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Adverse Selection and Risk Aversion in Capital Markets

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Abstract

We generalize the Boadway and Keen (2006) model of adverse selection in capital markets to allow for risk aversion on the part of entrepreneurs. We use this to analyze two types if policies. We first consider policies that would allow entrepreneurs to use a greater fraction of their total wealth in financing their projects, thus allowing them to reduce reliance on debt or equity finance by outside investors. We show that such policies may not be welfare improving because it exposes entrepreneurs to more down-side risk. This result highlights the importance of allowing for risk-aversion since policies that aim at alleviating inefficiencies associated with adverse selection may increase risk exposure and ultimately reduce welfare. We, then, consider how the tax treatment of losses affects social welfare. We show that if a society places a high value on distributional equity or if entrepreneurs are sufficiently risk averse, a full-loss offset system may be desirable even when there is excessive investment. *Keywords: Adverse selection, debt, equity, tax policy. JEL codes: H20, G14.*

1 Introduction

Since the 1970s, economists have recognized that capital markets do not always conform to the model of an idealized market where individuals and firms can always borrow funds at an interest rate that accurately reflects the degree of risk

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posed by their investment projects. One major source of capital market imperfection is asymmetric information between outside investors and entrepreneurs. In many instances, entrepreneurs have better information than outside investors about the probability that the investment will be successful, giving rise to a problem of adverse selection.

There is a good deal of controversy concerning the impact of adverse selection in economies with simple financial contracts, such as debt and equity. Two key papers in this literature—Stiglitz and Weiss (1981) and de de Meza and Webb (1987)—focused on economies with a competitive debt market and drew conflicting conclusions regarding the implications of adverse selection for investment. Stiglitz and Weiss concluded that adverse selection reduces investments and that resource allocation can be improved by subsidizing the interest rates on loans. Using a variation of that model, de Meza and Webb concluded that adverse selection results in excessive investment, in the sense that some projects with an expected return that is below the opportunity cost of capital will be financed. Thus, resource allocation could be improved by taxing the interest rate on loans.

Whether adverse selection coupled with debt/equity financial markets leads to excessive or inadequate private investment has potentially important implications for public policies, including tax policies. A symposium in the February 2002 issue of The Economic Journal contained a number of theoretical and empirical studies on the implications of asymmetric information for capital markets. In summarizing the debate, Cressy (2002) concluded that given our current knowledge about the performance of capital markets, the best advice that economists can give governments is "to do nothing," i.e., do not intervene by providing subsidies or imposing taxes.

Recent papers by Hellmann and Stiglitz (2000) and Boadway and Keen (2006) have extended the basic framework by allowing entrepreneurs to finance their projects using either debt or equity from outside investors.[1] Boadway and Keen (2006) showed that de Meza and Webb (1987)'s excessive investment result holds in an adverse selection model with debt and equity financing, in which entrepreneurs and outside investors are both risk neutral.

The Boadway and Keen's excessive investment result challenges the prevailing

[1]See also Fuest *et al.* (2003) and Fuest and Tillesen (2005) on adverse selection in capital markets. While Fuest and Tillesen (2005) explore occupational choice questions, Fuest *et al.* (2003) address the question of whether closed end subsidies may characterize optimal policies when individuals are risk neutral and markets present adverse selection problems very similar to ours.

view that capital market imperfections result in deficient investment, especially by small start-up firms. However, their assumption that entrepreneurs are risk-neutral is troubling because, at least in the conventional view, most entrepreneurs are risk averse and not able to hold diversified portfolio of assets. Their investment in their own firm represents the bulk of their wealth, and therefore they are exposed to a major source of risk if their firm should fail. Essentially, two sources of market failure could be operating at the same time: adverse selection in a debt/equity capital market structure, which leads to excessive investment in projects with negative net present values; and the entrepreneur's inability to diversify risks, which leads to inadequate investment in high-risk projects.

The risk-sharing possibilities in our model are exogenously restricted by the types of financial contracts allowed in the economy, namely debt and equity. This restricted contract space is sometimes justified by the existence of transaction costs that would prevent general mechanisms that were contingent on truthful announcements about the hidden characteristics of the productive projects. We claim that our framework is an appropriate one for representing capital markets in some economies, and it provides useful insights regarding economic policies that need to be assessed within the context of a model that incorporates those two sources of distortions, adverse selection and non-diversified risk.

There is now a large literature on the determination of entrepreneurship. Early contributions to this literature by Kihlstrom and Laffont (1979) and Kanbur (1979) contain models of entrepreneurship where individuals differ in their degree of risk aversion. In contrast, individuals in our model exhibit the same degree of risk aversion and have the same amount of wealth. They differ in the privately observed characteristics of their productive projects. This gives rise to an adverse selection problem that is not captured in these early models of entrepreneurship.

It is not our intention to review the recent literature on entrepreneurship, but we do note that the recent paper by Jaimovich (2010) most closely resembles our framework.[2] In his overlapping generations model, the only form of outside financing is equity investment. There are only two types of potential entrepreneurs, and the only risk that the high-type entrepreneurs face is the probability of failure. Consequently, although Jaimovich's model contains an interesting dynamic structure, it does not encompass the wider range of financing possibilities and investment outcomes that our model considers and does not capture the possibility

[2] See also the recent work by Hopenhayn and Vereshchagina (2009).

of equilibrium with excessive investment.

The remainder of the paper is organized as follows. In Section 2, we generalize the Boadway and Keen model by allowing agents to be risk averse. Simulations of the model in Section 3 show that their excessive investment result does not necessarily hold in this context. Total investment in risky projects declines with the degree of risk aversion, and there is a distortion in the mix of projects that are financed. Some projects with relatively low expected returns and negative net present values are financed, while others with positive net present values—but with high risk of failure—are not undertaken.

In Section 4, we consider two types of government policies that affect the volume of investment and the types of projects that are financed. First, in Section 4.1, we use the model to analyze the effects of policies that would enable agents to utilize a larger fraction of their total wealth to finance investments. We show that such policies will have an ambiguous effect on the expected utilities of entrepreneurs because, while their interest payments on debt financing will decline, they will be exposed to greater down-side risk if their project fails. Agents endowed with high-risk high-return projects are most likely to be made worse off when more of their wealth has to be invested in a risky project. Consequently, investment in risky projects may decline, and social welfare may or may not increase. These numerical results are aligned with the theoretical underpinnings in our companion paper, Braido *et al.* (2010), which considers an economy with debt financing only.

In Section 4.2, we consider how the tax treatment of losses can affect the level of investment and compare the social welfare gains of tax systems with a full-loss offset. Our simulations show that when entrepreneurs are risk neutral and there is excessive investment, social welfare can be improved by a reduction in the rate at which entrepreneurs are compensated for their losses combined with a revenue neutral reduction in tax on gains. (This tax policy reduces investment in projects with high risk and low return.) However, our results also indicate that if a society places a high value on distributional equity or if entrepreneurs are sufficiently risk averse, a full-loss offset system may be desirable even when there is excessive investment.

Our main results are then summarized and commented in Section 5, which serves as a conclusion.

