1

DIRECT CURRENT CIRCUITS

1.1 ELECTRIC CHARGE AND CURRENT

There are two kinds of electric charge, positive and negative. They are so named because they add and subtract like positive and negative numbers. All atoms contain charge; the usual picture of an atom is a small (10^{-13}cm diameter) positively charged nucleus around which negatively charged electrons move in orbits of the order of 10^{-8}cm diameter. Charge is measured in coulombs in mks units and in statcoulombs in the cgs system. The basic indivisible unit of charge is the charge on one electron which is -1.6×10^{-19} coulombs or -4.8×10^{-10} statcoulombs. All electric charges (positive and negative) are, in magnitude, integral multiples of the charge of the electron. However, in most electronic circuit problems the discrete nature of electric charge may be neglected, and charge may be considered to be a continuous variable. One of the most fundamental conservation laws of physics says that in any closed system the total net amount of electric charge is conserved or, in other words, is constant in time. For example, in a semiconductor if one electron is removed from a neutral atom then the atom minus the one electron has a net electric charge of $+1.6 \times 10^{-19}$ coulombs.

The flow of electric charge, either positive or negative, is called "current"; that is, the current I at a given point in a circuit is defined as the time rate of change of the amount of electric charge Q flowing past that point.

$$I \equiv \frac{dQ}{dt} \tag{1.1}$$

The direction of the current is taken by convention to be the direction of the flow of *positive* charge. If electrons are flowing from right to left in a wire, then this electron current is electrically equivalent to positive charge flowing from left to right; hence, we say the current is to the right. In a current carrying wire it is actually the electrons which flow, but nevertheless, the direction of current is taken as the direction of the equivalent positive charge flow, which is opposite to the direction of the electron flow.

Current is measured in "amperes"; one ampere of current is the flow of one coulomb of charge per second which is 6.25×10^{18} electrons per second. Other units of current are the milliampere (mA) which is 0.001 ampere, the microampere (μA) which is 10^{-6} ampere, the nanoampere (nA) which is 10^{-9} ampere, and the picoampere (pA) which is 10^{-12} ampere. Prefixes for various powers of ten are given in Table 1-1. Typical currents which flow in low

TABLE 1-1. POWERS-OF-TEN PREFIXES

Prefix	*Symbol*	*Meaning*	*Example*
giga	G	10^9	1 gigahertz = 1 GHz = 10^9 hertz
mega	M	10^6	1 megohm = 1 MΩ = 10^6 ohms
kilo	k	10^3	1 kilovolt = 1 kV = 10^3 volts
milli	m	10^{-3}	1 milliampere = 1 mA = 10^{-3} amperes
micro	μ	10^{-6}	1 micro volt = 1 μV = 10^{-6} volts
nano	n	10^{-9}	1 nanoampere = 1 nA = 10^{-9} amperes
pico	p	10^{-12}	1 picofarad = 1 pF = 10^{-12} farads

power transistor electronic circuits, such as in small radio receivers and amplifiers, are of the order of 1 to 10 mA; typical currents in low power vacuum tube circuits are of the order of 10 to 100 mA. The largest current normally encountered in vacuum tube circuits is about 500 mA, but specially designed high current transistors are available which carry currents from 1 to 100 amperes.

There are two general kinds of current, direct current ("dc") and alternating current ("ac"). Direct current is a flow of charge in which the direction of flow is always the same. If the magnitude of the current varies from one instant of time to another, but the direction of flow remains the same, then this type of current is called "pulsating" direct current. If both the direction and the magnitude of the flow of charge are constant, then ~~this current~~ is called "pure direct current" or simply "direct current." If the charge flows back and forth

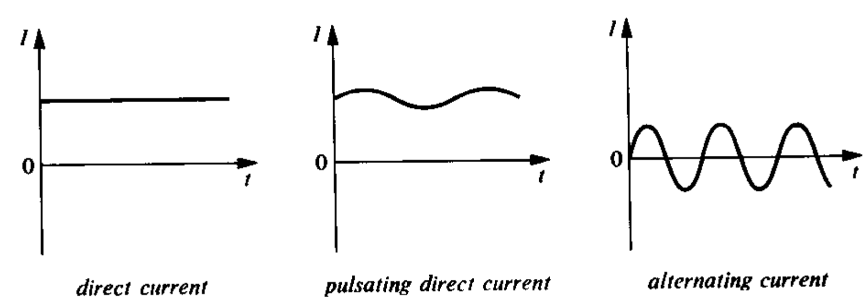


FIGURE 1.1 Types of current.

alternately, then this current is called "alternating current" (to be discussed further in Chapter 2). Graphs of these three kinds of currents are shown in Fig. 1.1.

It is sometimes useful to think of electric current in a wire as being similar to the flow of water in a pipe, with the water molecules being analogous to the electrons in the wire. A flow of water in grams/second is analogous to a flow of electric charge in coulomb/second or amperes.

