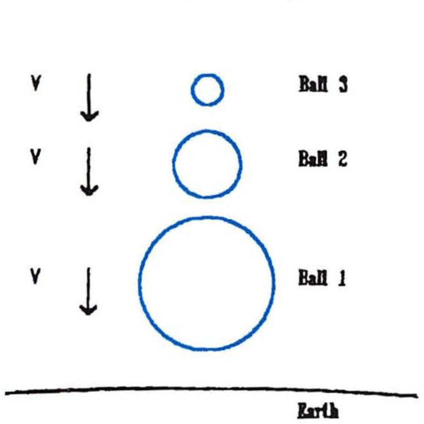

Fig. 1. Just before first collision (Earth's frame).

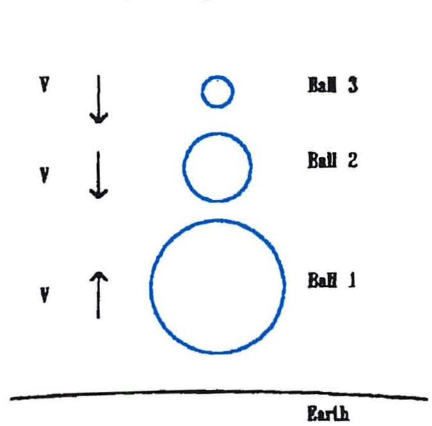

Fig. 2. Before second collision (Earth's frame).

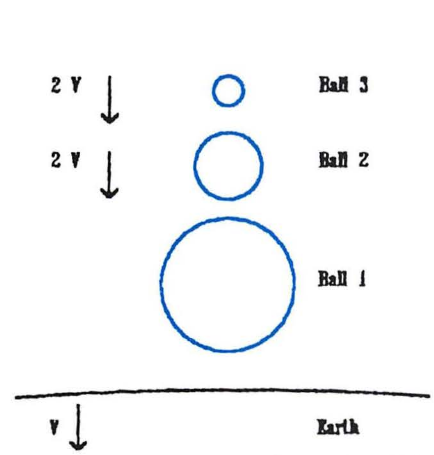

Fig. 3. Before second collision (Ball 1's frame).

equating the kinetic energy at the surface to its potential energy maximum, Mgh, and solving for h, which gives $h = (V^2/2g)$. Substituting the general form of V_n into this equation for V, the maximum height reached by the top n^{th} ball, h_n, may be seen to be $(2^n - 1)^2$ times the height the balls fell. This indicates that if balls are dropped 1 m, the second ball (of two) could reach a maximum height of 9 m, the third (of three) 49 m, etc. These calculations ignore the ball's diameters, which would introduce a small correction.

In our experience, it is also helpful to put these results into a general context by making comparisons with other collisions. Collisions are typically analyzed by considering the total energy and momentum of all interacting particles, quantities that must be conserved in nature. More detailed calculations still assuming perfectly elastic collisions and using the masses of a softball and basketball[2] dropped 1 m predict a maximum rebound of 6.47 m, and for a tennis ball, softball, and basketball, 23.9 m. Real collisions with these objects are, of course, inelastic. The basketball we used rebounds 0.75 m when dropped 1 m. With a tennis ball on it, the basketball rebounded only 0.45 m, showing that the extra energy acquired by the tennis ball came from the ball below it.

These demonstrations are also useful for explaining gravitationally assisted orbits (the so-called sling-shot effect) in which a satellite passing a moving planet acquires higher velocity at the expense of a small fraction of the planet's energy. The moving planet provides a moving frame for the collision. Students might initially find it odd to consider this a collision, since no direct contact is made between the satellite and planet, and the interaction occurs over an extended time. Gravitational forces do transfer energy and momentum between the interacting bodies, so these interactions must be considered collisions. The point here is that a moving frame analysis is useful whether the collision is between solid balls, between plasma layers in a star undergoing cataclysmic processes, or between a planet and a passing satellite.

Moving frames also are useful in explaining how heat engines extract energy and work from hot gases, and why compressing a gas adiabatically heats it. Consider the two-ball case, where the lower ball is identified as a piston, which is moving towards the gas so as to compress it, and the upper ball is one of many gas molecules. The resulting collision causes the gas molecule to rebound with greater speed, which is equivalent to an increased gas temperature, showing that the piston heats a gas as it is compressed. Alternatively, if the lower ball is considered to be a gas molecule and the upper ball a piston, which is moving away from the gas, the results described above for the basketball and tennis ball show that the gas molecule (the basketball) loses energy and therefore is cooled by the collision. This cooling is a result of work being done on the piston as energy is taken from the gas.

References

1. R.A. Egler, *Phys. Teach.* **28**, 558 (1990).
2. J.A. Cuddon, *The International Dictionary of Sports and Games* (Schocken Books, New York, 1979).

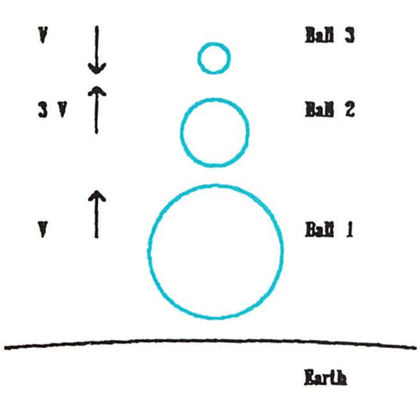

Fig. 4. After second collision (Earth's frame).

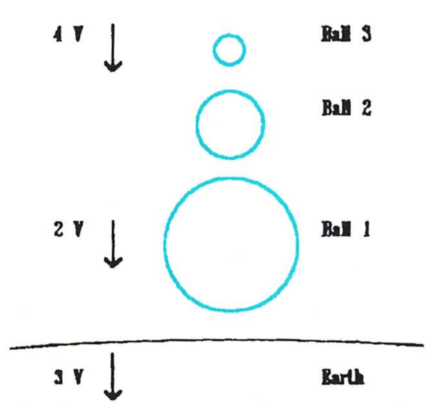

Fig. 5. After second collision (Ball 2's frame).

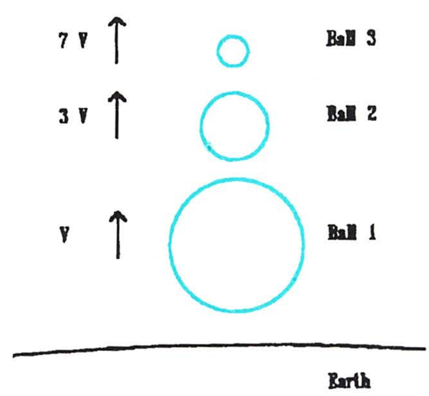

Fig. 6. After third collision (Earth's frame).