

# The whetstone of witte,

whiche is the seconde parte of  
Arithmetike: containyng the extrac-  
tion of Rootes: The Coslike praife,  
with the rule of Equation: and  
the woorkes of Surde  
Nombers.

*Though many stones doe beare greate price,  
The whetstone is for exercise  
As needefull, and in woork as straunge:  
Dulle thinges and harde it will so change,  
And make them sharpe, to right good vse:  
All artesmen knowe, thei can not chuse,  
But vse his helpe. yet as men see,  
Noe sharpenesse semeth in it to bee.*

*The grounde of artes did brede this stone:  
His vse is greate, and moare then one.  
Here if you list your wittes to whette,  
Aske sharpenesse therby shall you gette.  
Dulle wittes hereby doe greatly mende,  
Sharpe wittes are fined to their fulle ende,  
No to proue, and praise, as you doe finde,  
And to your self be not vnkinde.*

**T**hese Bookes are to bee sold, at  
the Weste doore of Poules,  
by Iohn Wyngston.



**T**o the right worshipfull, the gou-  
 uerners, Consulles, and the reste of the com-  
 paigne of venturers into Moscouia, Robert Re-  
 coyde Philitian, wissheth healtbe with  
 continuall increase of commodi-  
 tie, by their worthie and  
 famous trauell.



**I** wil not, nother ought I  
 so euilly to iudge of my  
 countrie, that learnyng  
 here can haue no liber-  
 tie: but by aide of frende-  
 shippe, or strength of po-  
 wer. For as Englande  
 did neuer wante learned  
 wittes, so at this tyme I doubt not, but there  
 be a great multitude, that desirously embrace  
 all kindes of knowledg, and frendely are af-  
 fected toward the furtherance of it. And ther-  
 fore I dare sate, thei can not malice me, whi-  
 che am so willyng to helpe the ignoraunte, ac-  
 cording to my gifte and simple talête. wher-  
 by also this moche praise I meite iustly craue,  
 to haue the commendation and rewarde of a  
 solliciter in this cause. For though my trauell  
 can not moche profite them, that be well lear-  
 ned, yet doeth it excite the beste learned, to re-  
 member their duetie to their countrie: and to  
 be a shamed, that thei hauyng so greate habi-  
 litie, shall be founde moare slacke to aide their  
 coutrie, then he that hath smaller knowledg,

a.ii,      and

## *The Epistle*

and lesse occasion otherwaies. Accoꝝdyng as men haue receiued, so are thei bounde to yeld. These excellent gifts are not lente vnto me, to be hidden. And there are a great multitude that thirst, and long moche for soche aid. For bothe these causes I saie, that naturalle bode to our countrie doeth challenge it: and for that the honeste desires of so many good natures so moche requireth it, I exhoꝛte them that be beste hable, to take from me this chargeable wooꝛke, and to further their countrie men, as equitie would. And in the meane reason while I see them so slacke, let them not bee offended with me, for pꝛeuentynꝝ them. For better it is that a simple Coke doe prepare thy bꝛekfast, then that thou shouldest goe a hungered to bedde. Yea better it is to haue some grosse repaste, then to sterue for hunger. And the common loꝛte will finde small faulte of wante, as long as thei see any man serue their expectation. So that for this cause also, that my paines for a tyme, doeth excuse other finer wittes, thei ought to render me some thanks again. But if thei staie for feare of tauntes, and barkynꝝ of curres, their corage is small. If thei misdoubte the gratefull acceptation of their studies, thei doe iniurie to their countrie. For whoe ca doubt but so ciuile a countrie, will thankfully receiue, and moste frendly recompense the trauelle, of soche as studie for their benifite,



## Dedicatorie

benefite, and serueth their necessarie commodities. This perswasion maketh me so bolde, that I can not thinke it needefull, to seke any protector, for this or any like woork. Sith euery good man will offer hymself, to defende that, wherby his native countrie is benefited. Excepte at some tyme, by excitation of the furies, some naughtie natures doe practice their fraude, to berefte the realme of some singular commoditie. But as I feare no soche, so at this tyme I seke no soche aide against the. Yet for testefying of frendshippe, and gratefull remembraunce, I could doe noe lesse, but sende this Booke to soche as I thought, not onely to deserue it, but also would gladly receiue it. And if I maie perceiue, that you doe accepte it (as I doubt not) with as good a wille, as I dooe sende it, I will for your pleasure, to your counforste, and for your commoditie, shortly set forth the soche a booke of Nauigation, as I dare saie, shall partly satisfie and contente, not onely your expectation, but also the desire of a greate number beside. Wherein I will not forgett specially to touche, both the olde attempte for the Northlie Nauigations, and the later good aduenture, with the fortunate success in discoueryng that voyage, whiche no men befoze you durste attempte, sith the tyme of kynge Alluredes his reigne. I meane by the space of, 700, yere. Nothor ever  
a. m. any

## *The Epistle*

any before that tyme, had passed that boiage,  
excepte onely Ohthere, that dwelte in Hal-  
golande:whoe reported that iorney to the no-  
ble Kyng Alured:As it doeth yet remaine in  
aunciente recozde of the olde Saxon tongue.  
So that if you continue with corage, as you  
haue well begon, you shall not onely winne  
greate riches to your selues, and byng won-  
derfull commodities to your coutrie. But you  
shall purchase therewith immortall fame, and  
be praised for ever, as reason would: for ope-  
nyng that passage, that shall profite so many.  
In that Booke also I will shewe certain mea-  
nes, how without greate difficultie, you maie  
saile to the Northe Easte Indies. And so to  
Camul, Chinchital, and Balor, whiche bee  
countries of greate commodities. As for Cha-  
tai lieth so farre within the lande, toward the  
Southe Indian seas, that the iorneye is not  
to be attempted, bntill you be better acquain-  
ted with these countries, that you must first ar-  
riue at. But these thynges come in this place  
bntimely. I praie you accepte frendely in the  
meane reason this Booke, whiche will bee a  
greate aide to the well vnderstandyng of the  
teste that is behinde. And as I shall vnder-  
stande your desire, so will I haste the other.  
God prospere well your endeuoure, and sende  
you soche good succeste, as so worthie aduen-  
ture doeth deserue:Whiche I double not will  
insue,

*Dedicatorie.*

insue, if cankered malice of some spitefull sto=  
mackes doe not preuaile, as thei can not cease  
to practice, to hinder your comoditie, and  
deface your trauel. But as it is euer seen, and  
therfoze commonly knowen, that enuie doeth  
still repine at glozie, so ought all honeste har=  
tes, to prosecute their good attemptes, and  
contempne the ballynge of dogged  
curses. So fare you well. And  
loue hym againe, that de=  
lighteth and studieth  
to farther your  
comoditie.

At London the .xii. daie of  
Nouember, 1557.



# THE PREFACE

to the gentle Reader.



Although number be infinite in increasynge: so that there is not in all the worlde, any thing that can excede the quantitie of it: Neither the grasse on the ground, neither the droppes of water in the sea, no not the small graines of Sande though the whole masse of the earth: yet make it seme by good reason, that noe man is so experte in *Aritmetike*, that can number the commodities of it. Wherefore I make truely saie, that if any imperfection bee in number, it is because that number, can scarcely number, the commodities of it self. For the moare that any experte man, doeth weigh in his mynde the benifites of it, the moze of them shal he see to remain behinde. And so shall he well perceiue, that as number is infinite, so are the commodities of it as infinite. And if any thyng doe or make excede the whole worlde, it is number, whiche so farre surmounteth the measure of the worlde, that if there were infinite worldes, it would at the full comprehend them all. This number also hath other prerogatiues, aboue all naturalle thynges, for neither is there certaintie in any thyng without it, neither good agremente where it wanteth. Whercof no man can doubt, that hath been accustomed in the Bookes of Plato, Aristotell, and other aunciente Philosophers, where he shall see, how they searche all secreete knowledge and hid misteries, by the aide of number. For not onely the constitution of the whole worlde, dooe they referre to number, but also the composition of

b. j. man,

manne, yea and the whole substance of the soule. Of  
 whiche thei professe to knowe no moare, then thei ca  
 by the benifite of number attaine. Furthermoze, for  
 knowledge and certaintie in any other thyng, that  
 mannes witte can reche vnto, there is noe possibilitie  
 without number. It is confessed amongst all men,  
 that knowe what learning moaneth, that beside the  
 Mathematicke artes, there is noe infallible knowe-  
 ledge, excepte it bee borrowed of them. And amongst  
 them, it is sufficiently knowne, and well declared by  
*Nicomachus*, and diuerse other writers, that *Arithme-  
 tike* is the fountaine of all the other, and their ground  
 and bonde, as he calleth it. If any man will saie, that  
 Diuinitie, Lawe, and Physike, maye be had without  
 it: or that thei take little aide thereby. Although I haue  
 before this tyme answered thereto, yet now I saie  
 again: that in Diuinitie there are greates hidde secre-  
 tes in numbers. So that diuerse excellent Diuines,  
 haue written whole Bookes of the misteries of num-  
 bers. And some of their Bookes intituled: *The Diui-  
 nitie of Numbers*. But what Chyssen manne is igno-  
 raunte, that betwene *Trinitie* and *vnitie*, doeth consist  
 the full grounde of al Diuinitie: Wherefore I neede  
 not to alleage the other hollie and sacred *Numbers*.  
 Saue that. 7. will not permitte me to passe it with si-  
 lence. In whiche is contained, not onely the secretes  
 of the creation of all thynges: and the consummation  
 of the whole worlde againe, with the state of eterni-  
 tie: But also by it is the *Sabbthes* rest, and thereby  
 the full life and conuersation of godlie persones, re-  
 presented and insinuate. In Lawe twoe kyndes of  
 Justice are the somme of the studie: *Iustice Distribu-  
 tute*, and *Iustice Commutative*, whiche termes I vse,  
 as beste knowne in that arte: But what is any of the  
 bothe without *number*: I haue said in an other place  
 (as I learned of that noble Philosopher *Aristotell*)  
 that

Diuinitie.

Lawe.

that if the knowledg and distinction, of *Geometricall* and *Aritmeticall* proportion bee not well obserued, there can noe Justice well bee executed. And how often the ministers of the Law vse aide of *Number*. I neede not repete, bicause none but madde men doubt of it. And as for *Physike*, without knowledg and aide of number is nothyng. Wee see that nature in generation, bethe of manne and beastes, yea and of all thynges els doeth obserue number exactly. As well in the tyme of formation, as in the monethes of quickenyng, and of birth. The misteries of the seuenth and nineth monethes are sufficiente testimonies therein. Beside that from the fourthe monethe til the seuenthy many thynges bee permitted, that els bee not conueniente. For the vse of the pulse, and for criticall dayes, beside the proportion in degrees in simple medicines, and mixture of compounde medicines, and othet infinite maters, what number can doe and what aide it giueth, onely the ignoraunte doe doubt.