2 A Model of Adverse Selection with Risk Averse Entrepreneurs

The economy is inhabited by a continuum measure one of agents, whose preferences are represented by expected utility functions with identical constant relative risk aversion Bernoulli functions, given by:

$$u(W) \equiv \frac{[W]^{1-\sigma}}{1-\sigma}, \text{ for } \sigma \geq 0 \text{ and } \sigma \neq 1; \tag{1}$$

and

$$u(W) \equiv \log(W), \text{ for } \sigma = 1; \tag{2}$$

where $W \in \mathbb{R}_+$ represents individual consumption wealth.

All agents are potential entrepreneurs. Each of them is endowed with a project that requires one unit of wealth if it is to be realized. Projects are characterized by their probability of success, $0 < p \leq 1$; and the magnitude of their return if they are successful, $R > 0$. If the project is unsuccessful, the return is zero. Returns on the individual projects are uncorrelated; and agents privately know the (p, R) characteristics of their own projects.

This economy also has a continuum of potential outside investors whose opportunity cost of capital is given by an exogenous risk-free real rate of return, $r \geq 0$. Outside investors know the joint distribution of p and R across all projects and are aware that each agent is privately informed about the characteristics of his project, (p, R).

The agent's wealth consists of two types of assets. One asset, L, cannot be used to finance investment in the project. For example, L could be the earning of some family member or an asset that cannot be pledged as security for a loan because of legal restrictions or the absence of full property rights. The other asset, K, can be used to finance investment in the risky project. We assume that $K < 1$ and, therefore, the agent requires outside financing in order to make the investment in his project. Define $\varphi \equiv 1 - K > 0$ as the level of outside investment that is required to finance a project.

Debt Financing Let us first consider the case in which entrepreneurs can finance their projects in a competitive debt market. The interest rate on debt financing for these projects is $i \in \mathbb{R}_+$. If the agent invests in the safe asset, he will have a secure amount of wealth given by:

$$W_s \equiv L + (1+r)K = L + (1+r)(1-\varphi). \tag{3}$$

If an agent borrows to finance a project, his wealth is:

$$W_{1B} = \max\left(L\ ,\ R + L - (1+i)\varphi\right), \tag{4}$$

in case of success; and

$$W_{0B} = L, \tag{5}$$

in case of failure. Notice however that agents with $R < (1+i)\varphi$ will never apply for a loan, since this would imply a consumption level below W_s in all scenarios. The expected utility of a debt-financed entrepreneur can be written as:

$$EU_{DF} = p\frac{\left[W_{1B}\right]^{1-\sigma}}{1-\sigma} + (1-p)\frac{\left[W_{0B}\right]^{1-\sigma}}{1-\sigma},\ \text{for } 0 \leq \sigma \neq 1; \tag{6}$$

and

$$EU_{DF} = p\log\left(W_{1B}\right) + (1-p)\log\left(W_{0B}\right),\ \text{for } \sigma = 1. \tag{7}$$

Agents will prefer debt financing their projects to investing K in the safe asset if $EU_{DF} \geq \left[W_s\right]^{1-\sigma}/(1-\sigma)$. We can use (3) and (6)-(7) to define a function $Z(p)$ that partitions the space $p \times R$, through (8), in two different sets: the *locus* (p, R) such that agents opt to become entrepreneurs (through debt financing their projects); and the *locus* for which they opt not to do so. That is, agents choose debt financing over investing in the safe asset whenever:

$$R \geq Z(p) - L + (1+i)\varphi; \tag{8}$$

where

$$Z(p) = \left[\frac{\left[W_s\right]^{1-\sigma} - (1-p)L^{1-\sigma}}{p}\right]^{\frac{1}{1-\sigma}},\ \text{for } 0 \leq \sigma \neq 1; \tag{9}$$

and

$$Z(p) = L\left(\frac{W_s}{L}\right)^{1/p},\ \text{for } \sigma = 1. \tag{10}$$

The term $Z(p) - L$ is always positive.[3] Moreover, $Z(\cdot)$ is decreasing in p and increasing in σ. It can be interpreted as the level of wealth that the agent needs if the project succeeds in order to make the expected utility with a debt-financed investment equal to the utility from investing in the safe asset.

[3]For that to be negative, one should have $\frac{[W_s]^{1-\sigma}}{1-\sigma} < \frac{p}{1-\sigma}L^{1-\sigma} + \frac{1-p}{1-\sigma}L^{1-\sigma}$, which is impossible since $W_s > L$.

If the agents are risk neutral, inequality (8) can be written as:

$$R \geq \frac{(1+r)(1-\varphi)}{p} + (1+i)\varphi. \tag{11}$$

In Figure 1, which shows the Boadway and Keen equilibrium with risk neutral agents, the BB' curve represents the projects with (p, R) values such that risk-neutral agents are indifferent between investing in their project using borrowed funds and investing their wealth in the safe asset.

In equilibrium, competition among lending institutions implies that the market interest rate on debt is determined by the following condition:

$$\hat{p} \equiv \mathbb{E}[p \mid R \geq Z(p) - L + (1+i)\varphi] = \frac{1+r}{1+i}, \tag{12}$$

where $\hat{p}$ is the average probability of success (and therefore repayment of a loan) of debt-financed projects, r is the risk-free rate of return that the lending institutions pay to their depositors, and $\mathbb{E}[\cdot]$ is the mathematical expectation. (It is implicit in this reasoning that lending institutions hold a well-diversified portfolio of loans so that they can be treated as risk neutral.)

Equity Financing Now consider equity financing by outside investors. Agents endowed with project (p, R) can issue equity shares against the project's return. The number of shares issued by each firm is normalized to one. The price of each share, V, is given by the stock-market value of the firm. All firms have the same equity value since they are *ex ante* identical for outside investors.

Equity investors receive $\min(\varphi/V\ ,\ 1)$ of the profits if the project is successful, and 0 otherwise. Equity-financed entrepreneurs, on the other hand, receive $1 - \min(\varphi/V\ ,\ 1)$ if the projects succeeds, and 0 otherwise. Their contingent wealth is:

$$W_{1E} = R(1 - \min(\varphi/V\ ,\ 1)) + L, \tag{13}$$

in case of success; and

$$W_{0E} = L, \tag{14}$$

in case of failure. The entrepreneur's expected utility with equity financing is given by:

$$EU_{EF} = p\frac{1}{1-\sigma}[W_{1E}]^{1-\sigma} + (1-p)\frac{[W_{0E}]^{1-\sigma}}{1-\sigma}, \text{ for } 0 \leq \sigma \neq 1; \tag{15}$$

and

$$EU_{EF} = p\log(W_{1E}) + (1-p)\log(W_{0E}), \text{ for } \sigma = 1. \tag{16}$$

Agents will use equity to finance their projects (instead of investing all their resources in the safe asset) whenever

$$EU_{EF} \geq [W_s]^{1-\sigma} / (1-\sigma) . \tag{17}$$

Notice then that projects such that $\varphi/V \geq 1$ will never be equity financed. Inequality (17) can then be written as:

$$R \geq \frac{V}{V-\varphi} [Z(p) - L] , \tag{18}$$

where $Z(p)$ is as defined in (9).

If the agent is risk neutral, (18) can be written as:

$$R \geq \frac{V}{V-\varphi} \frac{(1+r)(1-\varphi)}{p} . \tag{19}$$

The EE' curve in Figure 1, shows combination of (p, R) values such that risk-neutral entrepreneurs are indifferent between equity financing and investing their wealth in the safe asset.

