CHAPTER

TWO

TWO-TERMINAL RESISTORS

Two-terminal elements play a major role in electric circuits. As a matter of fact, many introductory texts on electric circuits consider circuits consisting only of two-terminal elements exclusively. In this chapter we give a comprehensive treatment of two-terminal resistors. However, unlike the usual terminology, a resistor may be linear, nonlinear, time-invariant, or time-varying. It is characterized by a relation between the branch voltage and the branch current. We speak of the v-i characteristic of a resistor, and we discuss the characteristics of various types of resistors such as a linear resistor which satisfies Ohm's law, an ideal diode, a dc current source, a *pn*-junction diode, and a periodically operating switch. All of these are resistors.

By interconnecting two-terminal resistors, we form a resistive circuit. The simplest forms of interconnection, i.e., series, parallel, and series-parallel interconnections, will be treated and illustrated with examples. These require the use of Kirchhoff's laws together with branch equations which characterize the elements. A one-port formed by the interconnection of resistors is characterized by its driving-point characteristics relating its port voltage and its port current. We introduce the concepts of equivalence and duality of one-ports by simple examples. These will be generalized in later chapters.

An important problem in nonlinear circuits is the determination of the dc operating points, i.e., the solutions with dc inputs. Various methods and techniques are introduced and illustrated.

Another important problem in nonlinear circuits is the small-signal analysis. Its relation to dc operating points and the derivation of the small-signal equivalent circuit are treated by way of a simple example. This subject will be discussed in a more general fashion in later chapters.

Finally, we discuss the transfer characteristic of resistive circuits and demonstrate the usefulness of the graphic method in analyzing nonlinear resistive circuits.

1 v-i CHARACTERISTIC OF TWO-TERMINAL RESISTORS

1.1 From Linear Resistor to Resistor

The most familiar circuit element that one encounters in physics or in an elementary electrical engineering course is a two-terminal resistor which satisfies Ohm's law; i.e., the voltage across such an element is proportional to the current flowing through it. We call such an element a *linear resistor*. We represent it by the symbol shown in Fig. 1.1, where the current i through the resistor and the voltage v across it are measured using the *associated reference directions*. *Ohm's law* states that, at all times

$$v(t) = Ri(t) \qquad \text{or} \qquad i(t) = G\,v(t) \tag{1.1}$$

where the constant R is the *resistance* of the linear resistor measured in the unit of ohms (Ω), and G is the *conductance* measured in the unit of siemens (S). The voltage $v(t)$ and the current $i(t)$ in Eq. (1.1) are expressed in volts (V) and amperes (A), respectively. Equation (1.1) can be plotted on the i-v plane or the v-i plane[1] as shown in Fig. 1.2a and b, where the slope in each is the resistance and the conductance, respectively.

While the linear resistor is perhaps the most prevalent circuit element in electrical engineering, nonlinear devices which can be modeled with nonlinear resistors have become increasingly important. Thus it is necessary to define the concept of nonlinear resistor in a most general way.

Consider a two-terminal element as shown in Fig. 1.3. The voltage v across the element and the current i which enters the element through one terminal and leaves from the other are shown using the associated reference directions. A two-terminal element will be called a *resistor* if its voltage v and current i

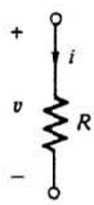

Figure 1.1 Symbol for a linear resistor with resistance R.

[1] When we say x-y plane, we denote specifically x as the horizontal axis and y as the vertical axis of the plane. This is consistent with the conventional usage where the first variable denotes the abscissa and the second variable denotes the ordinate.

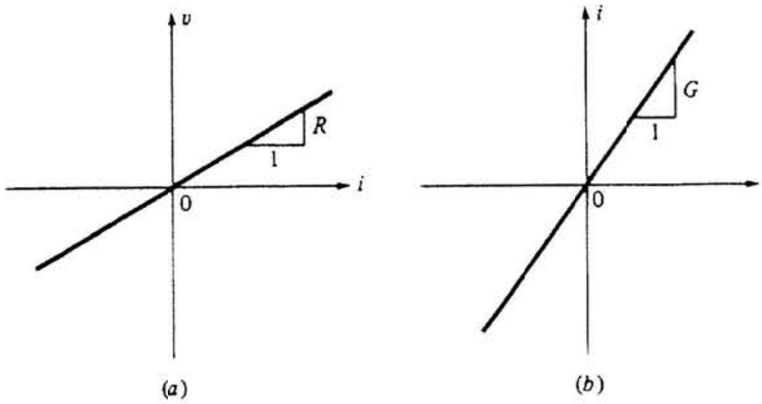

Figure 1.2 Linear resistor characteristic plotted (*a*) on the *i*-*v* plane and (*b*) on the *v*-*i* plane.

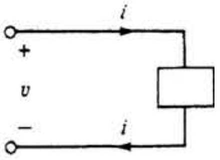

Figure 1.3 A two-terminal element with v and i in the associated reference directions.

satisfy the following relation:

$$\mathcal{R}_R = \{(v, i): f(v, i) = 0\} \tag{1.2}$$

This relation is called the *v-i characteristic* of the resistor and can be plotted graphically in the v-i plane (or i-v plane). The equation $f(v, i) = 0$ represents a curve in the v-i plane (or i-v plane) and specifies completely the two-terminal resistor. The key idea of a resistor is that in Eq. (1.2) the relation is between $v(t)$, the instantaneous value of the voltage $v(\cdot)$ and $i(t)$ the instantaneous value of the current $i(\cdot)$ at time t.