But where can there bee any better testimonie for *Astronomie*. *Number*, then that the celestiale bodies doe keepe an vnfallible number, in all their wonderfull motions: By meanes whereof, mannes witte is habled to attaine the knowledg of them. As by the *Aritmeticall* tables, of their motions it is easily known. Therefore and for that we see the yere, and all the distinction of times, beside the common vse of trafike betwennemenne, to depende of number, wee muste needes not onely confesse it to bee, as it were the onely staic of all natures woorkes, and of all ciuilitie: but we must also honoure and reuerence it, as often as wee duely remember the excellencie and benefite of it. Was not *Number*, thinke you, wonderfullie honoured, when noe name was thought moare meter for God, then the name of *Number*: I meane. 1. and. 3. the name of the *Trinitie*. But to come to moare familiare mat-

b. u.      ters,

## THE PREFACE.

Measure.  
weighte.

ters, I will saie, as Plato saith in his Booke De summo bono. *Take awaie Arithmetike, with measure and weightes, from all other artes, and the reste that remaineth is but base, and of noe estimation.* Where although Plato dooe name thre thinges in apperaunce, that is Number, Measure, and Weighte. What are Measure and Weighte, but number applied to seuerall uses: For Measure is but the nombryng of the partes of lengthe, bredthe, or depthe. And so weighte (as here it is taken) is the nombryng of the heuynesse of any thyng. So that if number were withdrawen, no manne could either measure, or weigh any quantitie. And therfore it must followe: that number onely maketh all artes perfecte, and woorthie estimation: seying that without it, all artes are but base, and without commendation. This maie suffice for the iuste comendation of *Arithmetike*. But yet one commoditie moare, whiche all menne that studie that arte, doe sele, I can not omitte. That is the flyng, sharpenyng, and quickenyng of the witte, that by practice of *Arithmetike* doeth insue. It teacheth menne and accustometh them, so certainly to remember thynges passe: So circumspectly to consider thynges presente: And so prouidently to foresee thynges that followe: that it maie truely bee called the *File of witte*. Yea it maie aptly bee named the *Scholehouse of reason*. The like iudgemente had Plato of it, as appeareth by his woordes in the seuenth booke Dere publica. Where he saith thus: *Thei that be apte of nature to Arithmetike, bee readie and quicke to attaine all kindes of learnyng. And thei that bee dulle witted, and yet be instructed and exercised in it, though thei gette nothyng els, yet this shall thei all obtaine that thei shall bee moare sharpe witted, then thei were before.* What a benifite that onely thyng is, to haue the witte whetted and sharpened, I neede not trauell to declare, with all menne comfesse it to be as greate as maie be. Excepte any witlesse per-

son



some thinke he maie bee to wise. But he that moste  
 feareth that, is leaste in daunger of it. Wherefore to  
 conclude, I see moare menne to acknowledge the be-  
 nifite of number, then I can espie willing to studie,  
 to attaine the benifites of it. Many praise it, but fewe  
 dooe greatly practise it: onlesse it bee for the bulgare  
 practice, concernyng Merchandes trade. Wherein  
 the desire and hope of gain, maketh many willing to  
 sustaine some trauell. For aide of whom, I did sette  
 forth the firste parte of *Arithmetike*. But if thei knewe  
 how farre this seconde parte, dooeth excell the firste  
 parte, thei would not accounte any tyme losse, that  
 were imploied in it. Yea thei would not thinke any  
 tyme well bestowed, till thei had gotten soche habilitie  
 by it, that it might be their aide in al other studies.  
 And if *Plato* doe require *Arithmetike*, as a specialle and  
 a necessarie qualitie in hym, whom he would admitte  
 as a citezein in his politike tounce: How maie wee  
 thinke of our selues, that desire to gouerne other, and  
 yet can scante skille of common number: So farre are  
 many, yea moste parte of vs from cunnyng in num-  
 ber. *Plato* thinketh noe manne hable to bee a good ca-  
 pitaine, excepte he bee skilfulle in this arte: And wee  
 accounte it noe parte of those qualities, that bee re-  
 quired in any soche manne. Howbeit for the better  
 triall thereof, I haue in this Booke framed some of  
 the questions in soche sorte, as thei maie approue the  
 vse of this arte, not onely good for capitaines, but al-  
 so moste necessarie for them. So that without it, thei  
 can not Marshall their battaile, nother beloe their e-  
 nemies campe or sorte. And if I shall saie as I thinke,  
 without it a capitaine is noe capitaine. In this booke  
 what I haue witten, for the aide of all menne, and  
 namely soche of my countrie menne, that vnderstand  
 nothyng but Englishe, I meade not to repecte particu-  
 larly, but remitte them to the booke it self, to see it at

large. Onely this male I sale: that as I haue doon in other artes, so in this I am the first venturer, in these darke maters. Wherefore I trust thet that be learned, and happen to reade this woꝝke, wil beare the moare with me, if thei finde any thyng, that thei doe mislike: Wherein if thei will vse this curtesie, either by wꝛitynge to admonishe me thereof, either theim selves to sette foꝛthe a moare perfecter woꝝke, I will thynke them pꝛaise woꝛthie. But if any manne will be so hasty, other to blame that, whiche he is not hable to amende, or to condemne that, whiche he did neuer vnderstande: As some ofte tymes doe of a sonde curiositie, I will wisshye hym a better witte, and moare modestie. And to pꝛeuent all soche seuerẽ Judges, I thought it good to admonishe you befoꝛe, that by occasion of trouble vpon trouble, I was hindered from accomplisshyng this woꝝke, as I did intende. But yet is here moare, then any manne might well looke foꝛ at my handes, if thei did knowe and consider myne estate. And this moche moare I sale: that if I male perceiue, that this Booke bee as well receiued, as the firste parte was, I will strue moche, to stele from my troubles so moche tyme, as to set out the reste of this arte, moare completely in Englishe, then euer I saue it in any tounge, hetherto doen: trust thereto assuredly. And wisshye hym good, that traueleth foꝛ thy benifite.

*Of the rule of Cose.*

One thyng is nothyng, the prouerbe is,  
whiche in some cases doeth not misse.  
Let here by woorkyng with one thyng,  
Soche knowledge doeth from one roote spryng,  
That one thyng maie with right good skille,  
Compare with all thyng: And you will  
The practice learn, you shall sone see,  
What thynges by one thyng known maie bee.

*To the curiouse scanner.*

If you ought finde, as some men maie,  
That you can mende, I shall you praie,  
To take some paine: so grace maie sende,  
This worke to growe to perfecte ende.

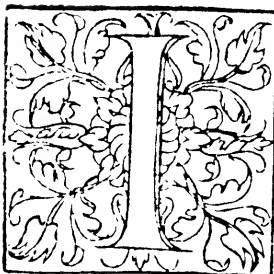
But if you mende not that you blame,  
I winne the praise, and you the shame.  
Therefore be wise, and learne before,  
Sith slaunder hurtes it self moſte ſore.



**The seconde parte of Arithmetike,**  
*contayning the extraction of Rootes in di-*  
*uerse kindes, with the Arte of Cossike*  
*numbers, and of Surde numbers*  
*also, in sondrie sortes.*

**The interlocutors, Master. Scholar.**

**The Master.**



**S**ee your desire can not  
 bee satisfied, neither your re-  
 quest staied, vntill I maie tu-  
 sly aunswere you, that I can  
 teache you no more : whiche  
 aunswere maie staie your re-  
 quest, although it content not  
 your desire.

**Scholar.** I beseeche God of  
 his mercie, to withstande all suche occasion: except it  
 maie be more to your owne contentation and profite,  
 then it would be pleasaunt to the louers of learning.

**Master.** Yet a iust excuse maie stande for my de-  
 claration : As if ignoraunce doe inforce me to staie  
 my trauell.

**Scholar.** Your owne ignoraunce, I trust, you will  
 not allege: and as for the ignoraunce of other, it ought  
 to bee no staie : sith the ignoraunte multitude doeth,  
 but as it was euer wonte, enuie that knoweledge,  
 whiche thei can not attaine, and wishe all men igno-  
 raunt, like vnto themself, but all gentle natures, con-  
 temneth suche malice : and despiseth theim as blinde  
 woymes, whom nature doeth plague, to stay the poi-  
 sone of their venemous synges.

**Master.** We shall not nede to stande on this talke,  
 but trauell with knowledgeto banquish ignoraunce:  
 And beleue that the *pricke* of knoweledge, is more of

A. i.

Geometric,

## The seconde parte

Geometrie, and the vnitie in *Arithmetike*, though bothe be vndiuifible, doe make greater woorkes, & increase greater multitudes, then the brutifhe bande of ignorance is hable to withftande.

*Vnitie.*

Scholar. Our talke groweth well to our mater. I befeke you therfore, with that vnitie beginne, and builde on it your worke, as a fozte againft ignorance

Mafter. Vnitie is of it self vndiuifible, and yet is it in al partes of the worlde, and in euery thing. Yea, the worlde it self confifteth of vnitie, is named of vnitie, was made by vnitie, and is preferved by vnitie, and onely ignorance with her broode fecluded from vnitie, fo that of it to repete the fulle force, would occupie muche time, and make greate volumes.

*Number.*

Scholar. Sith vnitie is fo mightie, and of fuche force (as you faie) what maie bethought then of number, whiche containeth a multitude of vnitie? And is nothynge els but a collection of vnitie.

Mafter. Vnitie is the fountaine and originall of number, yea vnitie by addition onely fhall make a greater number, then any numbers can doe by multiplication. But this is marueiloufe, that no number repineth againft diuifio, till it come to an vnitie: and then will it permit no farther diuifio. And therfore it is faid, that vnitie doeth neither multiplie nor diuide.

And as al numbers maie be moze or leffe, fo the leffer is euer a parte or partes of the greater.

*A parte.*

*Partes.*

As 5 vnto 10 is a parte, named a halfe: but vnto 7.5. is not a parte, but partes, and is called  $\frac{5}{7}$ . So 8 to .24. is a parte that is  $\frac{1}{3}$ ; but vnto .36. it is partes, that is  $\frac{2}{9}$ .

Scholar. I perceiue, you call it a parte, when the numerator in the fraction (reduced to the smalleffe) is an vnitie. And when the numerator is a number, then that fractio betokeneth partes of a number.

But I praie you, what varieties of numbers bee there principally to be confidered in this arte?

Mafter.

## of Arithmetike.

**Maſter.** Number is diuided into diuerſe kindes, *The firſt di-*  
 ſoꝛ ſome are *whole numbers*, and thei onely of *Euclide*, *uſion of num-*  
*Boetius*, and other good wꝛiters are called numbers. *bers.*  
 Other are *broken numbers*, and are commonly called  
*fractions*. Of theſe bothe I haue wꝛitten in the firſt  
 and ſeconde partes of *Arithmetike*: So that I mighte  
 ſeme to curioſe, to repete any parte of it again.