In equilibrium, investors bid up equity shares until:

$$V = \frac{\mathbb{E}\left[pR \mid R \geq \frac{V}{V-\varphi} [Z(p) - L]\right]}{1+r} . \tag{20}$$

In words, the value of a firm is equal to the present value of its expected return, conditional on equity financing, discounted at the risk-free rate of return.

The Choice Between Equity and Debt Financing Some entrepreneurs that would accept to use equity to finance their projects may actually prefer debt financing. Notice that, in our model, the entrepreneur's degree of risk aversion does not affect the preferred method of financing. This occurs because entrepreneurs must always invest $K > 0$ from their own wealth into the project.[4] For the same reason, agents have no motivation for mixing debt and equity to finance a given project.

Debt financing is preferred to equity financing whenever $EU_{EF} < EU_{DF}$. From (6)-(7) and (15)-(16), this is equivalent to saying that:

$$pR(1 - (\varphi/V)) < p(R - (1+i)\varphi). \tag{21}$$

[4] This is equivalent to assuming that, in case of bankruptcy, the entrepreneurs are legally responsible for debts up to the amount $K > 0$.

From this reasoning, entrepreneurs are indifferent between debt and equity financing when:

$$R = (1+i)\,V. \tag{22}$$

The curve MM' in Figure 1 represents the projects for which agents are indifferent between debt and equity financing. An agent with a project whose R lies below MM' will prefer equity financing to debt financing.

It is important to notice that the BB′, EE′, and MM′ curves intersect at the same point, namely:

$$p_{MEB} = \frac{L^{1-\sigma} - [W_s]^{1-\sigma}}{L^{1-\sigma} - [(V-\varphi)(1+i)+L]^{1-\sigma}},\ \text{for } 0 \leq \sigma \neq 1; \tag{23}$$

and

$$p_{MEB} = \frac{\log(W_s) - \log(L)}{\log[(V-\varphi)(1+i)+L] - \log(L)},\ \text{for } \sigma = 1. \tag{24}$$

Moreover, when $\sigma = 0$, we have:

$$p_{MEB} = \frac{1-\varphi}{V-\varphi}\hat{p}. \tag{25}$$

Therefore, entrepreneurs with projects whose (p, R) values that lie above the line segment BJM' in Figure 1 will finance their projects with debt. The projects with relatively high R values are debt-financed so that the owners do not have to share the high returns with outside shareholders. Entrepreneurs with projects in the region $JM'E'$ will be financed in the stock market. Equity financing is attractive for agents with projects with relatively low R values and relative high probabilities of success.

The projects where the expected rate of return is greater than or equal to the rate of return on the safe asset satisfy the condition $pR > (1+r)$. In Figure 1, all of the projects that satisfy this condition lie on or above the FF' line. These are the projects that would be financed under symmetric information by risk neutral investors. The FF' line intersects the MM' line at the probability level, p_{MF} defined by:

$$p_{MF} = \frac{\hat{p}}{V}, \tag{26}$$

where $p_{MEB} < p_{MF}$. Therefore, point A in Figure 1 lies to the right of point J.

Market Equilibrium Equilibrium in this economy is given by an allocation—namely, three *loci* of projects (p, R) that are debt-financed, equity-financed, and not undertaken—and a vector of prices (i, V) such that agents maximize expected utility and the debt and equity markets satisfy the zero-profit conditions (12) and (20). As previously described, the individual optimal decisions are summarized by conditions (8), (18), and (21).

Figure 1 illustrates Boadway and Keen's over-investment result with risk neutral agents. The debt-financed projects in the region $BJAF$ will have a negative expected net present value because the FF' and the BB' curves intersect at the probability level $\hat{p}$ and FF' is steeper than BB'. Also note that the EE' curve is always below the FF' curve. In equilibrium, V is greater than one. (V can be interpreted as Tobin's average q.) The equity-financed projects in the region $JE'F'A$ will have negative expected present values. Thus, the Boadway and Keen model predicts that all of the projects with positive expected net present values will be financed and that some projects with negative expected net present values will be also be financed either by debt or equity.

If agents are risk averse, it is possible that some projects with a positive net present value will not be financed. Intuitively, p_{MEB} increases as the degree of risk aversion increases—holding V and i constant—which shifts point J in Figure 1 towards point A. This reduces the region of over-investment. Note, however, that as the degree of risk aversion changes and poor projects are dropped, the equilibrium values for i and V will also change. Our intuition suggests that i will decrease and V will increase as risk aversion increases, which would have offsetting and therefore ambiguous effects on the position of the J and A. Therefore, an equilibrium analysis is required to assess the full effect of an increase in risk aversion on the level of over-investment.

In addition to that, the slopes of the BB' and EE' curves will also change when agents become more risk averse, creating the possibility that the BB' curve may intersect the FF' curve above the MM' line. Therefore, some projects with low probabilities of success—but high returns and positive net present values—will not be undertaken by agents because they are too risky.

In summary, when agents are risk averse, some projects which should not be undertaken will be undertaken, while some projects which should be undertaken will not be in equilibrium. The first type of inefficiency occurs for projects with high probability of success and low conditional return, and the second for projects with low probability of success and high conditional returns. Given the complexity of the model, it is not possible to pin down the exact conditions leading to under- or over-investment. We shall rely on numeric exercises to illustrate our results.

3 Equilibria with Excessive and Inadequate Investment

We present here some numeric exercises in which the equilibrium exhibits either excessive or inadequate levels of investment, for different parameters. The model is simulated with the following parameter values: $r = 0.05$, $\varphi = 0.60$, $L = 0.80$, $K = 0.40$, and $W_s = 1.22$. The distribution function is $f(p, R) = \theta e^{-\theta R}$, where $\theta = 1.25$. With this distribution function, $p(R > 1) = 0.287$, $p(R > 2) = 0.082$, $\mathbb{E}(p) = 0.5$, $\mathbb{E}(R) = 0.80$, $\mathbb{E}(pR) = 0.40$. The area above the FF' curve—which measures the proportion of the projects with positive net present values evaluated at the risk-free rate of return—is 0.095.

Table 1 shows the computed values of the key endogenous variables with three different levels of risk aversion. In the first column, for comparative purposes, we report the equilibrium values for the case with risk-neutral agents. In this case, the interest rate on debt is 63.1 percent. (One of the reasons why the interest rate is so high in this model is that we have assumed that the investment has no scrap value if it fails.) The market value of the shares in the firms that are financed by equity investment is 1.109. The average probability that a debt financed firm will repay its debt, $\hat{p}$, is 0.644. In other words, 35.6 percent of the debt-financed projects default on their loans. This also explains the high interest rate on debt in this model. Figure 2a shows the MM' curve and the relevant sections of the BB' and EE' curves, now labelled as BE', intersect at the 0.506 probability level. As noted in the previous sections, excessive investment is a characteristic of the equilibrium with risk-neutral agents. About 7.24 percent of all projects are debt financed and 5.54 percent are financed by equity. The total percentage of projects financed, 12.8 percent, exceeds the 9.5 percent of the projects with positive net present values.

