COPLANAR AIR LAUNCH WITH GRAVITY-TURN

LAUNCH TRAJECTORIES

THESIS

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AFIT/GAE/ENY/04-M04

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THESIS

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Abstract

The purpose of this study was to determine the feasibility of launching a vehicle based on the Boeing AirLaunch System in a coplanar, direct to rendezvous trajectory with gravity-turn. The focus of the research was to model the launch trajectory and determine the ability to reach different coplanar orbits. The launch trajectory was modeled using two-dimensional equations of motion and a boundary value problem was posed and solved for the gravity-turn trajectory. Trajectories were then created in an attempt to reach different altitudes through coasting and transfer orbits. Finally a specific orbital altitude was chosen and the trajectories were analyzed to find the most efficient route to the target orbit for fuel and time.

Acknowledgements

I would like to thank my thesis advisor, Dr. William Wiesel, for his time, patience and support in this research effort. By sharing his knowledge, humor, and interests with me, he made the thesis process much more enjoyable. I would like to thank my fellow students, Dennis McNabb, Chris Blackwell, and Brian Lutz. Without their help in some way or form, I would not have reached this point. Finally, I'd like to thank my family and closest friends (there are too many to name here); but without whose support, friendships and guidance, I would not be where I am today.

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I. INTRODUCTION

1.1 Motivation

The goal for this research was to investigate launching a vehicle direct to rendezvous with an orbiting satellite using a gravity-turn trajectory. Current launch platforms use launch windows for mission planning. This practice launches the vehicle into the desired orbital plane as it passes overhead. Once on orbit, the vehicle then has to play catch up with the target. In some cases the catch up period can take up to three days. The launch windows themselves are very restrictive and sometimes offer limited time for a successful launch. The trade off for this restrictive schedule is a savings in fuel and energy and provides maximum payload to orbit.

Currently the Air Force is investigating concepts to provide rapid, responsive access to space. Several new designs attempt to achieve these concepts through different means. One such possible design is air launch. An air launch system would be capable of flying to a desired point for launch, thus giving the ability to reach launch points at the user's discretion. The desired capability of rapid access to space with the ability to launch direct to rendezvous would offer a range of options such as re-supply, emergency repair,

constellation regeneration, satellite protection and many other opportunities desired by the modern military.

Gravity-turn trajectories are also an item of current interest. Launch vehicles must maintain a zero angle of attack during launch through the atmosphere due to structural strength. Even a small angle of attack can mean structural failure for the vehicle. In a gravity-turn trajectory, the vehicle takes advantage of the force of Earth's gravity in order to rotate from vertical to a horizontal flight orientation tangential to its orbit. This allows the vehicle to conserve fuel and the mass of extra engines. In a gravity-turn, roll and angle of attack are maintained at zero, so that no lift is generated. Space launch vehicles are made to be very strong along their longitudinal axis, however are very weak along the lateral axis. If lift is generated the vehicle will more than likely disintegrate. To avoid this, the vehicle's computer will compensate to keep the angle of attack and roll zeroed out while letting the earth's gravity-turn the vehicle. The vehicle is given a very small nudge from the vertical to begin the process. During this time a small amount of lift will be created, so the process is begun shortly after launch when the vehicle's speed is very slow. It cannot be done from the initial launch position as the vehicle does not have enough momentum and will simply fall over. Like generating lift at speed, this is a very bad situation for a rocket to find itself.

1.2 Overview

The rest of this chapter is devoted to discussing the Boeing AirLaunch vehicle used as the basis for the research. Chapter 2 describes setting up the trajectory problem

and introduces the derivation of the equations of motion and Hohmann transfer. The computer algorithms used in the research are discussed in Chapter 3. Results of the gravity-turn trajectory analysis are presented in Chapter 4. Chapter 5 then finishes with the conclusions and recommendations.

1.3 Vehicle Background

One of the most desired capabilities of current space launch is the ability to launch on demand into a first pass orbit. Current systems such as Expendable Launch Vehicles (ELVs) and the Shuttle Launch System (SLS) are based on timetables, schedules, and launch windows. These systems require months and possibly years of planning and executing for a launch. The only launch on demand systems currently operating is that of Orbital Sciences of Dulles, Va. In Orbital's system a Pegasus launch vehicle is carried aloft on a Lockheed L-1011 and flown to a specific point along the launch corridor. The Pegasus can lift small payloads of 1,000 lb into orbit. Boeing's proposed AirLaunch System is an attempt to support payloads on the order of 15,000 lb. The system will be carried on the back of a Boeing 747-400F freighter and have the ability to operate from any 10,000 to 12,000 foot runway. (Wilson, 2001:43-46)

The Boeing 747-400F would then carry the launch vehicle to a specific launch altitude and position. With an in-flight refueling capability, practically any launch point can be reached. AirLaunch's wings and tail assembly will provide the vehicle with glide ability so that a safe distance can be achieved between separation from the aircraft and engine ignition. Approximately five seconds after ignition the vehicle would jettison its wings and tail for the launch. It should be noted that the discussion on operations from

Boeing does not give a great amount of detail on the length of the pull up, or the lateral acceleration tolerable by the vehicle. For the purposes of this research, it is assumed that the vehicle can reach a vertical attitude safely at 20,000 feet and a velocity of 300 mph. ("Phantom Works", 2003)

Boeing's AirLaunch System is the basis of the vehicle used in this research. The launch vehicle of AirLaunch will consist of 3 stages plus payload. The first two stages are Thiokol Castor 120's with the third stage made up of a Thiokol Star-92. Castor 120's are off the shelf solid rocket motors with specifications shown in Table 1.

Table 1: Castor 120 Specifications ("Castor 120", 2003)

Length	347 in	881.38 cm
Diameter	93 in	236.22 cm
Propellant Weight	107,137 lbm	48,596.526 kg
Total Weight	116,159 lbm	52,688.836 kg
Average Thrust	370,990 lbf	1,650,245.737 N
Specific Impulse	280 sec	280 sec
Burn Time	82 sec	82 sec

Thiokol Star-92's solid rockets are not currently released, and no information was available from Thiokol or Boeing. Therefore, specifications were assumed from known solid rocket motor performance and limitations. The ratio of structure mass to payload mass was also assumed to be similar to those of the Castor 120's. A mass flow half that of the Castor 120 was used to produce the burn times and average thrust. For the mass of

the third stage, the given total vehicle weight was used minus the known payload mass and the masses of the first two stages. The specifications used for the Star-92 stage is given in Table 2.

Table 2: Star-92 Specifications

Propellant Weight	25,177.853 kg
Total Weight	27,298.096 kg
Average Thrust	813,932.545 N
Specific Impulse	280 sec
Burn Time	84 sec

The major motivation to use the Boeing AirLaunch System as the basis for this research was for the capabilities of air launch. The focus of the research was to look into the feasibility of a gravity-turn launch trajectory into a direct to rendezvous orbit for the Air Force's Space Maneuvering Vehicle (SMV). To achieve this goal with launch on demand, a movable launch pad or air launch was preferred. Boeing's AirLaunch is the closest feasible vehicle in achieving the ability to launch on demand in the payload class of the SMV of 7,500 lb (3,000 kg).

II. MODELING THE TRAJECTORY

2.1 Literature Search

A search of current literature produced articles on the Boeing AirLaunch System (ALS).   This information included stage types, total weight of the vehicle, specifications on the first and second stage engines, and some operational information. The ALS is piggy backed aloft on a Boeing 747. The operations of this carrier craft were discussed, allowing types of air fields capable of deploying from, and an in flight refueling capability giving the carrier aircraft a virtual unlimited range. The vehicle can then be flown to a launch point as necessary. However, no trajectory profiles were discussed in the research.

2.2 Reference Frames

The frame decided upon for this research was simplified to the downrange distance measured in the x-direction and altitude measured in the y-direction. As seen in Figure 1, the H axis is oriented to point away from the center of the Earth, and the frame rotates such that H-axis remains pointed away from the center of the Earth. The X-axis is defined as shown perpendicular to the H-axis in the direction of movement.
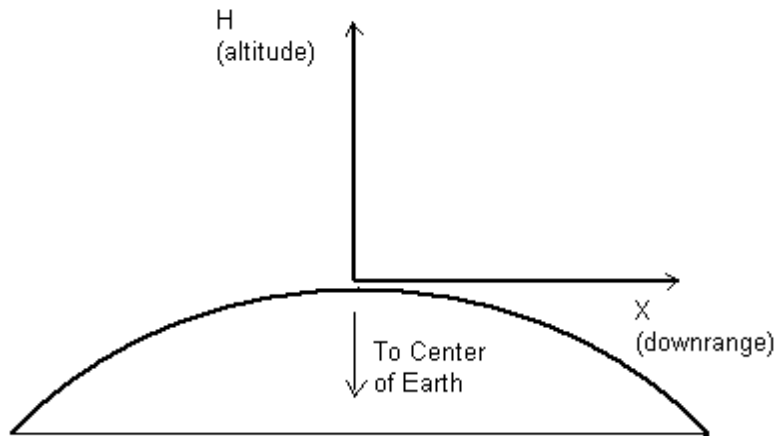
Figure 1: 2-D Reference Frame

This rotation is dependent on the horizontal velocity of the frame, such that:

$$\omega^{hi} = -\frac{\dot{X}}{(R_e + H)} \hat{h}_3$$ **(2-1)**

Where R_e is the radius of the Earth, and H is the altitude of the vehicle. $\dot{X}$ is the velocity in the X-axis, and $\hat{h}_3$ would be represented by the plane coming out of the paper. The rotation is negative as it is opposite of the standard right-handed notation (Wiesel, 1989:19).

The centripetal acceleration can then be found as:

$$\omega^{hi} \times (\omega^{hi} \times r) = -\frac{\dot{X}^2}{(R_e + H)} \hat{h}_2 \qquad (2\text{-}2)$$

where $r = (R_e + H)\hat{h}_2$ and $\hat{h}_2$ is in H-axis as drawn in the Figures 1 and 2. However, it is conducive in this situation to use the body frame in the derivations of the equations of motion, so the flight path of the vehicle can be taken into account (Wiesel, 1989:217). As shown in Figure 2 the flight path angle, γ, is the angle between the local horizon and the velocity vector.
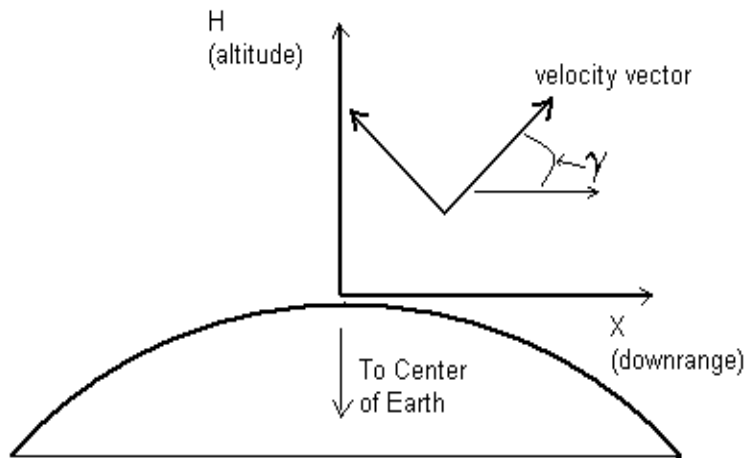
Figure 2: Reference frame with flight path angle

With the frames defined, we are ready for the equations of motion.

2.3 State Variables and Equations of Motion

The development of the equations of motion here is a modified form of those presented in (Wiesel, 1989: 216-219). First, the state space vector must be defined. The five elements used for this research are:

m – mass of the vehicle

H – altitude of the vehicle

X – distance downrange of the vehicle

V – velocity of the vehicle

γ – flight path angle of the vehicle

With the state vector defined, the equations of motion can be developed further. Using Figure 3 as a reference, we can make some observations. First, the vehicle stays on the H axis, therefore the vertical acceleration is $\ddot{H}$ and the downrange acceleration is represented as $\ddot{X}$. By geometry, the following equations can be produced:

$$\frac{dX}{dt} = V \cos \gamma \qquad \text{(2-3)}$$

$$\frac{dH}{dt} = V \sin \gamma \qquad \text{(2-4)}$$
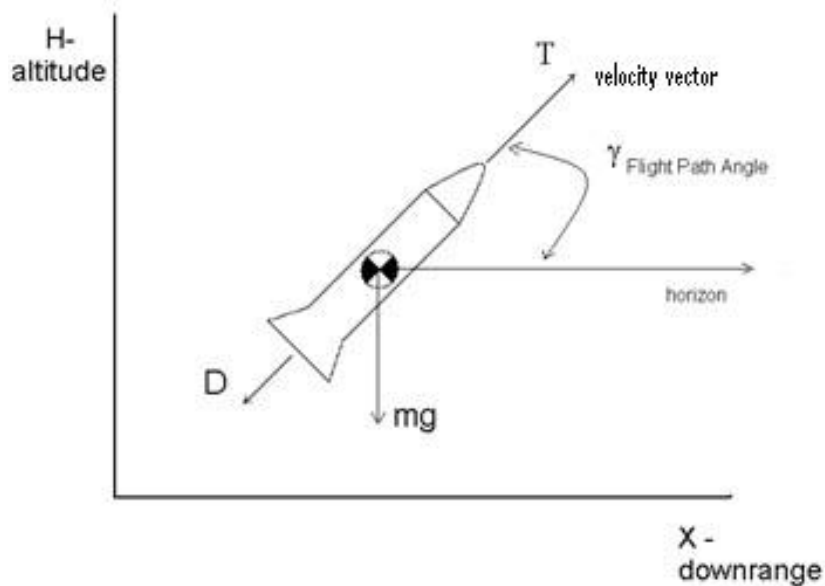
Figure 3: Diagram of Forces Acting on the Vehicle

As mentioned in the previous section, it is more conducive to use the body frame instead of resolving into the vertical and horizontal frames. By doing so, the accelerations of the vehicle can be summed up into acceleration along and transverse to the vehicle's axis. Considering that V is the velocity of the vehicle, and γ is the flight path angle measured counterclockwise from the horizon, then $\dfrac{dV}{dt}$ is the acceleration produced along the axis of the vehicle, and $V\dfrac{d\gamma}{dt}$ is the acceleration transverse to the vehicle's axis.