But now in the kinde of theſe, there are certaine *The ſeconde*  
 numbers named *Abſtraſte*: and other called numbers *diuiſion of*  
*Contraſte*. *numbers.*

*Abſtraſte* numbers are thoſe, whiche haue no deno- *Abſtraſte.*  
 mination annexed vnto them. And thoſe that haue a  
 ny denomination ioyned to theim, are called *Contraſte*.  
*numbers.*

**Scholar.** This I ſee to be a reaſonable diſtinctiō,  
 and agreeable to the ſignification of the names.

For as that number is cōtraſte, from his generall  
 libertie of ſignification, whiche is bounde to one deno-  
 mination, as in ſaiyng. 10. grotes (where. 10. is re-  
 ſtrained frō the libertie of valowynge any other thing  
 but grotes) ſo if it had no denomination adioined, it  
 might then ſignifie the number of daies, or of miles,  
 or any like thyng, as well as of grotes. For when I  
 ſaie. 10. and doe not limitte any denomination, then is  
 that. 10. abſtraſte and ſeuered frō all ſpecialties, and  
 ſtandeth free to any name of thing.

But this (me thinketh vnder your correccion) can *whether bio-*  
 not extend to broken numbers: whiche euermore car- *ken numbers*  
 ry with them their denomination: ſeyng thei conſiſte  
 of a numerator and a denominator. *be contraſte,*  
*or not.*

**Maſter.** Pou ſeme to ſaie well. And the like iudge-  
 mēt doeth appere to be in ſome wꝛiters of this arte.  
 But yet ſeyng that fractions maie haue all other ar-  
 tificiall denominations, that whole numbers maie  
 receiue: and maie alſo bee without them: therefore  
 muſt wee either make a more curioſe diſtinction of

## The seconde parte

that name of denomination : or els wee must seclude fractions, fro the necessitie of that name: or els thirdly, to avoied contention, cal them numbers contracte improperly.

Scholar. I assente thereto as reason would.

*Why fractions be not called numbers properly.* Yet one thyng more I must demaunde of you, why Euclide, and the other learned men, refuse to accompte fractions amongst numbers.

Master. Because all numbers doe consist of a multitude of unities : and euery proper fraction is lesse then an unitie, and therefore can not fractions exactly be called numbers: but maie bee called rather fractions of numbers.

Scholar. In deede now that I doe waite the matter more exactly, it appereth that a fraction is not properly a number, but a connerion and conference of numbers, declaring the partes of an unitie. For the numerator doeth signifie one nuber, and the denominator another: The denominator declaring into how many partes the unitie is diuided, and the numerator signifying that of those partes, not all, but so many onely are to be take, as the numerator importeth.

*The diuision of numbers Abstracte. Numbers Absolute. Numbers Relatiue.* Master. Well, then to procede, numbers abstracte are considered in .3. principall varieties: That is, first without comparison to any other number or figure. And that number maie well be called *number absolute*.

Secondarily, some numbers bee vsed onely in relation to other, and therefore ought to bee called *numbers relatiue*.

Thirdly, many numbers are referred to some figure, that maie rise by multiplicacion of their partes together, and that diuersly. And those numbers therefore maie bee called *figuralle numbers*.

*Numbers Figuralle.* Scholar. If I conceiue your wordes rightly, this is your meaynge: that when I saie. 10. 25. 100. or 200. &c. these numbers stand absolute from all denomination



## of Arithmetike.

minacion, and clere from all relatiō and comparisōn.

But when I saie. 6. is halfe of. 12. or. 15. is triple to 5. here the numbers beeyng compared together, are aptly called *numbers relatiue*: So if I saie, that. 16. is a *square number*, bicause it is made of. 4. multiplied by. 4. then is. 16. here to be called a *figuralle number*.

Master. You take it well. Therfoze will I brie fly touche the membez of euery kinde.

First of absolute nombzēs, some are *euē numbers*, and some are *odde*.

Scholar. All men knowe that. And farther, that *Numbers*, *euē numbers* are those, whiche maie be diuided into e: *euē*, & *odde* qualle halves: and so can not *odde numbers*, without a fraction.

Master. Of this plaine easie thyng, marke what foloweth: a greater doubtē dissolued. For if an *odde number* (as. 7. for example) can not bee parted into. 2. equalle numbers, eche beeyng halfe of. 7. then.  $3\frac{1}{2}$ . whiche is commonly called the halfe of. 7. is no nōber

Scholar. It can not be denied. And so (I see now) no fraction can bee a number. This greate doubtē is plainly dissolued, by a very certaine and moste known principle.

Master. Now farther. Of bothe these kindes of *Numbers* *euē* numbers, some bee *compounde*, and some bee *simple* and *poude*, and *vncompounde*. *Compounde numbers* are made by multi: *simple*. plication of. 2. nombzēs together, and not by additiō, though the name might seme to serue to bothe.

Scholar. So I perceiue, that 5. is no *compounde* nōber, although it bee made by addition of. 2. and. 3. but 6. whiche is made by multiplication of. 2. and. 3.

Likewates. 9. is *compounde*, bicause that. 3. multiplied by. 3. doeth make. 9.

And. 15. also is *compounde* by multipliyng. 5. and. 3. together.

One is no

And hereby I se that. 1. is not to be called a number *number*.

## The seconde parte

for then all nōbers aboue it, must nedes be *compounde*, bicause thei consist all of vnities.

Master. But yet by multiplication of .1. no other number is *compounde*.

Scholar. By those wordes I am taught to knowe more, and speake better.

Two is vn-  
compounde. Master. *Euen numbers* are yet diuersly to be considered in their diuisions. For although the greate multitude of *euen numbers* bee *compounde*, yet .2. is accomplished truely an *euen number*, originall, and *vncompounde*. So that it maie make other numbers, & is made of no nōbers, but of vnities onely, as al *odde numbers* are.

Euen nom-  
bers, euenly. All other *euen numbers* are *compounde*, and are diuersly diuided, for some are *euen numbers euenly*, and some are *euen numbers oddely*, and some are *euen numbers bothe euenly and oddely*. *Euen numbers euenly*, are suche numbers as maie bee parted continually into *euen halves*, till you come to an *vnitie*. As for example, 32. first is diuided into, 16. as his *euen halfe*: and again, 16. into, 8. as his *halfe*: And, 8. againe by, 4. is parted into, 2. *euen partes*: Then, 4. also by, 2. And that, 2. is diuided into, 2. vnities, as his *luste halves*.

Euen numbers  
vneuenly. But *euen numbers vneuenly*, are suche numbers as maie bee diuided into, 2. *equalle partes*: whiche are *odde numbers*. As, 18. is diuided into, 9. and, 9. as his *halves*, and thei are *odde*. So, 10. is diuided by, 5. And 30. by, 15. with a greate number more of *suche sorte*.

Euen nom-  
bers, euenly  
and oddely. Numbers *euen euenly and oddely*, bee commonly called *suche numbers*, as maie bee diuided into, 2. *equalle and euen halues*: but befoze you come to an *vnitie*, the *halves* will be *odde numbers*. As, 60. maie be first parted into, 30. and, 30. whiche are *euen*: And thei againe diuided by, 15. whiche is *odde*.

Likewaies, 24. is diuided first by 12. And that 12. by 6. & lastly, 6. is diuided by, 3. whiche is an *odde nōber*. So, 28. maie bee diuided into, 2. *equalle and euen halues*,

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halues, that is into 14. And that. 14. into. 7. whiche is the halfe of. 14. but is an odde number.

Scholar. This I perceiue well. And, as I iudge, the distinction into those. 3. kindes, is not onely reasonable, but also needfull. And yet you seeme to speake doubtfully, of this laste member. Bicause I remember not that you vse this worde commonly, but where you giue place rather to custome, then to reason.

Master. O; els to custome of the common sorte of writers, rather then to the iudgemente of the moſte aunciente writers.

And so in this case *Euclide* doeth not seeme to admitte this thirde member. But accompteth it vnder the seconde kinde. As niale well appere in his. 9. boke, and 24. proposition, where he calleth suche a number, *euently euē*, and *euently oddē* also, whiche place cōferred with the definitions in the same booke, doeth approue in many wise mennes opinions, that *Euclide* minded but 2. onely kindes of those nōbers. And yet in this thing (I thinke) he did rather approue. 3. varieties by his propositions, then establishe onely. 2. sortes by his first definitions.

But herein I will spende no more tyme. But saie briefly that the distinctiō of. 3. kindes, serueth to good vse, and ease in teachyng.

And now for farther knowledge of numbers, some are called *numbers perfecte*, & some are *numbers imperfect*.

*Perfekte numbers* are suche ones, whose partes ioyned together, will make exactly the whole number. *perfecke.*

And therefore are. 6. and. 28. accompted perfecte nōbers: bicause the partes of eche of theim added together, doe make the ful and intere number, whose partes thei bee. As of. 6. the halfe is. 3. the thirde parte is 2. the sirte parte is. 1. As for a quarter, and fiftē parte it hath not in whole number. Now put together. 1. 2. and. 3. and thei make iuste. 6. whose partes thei bee.

And

## The seconde parte

28. And therfoze is. 6. a perfecte number.  
 Likeliwaies. 28. hath foze his halfe. 14. foze his quar-  
 ter. 7. foze his seventh parte. 4. and foze his sovertenth  
 parte. 2. and foze his. 28. parte. 1. all whiche put toge-  
 ther, that is. 1. 2. 4. 7. and. 14. doe make. 28. of this foze  
 there are very fewe moze in cōparisō. And foze an exā-  
 ple, I sett here, as many as are vnder. 6000000000.  
 and thei are these. 6. 28. 496. 8128. 130816. 2096128.  
 33550336. 536854528.

*Numbers*  
*imperfekte.* But now of the contrary kind, *imperfekte numbers* be  
 suche, whose partes added together, doe make either  
 moze oze lesse, then the whole number it self : whose  
 partes thei bee.

*Numbers*  
*superfluouse.* And if the partes make moze then the whole nom-  
 ber, then is that nōber called *superfluouse*, oze *abundaunt*.  
 As 12. whose partes are. 1. 2. 3. 4. & 6. whiche make 16.  
 So. 20. hath foze his partes. 1. 2. 4. 5. 10. whiche  
 make. 22. Likeliwaies. 120. hath these partes.

1. 2. 3. 4. 5. 6. 8. 10. } whiche make 240.  
 60. 40. 30. 24. 20. 15. 12. }

*Numbers*  
*Diminute.* And if the partes make lesse then the whole nom-  
 ber, whose partes thei be, then is that number called  
*Diminute*, oze *Defectiue*. As. 8. hath these partes. 1. 2. 4.  
 whiche make but. 7.

So. 16. hath these partes. 8. 4. 2. 1. and thei make  
 onely. 15.

Likeliwaies. 32. whose partes are. 1. 2. 4. 8. 16. and  
 make but. 31.

Scholar. In all these numbers I note that you re-  
 ken one, foze a parte of eche one of theim : whiche be-  
 foze I thought you had denied.

Master. I canne neither multiplie noze deuide, and  
 therfoze compoundeth no number. But one maie in-  
 crease addition, and therfoze where partes be added  
 together, there. 1. maie well be called a parte.