The second column presents the case in which the coefficient of relative risk aversion is 0.90. In this case, the interest rate on debt drops to 49.7 percent, because the probability that a debt-financed entrepreneur will repay its debt increases to 0.701. The market value of shares in the firms that are financed by equity investment remains at 1.109. As Figure 2b shows, the MM' and the BE' curves now intersect at a much higher probability level, 0.623. This equilibrium is characterized by both excessive investment and under investment. (The FF' curve lies above the BE' curve in the range of p values from about 0.22 to 0.60 and below the BE' curve for $p < 0.22$.) In other words, there is under-investment in some projects with positive net present values and high risk because agents are unable to diversify the project's risk. In this equilibrium, 7.23 percent of all projects are debt-financed and 3.80 percent are financed by equity. The total number of projects financed, 11.0 percent, exceeds the fraction of projects with positive net present value, namely 9.5 percent.

In the third column, where the coefficient of relative risk aversion is 2.00, the interest rate on debt drops to 35.0 percent, because the probability that a debt-financed project will repay his debt increases to 0.778. The market value of the shares in the firms that are financed by equity investment falls slightly to 1.096. As Figure 2c shows, the MM' and BE' curves now intersect at a higher probability level, 0.756. Now the equilibrium is characterized by under investment. The FF' curve lies almost completely below the BE' curve indicating significant under-investment in projects with positive net present values. Only 6.68 percent of all projects are debt financed and only 1.98 percent of projects are financed by equity. The aggregate level of investment, 8.66 percent of the total number of potential projects, is below the fraction of projects with positive net present values.

These simulations show that the excessive investment result does not necessarily hold in a debt/equity capital market with adverse selection and risk averse agents. Our numerical simulations indicate that the number of projects that are financed can either exceed or fall short of the number that would be financed in a frictionless economy. What the model reveals is a distortion in the mix of projects that are financed—some projects with low expected returns are financed, while some high risk projects with positive net present values are not.

It is important to emphasize that this is not a calibration exercise. That is, we did not choose parameters to try and match equilibrium values of some target variables. Instead, we chose functional forms and parameters that facilitated both our exposition and the numeric computations. In particular note that our choice of a zero scrap value for the project induces a counterfactually high value for interest rates. We do not believe that any of these choices are driving the under-investment results. Indeed, under-investment is fundamentally linked to the downside risk faced by entrepreneurs, which is not directly affected by the equilibrium interest rate.

4 The Welfare Effect of Government Policies

We investigate now the effects on social welfare of two different policies. First, we exam the effects of increasing the fraction of wealth that agents can use in financing their projects. Our numerical exercise here consider an economy with debt and equity markets. (In a companion work, Braido *et al.* (2010), we derive similar theoretical results for an economy with debt financing only.)

Second, we consider how the tax treatment of losses affects the level of investment and social welfare in our setting.[5] Many economists have argued for generous tax treatment of losses in order to promote risk taking—see Domar and Musgrave (1944), Mossin (1968), Stiglitz (1969), Mintz (1981), and Gentry and Hubbard (2000). However, most tax systems treat gains and losses asymmetrically, with the tax rate on gains exceeding the rate at which losses are compensated. As our previous analysis has indicated, it is not clear whether public policy should promote risk-taking or discourage it with those types of capital market distortions.

4.1 Increasing the Proportion of Wealth Available to Invest

An increase in K, with an offsetting reduction in L, has two effects on the incentive to invest in the risky project. First, the opportunity cost of investing in the risky project increases by the rdK, and therefore some entrepreneurs with marginal projects will find investing in the safe asset more attractive. Second, entrepreneurs will not have to borrow as much and their interest payments will decline, but at the same time they will face greater down-side risk because of the reduction in L. Consequently, the expected utility of entrepreneurs may either increase or decrease.

To see this, we can write the expected utility of an entrepreneur with a debt-financed project as $EU_{DF} = pU\left(R + L - (1+i)\varphi\right) + (1-p)U(L)$, since in equilibrium debt-financed projects display $R > (1+i)\varphi$, as remarked in Section 2. Hence, the effect on expected utility of an increase in K—holding total wealth constant—is given by:

$$\left.\frac{d}{dK}EU_{DF}\right|_{W} > 0 \text{ iff } i - \varphi\frac{di}{dK} > \frac{1-p}{p}\frac{U'\left(R + L - (1+i)\varphi\right)}{U'\left(L\right)}, \tag{27}$$

[5] Kanbur (1982) and Kihlstrom and Laffont (1983) also deal with the effect of taxation on entrepreneurship, but their framework is quite different from ours. In particular, in their models, risk-averse entrepreneurs face a uncertain production shock, but all potential entrepreneurs have face the same *ex ante* return. Thus, the adverse selection problem does not arise in their models.

where di/dK is the change in the equilibrium interest rate on loans to entrepreneurs and $i - \varphi di/dK$ is the reduction in the entrepreneur's interest payments from a small increase in K and an offsetting reduction in L, which shifts wealth from the loss state to the success state of the world.

An entrepreneur's marginal rate of wealth substitution between the two states of the world is given by the right hand side of the second set of conditions in (27). An entrepreneur will be better off if the reduction in his interest payments exceeds his marginal rate of substitution of wealth between the two states of the world. For a given value of R, the entrepreneurs with low p projects have a higher marginal rate of substitution of wealth between the two states of the world and are more likely to be worse off when K increases and L declines. Therefore, entrepreneurs with low p projects are the ones that are most likely to drop their projects. Consequently, the default rate on loans will decrease, and the interest rate on loans will to decline, i.e. $di/dK < 0$.

Holding p constant, the entrepreneurs with projects with higher R values will have a higher marginal rate of wealth substitution, and therefore they are more likely to be made worse off. Thus, somewhat paradoxically, reduced reliance on outside financing can make some entrepreneurs (with positive net present value projects) worse off. Finally, the greater the degree of risk aversion, the higher marginal rate of wealth substitution, the more likely that an increase in K will make the entrepreneurs worse off.

It is important at this point to note the crucial role played by our assumption that liability is the same for all entrepreneurs. We are, in this sense, ruling out the use of different levels of liability as a screening device. From a purely technical perspective, leaving the space of contracts to be only limited by the informational structure of the economy leads to a series of difficulties in terms of existence, let alone characterization. From a more practical perspective, very complicated non-linear contracts do not seem to be the common practice, at least for the type of small start-up firms that justifies our risk-aversion assumption.

As we have seen, an increase in the amount of wealth that can be used to finance investment will likely make some individuals better off and others worse off. Therefore, to evaluate the distributional effects of a policy which increases K and reduces L, we will use the following social welfare function:

$$SWF = \int_p \int_R (1-\zeta)^{-1} \mathbb{E}\left[U\left(\tilde{W}\right)\right]^{1-\zeta} f(p,R)\, dpdR, \tag{28}$$

where ζ is the coefficient of inequality aversion. If $\zeta = 0$, then the social welfare function is utilitarian. Our measure of social welfare is the equally distributed equivalent (EDE), defined as:

$$SWF = (1-\zeta)^{-1} [U(EDE)]^{1-\zeta}. \tag{29}$$

As demonstrated in our simulations in Table 2, the level of investment may decline when K increases and L declines. Entrepreneurs will be exposed to more risk when they are responsible for financing a higher proportion of the investment, and the opportunity cost of investing in the risky project may increase.

In Case I, the entrepreneurs' total wealth is 1.2, where $L = 0.8$ and $K = 0.4$. They must then rely on outside investors for 60 percent of the initial investment. Assuming $\sigma = 0.90$, $r = 0.05$, and $\theta = 1.25$, the equilibrium interest rate on loans would be 0.497, the value of a firm would be 1.109, and 11 percent of the projects would be financed. In Case I, with $\zeta = 0$, the EDE wealth level is 1.254573. To put this figure in perspective, note that those entrepreneurs that invest in the safe asset and who have the lowest expected utility have secure wealth equal 1.22.