The dc voltage versus current characteristics of devices can be measured using a curve tracer.[2]

The linear resistor is a special case of a resistor in which

$$f(v, i) = v - Ri = 0 \qquad \text{or} \qquad f(v, i) = i - Gv = 0 \tag{1.3}$$

A resistor which is not linear is called *nonlinear*.

Before considering nonlinear resistors, we should first understand linear resistors. Equations (1.1) and (1.3) state that, for a linear resistor, the relation between the voltage v and current i is expressed by *linear* functions. The first equation in (1.1) expresses v as a linear function of i, and the second equation in (1.1) expresses i as a linear function of v. Figure 1.2 shows that the v-i

[2] See for example:

J. Mulvey, *Semiconductor Device Measurements*, Tektronix Inc., Beaverton, Oregon, 1968.

L. O. Chua and G. Q. Zhong, "Negative Resistance Curve Tracer," *IEEE Transactions on Circuits and Systems*, vol. CAS-32, pp. 569–582, June 1985.

characteristic of a linear resistor is a *straight line passing through the origin at all times*. The single number R (or G), i.e., the slope of the characteristic in the i-v plane (or v-i plane), specifies completely the *linear* two-terminal resistor.

Open circuits and short circuits There are two special cases of linear resistors which deserve special mention, namely, the open circuit and the short circuit. A two-terminal resistor is called an *open circuit* iff its current i is identically zero irrespective of the voltage v; i.e., $f(v, i) = i = 0$. The characteristic of an open circuit is the v axis in the v-i plane, or in the i-v plane, as shown in Fig. 1.4. In the i-v plane, it has an infinite slope, i.e., $R = \infty$; and in the v-i plane, it has a zero slope, i.e., $G = 0$.

Similarly, a two-terminal resistor is called a *short circuit* iff its voltage is identically zero irrespective of its current i; i.e., $f(v, i) = v = 0$. The characteristic of a short circuit is the i axis in the i-v plane, or in the v-i plane, as shown in Fig. 1.5. In the i-v plane, the characteristic has a zero slope, i.e., $R = 0$; and, equivalently, $G = \infty$. Comparing Figs. 1.4 and 1.5, we see that the curve of the open circuit in one plane is identical to the curve of the short circuit in the other plane. For this reason, the *open circuit* is said to be the *dual* of the short circuit; and, conversely, the short circuit is the dual of the open circuit. Generalizing to the nonlinear case, we say that the *dual* of a given resistor is another resistor whose v-i characteristic in the v-i plane is the same

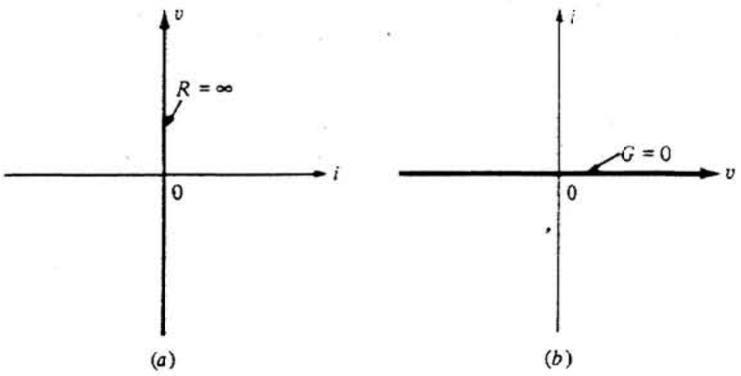

Figure 1.4 Characteristic of an open circuit.

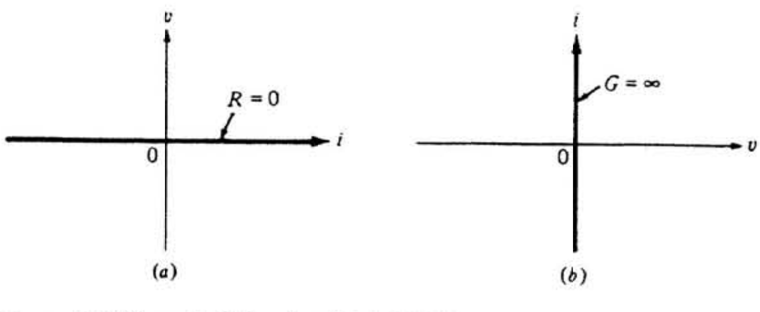

Figure 1.5 Characteristic of a short circuit.

curve as that of the given resistor in the i-v plane. The concept of duality is of utmost importance in circuit theory. It helps us in understanding and analyzing circuits of great generality. We will encounter duality throughout this book.

Exercises

1. A linear resistor of $100\,\Omega$ is given; what is its dual?
2. If $\mathcal{R}_R = \{(v, i): f(v, i) = v - i^3 = 0\}$ specifies a resistor, write down the relation of the dual resistor.
3. Given the v-i characteristic Γ of a resistor $\mathcal{R}$ on the v-i plane, show that the dual characteristic is obtained by reflecting Γ about the $45°$ line through the origin.