1.2 VOLTAGE

The voltage, or electric potential, V at a given point in a circuit is defined as the potential energy W a positive charge Q would have at that point divided by the magnitude of the charge, $V \equiv W/Q$. The voltage V is measured relative to some other specified reference point in the circuit where we say the energy of all charges is zero. This reference point is usually called "ground" or "earth" in British literature. A good ground is a cold water pipe or sometimes just the metal chassis or box enclosing the circuit. The point is that the circuit "ground" has a constant, unvarying potential which we set equal to zero volts by convention. The earth has a constant potential or voltage simply because it is so large and is a reasonably good conductor. Any charge taken from or added to the earth through a circuit's ground wire will not appreciably change the total charge on the earth or the earth's voltage. The symbols used for ground in circuit notation are shown in Fig. 1.2.

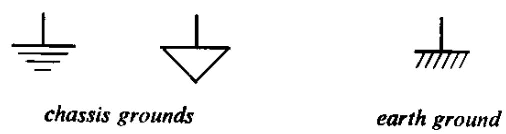
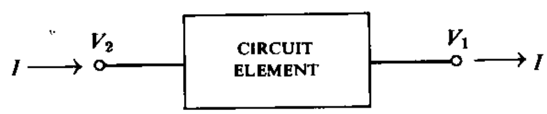


FIGURE 1.2 Ground symbols.

The units of voltage are joules/coulomb; one volt (V) is defined as one joule/coulomb. Other units of voltage are the kilovolt (kV) which is 1000 V, the millivolt (mV) which is 0.001 V, the microvolt (μV) which is 10^{-6} V, and the nanovolt which is 10^{-9} V. Notice that the voltage at a given point has no *absolute* meaning, but only means the potential energy per unit charge *relative to ground.* This energy per unit charge definition of voltage is rather difficult to understand intuitively until one considers the direction the charges will tend to move. For, in all natural processes, things tend to move in such a way as to minimize the potential energy; thus, positive charges will move from points of higher voltage toward points of lower voltage, for example from a point with a voltage of +15 V toward a point of voltage +12 V. Similarly, negative charges will tend to move from points of lower voltage toward points of higher voltage, because this direction of charge flow will minimize their potential energy. For example, negative charge (electrons) will move from a point of voltage $-12\,V$ to a point of voltage $-7\,V$. In terms of the analogy between current and water flow, the voltage is analogous to the water pressure, because water tends to flow from points of higher pressure toward points of lower pressure. It is also sometimes useful to think of the voltage as causing or forcing the flow of current, just as one thinks of the water pressure as causing or forcing the water flow. Thus, voltage is a "cause" and current an "effect."

1.3 RESISTANCE

If a current I flows through any two terminal circuit elements, then the static or dc resistance (usually referred to as simply the "resistance") of that circuit element is defined as the difference in voltage $V_2 - V_1$ between the two terminals divided by the current I (see Fig. 1.3). Strictly speaking, this definition applies



$$R \equiv \frac{V_2 - V_1}{I}$$

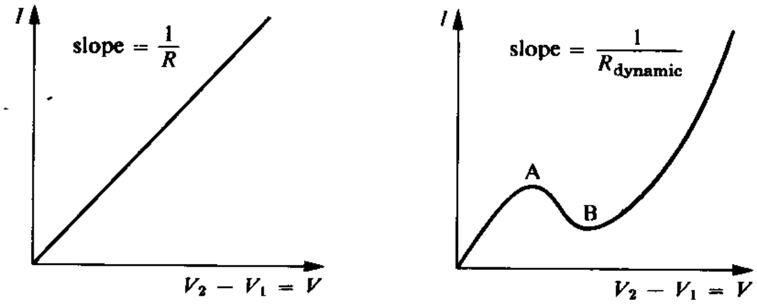
FIGURE 1.3 Definition of resistance R.

only to a circuit element which converts electrical energy into heat, but this situation occurs in the overwhelming majority of cases in electronic circuitry.

$$R \equiv \frac{V_2 - V_1}{I} \tag{1.2}$$

The current into the circuit element must exactly equal the current leaving from the conservation of charge law. V_2 is the voltage at the terminal where the current enters the circuit element; V_1 is the voltage where the current leaves. If the circuit element is "passive", that is, if there is no energy given to the charge by the element, then the charge loses energy in the element. Thus, V_2 must be greater than V_1. Thus, for all passive circuit elements, the static or dc resistance is positive. The current–voltage graph for an ordinary positive resistance is shown in Fig. 1.4(a).

In a formal sense, a battery has a negative resistance, because the battery gives energy to the charge making V_1 greater than V_2, but this is because chemical energy is converted into electrical energy in a battery. Some circuit elements



(a) *ordinary linear resistance*

(b) *nonlinear resistance (tunnel diode)*

FIGURE 1.4 Current-voltage curves.