Case II shows the effect of reducing L from 0.8 to 0.4 and simultaneously increasing K from 0.4 to 0.8. One could think of this increase in the amount of wealth that can be used to finance investments as arising from increasing property rights in informal housing in a developing country. Note that the entrepreneurs would still require outside financing for 20 percent of the projects cost. In the new equilibrium, the interest rate on loans would decline from 0.497 to 0.308, because the default rate on loans would decline by 10 percentage points (i.e., $\hat{p}$ would increase from 0.701 to 0.803). The decline in the default rate reflects the reduction in investment. In Case II, only 6.96 percent of the projects would be financed. Investment in the projects declines because the entrepreneurs' secure level of wealth will have increased to 1.24. Hence, some marginal projects with will no longer be financed because the opportunity cost of investing in the risky projects has increased. In addition, some entrepreneurs with positive net present value projects will now face a reduction in their expected utility from investing in their risky projects.

While the expected utilities of some of the entrepreneurs with good projects decline in Case II, overall social welfare increases with a utilitarian or pro-poor social welfare functions because some of the losers were initially better off than the winners.

The next two cases analyzed assume that $K > 1$ so that entrepreneurs must self-finance their project. Case III in Table 2 the total wealth of the entrepreneurs is the same as in the previous cases, but they can now self-finance the project. Only 5.24 percent of the risky projects are financed, in part because W_s increases to 1.26, but also because of the increase in the downside risk that the entrepreneurs face now that they finance the entire project from their own wealth. However, there is an overall social welfare gain because of the increase in W_s to 1.26.

Case IV shows what would happen if in converting L into K, the W_s remained constant so that there was no gain to those who only invest in the safe asset. The entrepreneurs can self-finance their investment because $K = 1.162$. The computations show that only 5.01 percent of the risky projects would be financed, even though the opportunity cost of the investment remains the same as in Case I. This occurs because of the greater downside risk that the entrepreneurs face. The computed values of EDE wealth also decline.

Figure 3 shows the equilibria in Cases I and IV. In Case IV, only the entrepreneurs with projects above the SS' curve will undertake their project, whereas all of the projects above the BE' curve are financed by either debt or equity in Case I. All of the entrepreneurs with projects between the SS' curve and the BE' curves are worse off in Case IV as compared to Case I. Furthermore, the entrepreneurs with debt financed projects in Case I that are above the SS' curve, but to the left of the curve labeled GG', would also be worse off. Only those entrepreneurs with debt financed projects in Case I that are to the right of the GG' curve and above the relevant sections of the SS' and MM' curves are better off in Case IV. These computations show that only entrepreneurs with debt-financed projects with high (p, R) values would be better in Case IV than in Case I. (There are also a small number of entrepreneurs with equity financed projects in Case I that would be better off in Case IV, but the area that represents those projects is too small to show in Figure 3.)

4.2 Tax Policy

Another type of public policy that affects investment decisions is the taxation of the returns on risky investments, and in particular, the tax treatment of losses. We study next the tax treatment of losses for a simple linear tax system. We first provide a brief description of how taxes can be introduced into our model. We then simulate the effects of varying the tax loss offsets and compare the welfare gains and losses with a full loss offset tax system.

Let investment income be taxed at the rate τ and investment losses reimbursed at the rate t. A tax system with full loss offsets is one where $\tau = t$.[6] The return on the safe asset is then taxed at the rate τ and, if the individual has invested in this asset, his wealth will be:

$$W_s' = [1 + (1 - \tau)r]\,(1 - \varphi) + L. \tag{30}$$

For debt-financed or equity-financed projects, the entrepreneur's wealth in state 0 (when the project fails) is given by:

$$W_{0B}' = W_{0E}' = L + (1 - \varphi)t, \tag{31}$$

Note that we assume that the tax loss is restricted to the actual amount that the entrepreneur has invested in the project.

If the entrepreneur debt-finance a project that succeeds, his wealth becomes:

$$W_{1B}' = \max\{0\ ,\ R - (1 + i)\varphi - \tau\,(R - 1 - i\varphi)\} + L, \tag{32}$$

where $R - (1 + i)\varphi$ represents the return that the entrepreneur obtains after paying the principal and interest on loans, and $\tau\,(R - 1 - i\varphi)$ is the tax payment. It is assumed that the tax system allows the investor to fully depreciate the initial investment and to deduct all of his interest payments. As we will show later, debt-financed projects will always have the property that $R > 1 + i\varphi$, and therefore a successful debt-financed project will always be in a taxpaying position.

However, this may not be the case for an equity-financed project. If the project is successful and $R \geq 1$. Then, the entrepreneur's wealth with an equity financed project will be:

$$W_{1E}' = \left(\frac{V - \varphi}{V}\right) R - \tau\,[R - 1]\left(\frac{V - \varphi}{V}\right) + L, \text{ for } R \geq 1, \tag{33}$$

where the first term on the right-hand side is the entrepreneur's share of the gross return on the project, and the second term is the entrepreneur's share of the taxes that have to be paid on the gain from the project. Again, we assume that the full cost of the project can be deducted from the return.

Note that it is also possible for an equity-financed project to be successful but incur a loss for tax purposes if $0 < R < 1$. Thus, the entrepreneur's wealth will be:

[6] In practice, losses are not actually reimbursed but the government allows the investor to carry them forward to reduce future tax liability. For multi-project firms, one can think that there is loss offset at the project level.

$$W'_{1E} = \left(\frac{V-\varphi}{V}\right) R + t\,[1-R] \left(\frac{V-\varphi}{V}\right) + L, \text{ for } 0 < R < 1. \tag{34}$$

The rest of the model follows the previous model with the addition of these tax variables. The BB' curve is implicitly defined by the equation $W'_{1B} = Z(p)$ and the EE' curve is implicitly defined by $W'_{1E} = Z(p)$, where as before:

$$Z(p) = \left[\frac{[W_s]^{1-\sigma} - (1-p)\,(L+(1-\varphi)t)^{1-\sigma}}{p}\right]^{\frac{1}{1-\sigma}}. \tag{35}$$

The MM' curve is given by:

$$R_M = \frac{(1+i)V - \tau(1+iV)}{1-\tau}. \tag{36}$$

By virtue of entrepreneurial selection, the return of a debt-financed project, R, will always exceeds R_M. Thus, since $\varphi < 1 < V$, one must have $R > 1 + i\varphi$.

Equilibrium in the debt market requires that the probability of success for a debt-financed project is equal to $(1+r)/(1+i)$ while equilibrium in the equity market requires that the expected return of an outside investor equals the after-tax return $(1-\tau)r$ that they can earn on the safe asset.

Table 3a illustrates how changes in the tax treatment of losses can affect investment in a debt/equity capital market with adverse selection. The first column shows the values of the key variables if entrepreneurs are risk-neutral and face a full loss offset tax system, where $\tau = t = 0.30$. Note that the level of total investment is virtually the same as in the equivalent no-tax case in Table 1. Since there is excessive investment with risk neutral entrepreneurs, it is interesting to examine the consequences of reducing t while holding total tax revenues constant, which implies a reduction in τ. When the tax loss offset is reduced to 0.2 and the tax rate on gains can be reduced to 0.287 while maintaining the same tax revenue as in the tax system with a full loss offset. Total investment is reduced from 12.72 percent of all potential projects to 12.06 percent. (Note that there is a slight increase in the number of debt-financed projects while the number of equity-financed projects declines sharply.) The interest rate and the value of a firm also decline.