Now, using Newton's Second Law:

$$\sum F = ma \qquad \text{(2-5)}$$

We can then proceed to sum the forces of the vehicle. Again referencing Figure 3, we find the forces acting on the vehicle are Thrust (T), Drag (D), gravity (mg), and centripetal acceleration (2-2). Rotating the gravity and centripetal accelerations by the flight path angle of the vehicle and summing the forces along the axis of the vehicle then gives:

$$\sum F = ma = T - D - mg\sin\gamma + \frac{m\dot{X}^2}{(R_e + H)}\sin\gamma \qquad \textbf{(2-6)}$$

simplifying to:

$$m\frac{dV}{dt} = T - D - (mg - \frac{m\dot{X}^2}{(R_e + H)})\sin\gamma \qquad \textbf{(2-7)}$$

Then similarly, summing the forces transverse to the axis of the vehicle produces:

$$\sum F = ma = -mg\cos\gamma + \frac{m\dot{X}^2}{(R_e + H)}\cos\gamma \qquad \textbf{(2-8)}$$

which then simplifies to:

$$mV\frac{d\gamma}{dt} = -(mg - \frac{m\dot{X}^2}{(R_e + H)})\cos\gamma \qquad \textbf{(2-9)}$$

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To find the mass flow rate of the vehicle the following equation was required:

$$\dot{m} = -\frac{T}{g_o I_{sp}}$$ (2-10)

where T is thrust, Isp is the specific impulse of the engine, and g_o is gravitational acceleration at sea level. For the third stage, equation (2-10) holds true, however it does not do so for the first and second stages. As the exact specifications are available for this solid rocket motor, a double check reveals that with a burn time of 82 seconds, the mass flow provided by (2-10) results in a total fuel burn that is greater than the actual available fuel. Therefore, in the first two stages, the mass flow is determined by:

$$\dot{m} = \frac{total\ mass\ of\ fuel}{burn\ time}$$ (2-11)

While it is most likely true that there is still some fuel left before the stage is jettisoned and the fuel is probably not burned at a constant rate, this provides a reasonable model.

Therefore, to form the scalar equations of motion for the system, we gather the equations:

$$\frac{dX}{dt} = V \cos \gamma$$

$$\frac{dH}{dt} = V \sin \gamma$$

$$m \frac{dV}{dt} = T - D - (mg - \frac{m \dot{X}^2}{(R_e + H)}) \sin \gamma \qquad \textbf{(2-12)}$$

$$mV \frac{d\gamma}{dt} = -(mg - \frac{m \dot{X}^2}{(R_e + H)}) \cos \gamma$$

$$\dot{m} = -\frac{T}{g_o I_{sp}}$$

In general, the equations of motion for non-linear time-dependent systems is written:

$$\dot{X} = f(x,u,t) \qquad \textbf{(2-13)}$$

Given x represents the state variables, u is representative of the control variables and t represents time (Sears, 1997:14-15). This form is used by MATLAB, which is discussed in the following chapter.

2.3 Hohmann Transfer

As will be discussed in Chapter 4, several options are available for achieving orbit. One possible augmentation to the launch is the addition of a transfer orbit to insert the vehicle into the proper orbit. The Hohmann Transfer is the most efficient use of available fuel with the disadvantage of being the longest transfer available in terms of time.

As derived and shown in (Wiesel, p74-75), there are two changes in velocities or ΔV's required. These two velocity changes are represented by equations 2-14 and 2-15.

$$\Delta V_1 = \sqrt{\frac{2\mu}{a_1} - \frac{2\mu}{a_1 + a_2}} - \sqrt{\frac{\mu}{a_1}} \qquad \text{(2-14)}$$

$$\Delta V_2 = \sqrt{\frac{\mu}{a_2}} - \sqrt{\frac{2\mu}{a_2} - \frac{2\mu}{a_1 + a_2}} \qquad \text{(2-15)}$$

where

ΔV_1= change in velocity at first maneuver

ΔV_2= change in velocity at second maneuver

μ = gravitational parameter of earth

a_1= radius of the first maneuver

a_2= radius of the second maneuver

The gravitational parameter of earth (μ) is defined as 3.98601 x 10^5 km^3/s^2 (Wiesel,

1989:323). A physical relationship between the variables in equations 2-14 and 2-15 can

best be seen in Figure 4 below:
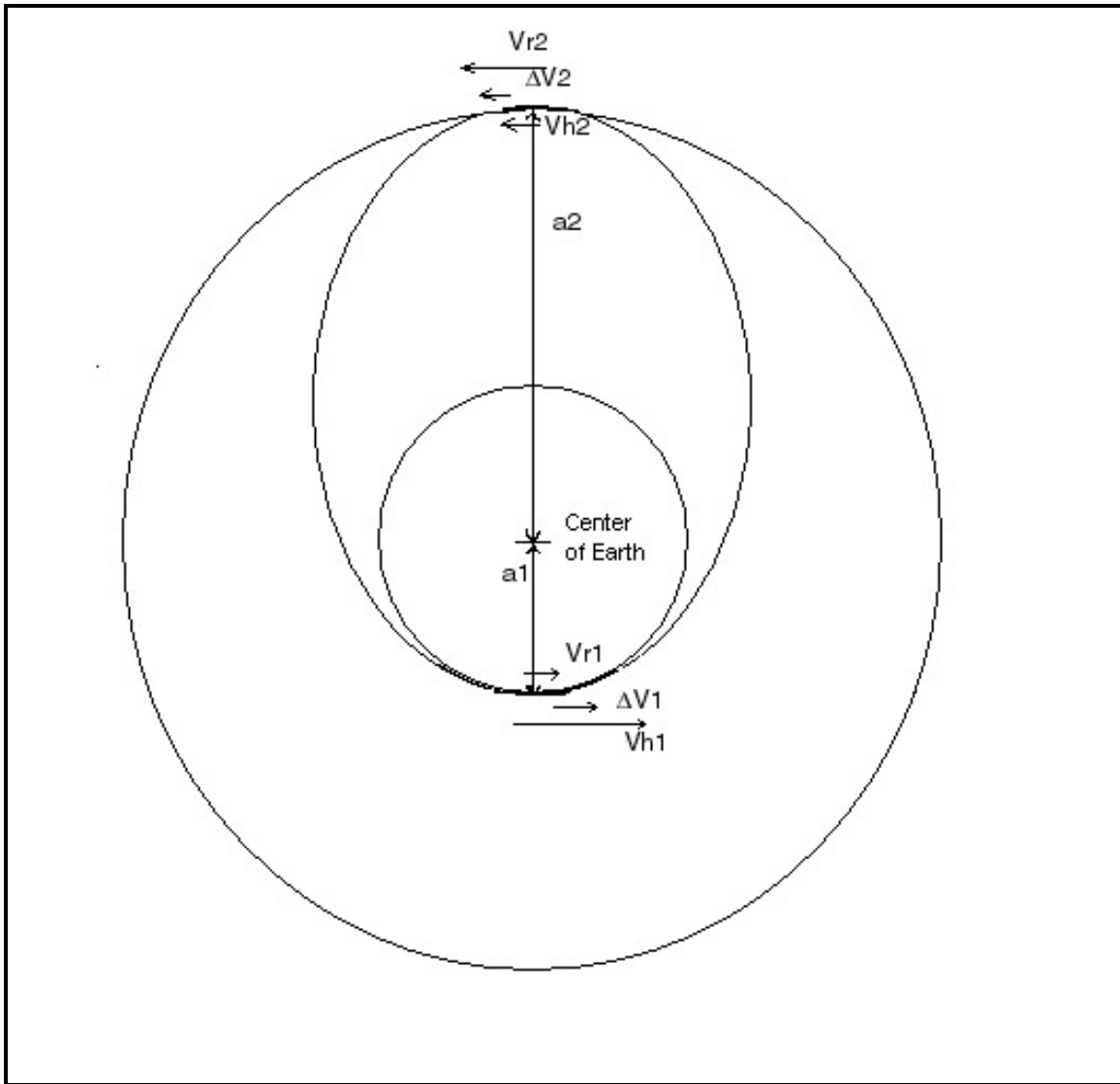
Figure 4: Hohmann Transfer between two orbits (Wiesel, 1989:74)

Also of interest is the time of flight of the transfer orbit. A Hohmann transfer is basically half of an elliptical orbit. So as seen in (Wiesel, 1989:75) the time between maneuvers is given as:

$$\Delta t = \pi \sqrt{\frac{a^3}{\mu}} \qquad \textbf{(2-16)}$$

where

$$a = \frac{a_1 + a_2}{2} \qquad \textbf{(2-17)}$$

$$\mu = 3.98601 \times 10^5 \ km^3/s^2$$

The time between maneuvers is especially of interest for mission planning.

III. ALGORITHMS

3.1 Introduction

In previous chapters the vehicle and equations of motion were discussed and developed. In this chapter we discuss the algorithms used in the computer programs to model the launch trajectories of the vehicle. Examples of the m-files can be found in Appendix B. Before the algorithms can be truly discussed, the assumptions for this problem need to be stated.

1. Drag is being neglected, i.e. D=0.

2. Vehicle specifications are as stated previously.

3. Third stage assumptions are accurate.

4. The vehicle has pitched to the full vertical position without expenditure of fuel at launch.

5. The vehicle begins with a 300 mile per hour velocity at 20,000 feet.

6. The derivations and assumptions stated in the preceding chapters are assumed correct.

The programs were written in the following units:

Mass = kilograms (kg)

Time = seconds (s)

Distance = meters (m)

Velocity = meters/second (m/s)

With the above assumptions, computer programs were written to solve the boundary value problem for the gravity-turn equations of motion. These programs were written in MATLAB 6.5 Student Edition and are commonly referred to as m-files in this research. The entire trajectory profile boundary value problem was broken into three parts. Each section represents a stage, as there is a discontinuity at the separation points. Each program consists of two m-files. The first file, INITIALCOND provides initial conditions and the initial state vector. The MATLAB command ODE45 then calls upon the second m-file, LAUNCHEOMS in this case, that contains the equations of motion. The routine continues for a set time period providing an output file of the state vector at each time step. The following stage's m-file then reads the last previous state vector and runs an almost identical algorithm, the major changes being the initial conditions and in the case of the third stage, the thrust. The end result is an output file with final conditions in mass, altitude, downrange distance, velocity, and flight path angle of the vehicle. The output file also contains the state vector at each of the iterations, providing data for manipulation to be presented in the following chapter. Other programs were used, but will be discussed later in the chapter. The purpose of these algorithms ultimately is to find the altitude, velocity, and initial flight path angle that will provide a zero flight path angle at burn out.

3.2 Initial Launch Trajectory

An initial trajectory was produced first. Several runs were made, and typically the first trajectory of each run was made with a flight path angle of 89.5°. This provided

a reference trajectory for each run.   Obviously an initial flight path angle of 90° would not be useful as there would not be a gravity-turn trajectory. The idea of gravity-turn is that the vehicle would be in a vertical orientation, and then nudged over a small amount so that the center of mass of the vehicle is no longer along the vertical axis, providing a force from gravity that will be used to turn the vehicle.

The first program, INITIALCOND, is an m-file with a section noted for the initial conditions. This file would provide the following:

- Initial 5 values of the state vector

- Time step and final time

- Prepare the output file to receive information

- Call on the second m-file through the ODE45 command

The second m-file for INITIALCOND would be LAUNCHEOMS. In LAUNCHEOMS, the equations of motion are written in terms of the initial state vector. The two files then work through the time period, which in this case is designated by the burn time of the rocket motor of 82 seconds, to produce a final state vector for the stage. The information is output to a text file at each time step so that the data can later be examined and plotted.