are said to have a negative resistance; but this always refers to *dynamic* resistance which is defined as the rate of change of voltage with respect to current:

$$R_{\text{dynamic}} \equiv dV/dI \tag{1.3}$$

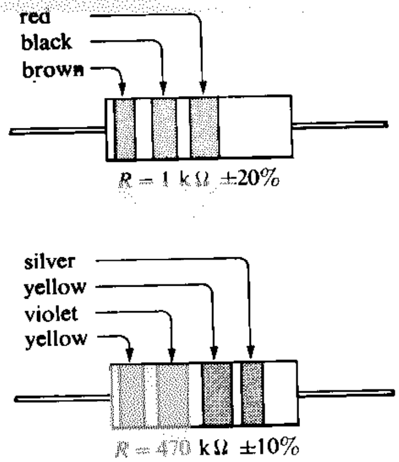
where dV is the change in the voltage across the circuit element and dI is the change in the current through the element. In Fig. 1.4(b), which is the current–voltage characteristic for a tunnel diode, the static resistance is always positive for all values of V, but between A and B the dynamic resistance is negative because I decreases as V increases. Also note that both the static and the dynamic resistance vary with voltage in Fig. 1.4(b). A circuit element for which the current voltage curve is not a straight line is said to have a "nonlinear" resistance.

The units of resistance are volts/ampere; one ohm (Ω) is defined as one volt per ampere. Other units of resistance are the kilohm (kΩ sometimes just written k) which is 1000 ohms and the megohm (MΩ or just M) which is 10^6 ohms. A "resistor" is a two terminal circuit element specifically manufactured to have a constant resistance. Thus a 4.7 kΩ resistor has a resistance of 4,700 ohms; a 2.2 MΩ resistor has a resistance of 2,200,000 ohms.

Usually the resistance of a resistor is given by a "color code" (see Fig. 1.5). Bands of different colors specify the resistance, according to the following rule:

R = (*first color number*) (*second color number*) × 10 (*raised to the third color number*)

The first digit of the resistance is given by the color band closest to the end of the resistor. The fourth color band gives the tolerance of the resistor. Silver means ±10% tolerance, gold means ±5% tolerance, and no fourth colored band means ±20% tolerance. A 2 kΩ 10% resistor will have a resistance somewhere between 1.8 kΩ and 2.2 kΩ.



COLOR	NUMBER
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Gray	8
White	9

FIGURE 1.5 Resistor color code.

If one has difficulty remembering the color code, then one can note that the colors follow the colors of the visible spectrum starting with red for the number two and going through violet for the number seven. Or, one can use the following scheme: "**B**lasphemous **B**oys **R**ape **O**ur **Y**oung **G**irls **B**ut **V**iolet **G**ives **W**illingly," the first letter of each word being the first letter of the colors for the numbers zero through nine. And it is extremely useful to note that if the third color is brown, the resistor is in the hundreds of ohms; red—thousands of ohms, orange—tens of thousands of ohms, yellow—hundreds of thousands of ohms, green—millions of ohms.

On certain precision metal film resistors, the resistance value is specified to three significant figures. These resistors have five colored bands; the first three bands represent the three significant figures, the fourth band the multiplier, and the fifth band the tolerance. Certain high reliability resistors are tested for failure rates under conditions of maximum power and voltage, and the results are expressed in percentage failure per thousand hours. The fifth colored band represents the failure rate per thousand hours according to the following scheme: brown 1%, red 0.1%, orange 0.01%, and yellow 0.001%.

1.4 OHM'S LAW

It is empirically true for many different kinds of circuit elements, that the resistance of the circuit element is constant if we keep the temperature and composition of the element fixed. This is true over an extremely large range of voltages and currents. That is, changing the voltage difference between the two terminals by any factor will cause the current to change by exactly the same factor, i.e., doubling the voltage difference $V_2 - V_1$ exactly doubles the current I.

Ohm's law is simply the statement that the resistance is constant. It can be written in three ways:

$$R = \frac{V_2 - V_1}{I} \qquad V_2 - V_1 = IR \qquad I = \frac{V_2 - V_1}{R}. \tag{1.4}$$

Any two terminal circuit element of constant resistance is shown in circuit diagrams as a zig-zag line [see Fig. 1.6(a)] and is called a resistor. The larger

(a) *schematic symbol* (b) *actual resistor*

FIGURE 1.6 Resistance.

the voltage difference, the larger the current for a fixed resistance; the larger the resistance for a fixed voltage drop, the smaller the current. Thus, "resistance" is a very appropriate name; it literally means opposition to current flow.

These three forms of Ohm's law can be thought of in the following terms. $R = (V_2 - V_1)/I$ means that if there is a voltage difference $V_2 - V_1$ across a circuit element through which a current I is flowing, then the circuit element must have a resistance $(V_2 - V_1)/I$. $V_2 - V_1 = IR$ means that if a current I is forced through a resistance R, then a voltage difference IR will be developed between the two ends of the resistance. $I = (V_2 - V_1)/R$ means that if there is a voltage difference $V_2 - V_1$ across a resistance then a current $(V_2 - V_1)/R$ must be flowing through the resistance. Perhaps the most important thing to remember about Ohm's law is that it is only the *difference* $(V_2 - V_1)$ in voltage across a resistor which causes current to flow. Thus a $5\,\text{k}\Omega = 5000\,\Omega$ resistor with one end at 35 V and the other end at 25 V will pass a current of 2 mA because $I = (V_2 - V_1)/R = 10\,\text{V}/5000\,\Omega = 0.002\,\text{A} = 2\,\text{mA}$. A $5\,\text{k}\Omega$ resistor with one end at 1078 V and the other end at 1068 V will also pass a current of 2 mA, since the voltage difference is also 10 V. It is useful to remember the short cut that the voltage difference in volts divided by the resistance in kilohms equals current in milliamperes; in the previous example $I = 10\,\text{V}/5\,\text{k}\Omega = 2\,\text{mA}$.