We compute the certainty-equivalent wealth to measure the social gains and losses from each of these revenue-neutral changes in the tax policy. When distributional equality is not important (i.e., $\varsigma = 0$), social welfare increases when the tax loss offset is reduced. The same result holds when there is a modest preference for distributional equity, namely $\varsigma = 0.5$. The improvement in social welfare from reducing the tax loss offset should not be surprising because in this case there is excessive investment in risky projects. Reducing the tax loss offset reduces investment in the low return, high risk projects while allowing an increase in the after-tax return on the safe asset. Note, however, that when there is a strong preference for equality, $\varsigma = 1.5$, social welfare declines when the tax loss offset is reduced because it adversely affects agents who have relatively low expected utilities (the marginal entrepreneurs) while the main beneficiaries of this policy are those entrepreneurs with high expected returns. Thus, somewhat surprisingly, if entrepreneurs are risk neutral a full loss offset tax system would be justified on the basis of its contribution to social equality and not on its efficiency-enhancing properties.

Of course these results were obtained for the case in which entrepreneurs are risk-neutral and there is excessive investment due to adverse selection in the capital market. It is interesting therefore to consider a case where entrepreneurs are risk averse. Table 3b considers the case where $\sigma = 0.9$. Recall that in this case, although there was excessive investment in aggregate, there was a distortion in the mix of investment projects with too many low return projects and too few high risk, high return projects. Again, we have simulated the model with a full loss offset system with $\tau = t = 0.3$, shown in the second column, and then we considered alternative levels of tax loss offsets while maintaining the same tax revenue as in the full loss offset case. The table shows that in this case when t is reduced below 0.3, social welfare declines even when there is no social preference for equality (a utilitarian social welfare function). We also show in this table that social welfare increases if losses are compensated at a more generous rate than gains are taxed, even though this implies that the tax rate on gains has to exceed 0.3 in order to maintain total tax revenues at the full loss offset level. Of course, such generous tax treatment of losses would not be practical if losses were affected by entrepreneurial effort or if losses could be shifted within a corporate group through transfer prices on intra-corporate sales. However, these computations do illustrate how important loss offset provisions can be in sheltering entrepreneurs from risk when they are not able to hold diversified portfolios.

5 Conclusion

This paper builds on the framework of adverse selection in capital markets analyzed by Boadway and Keen (2006). This class of models restricts the set of available financial contracts to include only debt and equity. We stick with this market structure and extend the basic setup to allow for entrepreneurial risk aversion. We also introduce a secondary modification into the model—useful for policy analysis—that agents are guaranteed to retain the illiquid part of their wealth in case of failure of their projects.

Boadway and Keen's main result states that adverse selection coupled with this capital market structure leads to excessive investment. We show that this result does not necessarily hold when entrepreneurs' are risk averse. This occurs because risk aversion introduces a new source of market failure into the problem: while adverse selection (coupled with the debt/equity market structure) leads to excessive investment in projects with negative net present values; the entrepreneur's inability to diversify risk leads to inadequate investment on high-risk projects with positive net present values.

This framework is also used to evaluate a policy of varying the wealth that entrepreneurs can use to finance investments. If institutional restrictions determine the share of individual wealth that can be used as collateral, then a natural policy question is how varying this fraction would affect welfare. We show that freeing more resources need not always be welfare improving. This may account for why many modern societies maintain restrictions on how one may use one's own wealth.

We have also investigated the welfare implication of the tax treatment of losses in this context. In debt/equity capital markets with adverse selection, there may be excessive investment in risky projects. Reducing the compensation for losses may help to discourage investment in such projects, resulting in a social welfare gain if entrepreneurs are not too risk averse or if society does not place an emphasis on distributional equality. However, our simulations show that full-loss offsets may be justified, even in situations where there is excessive investment, if entrepreneurs are sufficiently risk averse or if society places a high priority on distributional equality.

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A Tables and Figures

Table 1. Equilibrium Simulations

Degree of Risk Aversion	$\sigma = 0$	$\sigma = 0.9$	$\sigma = 2.0$
i	0.631	0.497	0.350
V	1.109	1.109	1.096
$\hat{p}$	0.644	0.701	0.778
p_{MEB}	0.506	0.623	0.756
Percentage of projects financed by:			
Debt	7.24	7.23	6.68
Equity	5.54	3.80	1.98
Debt and Equity	12.8	11.0	8.66

$L = 0.8,\ K = 0.4,\ \phi = 0.6,\ W_s = 1.22,\ r = 0.05,\ f(R) = 1.25e^{-1.25R}$

Table 2. Increasing the Proportion of Wealth Available to Invest

Case	I	II	III	IV
Key Parameter Values	$L = 0.8$	$L = 0.4$	$L = 0$	$L = 0$
	$K = 0.4$	$K = 0.8$	$K = 1.2$	$K = 1.162$
	$\varphi = 0.6$	$\varphi = 0.2$	$\varphi = 0$	$\varphi = 0$
	$W_s = 1.22$	$W_s = 1.24$	$W_s = 1.26$	$W_s = 1.22$
i	0.497	0.308	na	na
V	1.109	1.165	na	na
$\hat{p}$	0.701	0.803	na	na
p_{MEB}	0.623	0.782	na	na
Pctg. of Projects Financed				
Debt	7.23	5.49	na	na
Equity	3.80	1.47	na	na
Total	11.00	6.96	5.24	5.01
Equally dist. equivalent wealth				
$\zeta = 0$	1.254573	1.267364	1.281950	1.240943
$\zeta = 0.5$	1.254159	1.267005	1.281655	1.240656
$\zeta = 1.5$	1.253357	1.266311	1.281083	1.240101

$\sigma = 0.90$, $r = 0.05$, $f(R) = 1.25e^{-1.25R}$

Table 3a. Tax Loss Offsets: Risk Neutral Case ($\sigma = 0$)

Tax Loss Offset, t	0.300	0.200	0.100	0.000
Tax Rate on Gains, τ	0.300	0.287	0.276	0.265
i	0.624	0.594	0.567	0.543
V	1.075	1.072	1.070	1.067
Pctg. of Projects Financed				
Debt	7.45	7.51	7.54	7.53
Equity	5.27	4.55	3.96	3.47
Total	12.72	12.06	11.49	11.00
Equally dist. equivalent wealth				
$\zeta = 0$	1.229296	1.230061	1.230934	1.231852
$\zeta = 0.5$	1.223519	1.223777	1.224186	1.224679
$\zeta = 1.5$	1.213561	1.212612	1.211804	1.211070

