

# Dynamics of multiple stars: observations

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**Abstract.** The dynamics of hierarchical multiple stars is observed mostly by combination of various techniques that lead to the determination of orbits. It is found that the empirical limit of dynamical stability matches the theoretical limit for circular outer orbits but is more restrictive for eccentric ones. The relative orientation of orbits in triple stars shows some weak correlation, likely explained by their origin as decay products of small clusters. The combination of dynamics and tides leads to the formation of close binaries within higher-order multiples. Examples of such systems (Algol, Capella, 41 Dra) show that even distant tertiary components can significantly modify stellar evolution.

## 1. Introduction to multiple stars

The current version of the Multiple Star Catalog (MSC) (Tokovinin 1997) contains 943 physical systems ranging from triple to 6-component. The MSC is very incomplete even for dwarfs of spectral types F, G, K that are the easiest to detect, less massive stars being too faint and more massive stars too distant. Available statistics hints that up to 25% of all stellar systems with solar-mass primaries may be multiple, i.e. triple and higher order (Tokovinin 2004).

Most multiple stars are discovered by a combination of several observing techniques because orbital periods span from hours to few  $10^6$  yr. Close sub-systems are usually detected as spectroscopic binaries. However, most spectroscopic binaries discovered during last decade are cool stars (Pourbaix et al. 2004), so a systematic study of massive-star multiplicity is yet to be done.

Apart from catalogs and statistics, our knowledge of multiple-star dynamics comes from detailed investigations of specific systems, often with unusual properties. For example, A and B components of the visual binary ADS 11061 (41 and 40 Dra) are spectroscopic binaries (Tokovinin et al. 2003). The 10.5-day Bab binary is quite typical, while the 3.4-yr Aab pair has the highest known eccentricity,  $e = 0.9754$ . This system and the vast majority of other known multiples are *hierarchical*, as illustrated in Fig. 1a.

The motion of components in a hierarchical system is approximated by a combination of Keplerian orbits. Each close sub-system Aab and Bab behaves as a single body when the wide pair AB is considered. If these composite components (or super-components) had their own designations, the structure of a hierarchical multiple could be described by a binary graph as in Fig. 1a or by a chained list that links each (super)-component to a higher-level “parent” in the hierarchy. Some components believed today to be single will be resolved into sub-systems in the future and will thus become super-components.

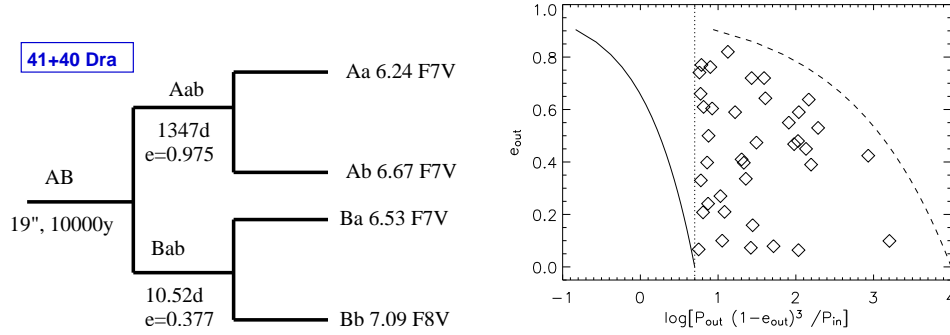


Figure 1. **Left (a):** Structure of the hierarchical quadruple system 41 and 40 Dra represented by a binary-tree graph. **Right (b):** The empirical stability criterion vs. outer orbit eccentricity for 38 systems with reliable inner and outer orbits. The full line shows a theoretical stability limit (fixed ratio of outer periastron to inner semi-major axis), the dashed line marks the maximum period ratio,  $P_{\text{out}}/P_{\text{in}} < 10^4$ .

The representation of hierarchical multiples by combinations of Keplerian orbits is an essential step in the study of their dynamics. The orbits are derived by such observing techniques as direct resolution (from visual micrometers to adaptive optics and long-baseline interferometry), radial velocities, photometry of eclipsing binaries, time-delay of their minima, and non-linear apparent motion (astrometric binaries). Not all published orbits are reliable, especially visual orbits with periods over 300 yr. Without critical attitude to observational data, wrong conclusions can be reached.

## 2. Dynamical stability

Dynamical stability of triple stars is a classical and well-studied subject. All known stability criteria require the periastron distance in the outer orbit to be larger than the semi-major axis of the inner orbit by a certain factor. The stability criterion can be formulated in terms of period ratio. For example, the most recent work of Mardling & Aarseth (2002) translates to the approximate stability criterion

$$\frac{P_{\text{out}}}{P_{\text{in}}}(1 - e_{\text{out}})^{1.8} \geq 4.7(1 + e_{\text{out}})^{0.4} \quad (1)$$

if we neglect a factor containing mass ratios. This criterion is valid for co-planar and co-rotating systems.