If two resistors R_1 and R_2 are connected in "series" (see Fig. 1.7), that is if

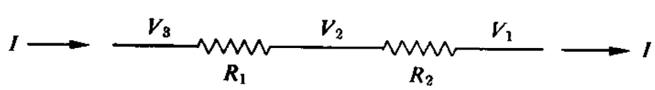


FIGURE 1.7 Two resistors in series.

they are connected "end-to-end" so that the same current flows through each of them, then the total effective resistance is simply the sum of the two individual resistances. This result follows from Ohm's law applied separately to R_1 and R_2.

$$R_{\text{total}} = \frac{V_3 - V_1}{I} = \frac{V_3 - V_2}{I} + \frac{V_2 - V_1}{I} = R_1 + R_2 \tag{1.5}$$

Thus, a 1 kΩ and a 3 kΩ resistor in series act like a single 4 kΩ resistor. This rule can be extended to N resistors in series, in which case the total effective resistance is equal to the sum of all the N individual resistances. $R_{\text{total}} = R_1 + R_2 + R_3 + \cdots + R_N$. Notice that the total resistance for a series connection always is greater than any of the individual resistances. Also notice that a straight line connecting the two resistances in a circuit diagram represents a wire or electrical connection of zero resistance. Thus all points of a straight line in a circuit diagram must be at exactly the same voltage; there is no voltage drop along a wire of zero resistance. In an actual circuit, the resistance of the wire used to connect various elements is usually negligibly small, e.g., a two-inch length of number 18 copper wire (0.04 in. diam) has a resistance of only 0.0011 Ω.

If two resistors are connected in "parallel" or "side by side" (see Fig. 1.8)

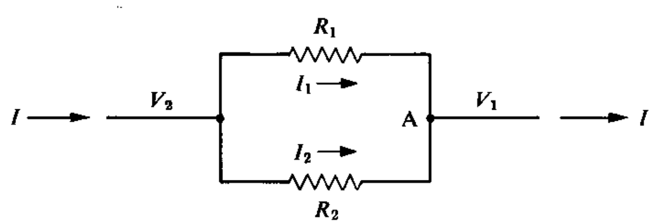


FIGURE 1.8 Two resistors in parallel.

so that the same voltage appears across each one, then the total effective resistance is given by

$$R_{\text{total}} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}. \tag{1.6}$$

This result follows from Ohm's law and the conservation of current. At the point A, the current I entering splits up into two parts, $I = I_1 + I_2$.

$$R_{\text{total}} = \frac{V_2 - V_1}{I} = \frac{V_2 - V_1}{I_1 + I_2} = \frac{V_2 - V_1}{\dfrac{V_2 - V_1}{R_1} + \dfrac{V_2 - V_1}{R_2}} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

For example, a 6 kΩ and a 4 kΩ resistor in parallel act like a single 2.4 kΩ resistor.

If we have N resistors $R_1, R_2, R_3, \ldots, R_N$ all connected in parallel, the total effective resistance is given by:

$$R_{\text{total}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}. \tag{1.7}$$

Notice that the total resistance for a parallel connection is always less than any of the individual resistances and that the voltage drop is the same across all of the resistors in parallel. From Ohm's law applied to each resistor, the current divides among the various resistors in such a way that the most current flows through the smallest resistance, and vice versa. For example, in the circuit of Fig. 1.8:

If $R_1 = 10\text{k}\Omega$ and $R_2 = 2\text{k}\Omega$, then $R_{\text{total}} = (10\text{k})(2\text{k})/12\text{k} = 1.67\text{k}\Omega$.
If $V_2 - V_1 = 8\text{V}$, then $I = 8\text{V}/1.67\text{k}\Omega = 4.8\text{mA}$.
The current I_1 flowing through $R_1 = 10\text{k}\Omega$ is $I_1 = 8\text{V}/10\text{k}\Omega = 0.8\text{mA}$.
The current flowing through $R_2 = 2\text{k}\Omega$ is $I_2 = 8\text{V}/2\text{k}\Omega = 4\text{mA}$.

In general $I_1/I_2 = R_2/R_1$ which follows from the fact that R_1 and R_2 have the same voltage drop across them: $I_1R_1 = I_2R_2$.

Variable resistors, often called "potentiometers" or "pots," are also available. They come in many sizes and styles, and are usually adjusted by manually turning a shaft, as shown in Fig. 1.9. They have three terminals, one at each end of the resistor and one for the variable position of the tap. The total resistance R_T between the two end terminals A and B is always constant and equals the resistance value of the pot. The resistance R_1 between A and the tap and R_2 between B and the tap varies as the shaft is turned. Notice that if the shaft is turned fully clockwise the tap is electrically connected to terminal A and $R_2 = R_T$, $R_1 = 0$. The variation may be "linear taper" with shaft rotation as shown in Fig. 1.9(c), or "logarithmetic taper" as in Fig. 1.9(d). For example, a $100\,\text{k}\Omega$ pot has $R_T = R_1 + R_2 = 100\,\text{k}\Omega$ regardless of the shaft rotation, but R_1 and R_2 depend on the shaft position—always subject to the condition $R_1 + R_2 = 100\,\text{k}\Omega$. A logarithmetic taper is usually used in volume controls for audio equipment; a linear tuner is more commonly used in scientific apparatus.