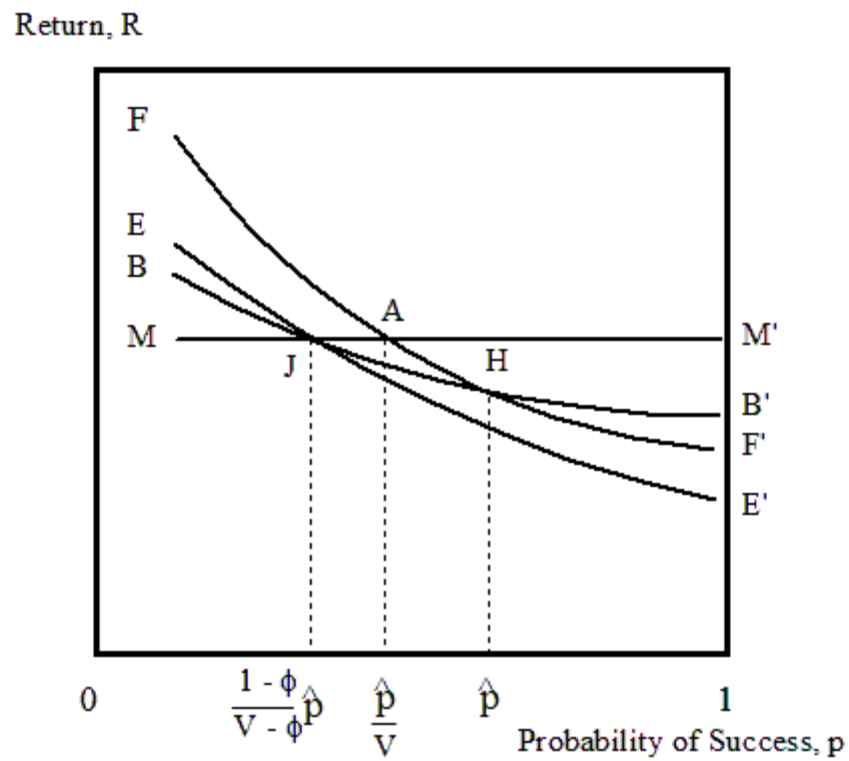
L= 0.80, $\phi = 0.60,\ r = 0.05,\ f(R) = 1.25e^{-1.25R}$

Table 3b. Tax Loss Offsets: Risk Averse Case ($\sigma = 0.9$)

Tax Loss Offset, t	0.400	0.300	0.200	0.100
Tax Rate on Gains, τ	0.313	0.300	0.290	0.282
i	0.570	0.528	0.492	0.460
V	1.078	1.075	1.072	1.069
Pctg. of Projects Financed				
Debt	7.45	7.46	7.41	7.32
Equity	4.96	4.05	3.33	2.76
Total	12.41	11.51	10.74	10.08
Equally dist. equivalent wealth				
$\zeta = 0$	1.210540	1.209368	1.208325	1.207188
$\zeta = 0.5$	1.209974	1.208678	1.207490	1.206174
$\zeta = 1.5$	1.208842	1.207289	1.205793	1.204090

$L = 0.80,\ \phi = 0.60,\ r = 0.05,\ f(R) = 1.25e^{-1.25R}$

Figure 1



Return, R
F
E
B
M
A
J
H
M'
B'
F'
E'
0
$\frac{1-\phi}{V-\phi}\hat{p}$
$\frac{\hat{p}}{V}$
$\hat{p}$
1
Probability of Success, p

Figure 2a

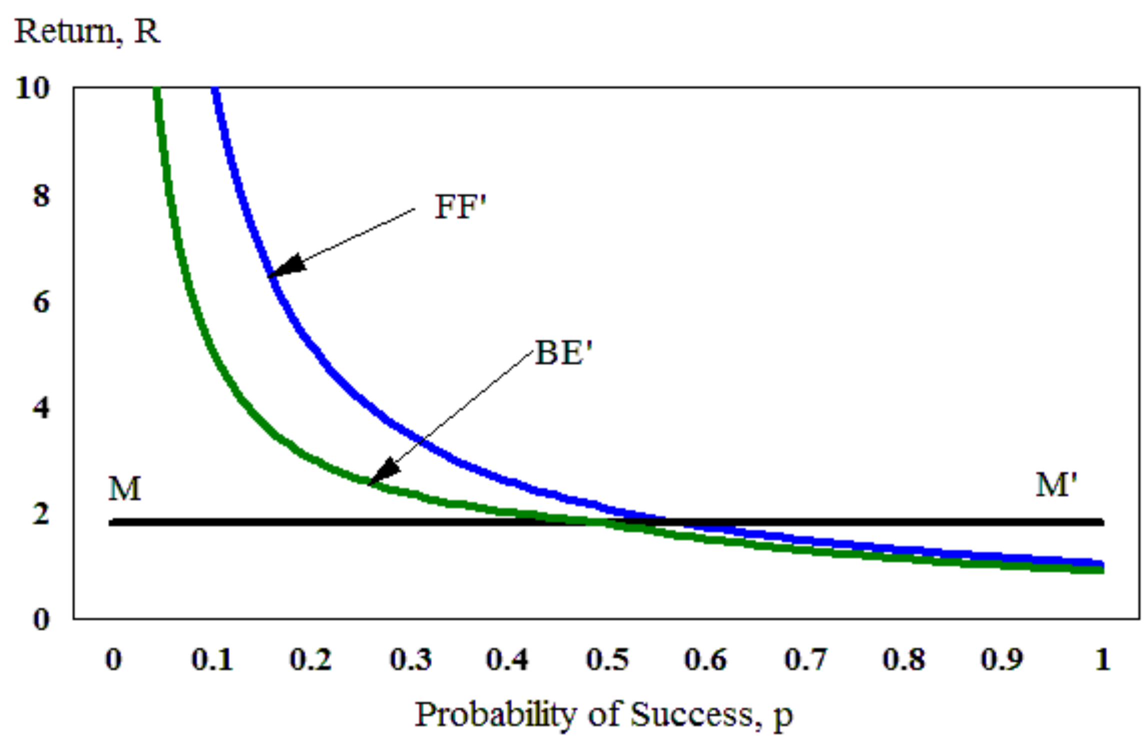
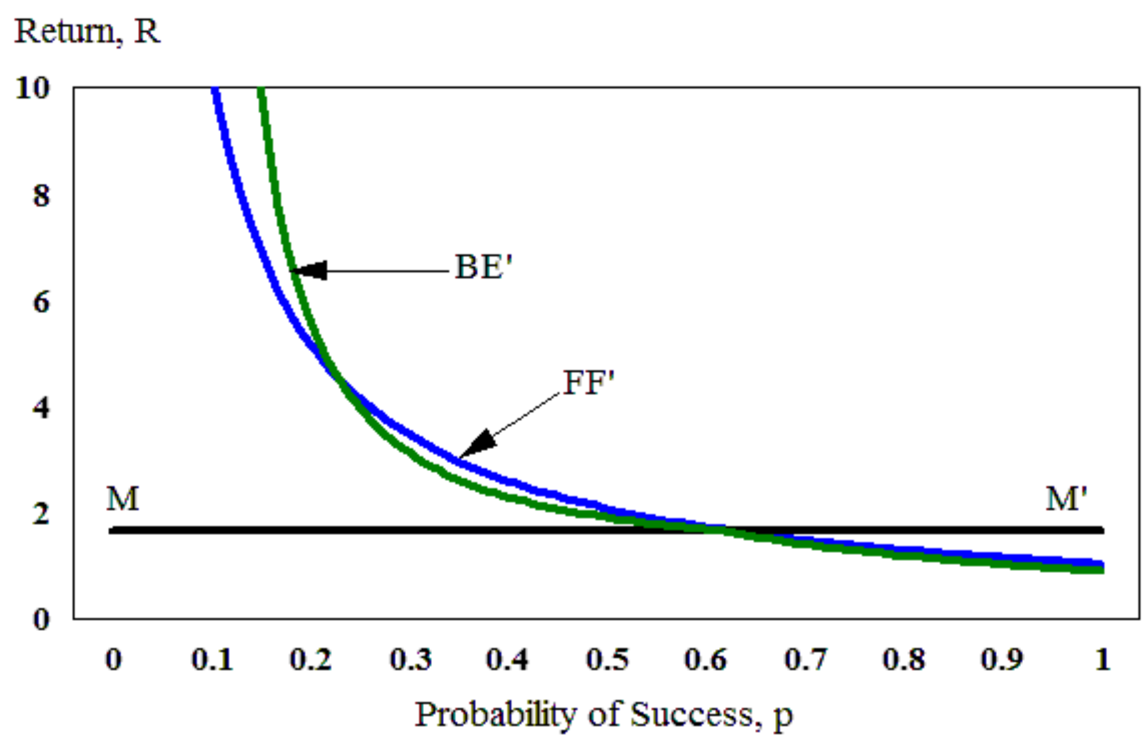