At the end of INITIALCOND's run, the vehicle has expended its first stage and jettisons the dead weight. This produces a discontinuity in the change of mass that dictates the need to write separate programs for each of the stages. The second program is SECONDSTAGE and is paired with SECONDEOMS. The program SECONDSTAGE is virtually identical to INITIALCOND except that it calls on a new initial mass, and then calls on the last line of INITIALCOND's output for the remainder of the initial state

vector. Then SECONDSTAGE calls SECONDEOMS, which again runs through the

equations of motion provided before. SECONDEOMS is identical to LAUNCHEOMS

except in the variables assigned to the state vector. The variables were named differently

to provide easier access for data manipulation. At the end of SECONDSTAGE, again

there is a discontinuity in mass from jettison of the second stage dead weight. Also note

that the third stage has a different thrust, mass flow rate, and burn time. Therefore

THIRDSTAGE is very similar to its predecessors but has very different values. It also

calls on the last vector provided by SECONDSTAGE. The output from THIRDSTAGE

is then provided in the output file, where a five-column matrix is given. Each stage has

it's own text file for easier differentiation of the data. Of primary interest in the results

for this research is the final flight path angle. Two graphs would be plotted using the m-

file, PLOTFILE. The first graph would plot altitude vs. downrange distance, which in

essence is a modeling of the actual flight path of the vehicle as shown in Figure 4. The

second graph (Figure 5) plots altitude vs. flight path angle, which provides a graphical

means to observe the behavior of the vehicle's attitude. With an initial reference

trajectory provided, we're ready to extrapolate to the desired point of a zero degree flight

path angle at burn out.
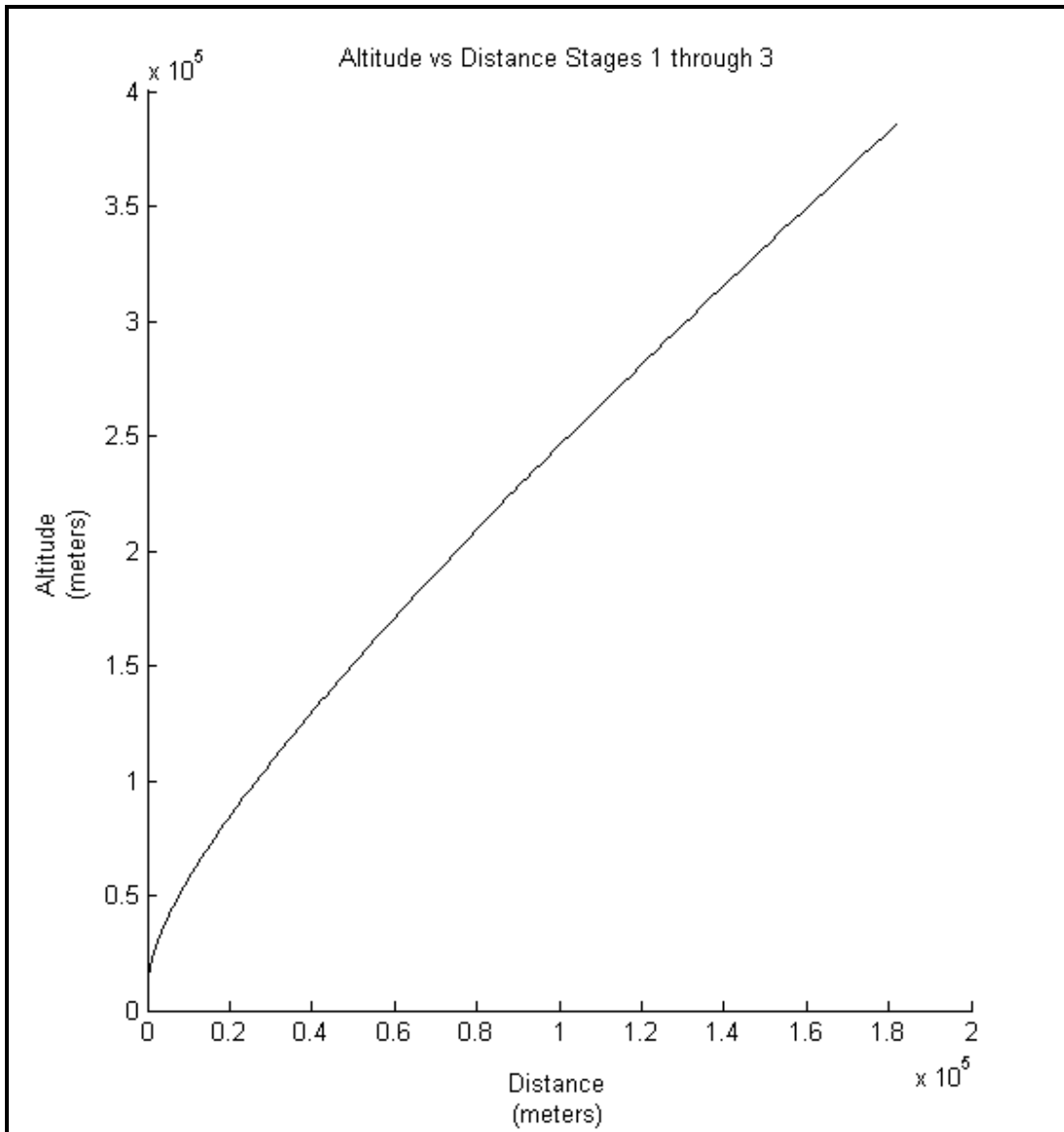
Figure 5: Altitude vs Downrange Distance for Initial γ_o of 89.5°
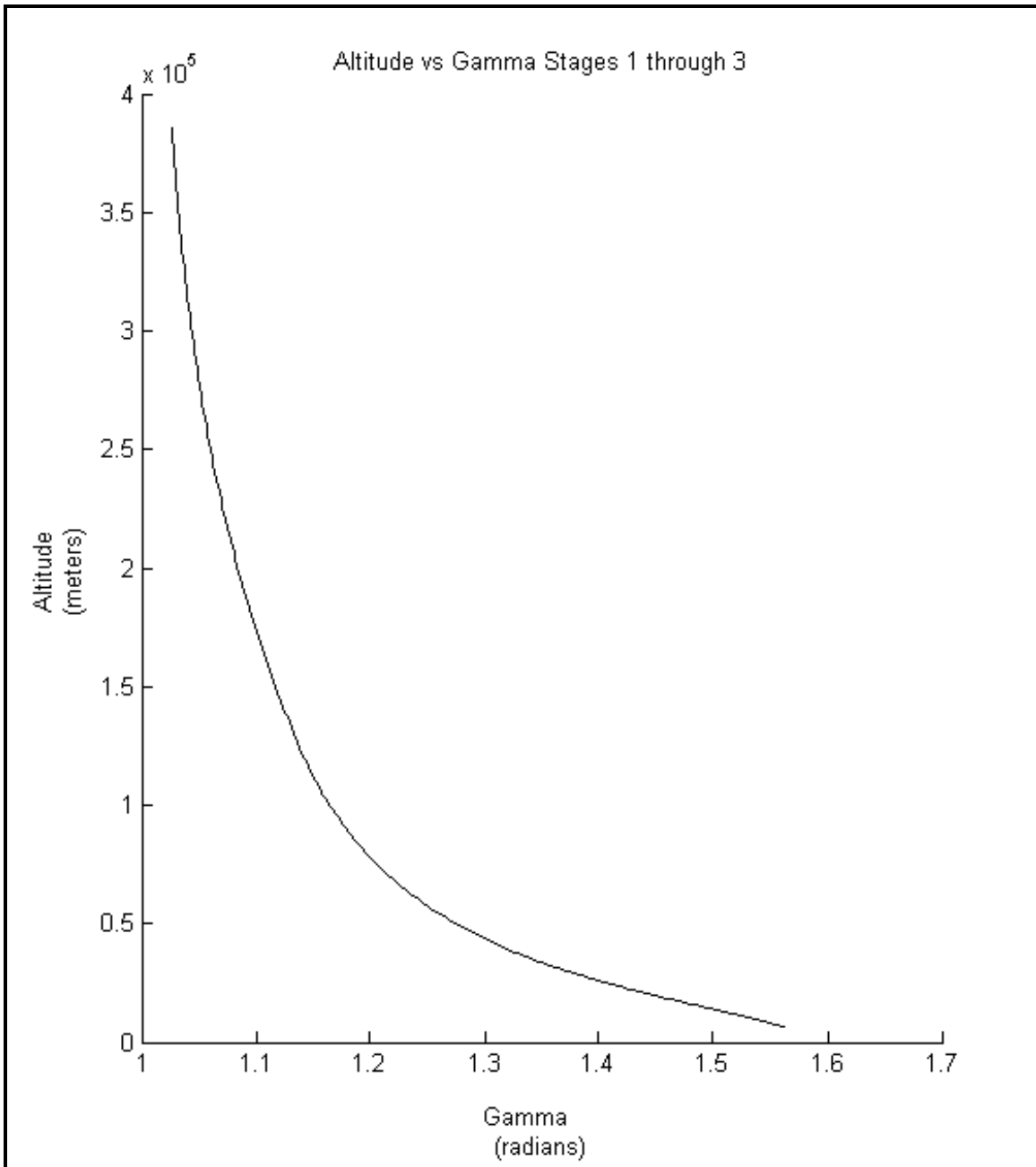
Figure 6: Altitude vs Gamma γ for γ_0 of 89.5^o

3.3 Extrapolation to Zero Flight Path Angle at Burn Out

The goal here is to find the correct initial conditions that provide for a zero degree flight path angle at burnout. To do this, a second trajectory is found using the same procedure as for the initial trajectory above. However, this trajectory begins with an angle of 88.5^o as it's initial flight path angle γ_o. After running the m-files through with this initial condition, a second trajectory is formed as shown in Figures 7 and 8 below.
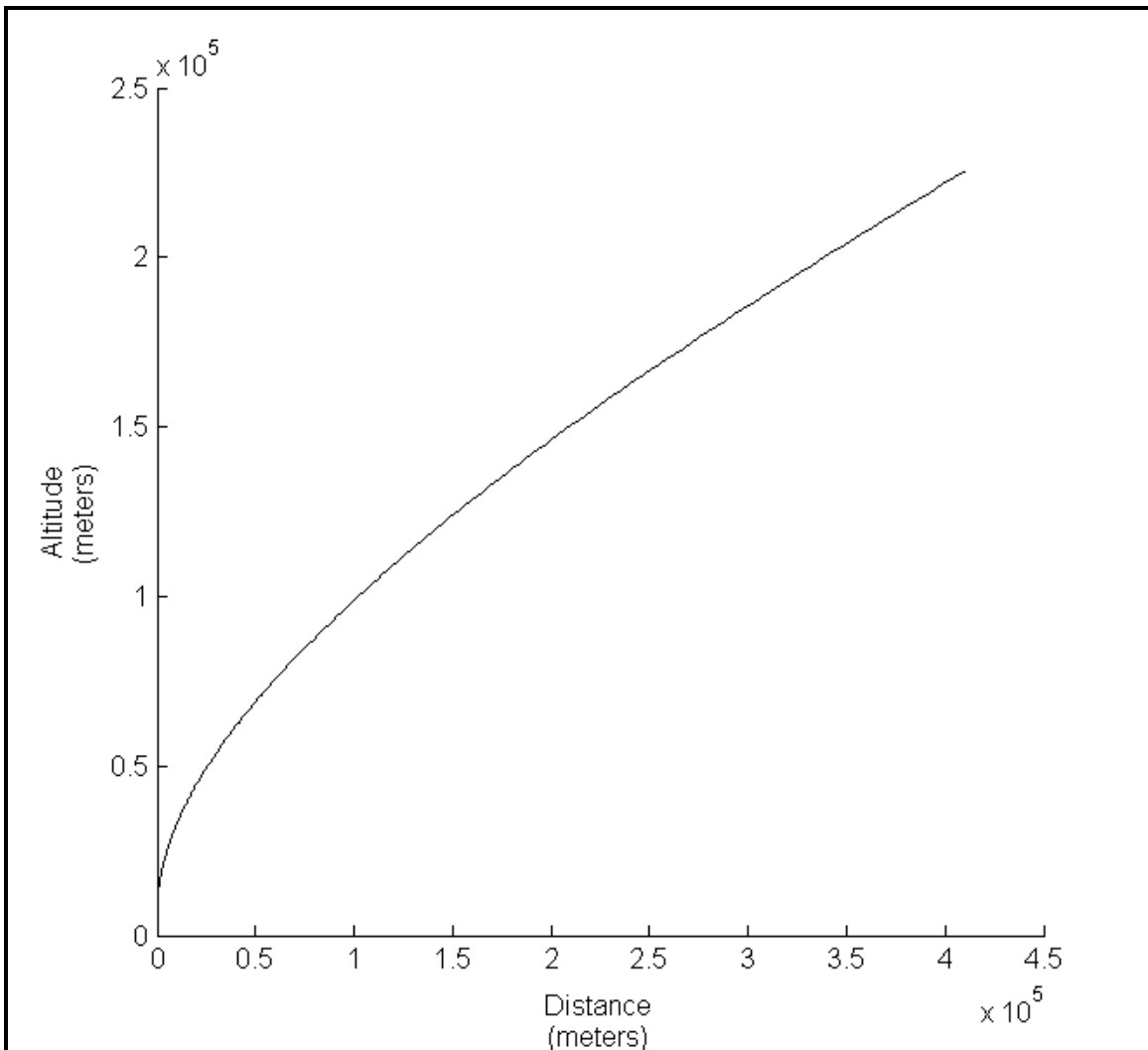
Figure 7: Altitude vs Distance for γ_o of 88.5^o
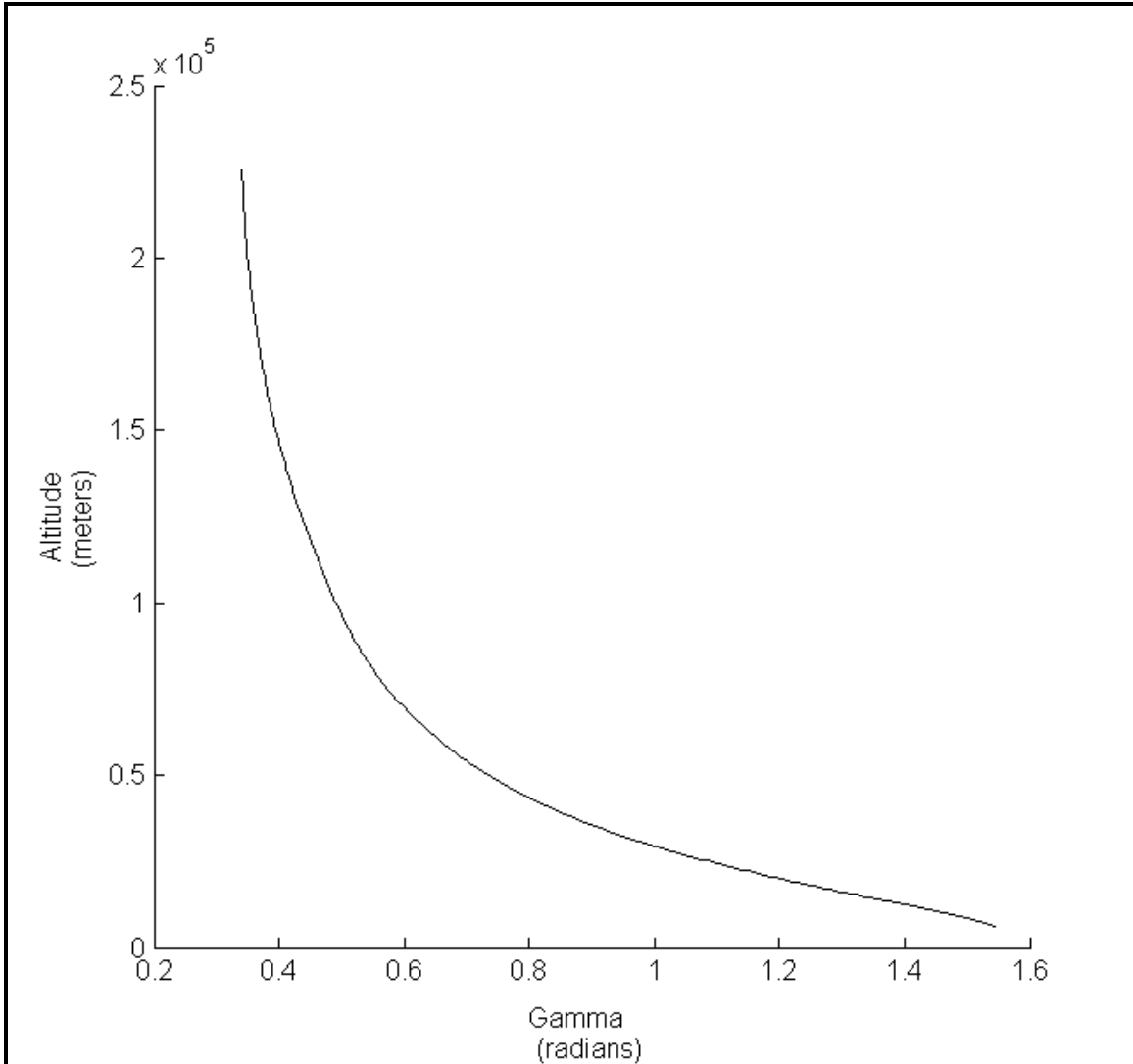
Figure 8: Altitude vs Flight Path Angle γ_o for γ_o of 88.5°