It is worthwhile to note that Ohm's law must apply to a resistance even though it is connected with another circuit element which does not obey Ohm's law. For example, a "zener diode" is a solid-state device which has the non-linear property that the voltage drop across its two terminals is essentially constant, regardless of the current through it over a wide range of currents. Thus Ohm's law does not apply to the zener diode, or in other words the resistance of a zener diode varies with the current through the diode. Consider a resistance and a zener diode connected in series with a battery as shown in Fig. 1.10. The current through R and the zener diode must be equal as they are connected in series, and the voltage of the battery must equal the voltage across

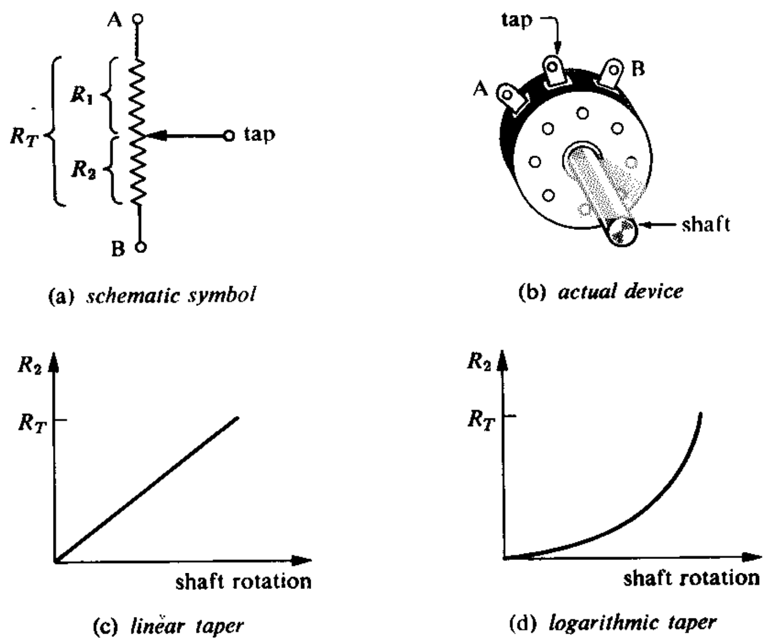


FIGURE 1.9 Variable resistor.

R, V_R, plus the voltage across the zener, V_z: $V_{bb} = V_R + V_z$. Ohm's law applied to the resistor alone implies $V_R = IR$. Thus $V_{bb} = IR + V_z$. If V_{bb} decreases, the constant zener voltage V_z implies that I must decrease so that the IR voltage drop across R always equals the difference between V_{bb} and V_z.

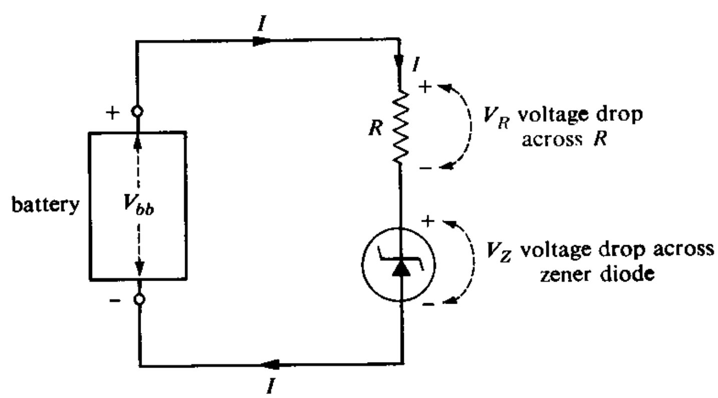


FIGURE 1.10 Zener diode circuit.

$$IR = V_{bb} - V_z \tag{1.8}$$

This circuit is often used to produce a constant output voltage which is taken from across the zener diode. If $V_{bb} = 12\text{V}$ and $V_z = 6.8\text{V}$, then $IR = 12\text{V} - 6.8\text{V} = 5.2\text{V}$. If $I = 10\text{mA}$, then $R = 5.2\text{V}/10\text{mA} = 520\Omega$ by Ohm's law. For $R = 520\Omega$, if V_{bb} falls to 10V, then I must decrease from 10mA to $(10\text{V} - 6.8\text{V})/520 = 3.2\text{V}/520 = 6.16\text{mA}$.