Power, passive resistors, active resistors, and modeling The symbol for a two-terminal nonlinear resistor $\mathcal{R}$ is shown in Fig. 1.6. The *power* delivered *to* the *resistor* at time t *by* the remainder of the circuit to which it is connected is, from Chap. 1,

$$p(t) = v(t)i(t) \tag{1.4a}$$

If the resistor is linear having resistance R

$$p(t) = Ri^2(t) = Gv^2(t) \tag{1.4b}$$

Thus, the power delivered to a linear resistor is always nonnegative if $R \geq 0$. We say that a *linear* resistor is *passive* iff its resistance is nonnegative. Thus a passive resistor always absorbs energy from the remainder of the circuit.

Also from Eq. (1.4*b*), the power delivered to a linear resistor is negative if $R < 0$; i.e., as current flows through it, the resistor delivers energy to the remainder of the circuit. Therefore, we call such a *linear* resistor with negative resistance an *active* resistor. The characteristic of a linear active resistor is shown in Fig. 1.7; note that the slope is negative. While linear passive resistors are familiar to everyone, linear active resistors are perhaps new to some readers. They are one of the basic circuit elements in the design of negative-

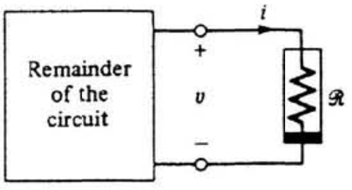

Figure 1.6 Illustrating power delivered to a nonlinear resistor from the remainder of the circuit.

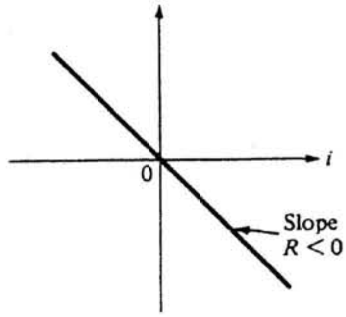

Figure 1.7 Characteristic of a linear active resistor with resistance $R < 0$.

resistance amplifiers and oscillators. We will illustrate their applications later. For the present we only wish to mention that the linear active resistor is useful in modeling nonlinear devices and circuits over certain ranges of voltages, currents, and frequencies.

We can easily generalize the above concept to nonlinear resistors. Obviously, from Eq. (1.4*a*), $p(t) \geq 0$ if and only if $v(t)$ and $i(t)$ have the same sign for all t. Thus we call a two-terminal resistor *passive* iff its v-i characteristic lies in the closed first and third quadrants of the v-i plane or the i-v plane. A resistor is said to be *active* if it is not passive.

At this juncture we wish to recall the concept of modeling introduced in Chap. 1. Let us use the term "physical resistor" to refer to the electric device in the laboratory or in a piece of equipment. This is not to be confused with the resistor we defined as a circuit element in Eq. (1.2). What is remarkable is that for most physical resistors made of metallic material, we can use the circuit element, a linear passive resistor, to model them almost precisely, i.e., they satisfy Ohm's law. The model is good over a large operating range. Only for excessive voltages or currents, or at very high frequencies, is a better model necessary. Often in such a situation the physical resistor fails to be of ordinary use; for example, a physical resistor will burn out if the current exceeds the specified normal operating range. Historically, and in most engineering usage, the term "resistor" is often loosely used to mean the physical resistor, a device which satisfies Ohm's law. In circuit theory we depart from the traditional practice and define a resistor as a *circuit element* which is specified by a voltage-current relation called the v-i characteristic. This way of defining a resistor has a special significance in modeling electric and electronic devices which, at low frequencies, behave like nonlinear resistors. We next turn our attention to nonlinear resistors.

1.2 The Nonlinear Resistor

Recall that a resistor that is not linear is said to be nonlinear. In this section we will first introduce some typical examples of nonlinear resistors and illustrate their properties. We will then point out some essential differences between circuits with linear resistors and those with nonlinear resistors.

Ideal diode A very useful two-terminal circuit element is the ideal diode. By definition, an *ideal diode* is a nonlinear resistor whose v-i characteristic consists of two straight line segments on the v-i plane (or the i-v plane), namely, the negative v axis and the positive i axis. The symbol of the ideal diode and its characteristic are shown in Fig. 1.8. Its relation can be expressed by

$$\mathcal{R}_{\text{ID}} = \{(v, i)\colon vi = 0, i = 0 \text{ for } v < 0 \text{ and } v = 0 \text{ for } i > 0\} \qquad (1.5)$$

Thus, if the diode is *reversed biased* ($v < 0$), the current is zero, i.e., the diode acts as an open circuit; if the diode is *conducting* ($i > 0$), the voltage is zero, i.e., the diode acts like a short circuit. Clearly, the power delivered to an ideal diode is identically zero at all times. In circuit theory there exist several ideal circuit elements having such a property, and we call them *non-energic* circuit elements.

The ideal diode is a very useful circuit element which plays a crucial role in device modeling and helps us in understanding how various electronic devices and circuits function. It is also essential in the method of piecewise-linear analysis to be introduced later.

There are many two-terminal electronic devices, called diodes, whose characteristics resemble that of an ideal diode to some extent. We will consider a few of them and discuss their characteristics. We shall consider the low-frequency properties of these devices because, at low frequencies, these devices can be modeled fairly accurately with nonlinear resistors alone.