As shown in these charts, the final γ is not equal to zero. Here another m-file routine called NEWGAMMA is applied. This program requires values to be input into the file directly. The initial flight path angles, γ_o, and the final angles, γ_f, from both runs are input here. The program then runs a simple formula to find the next γ_o:

$$\gamma_{o_{new}} = \gamma_{o_{old}} - \frac{\Delta \gamma_o}{\Delta \gamma_f} \gamma_{f_{old}}$$

(3-1)

Where $\gamma_{o_{old}}$ is the last initial flight path angle, $\Delta\gamma_o$ is the difference between the most recent two initial flight path angles, $\Delta\gamma_f$ is the difference between the two most recent final flight path angles, and $\gamma_{f_{old}}$ is the most recent final flight path angle of the vehicle. This last term would be a $\Delta\gamma$ of the difference between the given and desired final flight path angle except our final desired angle is zero. Then the new γ_o is input into the next run as the initial flight path angle. This process is iterative as the value that will bring γ_f to zero is found. The entire set of m-files is then run multiple times giving the results discussed in the following chapter.

IV. RESULTS AND DISCUSSION

4.1 Introduction

This chapter presents the results found in the gravity-turn trajectory modeling of the Boeing AirLaunch based vehicle. The only payload considered in this research was the Space Maneuver Vehicle (SMV) that is currently in development for the Air Force. This is considered to be approximately 3,000 kg (7,500 lb) of payload mass. With the vehicle described in previous chapters as the launch vehicle, the gravity-turn m-files can now be run.

4.2 Initial Results

Since the engines are solid rocket motors, they are not capable of throttling by design. This provides that there is no capability for changing thrust during the burn sequences. Intuition should prove that given these facts this method has a fixed altitude at which the final flight path angle γ_f goes to zero. Using the procedures described in the previous chapter, the first runs were found to go to zero at the state described in Table 3. This run is referred to as case 1 for the remainder of this thesis.

Table 3: Final conditions of case 1

Initial Flight Path Angle γ_o	87.51904^o
Altitude	81433.5009 meters
Downrange Distance	504090.589 meters
Velocity	7344.29656 m/s

As can be seen in Table 3, the final altitude is only 81.4 kilometers. It is immediately obvious that this is not high enough to be in low earth orbit. For this research, an orbit of 250 kilometers was chosen as the target altitude. Several ideas were approached to alter the final altitude. Since the vehicle is powered by solid rocket motors, there is no capability for variation of thrust or burn time. One possibility, therefore, is to look into coasting the vehicle between stages in order to gain a higher altitude.

4.3 Vehicle Coasting

In order to reach higher orbit altitudes, coasting the vehicle was investigated.   This involves waiting between stages before igniting the next stage.   After the previous stage has burned out, it will be jettisoned and the vehicle will then continue on for a specified amount of time under no thrust. This is referred to as coasting as the vehicle is under no power. By coasting, the vehicle will continue to gain altitude while conserving fuel and can therefore get to higher altitudes. To insert coasting, m-files were created that ran the equations of motion without thrust or mass flow. Similar to the routines described above,

the initial flight path angle had to be varied until a final flight path angle of zero was attained. Several runs were attempted while varying the coast times between stage ignitions. As discussed before, drag is assumed to be negligible, which could cause differences in real world scenarios. The results varied as presented in the remainder of this section.

The first cases attempted were made while keeping the coast times consistent with each other. In case 2, only a five second coast was considered. As seen in Figure 9 and Table 4 this showed that only a small change was attained. Obviously a much larger coast time is needed.

Table 4: Final conditions of case 2

Initial Flight Path Angle γ_o	87.88^o
Altitude	86441.6828 meters
Downrange Distance	505497.292 meters
Velocity	7263.80507 m/s
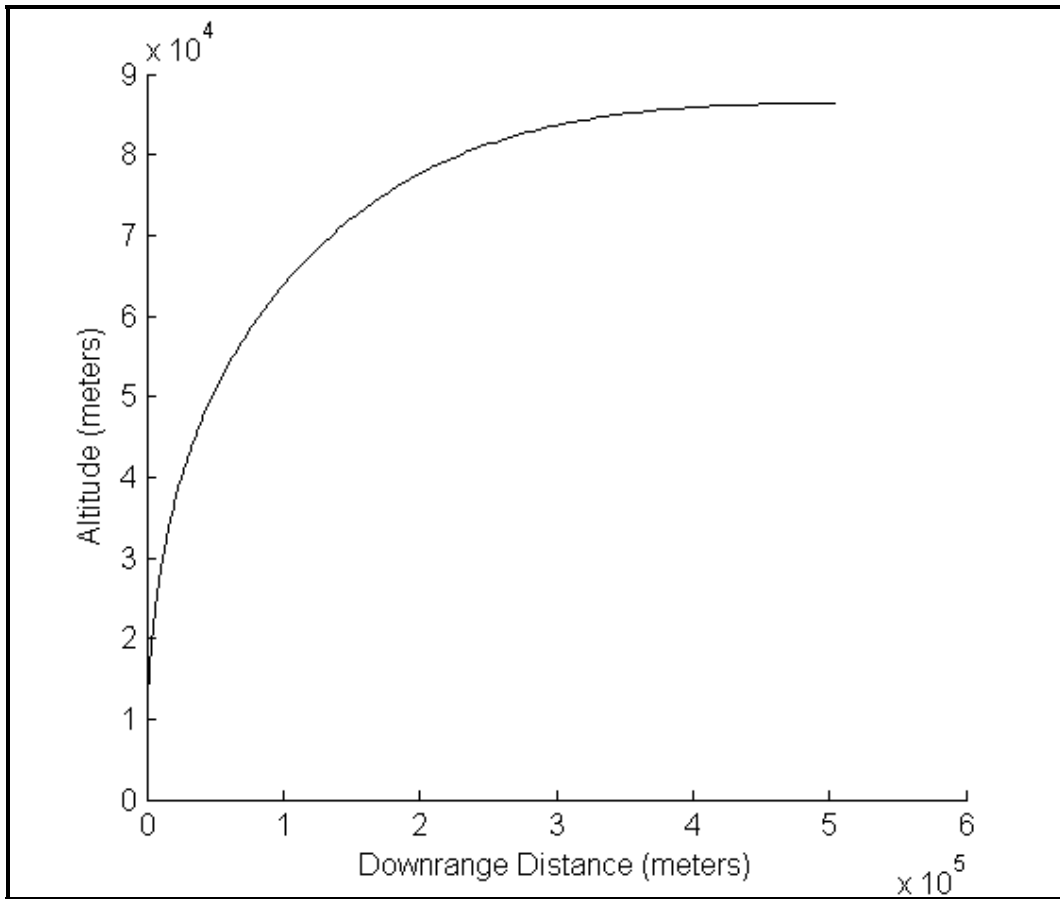
Figure 9: Launch trajectory for case 2

From case 2, a jump to a coast of thirty-five seconds between stages was performed in case 3 to see how large a difference could be found. As seen in Figure 10 and Table 5, a much higher altitude is reached than the previous runs. However, an altitude of 121 km is still fairly low for an orbit.
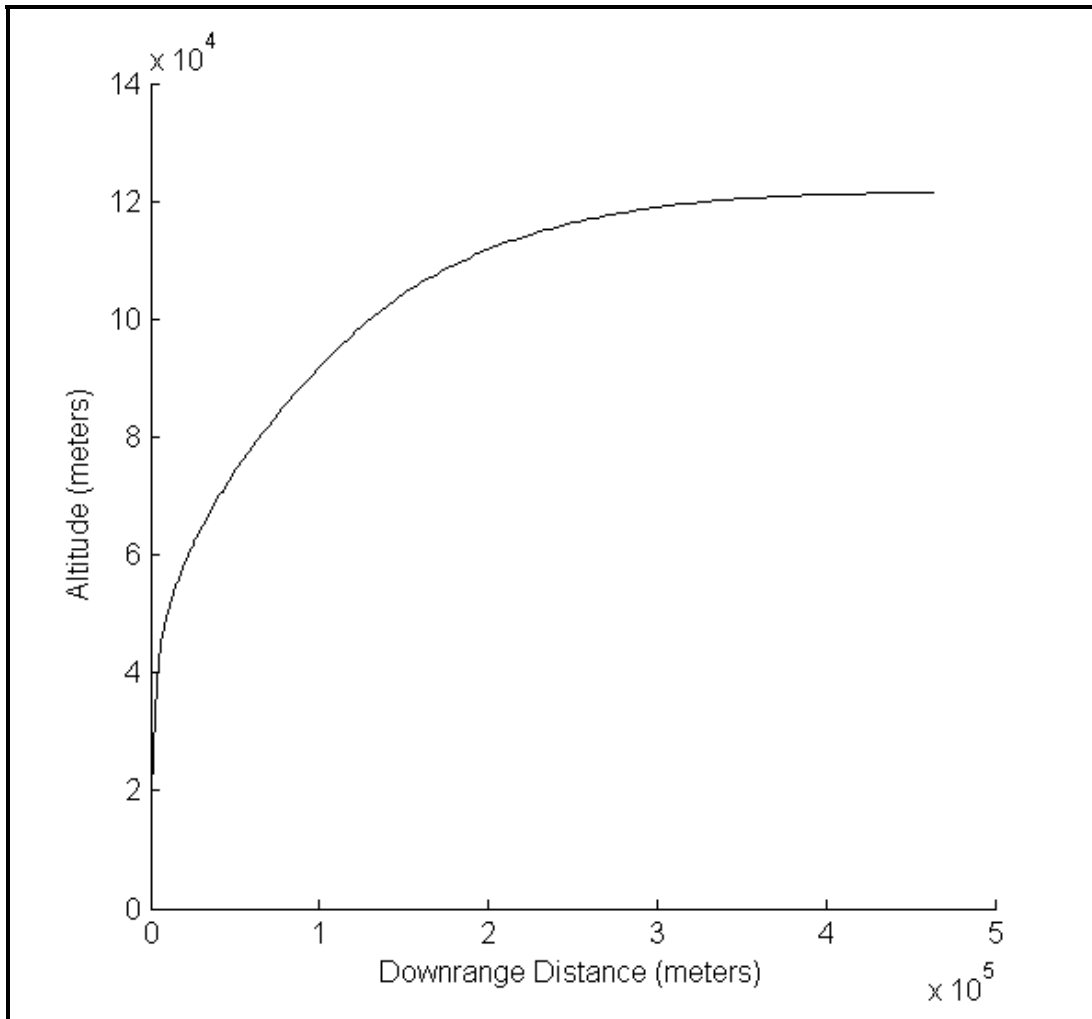
Figure 10: Launch trajectory for case 3

Table 5: Final conditions for case 3

Initial Flight Path Angle γ_o	89.57^o
Altitude	121398.648 meters
Downrange Distance	464387.699 meters
Velocity	6644.82810 m/s