The analysis of multiple systems from MSC with two known orbits shows that they follow a different, more strict stability criterion (Tokovinin 2004). We excluded uncertain visual orbits with  $P > 300$  yr and spectroscopic orbits with  $P < 10$  d likely modified by tides (see below). The remaining 38 systems follow the *empirical stability criterion*

$$\frac{P_{\text{out}}}{P_{\text{in}}}(1 - e_{\text{out}})^3 \geq 5, \quad (2)$$

as shown in Fig. 1b. Compared to the theoretical criteria, the power of the  $(1 - e_{\text{out}})$  factor changes from 1.5...1.8 to 3. Both criteria match for circular outer orbits.

I could not find any reasonable explanation to the increased power of  $(1 - e_{\text{out}})$ . If the outer periastron distance is replaced by another parameter like semi-latus rectum, angular momentum, angular velocity at periastron, etc., the resulting criterion is a combination of  $(1 - e_{\text{out}})^{1.5}$  with  $(1 + e_{\text{out}})$  to some power. The closest match to (2) is obtained when the linear velocity in the outer-orbit periastron is constrained, but this does not have any physical meaning. Thus, theoretical criteria of dynamical stability still miss some important factor when dealing with eccentric outer orbits.

Interestingly, close spectroscopic binaries show a very similar trend. The highest eccentricity at any given period (the outer envelope of the period-eccentricity relation) follows the  $P(1-e)^3 < \text{const}$  line (Pourbaix et al. 2004). If eccentricity were constrained by the distance at periastron (via contact between components or tides), that would translate to  $P(1 - e)^{3/2} < \text{const}$ .

### 3. Relative orbit orientation

Are the orbits in multiple stars co-planar? The statistics of the angle  $\Phi$  between the vectors of angular momentum in inner and outer sub-systems should answer this question. Of course, the elements of Keplerian orbits that describe component's motions in hierarchical multiple systems change with time owing to the orbit-orbit interaction. Generally,  $\Phi$  oscillates between certain limits. If the orbits were initially co-planar and co-rotating ( $\Phi = 0$ ), this condition is preserved. The statistics of  $\Phi$  is hence not washed out completely by the dynamical evolution and contains information on the initial conditions at the epoch of formation.

In practice, the angle  $\Phi$  is very difficult to measure: we need to know *both* inner and outer apparent (visual) orbits and, in addition, the ascending nodes of these orbits must be identified correctly by radial velocities. A bright triple  $\eta$  Vir has been observed with long-baseline interferometer by Hummel et al. (2003) who found  $\Phi = 30.8^\circ$ . Until interferometry becomes more common, however, such studies remain exceptional. Only 22 cases where both visual orbits are known were listed by Sterzik & Tokovinin (2002), but many of those have doubtful outer orbits ( $P_{\text{out}} > 300$  yr) and unknown nodes.

A better way to constrain the  $\Phi$  statistics consists in counting the apparently co- and counter-rotating systems among resolved triples. Such data on 135 systems lead to  $\langle \Phi \rangle = 67^\circ \pm 9^\circ$  (Sterzik & Tokovinin 2002). It was found that the tendency to co-planarity is more marked for weakly-hierarchical systems near stability limit. Moreover, simulations of small-cluster decay seem to show a similar trend. Such study would benefit from a dedicated program of high-resolution imaging (adaptive optics or speckle) to extend the number of accessible systems.

#### 4. Kozai cycles

It is known since the work of Kozai (1962) that in a triple system  $\Phi$  oscillates. At the same time the eccentricity of the inner orbit changes, preserving the *Kozai invariant*  $(1 - e_{\text{in}})^2 \cos^2 \Phi = \text{const}$ . Thus, inner and outer orbits exchange the angular momentum with a Kozai-cycle period of the order of  $P_{\text{out}}^2/P_{\text{in}}$ .

Stars are not material points. Whenever they approach to a distance of several stellar radii, tidal interactions come into play. This effect has been extensively simulated, e.g. by Eggleton & Kiseleva-Eggleton (2001). Tides slowly rotate the line of apsides of the inner orbit, disturb the Kozai cycles and “lock” the eccentricity in its high state. Subsequent dissipation of the inner-orbit energy by tides reduces the semi-major axis and period of the inner sub-system. The formation of the inner binary in Algol has been modeled in this way.

The distribution of orbital periods in late-type inner sub-systems of multiple stars shows a maximum in the range from 2 to 7 days and a sharp decrease in the longer-period bins (Tokovinin & Smekhov 2002). Of course, longer periods are more difficult to discover, but the observational selection is not sufficient to explain the observed feature. Most likely it results from the combination of Kozai cycles with tides. Inner orbits with initial  $P_{\text{in}} > 10$  d were modified by this process and migrated to shorter periods. The fraction of modified orbits slowly decreases with increasing  $P_{\text{in}}$  because the constraints on  $\Phi$  required to reach sufficiently high  $e_{\text{in}}$  become tighter. Contrary to intuition, the distance to the tertiary component and its mass have little influence on the efficiency of this mechanism, they only make the cycles slower.