***pn*-Junction diode** The *pn*-junction diode and its v-i characteristic are shown in Fig. 1.9. For most applications the device is operated to the right of point A, where A is near the "knee" of the curve. In the normal operating range, i.e., to the right of A, the current obeys the "diode junction law."

$$i = I_s\left[\exp\left(\frac{v}{V_T}\right) - 1\right] \tag{1.6}$$

where I_s is a constant of the order of microamperes, and it represents the *reverse saturation current*, i.e., the current in the diode when it is reversed biased with a large voltage. The parameter $V_T = kT/q$ is called the *thermal voltage*, where q is the charge of an electron, k is Boltzmann's constant, and T is temperature in Kelvins. At room temperature V_T is approximately 0.026 V. In Eq. (1.6) we have a nonlinear resistor whose current i is expressed as a function of its voltage v. This means that, for any given voltage v, the current i is *uniquely* specified. Any nonlinear resistor having this property is called a *voltage-controlled nonlinear resistor*. While Eq. (1.6) represents a good model

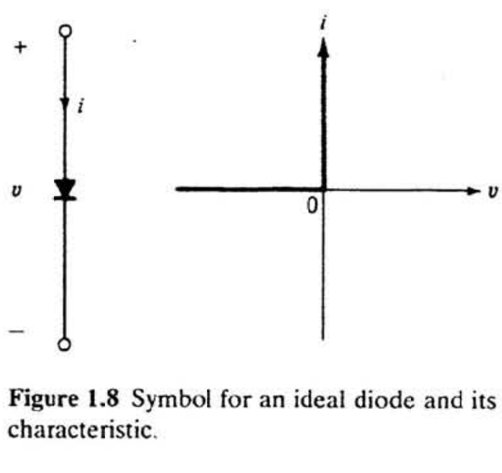

Figure 1.8 Symbol for an ideal diode and its characteristic.

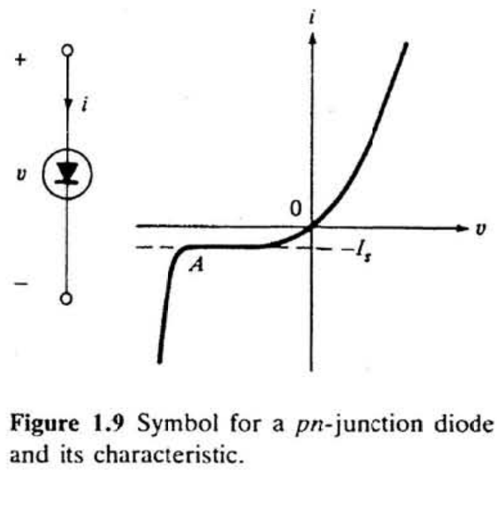

Figure 1.9 Symbol for a *pn*-junction diode and its characteristic.

for the *pn*-junction diode at low frequencies, we need to use additional circuit elements, capacitors, inductors, and linear resistors to model it at higher frequencies.

Tunnel diode The symbol of the tunnel diode and its v-i characteristic are shown in Fig. 1.10. The current i can be expressed as a function of the voltage v, hence we may write

$$i = \hat{i}(v) \tag{1.7a}$$

or, in terms of Eq. (1.2),

$$f(v, i) = i - \hat{i}(v) = 0 \tag{1.7b}$$

Note that the function $\hat{i}(\cdot)$ is *single-valued*, hence the nonlinear resistor given by Eq. (1.7) is voltage-controlled. Note, for $V_1 < v < V_2$, the slope of the curve is negative just like that of a linear active resistor having negative resistance introduced earlier. We shall see later that this negative slope makes the device useful in such applications as amplifiers and oscillators. Note also that, for $I_2 < i < I_1$, a given current i corresponds to three different values of v on the characteristic. This multivalued property makes the device useful in memory and switching circuits.

Glow tube The symbol of the glow tube and its characteristic are shown in Fig. 1.11. We can express the characteristic of such a nonlinear resistor by a single-valued function

$$v = \hat{v}(i) \tag{1.8a}$$

or, in terms of Eq. (1.2),

$$f(v, i) = v - \hat{v}(i) = 0 \tag{1.8b}$$

A nonlinear resistor whose voltage is a "single-valued" function of the current

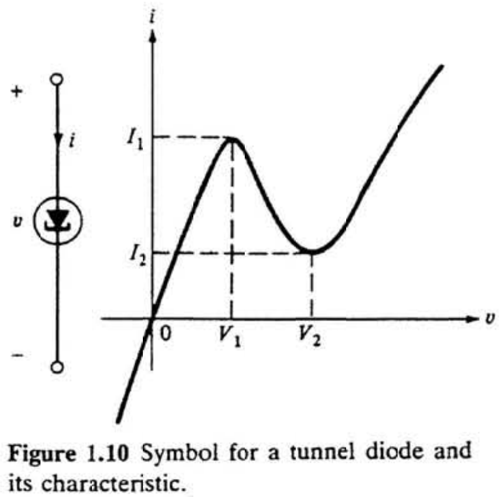

Figure 1.10 Symbol for a tunnel diode and its characteristic.

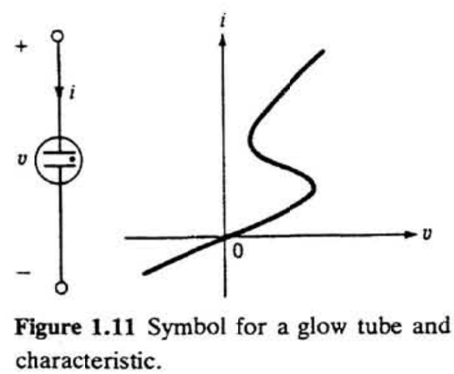

Figure 1.11 Symbol for a glow tube and its characteristic.

is called a *current-controlled resistor*. Thus, the glow tube is a current-controlled nonlinear resistor. Since the current is not a (single-valued) function of the voltage, it is not voltage-controlled.