In case 4, a coast time of sixty seconds between stages was investigated. This however had some serious problems as seen in Figure 11 and Table 6. It was found that the flight path did not converge to zero for this coast time, and the vehicle would impact the surface during the second stage burn. This is obviously a bad thing to happen to the launch vehicle.
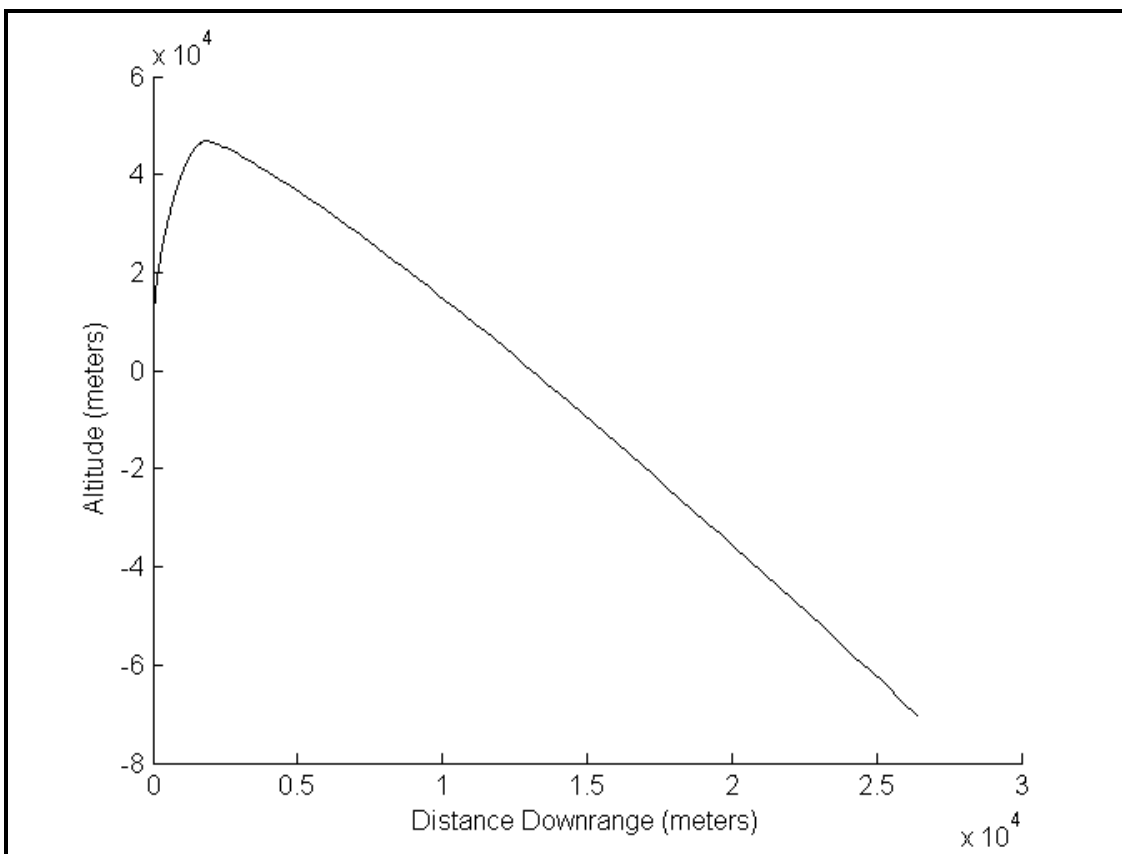
Figure 11: Launch trajectory for case 4

Table 6: Final conditions after case 4

Initial Flight Path Angle γ_o	89.9^o
Altitude	-70583.0107 meters
Downrange Distance	26483.1574 meters
Velocity	3.24739556 m/s

After several experimental runs, it was determined that a coast time larger than approximately 50 seconds during the first coast caused the vehicle's flight path angle to degrade too quickly. The decision was then made to confine the first coast time to 50 seconds or less. Then the second coast times were varied to attain higher altitudes. In case 5, the first coast stage was set to 40 seconds and then run with second coast times of 60 seconds, 90 seconds and 120 seconds which are represented in Figures 12, 13, and 14, respectively. These figures give a representation of possible altitudes to reach, as all altitudes between them are capable of being achieved through modification of the coast times. Also note the large difference in altitudes between the different coast times. This gives evidence that the vehicle has feasibility to reach more reasonable orbital altitudes.
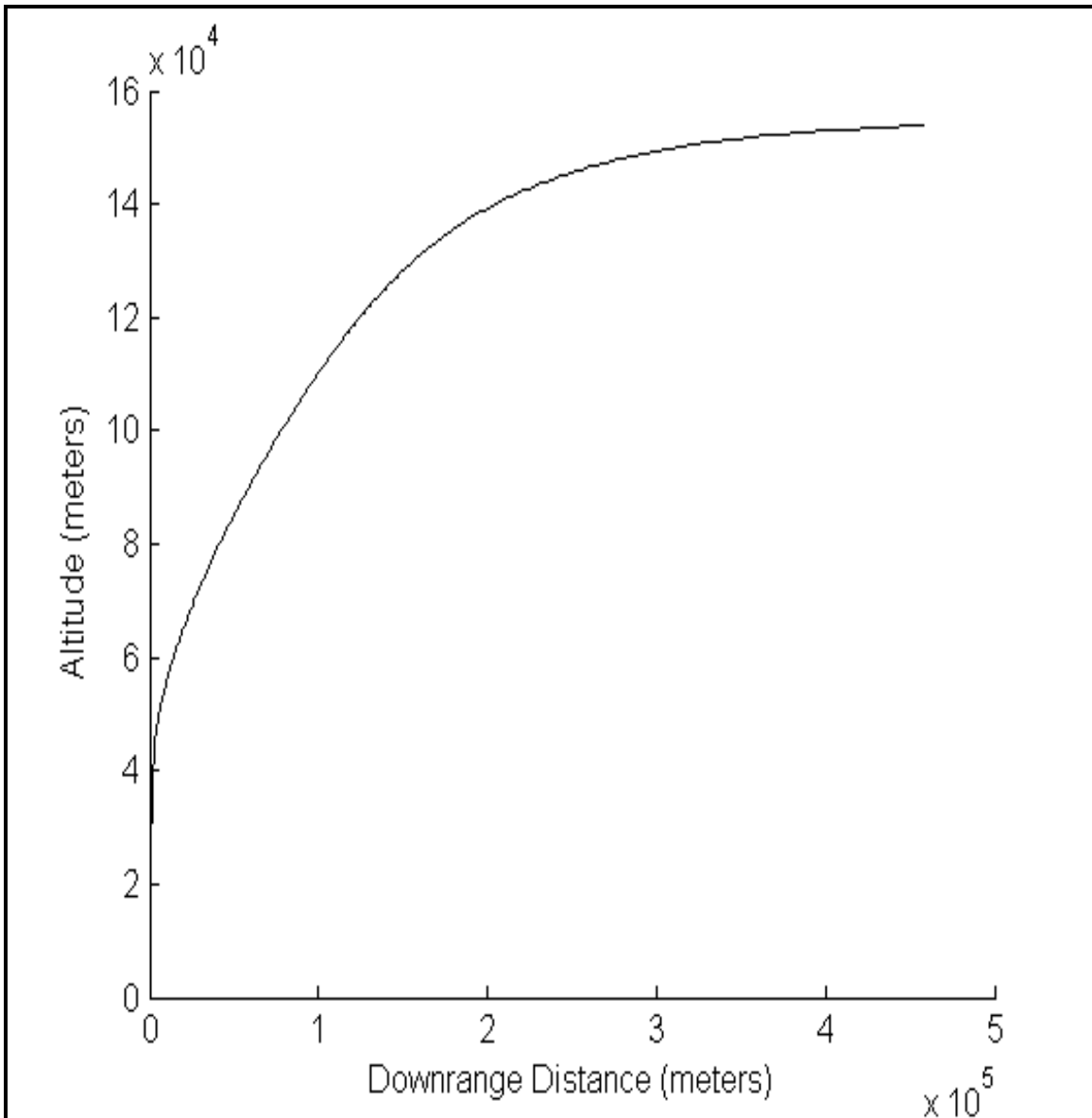
Figure 12: Launch trajectory of case 5 with second stage coast of 60 seconds
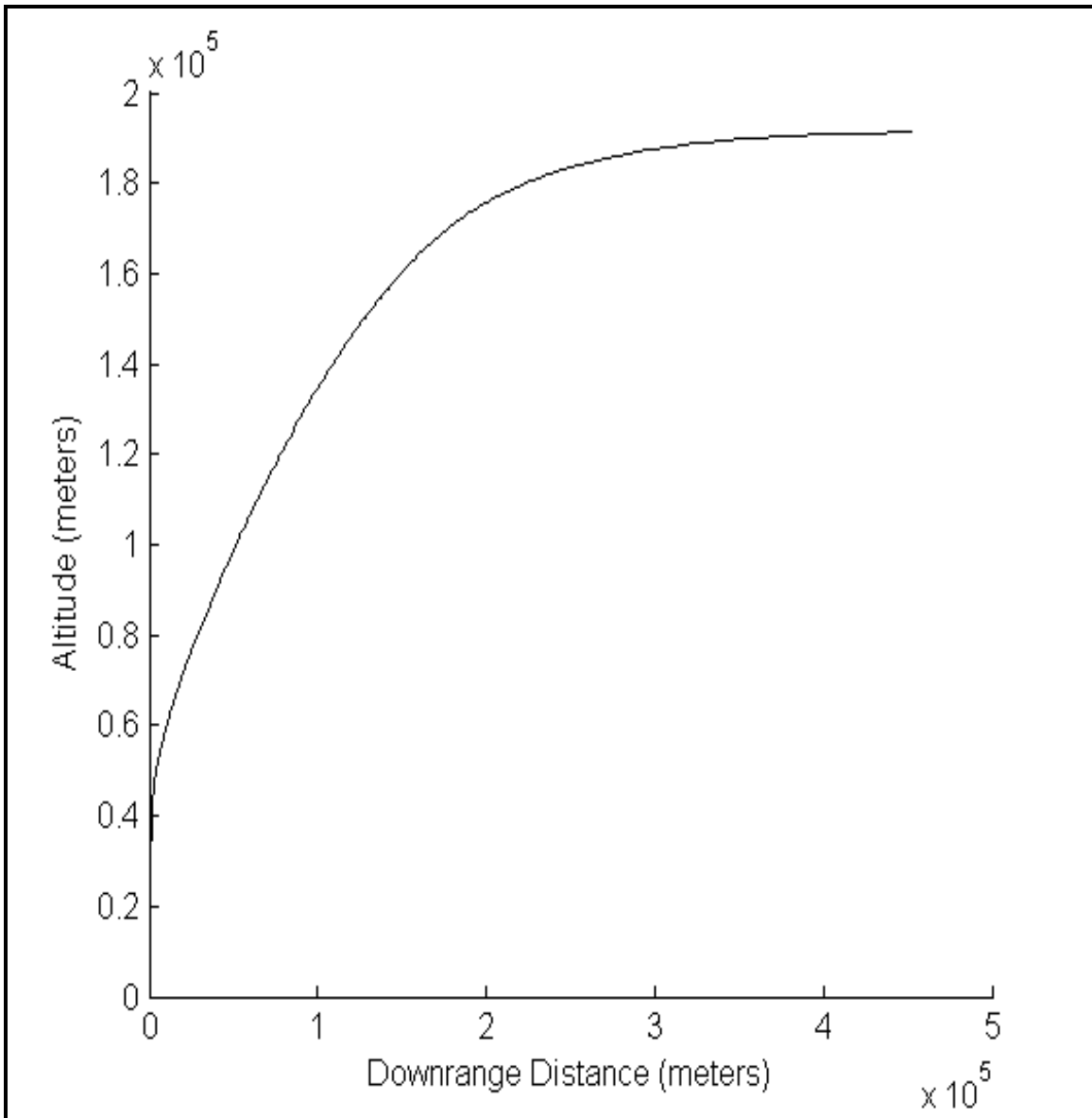
Figure 13: Launch trajectory of case 5 with a second stage coast of 90 seconds
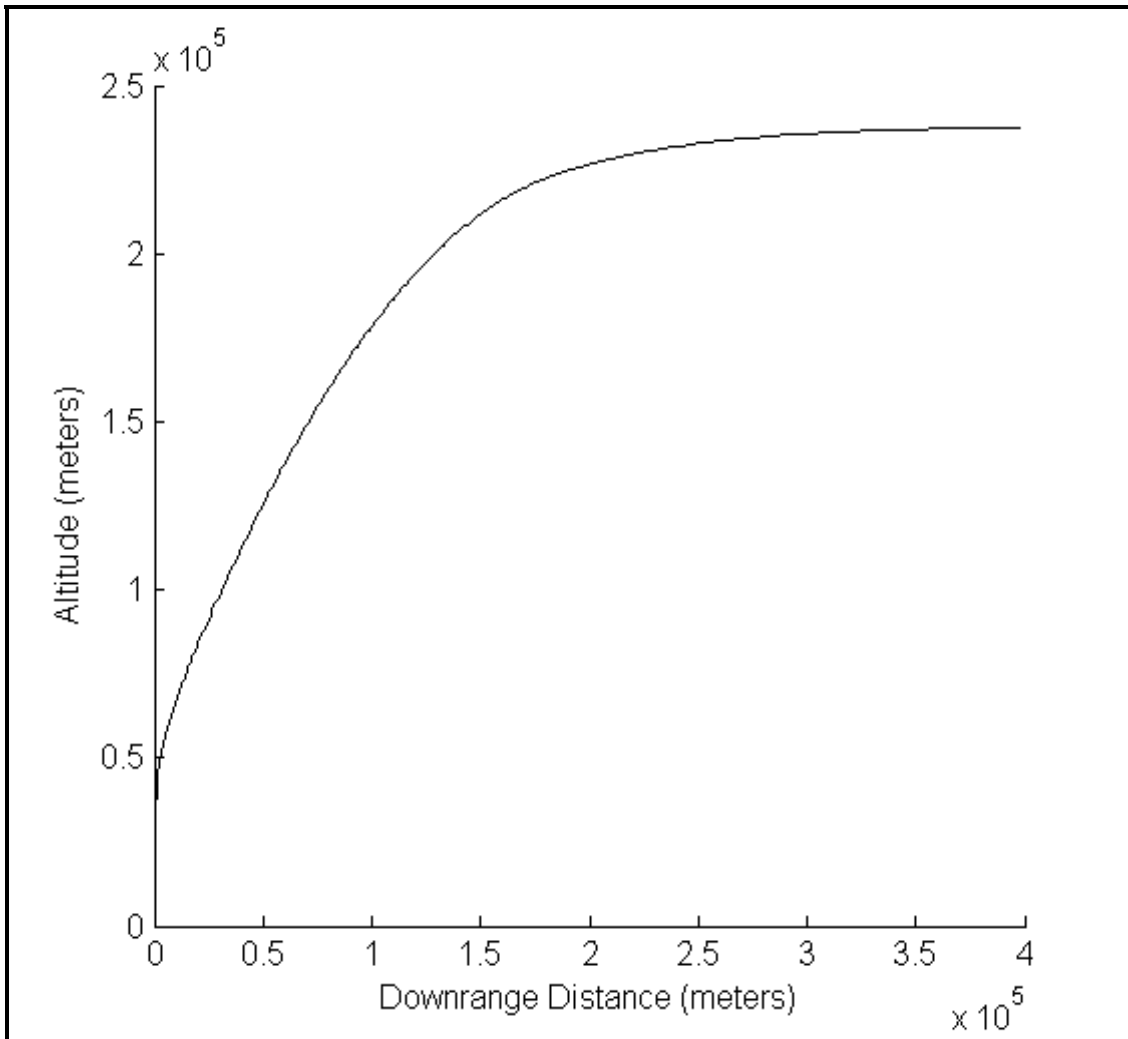
Figure 14: Launch trajectory of case 5 with a second stage coast of 120 seconds