Return, R
10
8
6
4
2
0
FF'
BE'
M
M'
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of Success, p

Figure 2b



Return, R
10
8
6
4
2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of Success, p
BE'
FF'
M
M'

Figure 2c

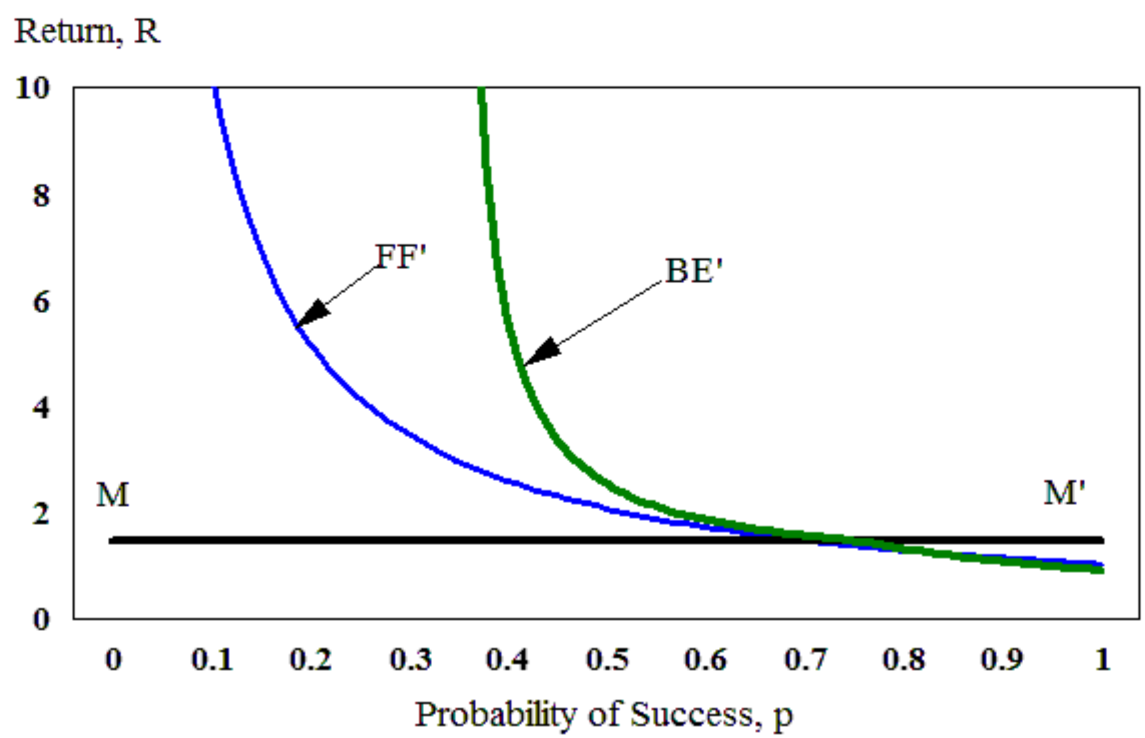
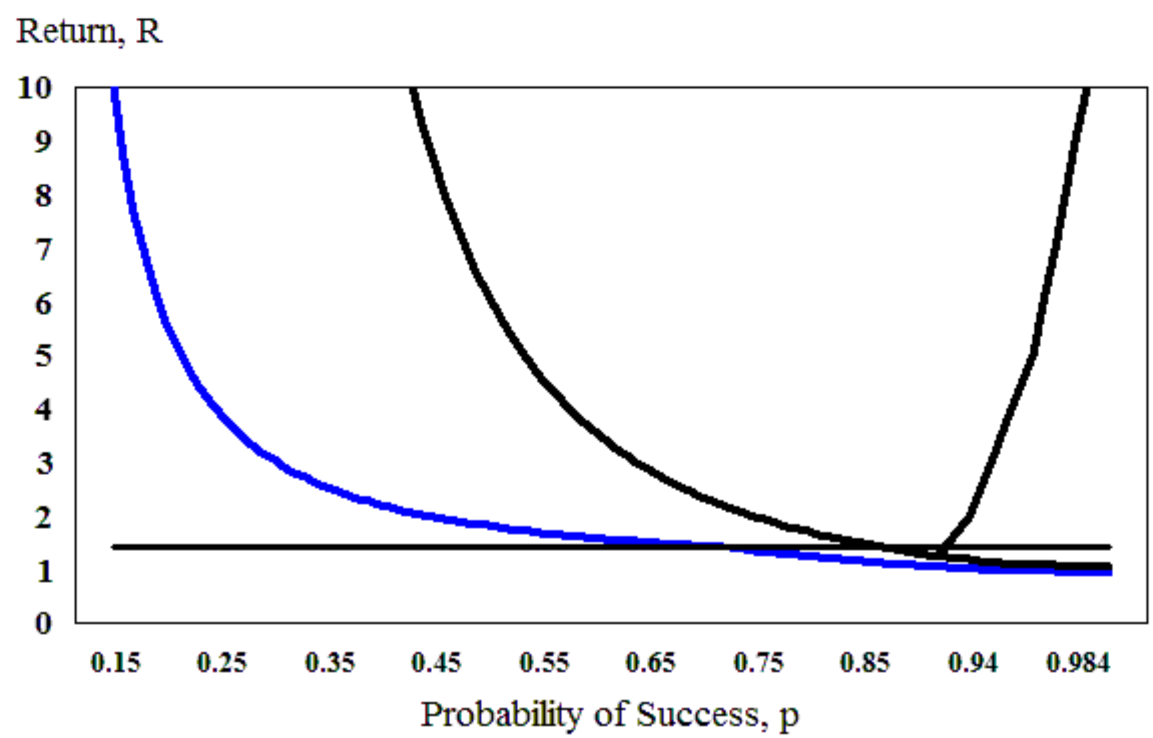

Return, R
10
8
6
4
2
0
M
M'
FF'
BE'
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of Success, p

Figure 3



Return, R
10
9
8
7
6
5
4
3
2
1
0
0.15
0.25
0.35
0.45
0.55
0.65
0.75
0.85
0.94
0.984
Probability of Success, p