1.5 BATTERIES

A battery is a two terminal device in which chemical energy is converted into electrical energy, and a voltage difference is generated between the two battery terminals. A battery tends to spew positive charge out of the positive terminal and draw it into the negative terminal. A battery also tends to spew negative charge out of the negative terminal and draw it into the positive terminal. In an actual battery negatively charged electrons are spewed out the negative terminal and are drawn in the positive terminal, even though we may for convenience speak of positive charge flowing. A "dry" battery is one in which the chemicals are essentially dry or in a paste form, e.g., a flashlight battery. A "wet" battery is one containing liquids, e.g., an automobile battery that contains sulfuric acid solution. The circuit symbol for battery is a series of long and short parallel lines, as shown in Fig. 1.11—with the longer line on the

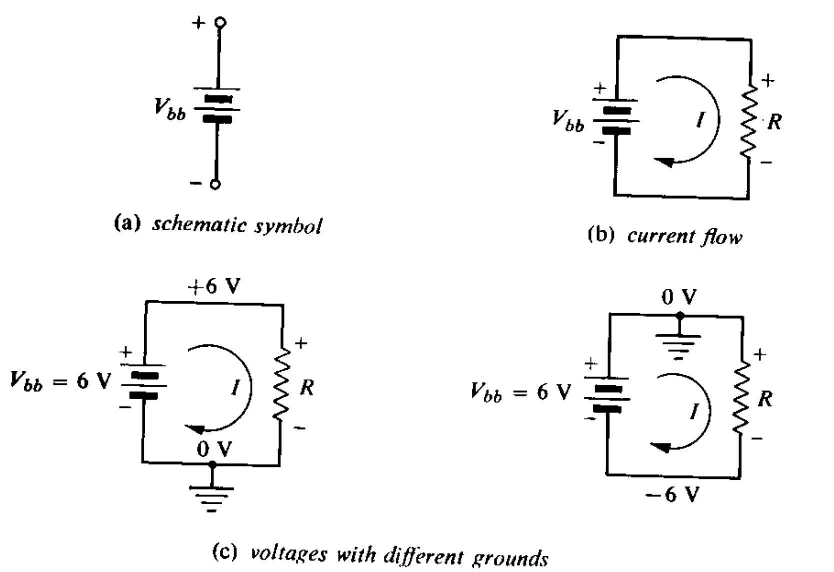


FIGURE 1.11 Batteries.

end representing the positive terminal and the shorter line on the other end representing the negative terminal. Thus, if a resistor R is connected between the two battery terminals as shown in Fig. 1.11(b), then positive current will flow out of the battery plus terminal, through the resistor, and back into the minus terminal of the battery. Notice that the (positive) current flows *out* of the battery "+" terminal and *into* the plus end of the resistor.

Notice again that the straight lines drawn in the circuit diagram represent wires with *zero resistance*. Thus there is no change in voltage along the wires represented by straight lines. The voltage at the positive battery terminal is exactly the same as the voltage at the top of the resistance in Fig. 1.11(b). In order to have a voltage difference between two points in a circuit, there must be some resistance between these two points from Ohm's law $V_2 - V_1 = IR$; that is, if $R = 0$, $V_2 - V_1 = 0$ even if $I \neq 0$. Notice also that the "+" and "−" signs represent the relative polarity of the voltages; that is, the top of the resistor is positive with respect to the bottom. Also notice that we may ground any *one* point in a circuit. Two such cases are shown in Fig. 1.11(c). In either case the voltage difference between the battery terminals is 6 V and the current is the same in each case. Notice that no current flows into the ground connection; the current flowing out of the positive terminal of the battery flows back into the negative terminal. The ground connection merely sets the zero voltage level.

A battery is rated in volts; its voltage rating V_{bb} indicates the difference in voltage which the battery will maintain between its two terminals. A perfect or ideal battery will always maintain the same voltage difference between its terminals regardless of how much current I it supplies to the rest of the circuit. However, the voltage of any real battery will decrease as more and more current is drawn from it. Thus for a real battery, V_{bb} represents the terminal voltage when no current is drawn from the battery. V_{bb} is often called the "open circuit" voltage. In general, the larger the battery's physical size (for the same voltage rating) the less the voltage will decrease as more current is drawn; in other words, a large battery can supply more current than a small battery at the same voltage. This behavior can be explained by the fact that a real battery has an internal resistance r as shown in Fig. 1.12(a). The actual current must flow through both r and R, which are in series, so $I = V_{bb}/(r + R)$. Notice that the battery terminal voltage $V_A - V_B$ must equal $V_{bb} - Ir$ because of the polarity difference between the Ir voltage drop across the internal resistance r and V_{bb}.

The larger the battery for a given voltage, the smaller r is ($r = 0$ for an ideal battery). The internal resistance of a battery can be determined by measuring the terminal voltage for various measured currents I drawn from the battery. The internal resistance r is then the negative slope of the graph of the terminal voltage plotted versus I as shown in Fig. 1.12(b). Or, if the terminal voltage is measured for two currents, then r is given by [see Fig. 1.12(b)] $r = (V_1 - V_2)/(I_2 - I_1)$.

When a battery goes "bad," its internal resistance r increases sharply.