Note that the ideal diode is neither voltage-controlled nor current-controlled.

Bilateral property In contrast to linear resistors, a nonlinear resistor in general has a v-i characteristic which is not symmetric with respect to the origin of the v-i plane. (See Figs. 1.9 and 1.10.) Consider the tunnel diode in Fig. 1.10. Let us change the reference direction of the current and of the voltage. We redraw the circuit as shown in Fig. 1.12, where $i_1 = -i$ and $v_1 = -v$.

The characteristic in the v_1-i_1 plane, which corresponds to that of the original two-terminal resistor with the two terminals interchanged, is shown in the figure. For this reason, it is important that the symbol for a nonlinear resistor indicate its orientation. Note that the general symbol for a nonlinear resistor in Fig. 1.13 is dissymmetric with respect to its two terminals: consequently, it is possible to specify the correct connection of the two terminals of a nonlinear resistor to a circuit.

The characteristic of a *linear* resistor is always symmetric with respect to the origin. A circuit element with this kind of symmetry is called *bilateral*. A *bilateral resistor* satisfies the property $f(v, i) = f(-v, -i)$ for all (v, i) on its characteristic. A nonlinear resistor may be bilateral, e.g., have the characteristic shown in Fig. 1.14.

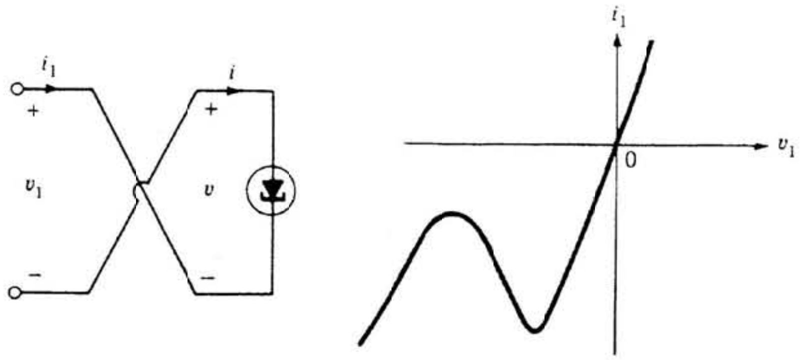

Figure 1.12 Characteristic of a tunnel diode with its terminals turned around.

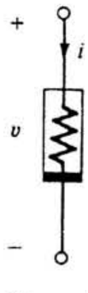

Figure 1.13 Symbol for a nonlinear resistor.

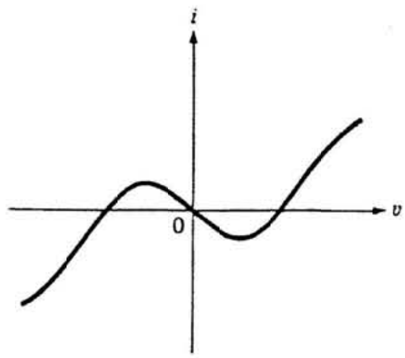

Figure 1.14 v-i Characteristic of a bilateral nonlinear resistor.

Simple circuits Circuits containing nonlinear resistors have properties totally different from those which have only linear resistors. The following examples illustrate some of the differences.

Example 1 (nonlinear resistors can produce harmonics) Consider a sinusoidal voltage waveform,

$$v(t) = 2 \sin \omega t \text{ (in volts)} \qquad t \geq 0$$

where the constant ω is the angular frequency in radians per second, i.e., $\omega = 2\pi f$ where f is frequency in hertz. If the waveform is applied to a linear resistor of $10\,\Omega$, the current is $i(t) = 0.2 \sin \omega t$ (in amperes), $t \geq 0$.

Let us apply the same voltage waveform to a nonlinear resistor which has the v-i characteristic shown in Fig. 1.15a, where