And this shall suffice foze the diuision of euen nom-  
 bers

## of Arithmetike.

bers Abstracte.

Now to speake of *odde numbers*, some of the are *com* *Odde nōbers*  
pounds, & some *vncompōūde*. Thei are *compōūde*, whiche *Compōūde*.  
maie bee diuided into any other partes then vnities.  
As. 9. whiche is cōpōūde of. 3. And. 15. that is made  
of. 5. and. 3. Also. 21. is compōūde of. 7. and. 3. And  
so furthe. But. 3. 5. 7. 11. 13. 17. 19. 23. 29. and suche  
like, bee *odde numbers vncompōūde*. For thei are not *Vncompōūde*.  
made of any other then of vnities.

Here must you vnderstande by *composition*, the mul-  
tiplication of the partes of numbers together, as you  
remembre, before was declared.

Scholar. I consider it so. And I remembre all that  
you haue taught me, for the diuisiō of *nōbers abstracte*  
and absolute. What saie you now of *nōbers relative*: *Numbers*

Master. Some tymes their *relation* hath regarde *Relative*.  
to their partes, namely, whether these. 2. that bee so  
compared, haue any common parte, that will diuide  
theim bothe. For if thei haue so, then are thei called  
*numbers commensurable*. As. 12. and. 21. bee *numbers com-* *Commensu-*  
*mensurable*: for. 3. will diuide eche of theim. *vable*.

Likewises. 20. and. 36. be *commensurable*, seying 4. is  
a commō diuisor for theim bothe. But if thei haue no  
suche common diuisor, then are thei called *incommensu-*  
*rable*. As 18 and 25. For 25 can bee diuided by no nou- *Incommen-*  
ber more then by. 5. And. 18. can not be diuided by it. *surable*.

In like maner. 36. and. 49. are *incommensurable*: For  
49. hath no diuisor but. 7. And 7. can not diuide. 36.

Scholar. Doe you meane then, that *incommensura-*  
*ble numbers*, haue no cōparison nor *proportion* together?

Master. Naie, nothyng lesse. For any. 2. numbers  
maie haue *comparison* & *proportion* together, although  
thei be *incommensurable*. As. 3. and. 4. are *incommensu-*  
*rable*, and yet are thei in a *proportion* together: as shall  
appeare anon.

But first I will declare vnto you, the varieties of  
13. i. *proportion*

## The seconde parte

**Proportion.** *proportion*, wherein there maie be double conferre:  $\mathfrak{A}$  meane of the lesser to the greater, or of the greater to the lesser.

**Of greater inequality.** *Of greater inequality.*  $\mathfrak{A}$  he the greater is compared to the lesser, it is called a *Proportion of the greater inequality*. As 6 to 2, or 5 to 3.  
**Of lesser inequality.** *Of lesser inequality.* And when the lesser is conferred to the greater, it is called a *proportion of the lesser inequality*. As. 3. to. 5. or. 2. to. 6.

**Scholar.** And what if  $\mathfrak{A}$  would compare two equall numbers together?

**Maister.** That is accounted also a proportion of many men: and is called the *proportion of equality*. And then ought the first diuision of proportion to be, thus

	}	Equality.
Proportion of		Inequality.
		<span style="font-size: 4em; vertical-align: middle;">{</span> <div style="margin-left: 10px;"> <p>The greater.</p> <p>The lesser.</p> </div>

**Multiplex.** So proportion of the greater inequality, is diuided into. 5. severall kindes: whereof. 3. be *simple*, and. 2. other *compound*. The firste kinde is, when a greater number containeth the lesser diuerse times: as twise, or thise, or oftener. So. 6. containeth. 3. twise: and it containeth. 2. thise. This proportion is called generally, *multiplex*, that is to saie, many folde: but specially it is named, according to the tymes that it containeth the lesser. So that if it contain hym twise, then is it named *dupla*, or double. As 2 to 1 and 4 to 2.

And if it containe it thise, As. 3. to. 1. and. 6. to. 2. it is called *tripla*, or triple.

If it containe it. 4. tymes, then is it *quadrupla*, or quadruple.

Of these and of diuerse other sortes in this kind also, here are the names briefly set doune with exâples.

*Dupla*

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<i>Dupla.</i>	4. to. 2. to. 6. to. 3. 10. to. 5. 18. to. 9.	1	2	Double.
<i>Tripla.</i>	6. to. 2. to. 9. to. 3. 12. to. 4. 18. to. 6.	1	3	Triple.
<i>Quadrupla.</i>	4. to. 1. 8. to. 2. 12. to. 3. 16. to. 4.	1	4	Fourfold.
<i>Quintupla.</i>	5. to. 1. 10. to. 2. 15. to. 3. 20. to. 4.	1	5	Fivefold.
<i>Sextupla.</i>	6. to. 1. 12. to. 3. 18. to. 3. 24. to. 4.	1	6	Sixfold.
<i>Septupla.</i>	7. to. 1. 14. to. 2. 21. to. 3. 28. to. 4.	1	7	Sevenfold.
<i>Octupla.</i>	8. to. 1. 16. to. 2. 24. to. 3. 32. to. 4.	1	8	Eightfold.
<i>Noncupla.</i>	9. to. 1. 18. to. 2. 27. to. 3. 36. to. 4.	1	9	Ninefold.
<i>Decupla.</i>	10 to 1 20. to. 2. 30. to. 3. 40. to. 4.	1	10	Tenfold.
<i>Vndecupla.</i>	11. to. 1. 22. to. 2. 33. to. 3.	1	11	Eleuenfold.
<i>Duodecupla.</i>	12. to. 1. 24. to. 2. 36. to. 3.	1	12	Twelffold.
And so infinitely.				

Beside this there is an other kinde of proportion, when the greater containeth the lesser, more then ones, and not twise: and that maie bein 2 sortes. For if the greater containe the lesser, and any one parte of hym, that proportion is called *Superparticulare*. For example, take 5. to. 4. With 5. doeth containe 4. and his quarter. Likewise. 6. to. 5. is in the same kinde of proportion: although, not of the same speciall sorte. For 6. comprehendeth 5. and his fiftte parte.

*Superparticulare.*

So that for a more speciall distinction, eche of these and many other, haue their seuerall names, according to that parte, whiche they doe containe. As if it containe the halfe more, it is named *Sesquialtera*. In whiche proportion are these numbers following.

*Sesquialtera*

3. to. 2. 6. to. 4. 9. to. 6. 12. to. 8. 15. to. 10. | 11.

But if the greater comprehend the lesser, and his thirde parte, then is that named *Sesquitercia* proportion. As in these.

*Sesquitercia*

4. to. 3. 8. to. 6. 12. to. 9. 16. to. 12. 20. to. 15. | 11.

And when the fiftte, sixte, seuenth, or eight part doeth make the proportion, or any other part els, the name is taken of that same parte. As for briefnesse I will here sette examples.

25. ii. *Sesquiquarta.*

## The seconde parte

<i>Sesquiquarta.</i>	5. to. 4: 10. to. 8: 15. to. 12.	$1\frac{1}{4}$	A quarter more.
<i>Sesquiquinta.</i>	6. to. 5: 12. to. 10: 18. to. 15.	$1\frac{1}{5}$	a fiftie more.
<i>Sesquisexta.</i>	7. to. 6: 14. to. 12: 21. to. 18.	$1\frac{1}{6}$	a sixtie more.
<i>Sesquisiptima.</i>	8. to. 7: 16. to. 14: 24. to. 21.	$1\frac{1}{7}$	a seuenth more.
<i>Sesquioc̃taua.</i>	9. to. 8: 18. to. 16: 27. to. 24.	$1\frac{1}{8}$	an eight more.
<i>Sesquinona.</i>	10. to. 9: 20. to. 18: 30. to. 27.	$1\frac{1}{9}$	a ninth more.
<i>Sesquidecima.</i>	11. to. 10: 22. to. 20: 33. to. 30.	$1\frac{1}{10}$	a tenth more.
<i>Sesquiundecima.</i>	12. to. 11: 24. to. 22: 36. to. 33.	$1\frac{1}{11}$	a leuenth more.
<i>Sesquiduodecima.</i>	13. to. 12: 26. to. 24: 39. to. 36.	$1\frac{1}{12}$	a tweluenth more.

And so as farre as you liste to procede in suche proportion: where one parte of the lesser, is the iustie difference and excesse, betwene it and the greater.

But if the difference be. 2. partes. 3. partes, or more *Superparties* partes: the proportion is named *superpartiente*. As. 5. to 3. And. 10. to. 6. For as. 5. containeth. 3. and.  $\frac{2}{3}$ . of it: so 10. holdeth. 6. and.  $\frac{2}{3}$ . of it.

Scholar. Now I perceiue some vse also, of the distinction betwene a parte and partes in number: Of whiche at the beginnyng you did speake. But how many kindes are there of this sorte?

Master. There are infinite kindes in this sorte of proportion, as well as in the other. But for examples sake, I will set furthe some of the moste common numbers: that therby you maie gather the formes of the reste. And these be they, with their names.

<i>Superbipartiens.</i>	{	<i>Tertias.</i>	5. to. 3: 10. to. 6: 15. to. 9: 20. to. 12.	$\frac{2}{3}$
		<i>Quintas.</i>	7 to 5: 14. to 10: 21. to 15: 28. to. 20	$\frac{2}{5}$
		<i>Septimas.</i>	9 to 7: 18 to 14: 27. to 21: 36. to 28.	$\frac{2}{7}$
		<i>Nonas.</i>	11 to 9: 22 to 18: 33. to 27: 44. to 36	$\frac{2}{9}$
		<i>Undecimas.</i>	13. to 11: 26 to 22: 39 to 33: 52 to 44	$\frac{2}{11}$