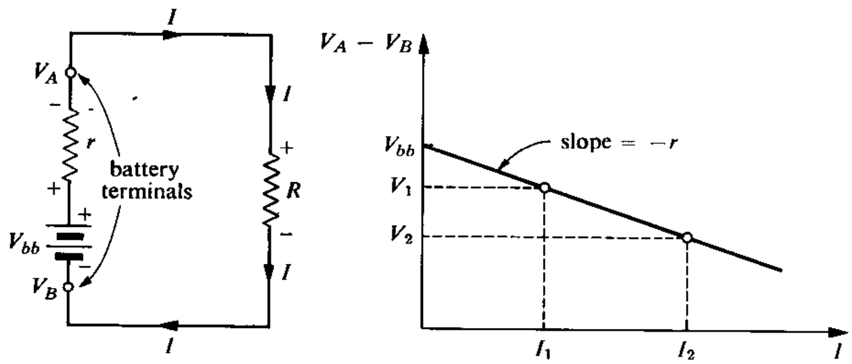


FIGURE 1.12 Internal resistance r of battery.

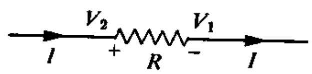
The typical good 12 V automobile battery has an internal resistance of about 0.03 ohm. A size D 1.5 V carbon–zinc dry cell such as is used in flashlights and in some portable transistor circuits has $r = 0.5$ ohm. A small $1'' \times \frac{1}{2}'' \times \frac{1}{4}''$ 9 V battery often used to power portable transistor radios has $r \cong 13\Omega$. Usually the internal resistance r is omitted from circuit diagrams, but this omission is valid only when r is much less than any other series resistances in the circuit.

In scientific circuitry, a battery which very gradually goes bad (that is, whose voltage slowly decreases) is a real disadvantage, because the circuit behavior may become erratic and difficult to diagnose as the battery slowly wears out. However, with a mercury battery which goes bad very abruptly (after a long life), the battery voltage is either all right or extremely low in which case the circuit will usually not function at all. Thus, in most scientific instruments mercury batteries are used, particularly if proper circuit behavior depends strongly upon a certain minimum battery voltage. The nickel-cadmium battery also goes bad abruptly after a long life, and it has the additional advantage of being rechargeable. It is, however, more expensive than the mercury battery. For a brief summary of the six different types of batteries, see Appendix B.

1.6 POWER

Power is defined as the time rate of doing work or the time rate of expending energy; that is, $P \equiv dW/dt$ where W is work or energy and t is time. The units of power are thus joules/sec; one "watt" is defined as one joule/sec. We will now show that a dc current I flowing through a resistor R develops a power of I^2R or VI or V^2/R, where V is the voltage drop across the resistor.

Recalling that voltage is electrical potential energy per unit charge, we see that a charge has less electrical potential energy when it leaves a resistor than it has when it enters because of the decrease in voltage, or voltage "drop" across the resistor. The time rate at which the flowing charge gives up electrical potential energy is thus the amount of charge flowing per second times the energy lost per unit charge, which is exactly equal to the current I times the voltage drop V. Thus, $P = IV$. But, from Ohm's law we know that $V = IR$; thus the power can also be expressed as $P = I^2R$ (see Fig. 1.13). And, again from



$$V \equiv V_2 - V_1$$
$$P = IV = I(IR) = I^2R$$
$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$$

FIGURE 1.13 Power dissipated in a resistor.

Ohm's law, $I = V/R$; thus another way of expressing the power is $P = V^2/R$. These three expressions for the power are equivalent and apply only to direct current flowing through a resistor. For alternating current the phase angle between the current and the voltage must be taken into account—more about this in Chapter 2.

The power developed in a resistor shows up as heating of the resistor. In other words, the loss of electrical potential energy (due to the IR voltage drop) of the charge flowing through the resistor is converted into random thermal motion of the molecules in the resistor. The kinetic energy of the flowing charges remains approximately constant everywhere in the circuit. Electrical potential energy is converted into heat energy in any dc circuit element across which there is a voltage drop and through which current flows. In the above three expressions for power, R represents the effective dc resistance of the circuit element, and V and I stand for the voltage drop across and the current through the circuit element, respectively.

If too many watts of power are converted into heat in a resistor or in any circuit element, the resistor may "burn up" in which case the resistor turns a brown or black carbonized color and may actually fragment into several pieces, thus breaking the electrical circuit. In other words the resistor is "open" or has an infinite resistance. Or, if the resistor is heated too much its resistance value may increase tremendously, thus changing the operation of the circuit drastically. For these reasons, resistors are rated by the manufacturer according to how much power they can safely dissipate without being damaged. Resistors are commonly available with wattage ratings of $\frac{1}{8}$ W, $\frac{1}{4}$ W (for low power transistor circuits), $\frac{1}{2}$ W, 1 W, 2 W, 5 W, 10 W, 20 W, 50 W, 100 W, and 200 W. The actual sizes of several commonly used resistors rated for various powers are shown in Fig. 1.14.