The highest altitude achieved through the case 5 trajectories was approximately

237 km. This is much closer to the more desirable orbits. Attempts to reach higher orbits

with case 5's first coast were unproductive. So to achieve slightly higher altitudes in case

6, the first coast time was extended to 50 seconds and the second coast stage time was set

at 120 seconds. This second stage coast time gave the highest altitude previously, and is therefore a logical starting place. The results of case 6 are shown in Figure 15 and Table 7 below.
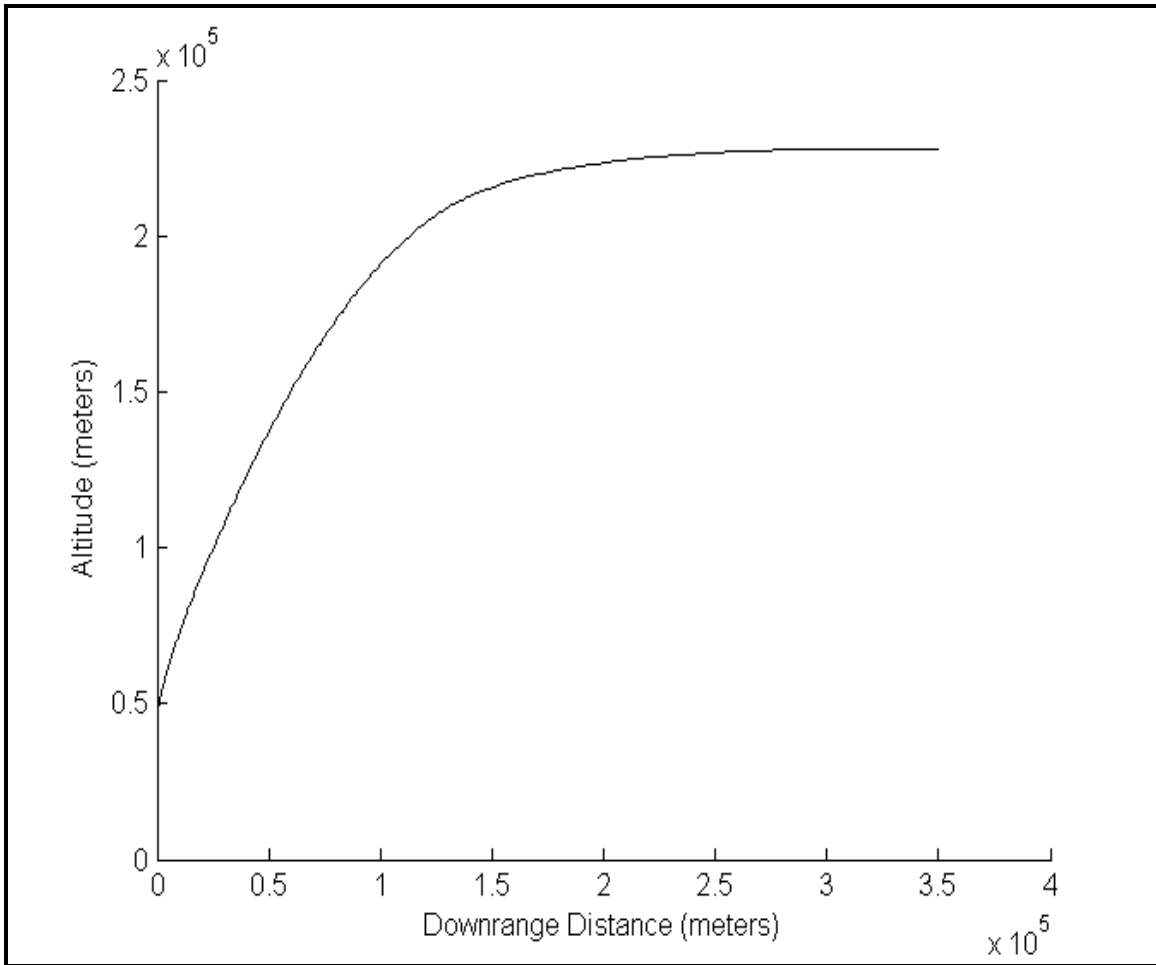
Figure 15: Launch trajectory of case 6

Table 7: Final conditions of case 6

Initial Flight Path Angle γ_o	89.98^o
Altitude	228092.170 meters
Velocity	5450.60642 m/s

As seen in Figure 14 and Table 7, the altitude reached is actually less than that of the highest achievable in case 5. After several iterations, a second coast stage of 160 seconds chosen for case 7. The trajectory and final conditions of case 7 are given in Figure 16 and Table 8, respectively.
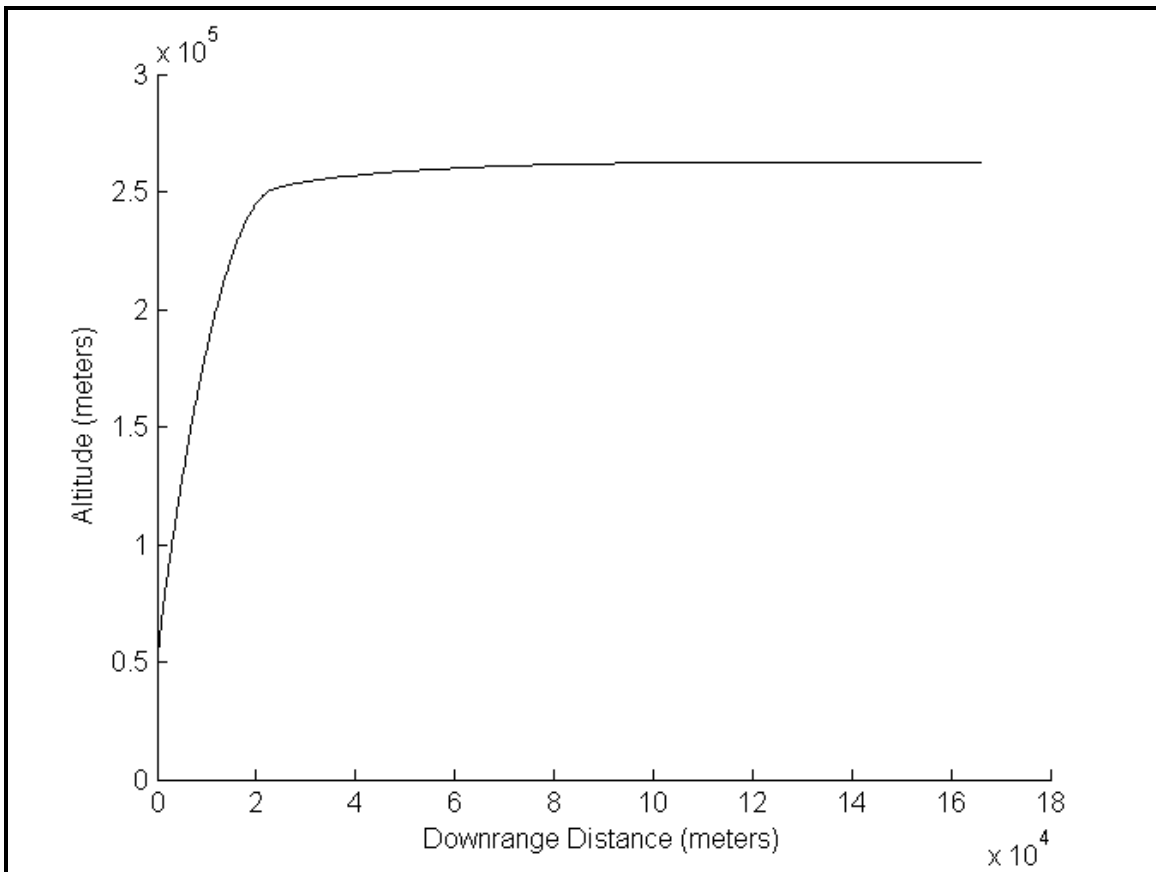
Figure 16: Launch trajectory with case 7

Table 8: Final conditions after case 7

Initial Flight Path Angle γ_o	89.9975^o
Altitude	262427.963 meters
Velocity	4584.99328 m/s

As seen in Table 8, the vehicle is finally above 250 km in altitude. Now that decent altitudes are capable of being reached, the vehicle was then set up to achieve a certain altitude.

For case 8, a goal was set for a 250 km orbit. A trajectory was then found that would approach 250 km as the flight path angle came to zero and then the vehicle was allowed to keep a zero flight path angle and accelerate in the orbit. The results of case 8 produce the trajectory and final conditions shown in Figure 17 and Table 9 below.
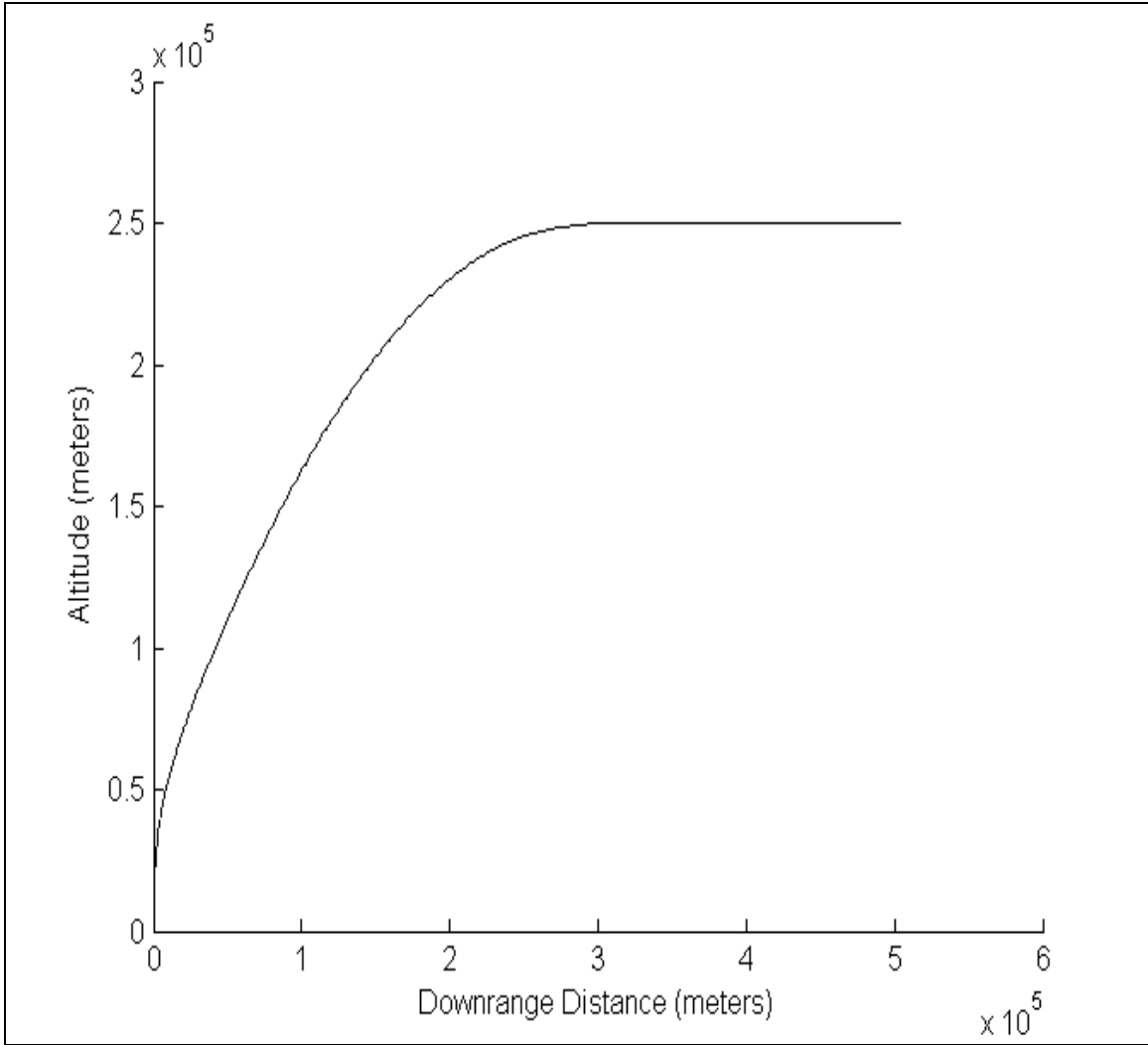
Figure 17: Launch trajectory of case 8

Table 9: Final conditions of case 8

Initial Flight Path Angle γ_o	89.9975^o
Altitude	250093.244 meters
Velocity	5945.16450 m/s

Of interest to the research at this point is the velocity needed to maintain a circular orbit. This velocity is governed by the equation:

$$V_c = \sqrt{\frac{\mu}{r}}$$

(4-1)

where μ is the gravitational parameter of the local earth system, and is given as 3.98601×10^5 km^3/s^2, and r = (R$_e$ + H), with R$_e$ given as 6378.135 km. Assuming an altitude (H) of 250 km, it is found that the velocity required to maintain this circular orbit is approximately 7.754 km/s. Given the final velocity shown in Table 9 of 5.945 km/s, it is obvious that the vehicle will not remain in this orbit. So an addition of velocity is required to maintain the desired orbit. As previously stated, the payload of the Boeing AirLaunch System for this research is assumed to be the Space Maneuvering Vehicle (SMV). The SMV is still in development, however by design it will be capable of 3200 m/s of additional velocity commonly referred to as ΔV ("Space Maneuver Vehicle Fact Sheet": 2004). For the purpose of reaching the desired orbital velocity, it is assumed that the SMV will be used to produce the required ΔV of 1809 m/s, which is well within its capabilities. Assuming the SMV accelerates at the same rate that the third stage did, about 100 m/s^2, this will add approximately 18 seconds to the launch time. This gives the time from ignition to on orbit as approximately 429 seconds or 7.15 minutes.