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<i>Suptriparties</i>	<i>Quartas.</i>	7. to. 4: 14. to. 8: 21. to. 12: 28. to. 16.	$\frac{1}{2}$
	<i>Quintas.</i>	8. to. 5: 16. to. 10: 24. to. 15: 32. to. 20.	$\frac{1}{5}$
	<i>Septimas.</i>	10 to 7: 20. to 14: 30 to 21: 40 to 28.	$\frac{1}{7}$
	<i>Octauas.</i>	11. to. 8: 22. to. 16: 33. to. 24.	$\frac{1}{8}$
	<i>Decimas.</i>	13. to. 10: 26. to. 20: 39. to. 30.	$\frac{1}{10}$
	<i>Vndecimas.</i>	14. to. 11: 28. to. 22: 42. to. 33.	$\frac{1}{11}$
	<i>Decimas tertias.</i>	16. to. 13: 32. to. 26: 48. to. 39.	$\frac{1}{13}$
<i>Superquadrupartiens.</i>	<i>Decimas quartas.</i>	17. to. 14: 34. to. 28: 51. to. 42.	$\frac{1}{14}$
	<i>Quintas.</i>	9. to. 5: 18. to. 11: 27. to. 15: 36. to. 20.	$\frac{1}{9}$
	<i>Septimas.</i>	11 to 7: 22 to 14: 33. to. 21: 44. to 28.	$\frac{1}{11}$
	<i>Nonas.</i>	13 to 9: 26. to. 18: 39. to. 27: 52. to 36.	$\frac{1}{13}$
	<i>Vndecimas.</i>	15. to. 11: 30. to. 22: 45. to. 33.	$\frac{1}{15}$
<i>Superquintupartiens.</i>	<i>Decimas tertias.</i>	17. to. 13: 34. to. 26: 51. to. 39.	$\frac{1}{17}$
	<i>Decimas quintas.</i>	19. to. 15: 38. to. 30: 57. to. 45.	$\frac{1}{19}$
	<i>Sextas.</i>	11 to 6: 22 to 12: 33. to. 18: 44. to 24.	$\frac{1}{11}$
	<i>Septimas.</i>	12. to. 7: 24. to. 14: 36. to. 21.	$\frac{1}{12}$
	<i>Octauas.</i>	13. to. 8: 26. to. 16: 39. to. 24.	$\frac{1}{13}$
	<i>Nonas.</i>	14. to. 9: 28. to. 18: 42. to. 27.	$\frac{1}{14}$
	<i>Vndecimas.</i>	16. to. 11: 32. to. 22: 48. to. 33.	$\frac{1}{16}$
	<i>Duodecimas.</i>	17. to. 12: 34. to. 24: 51. to. 36.	$\frac{1}{17}$
<i>Supersextupartiens.</i>	<i>Decimas tertias.</i>	18. to. 13: 36. to. 26: 54. to. 39.	$\frac{1}{18}$
	<i>Decimas sextas.</i>	19. to. 14: 38. to. 28: 57. to. 42.	$\frac{1}{19}$
	<i>Decimas sextas.</i>	21. to. 16: 42. to. 32: 63. to. 48.	$\frac{1}{21}$
	<i>Septimas.</i>	13. to. 7: 26. to. 14: 39. to. 21.	$\frac{1}{13}$
	<i>Vndecimas.</i>	17. to. 11: 34. to. 22: 51. to. 33.	$\frac{1}{17}$
	<i>Decimas tertias.</i>	19. to. 13: 38. to. 26: 57. to. 39.	$\frac{1}{19}$
	<i>Decimas septimas.</i>	23. to. 17: 46. to. 34: 69. to. 51.	$\frac{1}{23}$
<i>Supersextus partiens.</i>	<i>Decimas nonas.</i>	25. to. 19: 50. to. 38: 75. to. 57.	$\frac{1}{25}$
	<i>Vicesimas tertias.</i>	29. to. 23: 58. to. 46: 78. to. 69.	$\frac{1}{29}$

Scholar. I vnderstande by these examples, some-  
what of their reasons: but I perceiue, you doe not fo-  
lowe their naturall order, without interruption, in  
these

## The seconde parte

these of the lasse kinde.

Maister. To thintente you maie the better vnderstande good ground in that omission, I will set furthe here those omitted numbers: That you maie see how thei would expresse some other propoziti<sup>o</sup> here named And therfore thei doe seme rather to be omitted, then in deede so to be.

Marke theim well.

<i>Superbipartiens.</i>	{	<i>Secundas.</i>	4. to . 2:	8. to. 4.	$2\frac{1}{2}$
		<i>Quartas.</i>	6. to . 4:	12. to. 8.	$1\frac{1}{2}$
		<i>Sextas.</i>	8. to. 6:	16. to. 12.	$1\frac{1}{3}$
		<i>Octauas.</i>	10. to. 8:	20. to. 16.	$1\frac{1}{4}$
		<i>Decimas.</i>	12. to. 10:	24. to. 20.	$1\frac{1}{5}$

Scholar. In deede here I see, the firste is double propozition. The seconde *sesquialtera*, the thirde *sesquitercia*, the folwerth *sesquiquarta*, & the fift *sesquiquinta*.

Maister. So marke these other.

<i>Supertripartiens</i>	{	<i>Secundas.</i>	5. to. 2:	10. to. 4.	$2\frac{2}{3}$
		<i>Tertias.</i>	6. to. 3:	12. to. 6.	$\frac{5}{2}$
		<i>Sextas.</i>	9. to. 6:	18. to. 12.	$1\frac{1}{2}$
		<i>Nonas.</i>	12. to. 9:	24. to. 18.	$1\frac{1}{3}$
		<i>Duodecimas.</i>	15. to. 12:	30. to. 24.	$1\frac{1}{4}$

Scholar. The firste of these I knewe not, but all the other are named before.

Maister. The firste is a compo<sup>u</sup>nde propozition (as anon I will declare) and is named *dupla sesquialtera*.

But now will I sette furthe all the other omitted names.

*Secundas.*

# of Arithmetike.

superquadru- partiens.	Secundas.	6. to. 2: 12. to. 4.	$\frac{1}{2}$	Tripla.
	Tertias.	7. to. 3: 14. to. 6.	$2\frac{1}{2}$	Dupla sesquitertia.
	Quartas.	8. to. 4: 16. to. 8.	$2\frac{1}{2}$	Dupla.
	Sextas.	10. to. 6: 20. to. 12.	$1\frac{1}{2}$	superbiparties tertias
	Octauas.	12. to. 8: 24. to. 16.	$1\frac{1}{2}$	sesquialtera.
	Decimas.	14. to. 10: 28. to. 20.	$1\frac{1}{2}$	superbiparties quintas
	Duodecimas.	16. to. 12: 32. to. 24.	$1\frac{1}{2}$	Sesquitertia.
superquin- supartiens.	Decimas quartas.	18. to. 14: 36. to. 28.	$1\frac{1}{2}$	superbiparties septimas
	Decimas sextas.	20. to. 16: 40. to. 32.	$1\frac{1}{2}$	Sesquiquarta.
	Secundas.	7 to 2: 14 to 4.	$3\frac{1}{2}$	Tripla sesquialtera.
	Tertias.	8 to 3: 16 to 6.	$2\frac{2}{3}$	Dupla superbipartiens tertias.
	Quartas.	9 to 4: 18 to 8.	$2\frac{1}{4}$	Dupla sesquiquarta.
supersextu- partiens.	Quintas.	10 to 5: 20 to 10.	$2\frac{1}{5}$	Dupla.
	Decimas.	15 to 10: 30 to 20.	$1\frac{1}{2}$	Sesquialtera.
	Decimas quintas.	20 to 15: 40 to 30.	$1\frac{1}{3}$	Sesquitertia.
	Secundas.	8 to 2: 16 to 4.	$\frac{4}{1}$	Quadrupla.
	Tertias.	9 to 3: 18 to 6.	$\frac{3}{1}$	Tripla.
supersextu- partiens.	Quartas.	10 to 4: 20 to 8.	$2\frac{1}{2}$	Dupla sesquialtera
	Quintas.	11 to 5: 22 to 10.	$2\frac{1}{5}$	dupla sesquiquinta.
	Sextas.	12 to 6: 24 to 12.	$\frac{2}{1}$	Dupla.
	Octauas.	14 to 8: 28 to 16.	$1\frac{1}{2}$	supertripartiens quartas.
	Nonas.	15 to 9: 30 to 18.	$1\frac{2}{3}$	superbipartiens tertias.
	Decimas.	16 to 10: 32. to 20.	$1\frac{1}{5}$	supertripartiens quintas.
	Duodecimas.	18 to 12: 36 to 24.	$1\frac{1}{2}$	sesquialtera.
	Decimas quartas.	20 to 14: 40 to 28.	$1\frac{1}{2}$	supertripartiens septimas.
	Decimas quintas.	21 to 15: 42 to 30.	$1\frac{1}{3}$	superbipartiens quintas.
	Decimas sextas.	22 to 16: 44 to 32.	$1\frac{1}{2}$	supertripartiens octauas.
	Decimas octauas.	24 to 18: 48 to 36.	$1\frac{1}{3}$	sesquitertia.
	Vicesimas.	26 to 20: 52 to 40.	$1\frac{1}{5}$	supertripartiens decimas.
	Vicesimas secundas.	28 to 22: 56 to 44.	$1\frac{1}{7}$	supertripartiens undecimas.

Scholar. I see well that these proportions, bee agreeable with some other name: and therefore might some superfluous in this place,

Master.

## The seconde parte

Maſter. Not onely ſuperfluouſely, but alſo falſely ſhould thei bee placed here: ſeynge thei doe belong to other places of right.

Scholar. Why doe you not name theim all by Engliſhe names?

Maſter. Becauſe there are no ſoche names in the Engliſhe tongue. And if I ſhould giue theim newe names, many would make a quarrelle againſt me, for obſcuriſing the olde Arte with newe names: as ſome in other caſes all redy haue doen.

Scholar. Yet I praye you declare thoſe doubtfull names of compoſunde propoſitions.

Maſter. As there is one kinde of propoſition, that is named *multiplex*, or manyſolde, whiche doeth containe the leſſer diuerſe times exactly. And two other whiche doe containe the leſſer ones, and ſome parte or partes of theſame: So thoſe kindes maie be compounded together. As when the greater number containeth the leſſer, twice, or thriſe, or oftener: and yet moze ouer ſome parte or partes of theſame. So .8. containeth 3 twice, and his  $\frac{2}{3}$ . And 10 comprehendeth 3. thriſe and his  $\frac{1}{3}$ .

The firſt example is generally called *multiplex ſuperpartiens*: becauſe the greater containeth the leſſer certaine tymes, and ſome partes of it beſides. But moze ſpecially it is called *dupla ſuperbipartiens tertias*, that is, double with  $\frac{2}{3}$  moze.

The ſeconde example is generally referred to *multiplex ſuperparticularis*: becauſe in it the greater comprehendeth the leſſer oftentymes, (as here thriſe) and his  $\frac{1}{3}$  moze. And therfore ſpecially it is called *tripla ſeſquitertia*.