$$i = \hat{i}(v)$$

We wish to determine the current waveform $i(t)$ for $t \geq 0$. In simple

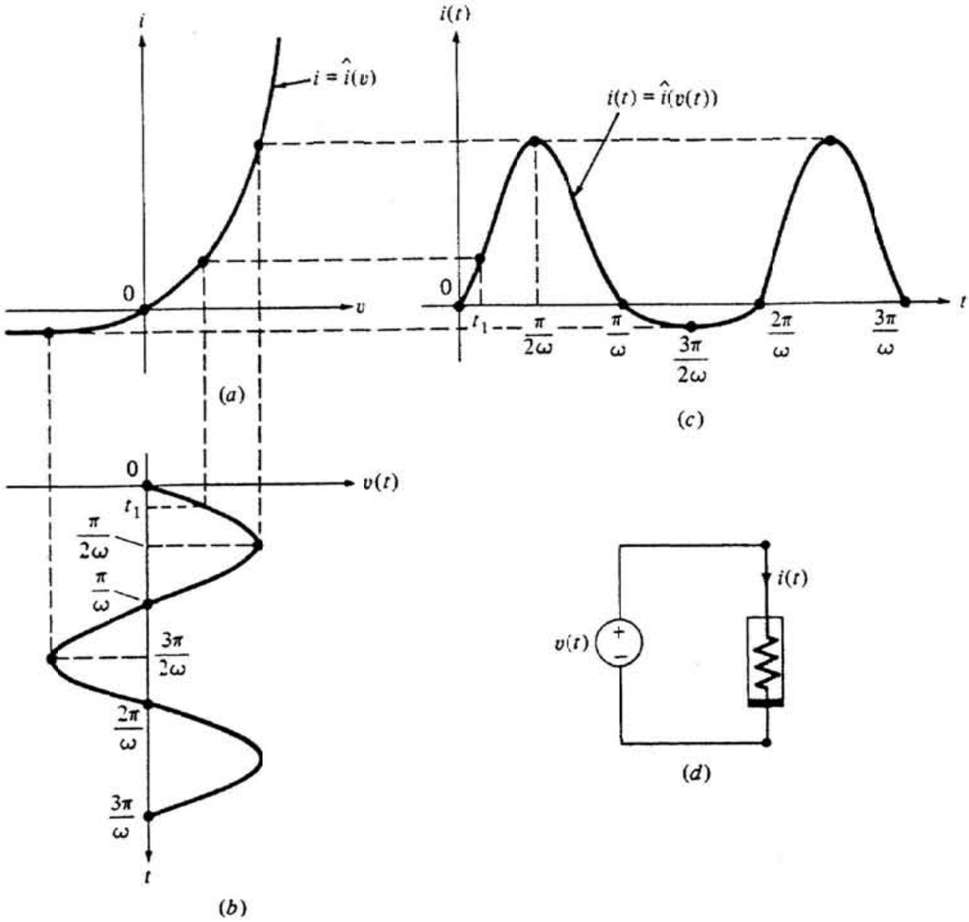

Figure 1.15 An example illustrating a special clipping property of nonlinear resistors; the negative half of the waveform has been clipped.

nonlinear circuit analysis a graphic method is often useful. Let us plot the voltage waveform $v(\cdot)$ as shown in Fig. 1.15*b* with the t axis lined up with the i axis of Fig. 1.15*a*. Then it is easy to obtain the current $i(t)$ shown in Fig. 1.15*c* by transcribing points on the voltage waveform $v(t)$ at $t = 0, t_1, \pi/2\omega, \pi/\omega, 3\pi/2\omega$, etc., through the v-i curve of the nonlinear resistor. The resulting current waveform $i(\cdot)$ is again a periodic function with period $2\pi/\omega$; hence it can be expressed in terms of a Fourier series

$$i(t) = I_0 + \sum_k I_k \sin k\omega t \tag{1.9}$$

It contains, besides the fundamental sinusoid, higher harmonics, i.e., the sinusoidal terms in Eq. (1.9) with $k > 1$. What is especially important to note is that, unlike the voltage waveform $v(\cdot)$, the current waveform $i(\cdot)$ has a *direct current* (*dc*) component I_0. The term I_0 in Eq. (1.9) is equal to the average value of the current waveform $i(\cdot)$ over a complete period. This dc component can be filtered out with a simple low-pass filter. This, of course, is the basis of all rectifier circuits which convert alternating current to direct current.

Example 2 (piecewise-linear approximation) Another basic method of nonlinear circuit analysis is the use of piecewise-linear approximation. The v-i curve in Fig. 1.15*a* can be roughly approximated by two linear segments as shown in Fig. 1.16. With this simple approximation, the current waveform can be obtained immediately. For v negative, i is identically zero. For v positive, the nonlinear resistor behaves like a linear resistor with conductance G. Thus the waveform is given by

$$i(t) = \begin{cases} 2G \sin \omega t & \text{for} \quad \dfrac{2n\pi}{\omega} \le t \le \dfrac{(2n+1)\pi}{\omega} \\ 0 & \text{for} \quad \dfrac{(2n+1)\pi}{\omega} \le t \le \dfrac{(2n+2)\pi}{\omega} \end{cases}$$

where n runs through nonnegative integers. The result is of course different from that obtained by the graphic method. However, had we used

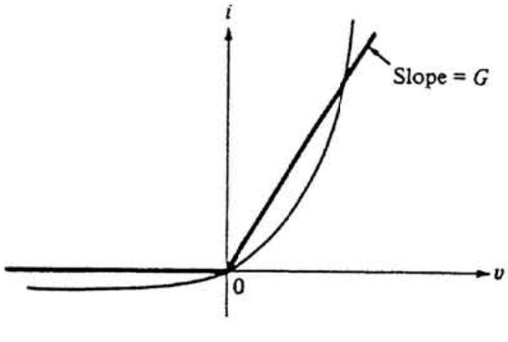

Figure 1.16 Piecewise-linear approximation of a nonlinear characteristic.

more linear segments to approximate the v-i curve, we could have obtained a solution closer to the current waveform shown in Fig. 1.15. The subject of piecewise-linear approximation will be treated in greater detail later in the chapter.

Example 3 (homogeneity and additivity) In this example we will compare the currents in a linear resistor whose characteristic is

$$i_\ell = \hat{i}_\ell(v) = 0.1v \tag{1.10a}$$

and in a nonlinear resistor whose characteristic is

$$i_n = \hat{i}_n(v) = 0.1v + 0.02v^3 \tag{1.10b}$$

The subscript ℓ denotes linear and the subscript n denotes nonlinear. Let us consider three different constant input voltages: $v_1 = 1$ V, $v_2 = k$ V, and $v_3 = v_1 + v_2 = 1 + k$ V. For the linear resistor, from Eq. (1.10a), we have $i_{\ell 1} = 0.1$ A, $i_{\ell 2} = 0.1 \times k$ A, and $i_{\ell 3} = 0.1 + 0.1 \times k$ A. Clearly

$$i_{\ell 2} = \hat{i}_\ell(kv_1) = k\hat{i}_\ell(v_1) \tag{1.11a}$$

and
$$i_{\ell 3} = \hat{i}_\ell(v_1 + v_2) = \hat{i}_\ell(v_1) + \hat{i}_\ell(v_2) \tag{1.11b}$$

Equation (1.11a) states the *homogeneity property* of a linear function, while Eq. (1.11b) states the *additivity property*.