It is important to emphasize that the actual physical size does not depend upon the resistance, but only on the power rating; e.g., a $\frac{1}{2}$ W 2.2 kΩ resistor is the same size as a $\frac{1}{2}$ W 470 kΩ resistor. In designing circuits a resistor power

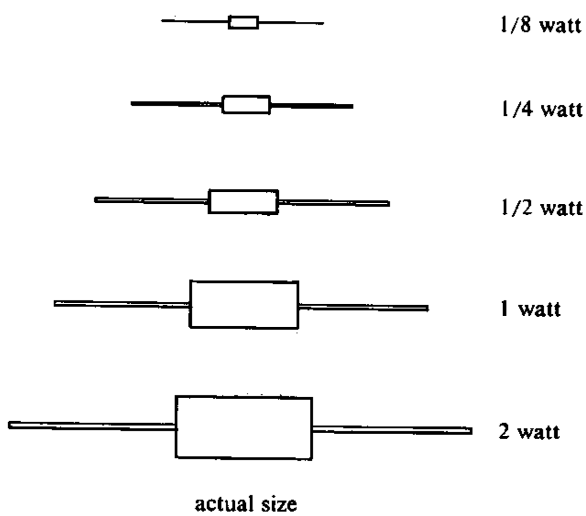


FIGURE 1.14 Resistor sizes for different power ratings.

rating of at least 3 or 4 times the expected power is usually chosen. For example, if a 1.5 kΩ resistor is to carry 20 mA of direct current, then the power dissipated as heat in the resistor will be $P = I^2R = (20 \times 10^{-3}\text{A})^2(1.5 \times 10^3\Omega) = 0.6\text{W}$. In the actual circuit, a 1.5 kΩ 2 W resistor would be used, or perhaps even a 1.5 kΩ 5 W resistor if the circuit were very sensitive to heat. If a large wattage is developed in a certain part of a circuit, care should be taken to provide an adequate vertical flow path for air around the hot element, so that the heat can be carried away by the resulting convection air currents. A resistor dissipating a large amount of power should never be placed in a closed chassis. Heat is an enemy of transistors as well as other circuit elements.

We will now derive an important theorem about power. If a battery has a certain fixed internal resistance r and we connect a load resistor R_L across its terminals, as shown in Fig. 1.15, how do we maximize the power $P_L = IV_L$ dissipated in R_L? If R_L is very small, I is large but $V_L = IR_L$ is small so the power in R_L is small. If R_L is very large, the voltage V_L across R_L is nearly equal to V_{bb}, but then the current is small so again the power in R_L is small. It seems reasonable that for some intermediate value of R_L the power dissipated in R_L is maximized. Let us set up an expression for P_L as a function of R_L, and maximize it.

$$P_L = IV_L \tag{1.9}$$

But

$$I = \frac{V_{bb}}{(r + R_L)} \quad \text{and} \quad V_L = IR_L = \left(\frac{V_{bb}}{r + R_L}\right)R_L.$$

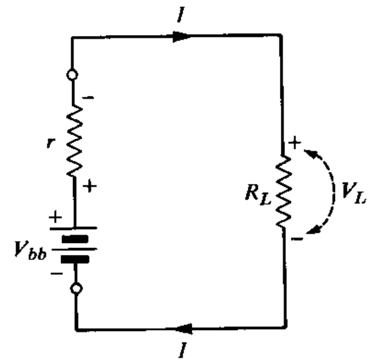


FIGURE 1.15 Circuit illustrating power transfer from source to load R_L.

Therefore,

$$P_L = \frac{V_{bb}}{(r + R_L)} \cdot \frac{V_{bb}}{(r + R_L)} \cdot R_L = \left(\frac{V_{bb}}{r + R_L}\right)^2 R_L \tag{1.10}$$

$$\frac{\partial P_L}{\partial R_L} = V_{bb}^2 \frac{(r + R_L)^2 - 2R_L(r + R_L)}{(r + R_L)^4}.$$

Set $\partial P_L/\partial R_L = 0$ to find the value of R_L for which P_L is an extremum.

$$\frac{\partial P_L}{\partial R_L} = 0 = \frac{V_{bb}^2}{(r + R_L)^4}[(r + R_L)^2 - 2R_L(r + R_L)]$$

$$(r + R_L)^2 = 2R_L(r + R_L)$$

$$R_L = r \tag{1.11}$$

It can be shown that P_L is a maximum not a minimum when $R_L = r$ by showing $\partial^2 P_L/\partial R_L^2$ is negative at $R_L = r$. In words, the maximum power is dissipated in the load resistance R_L when it equals the internal resistance of the battery. Under this condition, the voltage V_L across the load equals one half of V_{bb} which is the open circuit battery voltage. Also half of the total power is dissipated in r and half in R_L.

However, suppose we have a *fixed load resistance* R_L and V_{bb}, and variable r, and we ask the question: "How can we maximize the power dissipated in R_L?" The answer is *not* $r = R_L$, but $r = 0$! This answer is really obvious when we realize that any power dissipated in r is wasted as far as the load is concerned. It also follows mathematically from maximizing $P_L = V_{bb}^2 R_L/(r + R_L)^2$ with respect to r.

To sum up, if the load R_L is fixed, the smaller the internal resistance r the better. If we are presented with a fixed internal resistance r, then the load R_L gets maximum power when $R_L = r$. Also note that when r is fixed, the load *voltage* is very large when R_L is very large, and the load *current* is very large when R_L is very small.