4.4 Hohmann Transfer Results

Another possibility for orbital insertion is a transfer orbit into the desired plane. Since this vehicle has a limited amount of fuel available to it, a Hohmann transfer is the best choice to make the transfer when time to orbit is not an issue. As discussed in Chapter 2, the Hohmann transfer consists of two changes in velocity to first put the vehicle onto an elliptical orbit and then another to place the orbiter into the desired orbit.

For the first consideration, case 1 was taken as the launch trajectory and a 250 km orbit again was chosen as the target. This investigation is here dubbed case 9. First, the vehicle at the end of case 1 is at approximately 81.43 km and traveling at 7.344 km/s. In order to take advantage of equations 2-14 and 2-15, the vehicle must first be accelerated to the correct velocity to maintain the given orbit. This can be found using equation 4-1 and determining the difference in velocities. Equations 2-14 and 2-15 can then be used to determine the ΔV's required for the transfer, and equation 2-16 is used to find the time of flight of the vehicle. The results are shown below in Table 10.

Table 10: Case 9 results for Hohmann transfer

ΔV required for inner orbit	0.511 km/s
ΔV_1 for the Hohmann transfer	0.0504 km/s
ΔV_2 for the Hohmann transfer	0.0501 km/s
Total ΔV required	0.6115 km/s
Time of flight, Δt	2634.09 s or 43.9 minutes

With a total ΔV required of 0.6115 km/s, this maneuver is well within the range of the SMV's capability of a 3.2 km/s ΔV. This shows an excellent possibility for launching to orbit.

In case 10, the vehicle is assumed to be at the end of case 6. At an altitude of approximately 228.092 km and velocity of 5.45 km/s, the ΔV's required to achieve a 250 km orbit are shown in Table 11.

Table 11: Case 10 results for Hohmann transfer

ΔV required for inner orbit	2.3177 km/s
ΔV_1 for the Hohmann transfer	0.00643 km/s
ΔV_2 for the Hohmann transfer	0.00642 km/s
Total ΔV required	2.3305 km/s
Time of flight, Δt	2678.49 s or 44.6 min

The results of case 10 are still within the range of the available ΔV of the SMV, however, case 9 is a much more efficient profile. Interestingly this poses an unexpected scenario. The initial launch trajectory posed in case 1 barely reaches the altitude of 80 km but is far more efficient using a Hohmann transfer than coasting is, as displayed in case 8. Also of interest here is that the higher trajectories reached by coasting have a higher total ΔV than the trajectory that has no coasting. This is primarily due to the fact that as the vehicle coasts it is slowing down due to gravity losses while the launch trajectory without coasting provides a velocity very near the orbital velocity needed.

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The results shown in this chapter demonstrate that the vehicle represented in this research is capable of reaching a prescribed altitude through a gravity-turn trajectory with supplementation. Surprising to this researcher, the initial launch augmented with a Hohmann transfer is the most fuel efficient of the cases investigated. Using the capability of an air launch platform, this vehicle is capable of intercepting the proper orbit of a target and launching at such a time that at engines cut off, the target is alongside. For a fuel-efficient attempt, the vehicle could rendezvous at the end of the Hohmann transfer, and for a faster intercept the vehicle can use coasting to achieve the proper orbital altitude and use the SMV to accelerate into the orbital plane, as was shown in case 8. The following chapter discusses this as well as recommendations for future investigation.

V. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The goal of this research was to evaluate the feasibility of a coplanar, gravity-turn launch trajectory using a vehicle based heavily upon the Boeing AirLaunch System. The results from the previous chapter demonstrate that the vehicle described in this research is capable of reaching reasonable altitudes in low earth orbit with modifications to its trajectory through coasting and transfer orbits. Since the payload of the vehicle can be considered in essence a fourth stage, there are a considerable number of options that can be used to achieve a desired orbit. This research limited these options to coasting and Hohmann transfers.

Since the vehicle can be positioned literally anywhere in the world, the launch scenarios investigated are quite possible. With actual information on the third stage and the Space Maneuver Vehicle (SMV), calculations of a direct to rendezvous launch is possible. This would simply require determining the correct launch time as dictated by the target vehicle. The mission could be completed in however long it would take the Boeing 747 launch vehicle to arrive at the proper location.

This research provides an interesting and insightful look at gravity-turn launch trajectories and the use of solid rocket motors in a responsive launch situation. It shows that with coasting and transfer orbits, most low earth orbits are attainable, and direct to rendezvous is feasible. As an interesting development, it also demonstrated that the most efficient launch was to a lower altitude followed by a Hohmann transfer to the desired

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altitude. The tradeoff here of course is time to rendezvous at the correct orbit. Case 8 achieves engines off on orbit at just over 7 minutes, where as case 9 reaches the target orbit at approximately 48 minutes after engine ignition.

5.2 Recommendations

The most obvious next step for this system is to find actual specifications for the third stage and SMV. This may greatly affect the system if actual specifications are much greater or less than the assumed values. Also of interest is to add drag into the equation. The introduction of drag will primarily affect the first stage and possibly the first coasting stage. Another point of interest would be to reproduce the equations of motion in the three dimensional earth centered reference frame. With the three-dimensional models, a launch to rendezvous can be examined in depth for the purposes of mission planning.

As the field of space launch continues to evolve, the desirable capabilities of launch on demand and direct to rendezvous launch will become more and more feasible in reality. Platforms like the Boeing AirLaunch System are not too far into the future. Orbital already operates a much smaller payload class on a similar platform called Pegasus. The results of this thesis offer an insight into the possible next evolution of this developing technology. The ability of launching directly to rendezvous has significant advantages in civilian and military aspects as well as peacetime and wartime missions.

The portability of an air launch system allows the concept of reaching virtually any orbital plane desirable. The research presented here poses some interesting possibilities for this type of technology in the future.

APPENDIX A: Code Summary

FIRSTCOASTEOM

TYPE: Subroutine m-file

PURPOSE: This routine provides the equations of motion to iterate through the first

                coasting stage conditions. This file provides the changes in the state

                variables per time.

INPUTS: State vector

OUTPUTS: State vector

CALLS: None

AUTHOR: David W. Callaway

FIRSTSTAGECOAST

TYPE: Main Program m-file

PURPOSE: This program solves a boundary value problem for the gravity-turn

                trajectory of the first coasting stage.

INPUTS: Final state vector from first stage burn and new initial mass

OUTPUTS: Final state vector after coasting

CALLS: FIRSTCOASTEOM

                INITIALCOND

AUTHOR: David W. Callaway

INITIALCOND

TYPE: Main Program m-file

PURPOSE: This program solves a boundary value problem for the gravity-turn trajectory of the first stage burning.

INPUTS: Initial state vector

OUTPUTS: Final state vector for first stage

CALLS: LAUNCHEOMS

AUTHOR: David W. Callaway

LAUNCHEOMS

TYPE: Subroutine m-file

PURPOSE: This routine provides the equations of motion to iterate through the first stage burn conditions. This file provides the changes in the state variables per time.

INPUTS: State vector

OUTPUTS: State vector

CALLS: None

AUTHOR: David W. Callaway

NEWGAMMA

TYPE: Main Program m-file

PURPOSE: This program determines the change in final flight path angle per change in initial flight path angle. It then finds the new initial flight path angle for the next iteration.

INPUTS: Initial flight path angles from the two previous iterations
Final flight path angles from the two previous iterations

OUTPUTS: Initial flight path angle

CALLS: None

AUTHOR: David W. Callaway

PLOTFILE

TYPE: Main Program m-file

PURPOSE: This program calls the iterated state vectors from all stages of the launch and plots them for data analysis.

INPUTS: All Iterations of state vectors

OUTPUTS: Downrange distance vs. Altitude plot
Flight path angle vs. Altitude plot

CALLS: output from:
INITIALCOND
FIRSTSTAGECOAST
SECONDSTAGE
SECONDSTAGECOAST
THIRDSTAGE

AUTHOR: David W. Callaway

SECONDCOASTEOMS

TYPE: Subroutine m-file

PURPOSE: This routine provides the equations of motion to iterate through the second coasting stage conditions. This file provides the changes in the state variables per time.

INPUTS: State vector

OUTPUTS: State vector

CALLS: None

AUTHOR: David W. Callaway

SECONDEOMS

TYPE: Subroutine m-file

PURPOSE: This routine provides the equations of motion to iterate through the second stage burn conditions. This file provides the changes in the state variables per time.

INPUTS: State vector

OUTPUTS: State vector

CALLS: None

AUTHOR: David W. Callaway

SECONDSTAGE

TYPE: Main Program m-file

PURPOSE: This program solves a boundary value problem for the gravity-turn trajectory of the second stage burning.

INPUTS: Final state vector of either first stage or first coasting stage, dependent on case.

OUTPUTS: Final state vector for second stage

CALLS: SECONDEOMS

AUTHOR: David W. Callaway

SECONDSTAGECOAST

TYPE: Main Program m-file

PURPOSE: This program solves a boundary value problem for the gravity-turn trajectory of the second coasting stage.

INPUTS: Final state vector of second stage

OUTPUTS: Final state vector for second coasting stage

CALLS: SECONDCOASTEOMS

AUTHOR: David W. Callaway

THIRDEOMS

TYPE: Subroutine m-file

PURPOSE: This routine provides the equations of motion to iterate through the third stage burn conditions. This file provides the changes in the state variables per time.

INPUTS: State vector

OUTPUTS: State vector

CALLS: None

AUTHOR: David W. Callaway

THIRDSTAGE

TYPE: Main Program m-file

PURPOSE: This program solves a boundary value problem for the gravity-turn trajectory of the third stage burning.

INPUTS: Final state vector of second stage or second coasting stage, dependent on case

OUTPUTS: Final state vector for third stage

CALLS: THIRDEOMS

AUTHOR: David W. Callaway

APPENDIX B: M-Files

The m-files presented here are from case 7. It should be noted that they are

provided here in alphabetical order; however, they must be run from the same file in the

order INITIALCOND, FIRSTSTAGECOAST, SECONDSTAGE,

SECONDSTAGECOAST, THIRDSTAGE, and PLOTFILE.