Next, let us consider the nonlinear resistor defined by Eq. (1.10b). Setting $v_1 = 1$ V, $v_2 = k$ V, and $v_3 = v_1 + v_2$ V, we obtain successively $i_{n1} = 0.12$ A, $i_{n2} = 0.1 \times k + 0.02 \times k^3$ A, and $i_{n3} = 0.1(1 + k) + 0.02(1 + k)^3$ A. Clearly

$$i_{n2} = \hat{i}_n(kv_1) \neq k\hat{i}_n(v_1)$$

and
$$i_{n3} = \hat{i}_n(v_1 + v_2) \neq \hat{i}_n(v_1) + \hat{i}_n(v_2)$$

Thus in general for nonlinear resistors, neither the homogeneity property nor the additivity property holds.

1.3 Independent Sources

In circuit theory, independent sources play the same role as external forces in mechanics. Independent sources are circuit elements which are used to model such devices as the battery and the signal generator. The two independent sources we will introduce in this section are the independent voltage source and the independent current source. For convenience, however, we shall often omit the adjective "independent" and simply use the terms "voltage source" and "current source."

Independent voltage source A two-terminal element is called an *independent voltage source* if the voltage across it is a given waveform $v_s(\cdot)$ irrespective of the current flowing through it. Note that the waveform $v_s(\cdot)$ is part of the specification of the voltage source. Commonly used waveforms include the *dc voltage source*, i.e., $v_s(t)$ is a constant E for all t, the sinusoid, the square wave, etc. The symbol of an independent voltage source with waveform $v_s(\cdot)$ is shown in Fig. 1.17*a* where the signs + and − specify the polarity. The symbol for a dc voltage source is shown in Fig. 1.17*b* with $E>0$.

At any time t, the independent voltage source can be expressed by the relation

$$\mathcal{R}_{vs} = \{(v, i): v = v_s(t) \text{ for } -\infty < i < \infty\} \tag{1.12}$$

Consequently, an independent voltage source is a two-terminal resistor. Its characteristic in the v-i plane is a straight line parallel to the i axis. This is shown in Fig. 1.18*a*, where $v_s(t)$ denotes the voltage of the independent voltage source at time t.

Consider a sinusoidal voltage source $v_s(t) = V_m \sin \omega t$, where V_m, the amplitude of the sinusoid, is a constant and ω is the angular frequency. Then, for each t, the v-i characteristic is a vertical line as shown in Fig. 1.18*b*, depicting the value of the voltage waveform at time t.

Since the v-i characteristic is a straight line parallel to the i axis, the independent voltage source is a current-controlled nonlinear resistor. It is nonlinear because the straight line does not go through the origin unless $v_s = 0$; in that case, it becomes identical to the characteristic of a short circuit. The

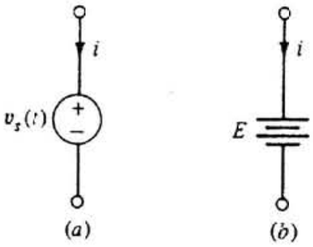

Figure 1.17 Symbols for an independent voltage source and a dc voltage source.

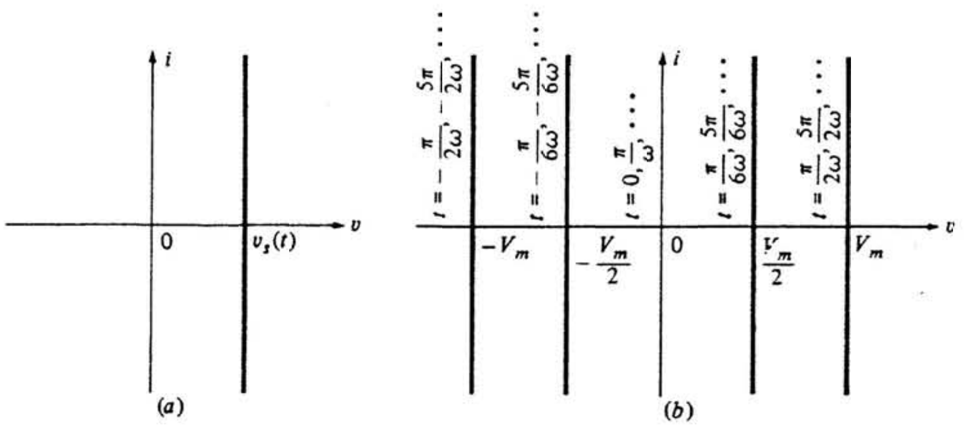

Figure 1.18 (*a*) Characteristic of an independent voltage source at time t, $v_s(t)$. (*b*) Characteristic at different values of t for $v_s(t) = V_m \sin \omega t$.

property that an independent voltage source v_s becomes a short circuit (zero resistance) if $v_s = 0$ is very important in circuit analysis. We shall use this property from time to time throughout the book.