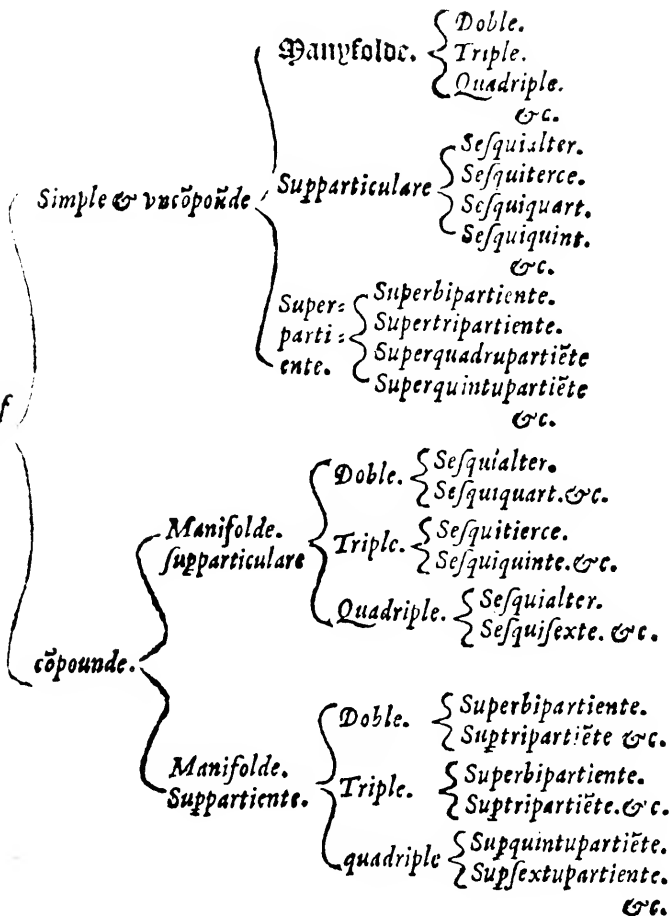
But as I doe intende hereby to ouer runne this parte: ſo will I by tables ſet forth the kindes of the with their examples.

The

# of Arithmetike.

## The table of proportion of the greater inequality.

Proportion of  
the greater  
inequality.



C. j. Examples

## The seconde parte

*Examples of eche compounde kinde,  
mentioned in the former table.*

<i>Manifolde Superparticulare.</i>	{	Double.	{	<i>Sesquialter.</i>	5 to 2.
			{	<i>Sesquiquarte.</i>	9 to 4.
	{	Triple.	{	<i>Sesquiterce.</i>	10 to 3.
			{	<i>Sesquiquinte.</i>	16 to 5.
	{	quadriple	{	<i>Sesquialter.</i>	9 to 2.
			{	<i>Sesquisexte.</i>	25 to 6.

<i>Manifolde fuppartiente</i>	{	Double.	{	<i>Superbipartientetierces.</i>	8 to 3.
			{	<i>Superipartiente quartas</i>	11 to 4.
	{	Triple.	{	<i>Superbipartiente tierces.</i>	11 to 3.
			{	<i>Suptripartiente quartes.</i>	15 to 4.
	{	quadriple	{	<i>Supquītupartiente quartas</i>	29 to 6.
			{	<i>Supfextupartiente septimas.</i>	34 to 7.

**Scholar.** What more is there to bee learned of these proportions: For by these formes, I may easily gather the value or rate of any proportion.

**Maſter.** This may ſtande for their numeration: ſaue that moſte aptly they ought to bee ſette as fractions, in their leaſte termes: as you haue here diuerſe examples.

**Scholar.** You meane that double ſesquialter muſt be written thus  $\frac{5}{2}$ , and ſo of the reſte.

**Maſter.** Or els thus  $2\frac{5}{2}$ , and ſo triple ſesquiquinte in this ſorte:  $3\frac{1}{3}$ , or thus  $\frac{10}{3}$  and ſo of all other.

And for farther worke, you ſhall vnderſtande that proportions may bee added, ſubtracted, multiplied and diuided; and verie ſtraunge workes thereby achieved:

## of Arithmetike.

achieued. For of the Arte of Proportions, dependeth all the subtilties, and fine workes, not onely of *Arithmetike*, but also of *Geometrie*: besides farther matter that as now I will not touche. But as for the workes of *Proportions*, I will omitte them til an other time: considering not onely the troublesome condition, of my vnquiete estate: but also the conuenient order of teaching, whereby it is required that the extraction of rootes, should go orderly before the arte of *Proportions*: whiche without those other, can not be wrought.

Therefore will I now onely declare these kindes of proportion, whiche yet are not spoken of: to the intent that you maie haue here, the generall diuision of numbers, somewhat sufficiently touched.

As you see that betwene any two numbers, there maie be a conference of proportion: so if any one proportion be continued in more then .2. numbers, there maie be then a conference also of these proportions, in their seuerall termes: and that conference or comparison is named *Analogie*: whiche some delight to call proportionalitie: As in this example, where 3 numbers beare like proportion in their progression: 4. 6. 9. You see that 6. to 4. is in proportion *sesquialter*: and so is 9. to 6. and therefore is there a like proportion betwene the .2. laste, as there is betwene the .2. firste.

Scholar. This I consider well by progression in *Arithmetike*.

Master. Likewises where soeuer termes or more be set in order of proportion, as here 2. 6. 18. 54.

Scholar. I perceiue this wel: for here the proportion is triple. But what saie you to this forme of comparison in Proportion: As 6. is to .2: So, 3. is to .20. Is it not all one kinde of *Analogie*?

Master. It is one kinde of *Analogie* generall, whiche maie be called *directe Analogie*: because the first *Direc*te one is compared to him that doeth followe nexte: & so eche *logie*.

C. n.

other

## The seconde parte

other is still referred to that, that foloweth nexte. But this is the difference: that in the firste, there is a continuance of collation: and one terme is compared with twoo numbers: But in that forme of example, whiche you put, there is no number compared twise: For the first is referred to the seconde, and the seconde to the thirde. And so haue thei seuerall names to distinguish thein a sonder.

*Continuall  
Proportion.*

Wherefore whē the first number is referred to the seconde, and that seconde to the thirde: the proportion is called *continuale*: and it maie consist betweene 3. termes. As 5. 15. 45. doe procede in a continuall triple proportion. For as 5. is to 15: so is 15. to 45. as you doe see. But when I saie thus: as 5. is to 15. so 6. is to 18. Here is a triple proportion, but not continuale. For the seconde terme beyng 15. is not compared with the thirde terme, that is 6. And therefore is it called a proportion *discontinuale*.

*Discontinual  
Proportion.*

Scholar. Now I perceiue certainly their distinction: For in twoo pointes these examples doe agree, and differ in a thirde pointe.

Firste thei agree in that (as you saied) that the firste is referred to the other that foloweth it nexte: And secondly, thei agree in this also, that bothe are compared in a triple proportion. But in this thei differ, that the seconde terme, doeth not beare like proportion to the thirde, as the thirde doeth to the fourth or the firste to the seconde.

Master. Farther more there is to bee noted, that in discontinuall proportion, there can bee no fewer then fower termes, or numbers: and so by such formes still, as 6. or 8. and so forth. Where as in continuall proportion, your termes maie bee of any number, euen or odde: aboue 2.

And although I might saie more of the diuersities of proportion: as of *Proportion conuersed* or *indirecte*, *Proportion*,



## of Arithmetike.

portion interchaunged, compounde Proportion, parted Proportion, reuerſed Proportion, and Proportion by equalitie. Yet I thinke better to procede for this tyme, to the other kindes of number, and to reſerue the explication of proportions to their peculiare place.

Scholar. As you knowe the beſt order, ſo it ſhalbe mete that you doe vſe your owne iudgement therein.

## Of figurall numbers.

Maſter.



**T**he nexte kinde of numbers are called *Figurall numbers*: becauſe thei doe, or maie repreſente ſome figure: And are euer conſidered in relation to thoſe formes.

Some of them haue a compariſon and relation to length onely, and therefore are named

*Linearie numbers*: whiche name, although it maie be referred moſte aptly to ſuche numbers, as will make no other forme due, yet maie it alſo be applied to any number abſtract. Sith all ſuche numbers maie be conſidered as the ſides of other figurall numbers.

Secondly, numbers maie be conſidered, according to ſuche formes as thei make other in progression, or in multiplication: And thoſe maie well be named *Superſiciall numbers*, or *Flatte numbers*. Whereof there be as many varieties, as there be diuerſities of figures in Geometrie. As numbers *Triangulare*, *Quadrata*, *Cinque-angeled*, *ſiſeangeled* and ſo forth. Alſo numbers *circulare*, *diametrall*, & like flatte, all whiche numbers haue bothe lengthe and breadth: and thereof be named *ſuperſiciall numbers*.

C. liij.

Beſide

## The seconde parte

Sounde  
numbers.

Beside these there are other numbers, whiche are made of many multiplications, and thei are called *sounde numbers*: because that as by the firste multiplication, thei take lengthe and breadthe, like flatte numbers, so by the second multiplication, thei take depthe also: And thei of be thei named *bodily numbers*, or *sound numbers*.

The leaste of them all is commonly called a *Cube*, or a *Cubike number*: And the other in their degrees seuerally named, as thei bee made by seuerall multiplications. For accordynge to the number of their multiplications, thei take their names. And all that haue like number of multiplications, are of one kinde, and bere one name: as well in flatte numbers, as in sounde.

But considering the infinite multitude of those figurall numbers, I thinke beste to speake of them onely in this place, whiche haue muche profitable vse in this arte. And, of those, among infinite flatte numbers, I will take onely folwer. That is to saie, *square numbers*, *longesquares*, *diametrall numbers*, and *likeflattes*.

Square  
numbers.

*Square numbers* are those, whiche may be diuided by some one number, and haue thesame number for the quotient: that is to saie, that a square nōber is made by the multiplication of any number into it self, as 10 multiplied by. 10. maketh. 100. That. 100. is a square number: whiche. 100. if I doe diuide by. 10. the quotient will be. 10. also.

Scholar. So, 4. multiplied by. 4. maketh. 16: and that must be a square number by like reason.

Master. So it is.

Scholar. And if I multiplie. 9. by. 4. is not that a square number: Seyng folwer semeth to make all nōbers square by multiplication.

Master. Consider this well, that a square number doeth make suche a square in number, as a *iuste square* doeth make in *Geometrie*: That is suche a one whose

## of Arithmetike.

whose sides are equalle. For and if the one side be longer then the other, that figure in *Geometrie* is called a *long square*, and so it is named in number, a *long square* also.

Now if I sette downe the figure of your number, as you termed it, and sette. 4. for the one side, and. 9. for the other, this will the figure shewe.

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: : : : : : : :
: : : : : : : :

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Here you se a plain longsquare: yet is the whole number that amounteth of this multiplication: truly named a square number, as here you may see. But then is the side or roote of it, neither. 4. nor. 9. but. 6.

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: : : : :
: : : : :
: : : : :
: : : : :

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Scholar. Now I vnderstande it: and the better by this figurall example. And here also I haue learned what a *Roote* is: so you seue to expounde it, to bee the *Roote* of a figurall number.

Master. Euery flatte number, and euery sounde number also haue their sides: But no flatte number, save onely squares haue a roote: because a roote in flatte numbers, is a number multiplied by it self.

And in sounde numbers, thei onely haue rootes, whiche bee made by many multiplications, of some one number by it self: other by that, whiche riseth of it.

As when I saie, twoo tymes, twoo twise, maketh 8. that number is a sounde number: and is named a *Cube*. And so. 3. tymes. 3. thise, doeth make. 27. whiche is also a *Cube*.

And generally, any number that is made by suche 2. multiplications, is called a *Cube*, or *Cubike* number. And the number of that multiplication, whiche commonly is named the multiplier, is in this point called the *Cubike roote* of that number.