The extreme eccentricity of 41 Dra is most likely caused by the Kozai cycles. Although components Aab approach to 7 stellar radii at periastron, the tides were not strong enough to circularize the inner orbit in its lifetime of 2.5 Gyr (Tokovinin et al. 2003), while the 10.5-day Bab has likely undergone such a transformation. The Aab will follow the same path because its components are now above the Main Sequence and increase their radii, the orbit being already locked in the high- $e$  state. The combination of nuclear evolution, tides and multiple-star dynamics will lead to the fast shrinking of the Aab orbit and, possibly, to subsequent merger. The merger product will owe its existence to the weak but persistent gravitational influence of the visual component B on the Aab orbit. This example shows that even a distant component can significantly influence stellar evolution. The actual change of the 41 Dra period may be observable by precise timing of periastron passages. It will permit a direct measure of the orbit’s evolution rate, checking the theories of tidal interaction.

Dissipative Kozai cycles are just one possible way to extract the angular momentum from an inner system of a multiple star. Other possibilities include direct close encounters or interactions during accretion stage (Bate et al. 2002). A presence of tertiary may be critical to the formation of any close binary. *Are all close binaries triple?* Among nearby solar-type binaries with periods below 10 d 40% are already known to have tertiaries. We are examining the remaining systems by a combination of modern techniques to find if short-period binaries without tertiaries exist at all.

## 5. Formation of multiple stars

It becomes clear that disintegration of small clusters is the leading mechanism of multiple-star formation. Small clusters where multiple cores in a protostellar cloud accrete and interact dynamically at the same time are a natural stage of star formation (Bate et al. 2002). Stellar aggregates like open clusters initially form as small sub-structures that evolve in much the same way as small isolated clusters and dissolve later in the large- $N$  systems.

Hierarchical multiples observed today in the field and clusters are the tip of the “iceberg” of primordial non-hierarchical multiples. Most of those systems were below the “waterline” of the stability limit and have disintegrated into single and binary stars (Fig. 2a). It has been conjectured that  $\iota$  Ori and two runaway stars are products of a close disruptive encounter (Gualandris et al. 2004). Even if this particular case were questioned, little doubt remains that at least part of runaway stars result from dynamical decay of unstable multiples.

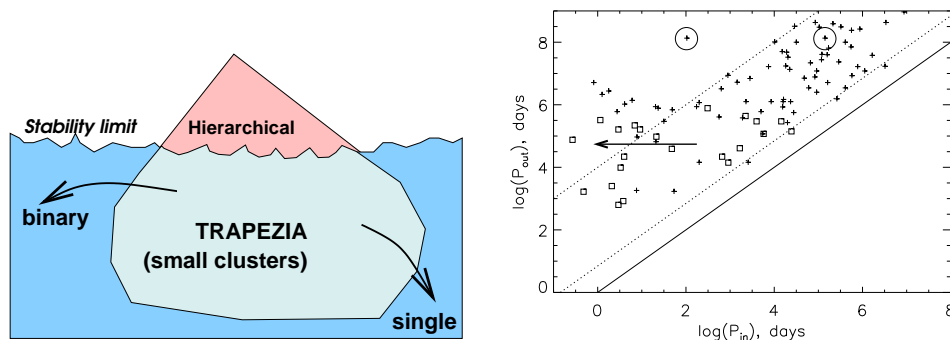


Figure 2. **Left (a):** Cartoon illustrating the formation of multiple stars. **Right (b):** The  $P_{in} - P_{out}$  diagram for nearby solar-type multiples. Periods known from orbital solutions are marked by squares, periods estimated from separations – by pluses. The full and dotted lines mark period ratios of 1, 5, and  $10^4$ . The arrow indicates dissipative Kozai evolution, the two circles mark the sub-systems of Capella.

Hierarchical multiples observed today in the field bear traces of formation and subsequent evolution. The plot of inner and outer periods (Fig. 2b) can be interpreted from this angle. Indeed, for  $P_{in} > 100$  d there is no tidal evolution and the period ratios are confined between 5 (stability limit) and  $10^4$ . Apparently, larger period ratios are not produced by small clusters. However, larger ratios are found in systems with  $P_{in} < 100$  d – a likely result of dissipative Kozai evolution.

There seems to be one exception to this rule. This is the 100-d Capella system with its distant 390-yr pair of M dwarfs. The orbit of Capella is circular and probably results from tidal interactions. It is likely that the primordial orbit of Capella had a much longer period, its eccentricity reached high values owing to Kozai cycles until the increased component radii caused orbit shrinkage. We observe the beginning of the same process in 41 Dra.

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