FIRSTCOASTEOM

```
% eoms for firststage coast
%LT Callaway, Thesis
%Trajectory model

function [ycdot]=firstcoasteom(t,yc)
global mdot T Re ge tstep

%EOM's for 2-D launch
%y0=[mnot Hnot Xnot Vnot gammanot};

%change in mass mdot
ycdot(1,1)=0;

%change in altitude Hdot
ycdot(2,1)=yc(4)*sin(yc(5));

%change in horizontal distance Xdot
ycdot(3,1)=yc(4)*cos(yc(5));

%change in Velocity
ycdot(4,1)=(1/yc(1))*(T-(yc(1)*ge-yc(1)*((yc(4)*cos(yc(5)))^2)/(Re+yc(2)))*sin(yc(5)));

%change in gamma
ycdot(5,1)=-(1/yc(1))*(1/yc(4))*(yc(1)*ge-
yc(1)*((yc(4)*cos(yc(5)))^2)/(Re+yc(2)))*cos(yc(5));
```

<u>FIRSTSTAGECOAST</u>
% coast stage 1
%LT Callaway, Thesis
%Trajectory model
%Intial Conditions:

close all
 file=fopen('C:\thesis\withcoast\firststagecoast.txt','w+');
 fprintf(file,'mass \t \t \t altitude \t \t \t X \t \t \t V \t \t \t gamma \n');

global mdot T Re ge tstep

%Times
tstep=1;   %sec
tfinal=50;   %sec

% Constants -------------------------

T=0;   %Thrust in N
mdot=0;   %mass flow is considered constant (kg/s);
Re=6378135;   %Radius of earth in meters;
ge=9.81;   %gravity g;

%%
% Initial Conditions

mnot=83388.874893;   %total mass - first stage

%%

% yyy0=[mnot Hnot Xnot Vnot gammanot];
yc0=[mnot y(83,2) y(83,3) y(83,4) y(83,5)];

% Integrate the equations of motion
options = odeset('RelTol',1e-7,'AbsTol',1e-10*ones(1,5));
[t,yc] = ODE45('firstcoasteom',0:tstep:tfinal,yc0,options);

out=[yc(:,2) yc(:,3) yc(:,4) yc(:,5)];
 fprintf(file,'%2.8e \t %2.8e \t %2.8e \t %2.8e \n',out');
% fprintf(file,'%2.8e \t %2.8e \t %2.8e \t %2.8e \t %2.8e \n',yc');
 fclose(file);

<u>INITIALCOND</u>
%LT Callaway, Thesis
%Trajectory model
%Intial Conditions:

```
clear
close all
 file=fopen('C:\thesis\withcoast\firststage.txt','w+');
 fprintf(file,'mass \t \t \t altitude \t \t \t  X \t \t \t V \t \t \t gamma \n');

global mdot T Re ge tstep

%Times
tstep=1;                                                %sec
tfinal=82;                                              %sec

%  Constants -------------------------

T=1650245.73705;         %Thrust in N
mdot=592.64056;          %mass flow is considered constant (kg/s);
Re=6378135;              %Radius of earth in meters;
ge=9.81;                 %gravity g;


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Initial Conditions

mnot=136077.711;               %total mass in kg
gammanot=89.9975*pi/180;       %gamma must be in radians;
Vnot=134.112;                  %Initial velocity 300 mph, but in meters/sec:
Hnot=6096;                     %Initial Altitude... 20kft...in meters
Xnot=0;                        %Initial X position

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
y0=[mnot Hnot Xnot Vnot gammanot];

% Integrate the equations of motion
options = odeset('RelTol',1e-7,'AbsTol',1e-10*ones(1,5));
[t,y] = ODE45('launcheoms',0:tstep:tfinal,y0,options);

out=[y(:,2) y(:,3) y(:,4) y(:,5)];
 fprintf(file,'%2.8e \t %2.8e \t %2.8e \t %2.8e \n',out');
%fprintf(file,'%2.8e \t %2.8e \t %2.8e \t %2.8e \t %2.8e \n',y');
 fclose(file);
```

LAUNCHEOMS
% Launcheoms for first stage
% LT Callaway, Thesis
%Trajectory model

function [ydot]=launcheoms(t,y)
global mdot T Re ge tstep

%EOM's for 2-D launch
%y0=[mnot Hnot Xnot Vnot gammanot};

%change in mass mdot
ydot(1,1)=-592.64056;

%change in altitude Hdot
ydot(2,1)=y(4)*sin(y(5));

%change in horizontal distance Xdot
ydot(3,1)=y(4)*cos(y(5));

%change in Velocity
ydot(4,1)=(1/y(1))*(T-(y(1)*ge-y(1)*((y(4)*cos(y(5)))^2)/(Re+y(2)))*sin(y(5)));

%change in gamma
ydot(5,1)=-(1/y(1))*(1/y(4))*(y(1)*ge-y(1)*((y(4)*cos(y(5)))^2)/(Re+y(2)))*cos(y(5));

NEWGAMMA
%dgamma/dgammanot program
%dgamma= old gamma - new gamma
gammao=5.25671725e-001;
gamman=1.15971452e-002;
dgamma=gammao-gamman %radians

% dgammanot= |new gamma - previous gamma|
gammanotnew=89.78;
gammanotold=89.9;
dgammanot=gammanotold-gammanotnew %degrees

deltagammanot=(dgammanot)*(gamman)/(dgamma) %degrees

newgammanot=gammanotnew-deltagammanot

<u>PLOTFILE</u>
hold on;
%plotting altitude vs X
plot(y(:,3),y(:,2));
plot(yc(:,3),yc(:,2));
plot(yy(:,3),yy(:,2));
plot(ycc(:,3),ycc(:,2));
plot(yyy(:,3),yyy(:,2));

%plotting altitude vs gamma
% plot(y(:,5),y(:,2));
% plot(yc(:,5),yc(:,2));
% plot(yy(:,5),yy(:,2));
% plot(ycc(:,5),ycc(:,2));
% plot(yyy(:,5),yyy(:,2));

<u>SECONDCOASTEOMS</u>
% eoms for second stage coast

function [yccdot]=secondcoasteoms(t,ycc)
global mdot T Re ge tstep

%EOM's for 2-D launch

%y0=[mnot Hnot Xnot Vnot gammanot};

%change in mass mdot
yccdot(1,1)=0;

%change in altitude Hdot
yccdot(2,1)=ycc(4)*sin(ycc(5));

%change in horizontal distance Xdot
yccdot(3,1)=ycc(4)*cos(ycc(5));

%change in Velocity
yccdot(4,1)=(1/ycc(1))*(T-(ycc(1)*ge-
ycc(1)*((ycc(4)*cos(ycc(5)))^2)/(Re+ycc(2)))*sin(ycc(5)));

%change in gamma
yccdot(5,1)=-(1/ycc(1))*(1/ycc(4))*(ycc(1)*ge-
ycc(1)*((ycc(4)*cos(ycc(5)))^2)/(Re+ycc(2)))*cos(ycc(5)));

SECONDEOMS
% Launcheoms for second stage

```
function [yydot]=secondeoms(t,yy)
global mdot T Re ge tstep

%EOM's for 2-D launch

%y0=[mnot Hnot Xnot Vnot gammanot};

%change in mass mdot
yydot(1,1)=-592.64056;

%change in altitude Hdot
yydot(2,1)=yy(4)*sin(yy(5));

%change in horizontal distance Xdot
yydot(3,1)=yy(4)*cos(yy(5));

%change in Velocity
yydot(4,1)=(1/yy(1))*(T-(yy(1)*ge-
yy(1)*((yy(4)*cos(yy(5)))^2)/(Re+yy(2)))*sin(yy(5)));

%change in gamma
yydot(5,1)=-(1/yy(1))*(1/yy(4))*(yy(1)*ge-
yy(1)*((yy(4)*cos(yy(5)))^2)/(Re+yy(2)))*cos(yy(5));
```

SECONDSTAGE
% SECOND STAGE INIITIAL CONDITIONS
%Intial Conditions:

```
close all
 file=fopen('C:\thesis\withcoast\secondstage.txt','w+');
 fprintf(file,'mass \t \t \t altitude \t \t \t  X \t \t \t V \t \t \t gamma \n');

global mdot T Re ge tstep

%Times
tstep=1;                                                          %sec
tfinal=82;                                                        %sec


%  Constants -------------------------

T=1650245.73705;              %Thrust in N
mdot=592.64056;               %mass flow is considered constant (kg/s);
Re=6378135;                   %Radius of earth in meters;
ge=9.81;                      %gravity g;


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Initial Conditions

mnot=83388.874893;            %total mass - first stage mass in kg
% gammanot=1.36707253;        %gamma must be in radians;
% Vnot=564.206512;            %Velocity from end of 1st stage, but in meters/sec:
% Hnot=30732.3097;            %Altitude from previous run
% Xnot=2685.21896;            %Initial X position


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% yy0=[mnot Hnot Xnot Vnot gammanot];
yy0=[mnot yc(51,2) yc(51,3) yc(51,4) yc(51,5)];

% Integrate the equations of motion
options = odeset('RelTol',1e-7,'AbsTol',1e-10*ones(1,5));
[t,yy] = ODE45('secondeoms',0:tstep:tfinal,yy0,options);

out=[yy(:,2) yy(:,3) yy(:,4) yy(:,5)];
 fprintf(file,'%2.8e \t %2.8e \t %2.8e \t %2.8e \n',out');
%fprintf(file,'%2.8e \t %2.8e \t %2.8e \t %2.8e \t %2.8e \n',yy');
fclose(file);
```

SECONDSTAGECOAST
%   coast stage 2
%LT Callaway, Thesis
%Trajectory model
%Intial Conditions:

close all
 file=fopen('C:\thesis\withcoast\secondstagecoast.txt','w+');
 fprintf(file,'mass \t \t \t altitude \t \t \t X \t \t \t V \t \t \t gamma \n');

global mdot T Re ge tstep

%Times
tstep=1;   %sec
tfinal=160;   %sec

% Constants -------------------------

T=0;   %Thrust in N
mdot=0;   %mass flow is considered constant (kg/s);
Re=6378135;   %Radius of earth in meters;
ge=9.81;   %gravity g;

%%%
% Initial Conditions

mnot=30700.027786;   %total mass - stages 1 and 2

%%%
% yyy0=[mnot Hnot Xnot Vnot gammanot];
ycc0=[mnot yy(83,2) yy(83,3) yy(83,4) yy(83,5)];

% Integrate the equations of motion
options = odeset('RelTol',1e-7,'AbsTol',1e-10*ones(1,5));
[t,ycc] = ODE45('secondcoasteoms',0:tstep:tfinal,ycc0,options);

out=[ycc(:,2) ycc(:,3) ycc(:,4) ycc(:,5)];
 fprintf(file,'%2.8e \t %2.8e \t %2.8e \t %2.8e \n',out');
% fprintf(file,'%2.8e \t %2.8e \t %2.8e \t %2.8e \t %2.8e \n',ycc');
 fclose(file);

% Launcheoms for third stage
% Corrected for mdot=50% of mdot stages 1 and 2

```
function [yyydot]=thirdeoms(t,yyy)
global mdot T Re ge tstep

%EOM's for 2-D launch

%y0=[mnot Hnot Xnot Vnot gammanot};

%change in mass mdot
yyydot(1,1)=-296.32028;

%change in altitude Hdot
yyydot(2,1)=yyy(4)*sin(yyy(5));

%change in horizontal distance Xdot
yyydot(3,1)=yyy(4)*cos(yyy(5));

%change in Velocity
yyydot(4,1)=(1/yyy(1))*(T-(yyy(1)*ge-
yyy(1)*((yyy(4)*cos(yyy(5)))^2)/(Re+yyy(2)))*sin(yyy(5)));

%change in gamma
yyydot(5,1)=-(1/yyy(1))*(1/yyy(4))*(yyy(1)*ge-
yyy(1)*((yyy(4)*cos(yyy(5)))^2)/(Re+yyy(2)))*cos(yyy(5));
```

THIRDSTAGE
% Third STAGE INIITIAL CONDITIONS
%Intial Conditions:

close all
 file=fopen('C:\thesis\withcoast\thirdstage.txt','w+');
 fprintf(file,'mass \t \t \t altitude \t \t \t X \t \t \t V \t \t \t gamma \n');

global mdot T Re ge tstep

%Times
tstep=1;   %sec
tfinal=84;   %sec   rounded down from 84.9 seconds

% Constants --------------------------

T=813932.5451;   %Thrust in N
mdot=592.64056;   %mass flow is considered constant (kg/s);
Re=6378135;   %Radius of earth in meters;
ge=9.81;   %gravity g;

%%%
% Initial Conditions

mnot=30700.027786;   %total mass - first stage mass in kg
% gammanot=1.13822111;   %gamma must be in radians;
% Vnot=2241.82678;   %Velocity from end of 2nd stage, in meters/sec:
% Hnot=124507.332;   %Altitude from previous run
% Xnot=37516.3229;   %Initial X position third stage

%%%

% yyy0=[mnot Hnot Xnot Vnot gammanot];
yyy0=[mnot ycc(161,2) ycc(161,3) ycc(161,4) ycc(161,5)];

% Integrate the equations of motion
options = odeset('RelTol',1e-7,'AbsTol',1e-10*ones(1,5));
[t,yyy] = ODE45('thirdeoms',0:tstep:tfinal,yyy0,options);

out=[yyy(:,2) yyy(:,3) yyy(:,4) yyy(:,5)];
 fprintf(file,'%2.8e \t %2.8e \t %2.8e \t %2.8e \n',out');
% fprintf(file,'%2.8e \t %2.8e \t %2.8e \t %2.8e \t %2.8e \n',yyy');
 fclose(file);

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Vita

Lieutenant David "Walker" Callaway was born in Harlingen, Texas. He graduated in 1995 from La Feria high School, La Feria, TX, and attended Embry-Riddle Aeronautical University, Daytona Beach, Florida, on an Air Force R.O.T.C. Scholarship. He received his Bachelor of Science in Engineering Physics with a minor in Mathematics in April 2000. Upon graduation, he was commissioned a Second Lieutenant in the U.S. Air Force and reported to Davis-Monthan AFB, Tucson, AZ, where he served as assistant to the squadron executive officer of the 41st ECS. Lt. Callaway entered Undergraduate Flight Training at Laughlin AFB, Del Rio, TX in November 2000. After injury to an ear due to flying, Lt Callaway was medically grounded and reassigned to Wright-Patterson AFB, Dayton, OH, where he performed the duties of project engineer for the Space Operations Vehicle Integrating Concept Office, AFRL/VAS. He entered the School of Engineering, Air Force Institute of Technology, in June of 2002. His next assignment is at AFRL/VA, Wright-Patterson AFB, Dayton, Ohio.

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14. ABSTRACT

The purpose of this study was to determine the feasibility of launching a vehicle based on the Boeing AirLaunch System in a coplanar, direct to rendezvous trajectory with gravity-turn. The focus of the research was to model the launch trajectory and determine the ability to reach different coplanar orbits. The launch trajectory was modeled using two-dimensional equations of motion and a boundary value problem was posed and solved for the gravity-turn trajectory. Trajectories were then created in an attempt to reach different altitudes through coasting and transfer orbits. Finally a specific orbital altitude was chosen and the trajectories were analyzed to find the most efficient route to the target orbit for fuel and time.

15. SUBJECT TERMS
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