Wherefore, thus also maye you define a *Cubike* number, *A cubike number, number.*

*A cube.*

*A cubike roote.*

## The seconde parte

ber: to bee suche a number, as beeyng diuided by his roote, shall haue for the quotiente the square of the same roote.

Scholar. Hereby I perceiue, that one multiplication, of any number by it selfe, doeth make a square number. And twoo multiplications in that sorte, doe make a Cubike number.

What if I doe multiplic any number thise, or forwer tymes, or oftener in that sorte, are there proper names for suche numbers?

Master. Yes in dedde: as I will declare anon.

But firste before we attempte the other sounde numbers, it shall bee mete, that we doe declare those twoo sortes of flatte numbers, whiche I named before: that is diametralle numbers, and like flattes.

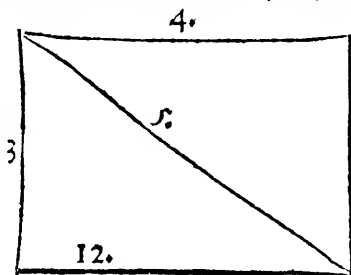
*A diametral number.* A *Diametralle number*, is suche a number as hath twoo partes of that nature: that if thei bee multiplied together, thei will make the *sated diametralle number*:

And the squares of those twoo partes, beeyng added together, will make a square nuber also: whose roote

*A diameter.* is the *diameter* to that *diametralle number*.

As 12 is named a *diametralle number*, for that he hath twoo partes, that is. 3. and. 4, whiche beeyng multiplied together, doe make 12. that is the firste number. And if their squares be added together, thei wil make a thirde square: and the roote of that number will bee the *diameter* to that platte forme of 12. As in this example you see.

The one side is. 4. and the other side is 3 whiche bothe multiplied together, doe make 12. When take the square of forwer whiche is 16 and the square of. 3, whiche is. 9. and put them



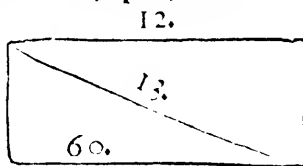
together

## of Arithmetike.

together, and they will make .25. whose roote, beyng 5. is the *diameter* of that platte forme.

Scholar. That doe I perceiue well, bicause it is confirmed by the .33. theoreme of the pathwaie.

Maister. Yet take another example. In this platte forme of .60. you see the one side to bee .5. and the other side to bee 12. Now take the square number of .12. whiche is. 144. and the square of .5. whiche is. 25. and put them together : so will it make 169. whiche is a square number: and hath .13. for his roote.



Likewises. 120. is to be accounted a *diametrall* number. For so much as it hath two partes: that is 8. and .15. whiche beyng multiplied together, doe make the firste number. 120. And the square of those two partes (that is. 64. for .8. and. 225. for .15.) beyng bothe added together, doe make .289. whiche is a square nōber: and hath for his roote. 17. And therfore that. 17. is the *diameter* to that *diametrall* number. 120.

Like examples infinite might I giue you. But these for explication of the name, maie suffice.

Scholar. I doe well vnderstande the examples: saue that I knowe not how to finde the roote of the laste square number, whiche amounteth by the addition of the former two squares together.

Maister. That arte will I teache you anon. But we maie not forgette firste to ende all the definitions of soche names, as I minde to write of.

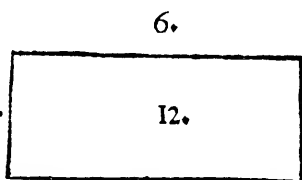
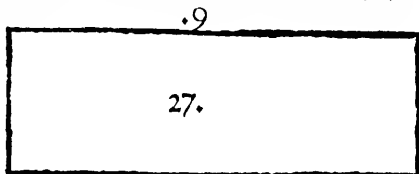
Whereof yet there resteth *like flattes*: whiche maie bee as well taken for *trianguler* figures, as for *quadrate* figures. *Like flattes.*

So that of any of them, when the sides of one platte forme, beareth like proportion together, as the sides

## The seconde parte

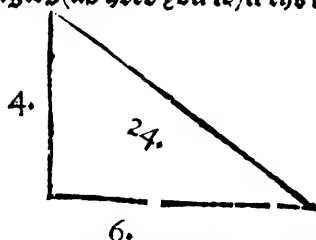
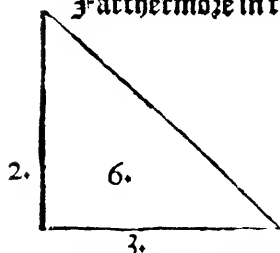
of any other flatte forme of thesame kinde doeth, then are those formes called *like flattes*. As in these. 2. longe 3.

Squares: because the sides of them bothe, are in one proportion (for. 6. is triple to. 2: as well as. 9. is triple to 3.) Therefore are 2. the whole figures called *like flattes*.



And so of due conueniencie, their numbers (that expresse their quantities, whiche here are. 27. and 12) be called by the like names, *like flattes*.

Farthermoze in triangles (as here you se) if the si-



des of the one beare like proportion together, as the sides of the other doe: then are they called *like flattes* also. And their numbers, that declare their quantities, in like sorte are named *like flattes*.

Scholar. I perceiue here: As 4 is to 2: so. 6. is to 3. bothe beyng in a double proportion. And therefore 6 and. 24. are to be called *like flattes*.

Master. You vnderstande it well.

And thus haue we briefly ouer runne the diuision of number, into his principalle kindes: And haue set forth the definitions of eche of them, with examples.

The

## of Arithmetike.

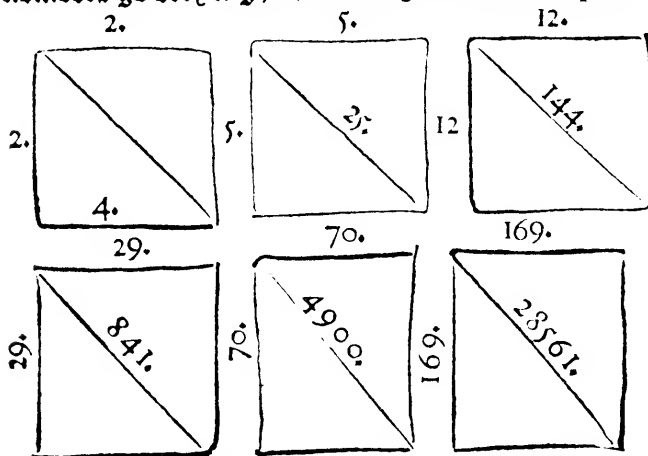
The vse of them you shall se largely in the practise of this arte.

• But to the intent you maie the better obserue and regarde these twoo laste kindes of numbers : whiche are commonly neglected of artes men , I will shew you some vse of them, with their properties.

Firste, all *diametralle numbers* doe sette forth a triangle, hauyng all thre sides knowne: whiche thing as it doeth serue to many and wonderfull purposes: so can it be found in no other numbers, then onely in *diametralle numbers*. *Of diametralle numbers.*

For although in figures Geometricalle, you maie neuer more unfallibly finde one line, that will make a square, equall to the twoo squares of any other twoo lines (as in the pathe waie you doe see it taught) yet the measure certaine of those sides, are not knowne.

Wherefoze in number that is not possible alwaies to be doen : neither can it be doen with any other numbers, then onely *diametricall numbers*. Yet maie other numbers go very nigh. As namely in these examples

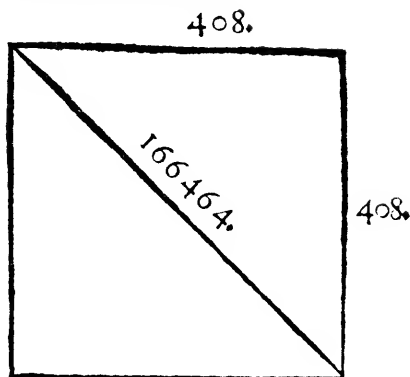


of square numbers: whose double, I take for the squares  
D. ij. res

## The seconde parte

res of the sides,  
bicause thei are  
equall: and thei  
make. 8. 50. 288.  
1682 . 9800.  
57122. &. 332928.  
All whiche dif-  
fer onely by an  
unitie, from a  
square number.

For nine is a  
square number  
and so are these  
other folowynge.



49. 289. 1681. 9801. 57121. &. 332929.  
whose rootes be. 7. 17. 41. 99. 239. 577.

Whiche examles if you doe consider well hereaf-  
ter, thei will helpe you to gesse at the nigheste rootes  
of numbers that be not square. And also for doblving  
of squares, in a square forme: within an vnspake-  
able nerenesse.

For as in doblving of this greater square. 166464.  
there riseth. 332928. whiche wanteth one of a iuste  
square. You se easely, that as that one is but a smalle  
portion to the whole square: So yet, that one wan-  
teth not in the roote, but in the whole square: where  
by you maie perceiue, that it is a very smalle and vn-  
sensible parte of one, that wanteth in the roote.

Scholar. It must seme by reason of multiplicati-  
on: that it is scarce the. 10000. parte of one.

Master. You saie truthe.

Scholar. But how shall I finde the diameter of  
suche numbers?

Master. What is easily doen, if you knowe firste  
certainly that your number is a diametrall number.

And secondarily, if you knowe the true partes of  
it:



## of Arithmetike.

it: whiche you should vse in this case.

Scholar. Will not any twoo soche partes serue, whiche by multiplication will make the whole number?

Maister. You maie by the former examples, easily se the contrary. For 12 is a *diametrall number*: and hath these partes (as it is sone perceiued). 2. 3. 4. 6. Yet if you take . 2. and . 6. for the sides of it, thei will not make a *diameter* in knowen number.

Scholar. That I vnderstande: for the square of 2. beyng. 4. added to. 36. whiche is the square of 6. doeth make. 40. whose roote must bee greater then. 6. and lesse then. 7. And therfore. 40. can haue no roote in whole number.

Maister. Neither yet in broken numbers: for that is a generall rule: that if any whole number haue a roote, that roote shall be a whole number. So that if the roote can not bee founde in whole number: you shall neuer finde it in broken numbers.

And for moze certaintie of that I saied before, that all partes be not apte for the sides of a *diametrall number*, to finde out the *diameter*: marke well the seconde example, whiche is. 60. and hath these partes.

2. 3. 4. 5. 6. 10. 12. 15. 20. 30.

So that beginnyng with the two extreme, that is. 2. and. 30. thei will by multiplication make. 60.

And likewise any two numbers, equally distant from those extremes: As. 3. and 20. Likewise. 4. and 15: other. 5. and. 12. And in like maner. 6. and. 10. All those couples by multiplication doe make. 60. Yet none of them are apte sides to finde the *diameter* by, but onely 5 and. 12. For of the other sides beyng multiplied squarely (that is by the selves) and those squares beyng added together, there wil not rise a square number. As you shall better vnderstande, when you

## The seconde parte

haue learned to knowe square numbers, by extractiō of their rootes.

¶ Yet in the meane reason I will set forth the certaine notes, to knowe the *diameter*, and the apte sides, in all *diametralle numbers*.