In Fig. 1.19 we show an independent voltage source connected to an arbitrary external circuit. The physical significance of the definition of the independent voltage source is that the voltage across the source is maintained to the prescribed waveform $v_s(\cdot)$ no matter what the external circuit may be. The nature of the external circuit only affects the current i flowing through the source. This is because an independent voltage source has "zero internal resistance" in contrast to a real battery which has a finite nonzero resistance. This will be illustrated later in the chapter after we introduce the series connection of resistors.

So far, we have used the associated reference directions for all circuit elements for consistency. With the associated reference directions, the power delivered *to* the voltage source is $v_s \times i$; therefore, the power delivered *from* the voltage source to the external circuit is

$$p = v_s \times (-i) = v_s i'$$

where i' is the current entering the external circuit. Note that as far as the external circuit is concerned v_s and i' are measured using the associated reference directions.

Independent current source An *independent current source* is defined as a two-terminal circuit element whose current is a specified waveform $i_s(\cdot)$ irrespective of the voltage across it. An independent current source has the symbol shown in Fig. 1.20, where the arrow gives the positive current direction, i.e., $i_s(t) > 0$ means that the current flows through the source from terminal ① to terminal ②. At any time t, the v-i characteristic for an independent current source is expressed by the relation

$$\mathscr{R}_{is} = \{(v, i)\colon i = i_s(t) \text{ for } -\infty < v < \infty\} \tag{1.13}$$

In terms of the v-i plane, an independent current source is represented by a straight line parallel to the v axis. It is a voltage-controlled nonlinear resistor. If $i_s = 0$, the characteristic is the v axis. Therefore an independent current source becomes an open circuit (infinite resistance) when the source current is zero.

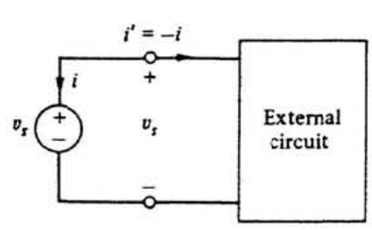

Figure 1.19 An independent voltage source connected to an external circuit.

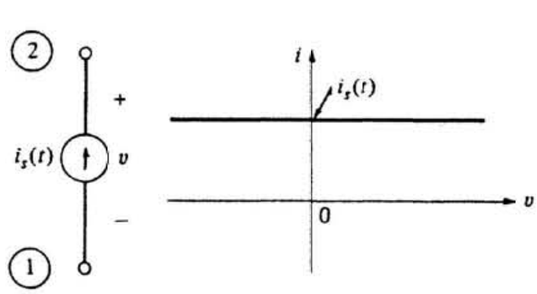

Figure 1.20 Symbol for an independent current source and its characteristic.

In Fig. 1.21 we show an independent current source connected to an arbitrary external circuit. The significance of the definition of the independent current source is that the current of the current source maintains its prescribed waveform $i_s(\cdot)$ and the voltage across it is determined by the external circuit. This is because an independent current source has "infinite internal resistance." The power delivered *from* the source *to* the external circuit is $p(t) = i_s(t)v'(t)$, for all t. Note that as far as the external circuit is concerned, v' and i_s are in the associated reference directions.

Exercise Under what condition is an independent current source i_s the dual of an independent voltage source v_s?

1.4 Time-Invariant and Time-Varying Resistors

We have discussed the independent voltage source, whose voltage is a sinusoidal waveform. It is a nonlinear resistor whose v-i characteristic is a function of time as shown in Fig. 1.18b. We shall introduce a formal definition of a time-varying resistor: A resistor is said to be *time-varying* iff its v-i characteristic varies with time; otherwise, it is said to be *time-invariant*. *A linear time-varying resistor* is characterized by Ohm's law

$$v(t) = R(t)i(t) \qquad \text{or} \qquad i(t) = G(t)v(t) \tag{1.14}$$

where $G(t) = 1/R(t)$. The function $R(\cdot)$ or $G(\cdot)$ is part of the specification of the resistor. Thus $R(t)$ gives the value of the time-varying resistance at time t. From Eq. (1.14) it is clear that an independent voltage source or current source having a nonconstant waveform is a nonlinear time-varying resistor. In this section we will first introduce examples of useful time-varying resistors and then bring out a unique property of time-varying resistors, which is important in communication engineering.

Periodically operating switches A periodically operating ideal switch is a linear time-varying resistor. Its symbol, property, and v-i characteristic are shown in Fig. 1.22. For $0 \le t < t_1$, the switch is open, $S = 1$, the current is zero, and the v-i characteristic is the v axis. For $t_1 \le t < T$, the switch is closed, $S = 0$, the voltage is zero, and the v-i characteristic is the i axis. After a period T, the switch repeats its operation; i.e., $S = 1$ and $S = 0$ alternately for $nT \le t < nT + t_1$ and $nT + t_1 \le t < (n+1)T$, respectively, where n is any integer >1. The periodically operating switch is a key circuit element in digital circuits and in both digital and analog communication systems.

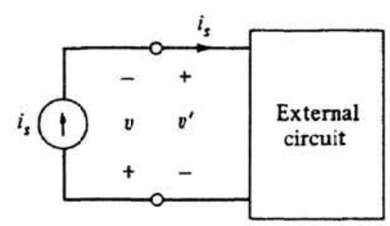

Figure 1.21 An independent current source connected to an external circuit.