1. And firste I saie: that as thei are thzee numbers in all (I meane the twoo sides, and the *diameter*) so all waies if the firste or leaste side bee odde, then shall there be twoo of them odde numbers. And the *diameter* shall euer bee the other of the odde numbers: that is to saie, the greatesse of them.

2. Secundarily. It is true that all *diametrall numbers* are euen numbers. And no odde number can bee a *diametralle number*.

3. Thirdly. I saie, that all odde numbers aboue one, maie be the lesser side in soche *diametralle numbers*.

But euen numbers doe not serue so generally: for thei onely maie stand in soche place, whiche be greater then. 4: As. 6. 8. 10. 12. 14. 16. 18. 20. &c. And none other euen numbers then soche as maie be diuided by 4. maie be the greater side in any *diametralle number*.

4. Fourthly. If the lesser side bee an odde number, then ordinarily, the square of it is iuste equalle with that that amounteth by the addition of the *diameter*, to the greater number. As in the firste erample, 3. is the lesser number, and. 4. is the greater: vnto them bothe the *diameter* is .5. Now. 3. hath for his square 9. and so moche is made by the addition of. 4. and. 5.

Again in the seconde erample, the lesser number is 5. and his square is 25. The greater number is 12. and the *diameter*. 13. Put. 12. and. 13. together, and thei make. 25. whiche is equalle with the square of the lesser.

Like waies. 7. and 24. multiplied together maketh 168. whiche is a *diametralle number*. And bicause the square of the lesser side (whiche here is. 49.) must bee equalle

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equalle to the greater side, and the *diameter* added together:therfore seying. 25. added to. 24. maketh. 49. that. 25. must nedes bee the *diameter* to the foresaid number.

By these rules ( if you doe marke them well ) you maie sone perceiue, how to make any *diametralle number* : if the lesser side bee giuen vnto you, and bee an odde number. Yet for your ease, I will giue you this plaine rule.

When any odde number is propounded : as the lesser side of a *diametralle number*, and you would finde the other side, and the *diameter* also : or els the *diametralle number*, that maie haue soche a side: multiplie that proponed number by it selfe, and it will make a square number, and will be an odde number : so that of it you shall finde no iuste halfe. Therfore take you those twoo numbers, that are nexte vnto the halfe of it: The lesser shall alwaies bee an enen number, and shall be the seconde side of the *diametralle number*: The other number whiche is the greater, shall alwaies be an odde number: and shall bee the *diameter* of that number whiche you desire. For example marke wel these formes that doe folowe.

If thre bee propounded as the one side of a *diametralle number*: And you would knowe, what maie bee the other side: and what is the *diametralle number*: And thirdly, what is the *diameter* to that number : Doe, as I saied before: multiplie. 3. by it selfe, and it will make 9. whiche is a square number, and an odde number: and therfore hath no iuste halfe. But the nighest numbers to the halfe, are. 4. and. 5.

Therfore I saie, that. 4. whiche is the lesser of the twoo, is the seconde side of the *diametralle number*: and 5. being the greater of them, is the *diameter* it self.

Scholar. Now is it light inough to perceiue that the *diametralle number* is. 12; seying. 3. multiplied by  
folwer

## The seconde parte

4. maketh. 12.

Master. So is it.

Again, if. 5. be assigned for one side of a *diametralle* number, and you obserue the former worke you maie easily finde the other side, and the *diameter*.

First you see, that the square of 5. is. 25. and it hath no halfe. But. 12. and. 13. are the. 2. numbers nighest his halfe: wherfoze. 12. shall bee the seconde side: and 13. must be the *diameter*. And the *diametralle* nōber is. 60.

Like waies, if. 7. be set for the lesser side, the greater side shall be. 24. and the *diameter*. 25.

Scholar. Touching this I neede no more instruction: the thynge is so manifeste.

Master. Then shewe your knowlege by an example, or twoo.

And first I appointe 9 for the lesser side of a *diametralle* number, whereunto I would haue you to assigne the other side, and the *diameter*, &c.

Scholar. I followe your precepte, and multiplie 9. by it self, whereof commeth. 81. whose halfe is betwene. 40. and. 41. Therfoze must. 40. be the other side: and 41. the *diameter*. And here the *diametralle* number is. 360.

Master. Wone the like: where. 15. is the lesser number.

Scholar. 15. multiplid square maketh. 225: whose nighest halfe is. 112. and. 113. of whiche the first is the seconde side, and the later is the *diameter*: and the *diametralle* number is. 1680.

Master. What shall be the other numbers: where 21. is the lesser side?

Scholar. 21. yeldeth in square. 441. whose portions nighest his halfe, are. 220. and. 221: And so appereth their offices, and the *diametralle* number is 4620.

Master. So maie you see that vnto. 27. being the lesser side; the greater side shall be. 364. and the *diameter*

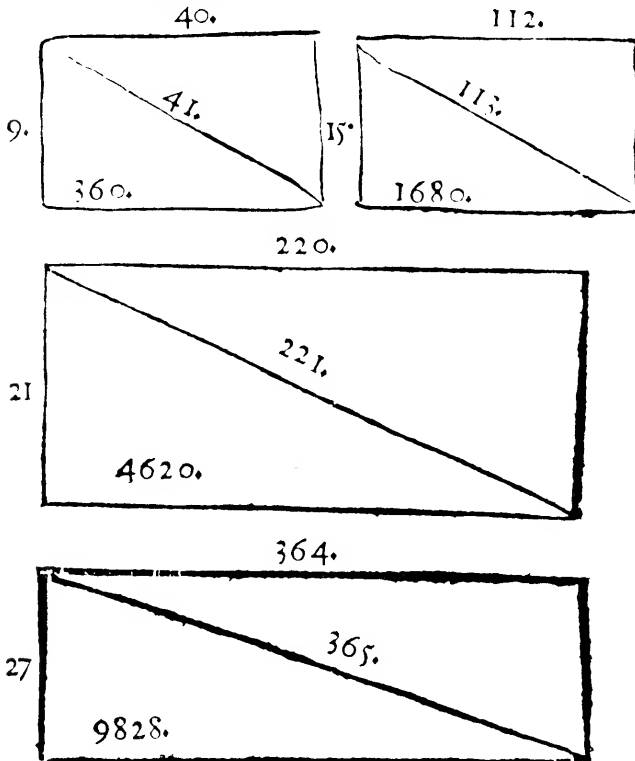
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ter. 365. because the square of. 27. is. 729. And the *diametralle* number is. 9828.

Scholar. So must it be, by your rule.

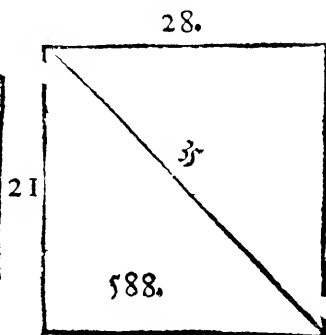
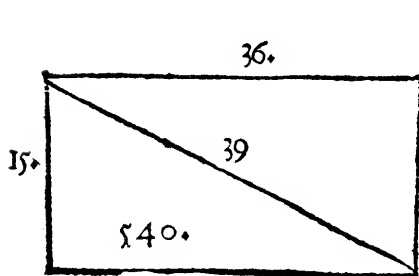
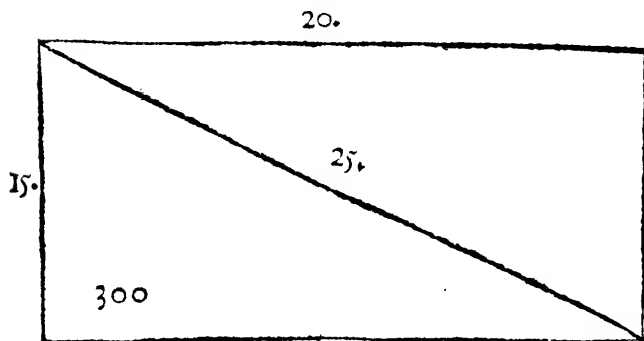
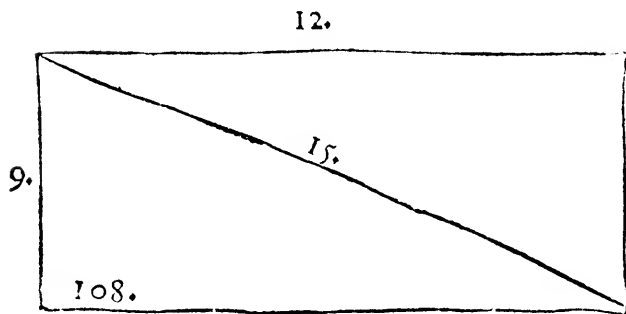
Master. Not onely the rule doth teache you that it is so, but also the nature and figure of soche *flatte* numbers. As here you see.



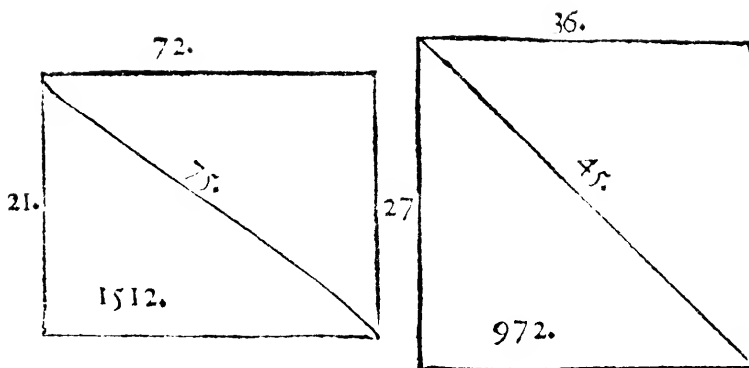
But to the Intente you made the better vnderstand the nature of these numbers: I wil set forth here the like sides with other numbers: whereby you may knowe, that one side may serue to diuerse *diametralle*

C. j. numbers

*The seconde parte*  
*numbers. Therfore marke these formes well.*



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Scholar. Here I see the same 4 numbers. 9. 15. 21. and. 27. set as the lesser sides: And their greater sides are soche as disagree frō the former rule. And in. 15. 21. and. 27. I see two varieties, unlike to the former example. But seeing the sides doe disagree, I doe not marvel that the *diametralle* numbers are diverse from the former.

Master. Examine these numbers, whether they be true.

Scholar. I must multiply eche side by it self, and then adde the together: and if they make as moche truly, as the *diameter* being multiplied square, then are they true numbers. So I see, that. 9. maketh. 81. and 12 doeth yelde 144 whiche bothe added doe make. 225. And so moche doth 15 make, being multiplied square.

Likewises, for the second figure 15. bringeth forth

C. ij. 225.

## *The seconde parte*

225. and . 20. giueth. 400. that is by addition. 625. whiche somme doeth amounte also, when. 25. is multiplied square.

The thirde figure hath. 15. also for the one side, whose square is. 225. and for the other side. 36. whiche maketh in square. 1296. And thei bothe together giue 1521. And so many commeth of 39 multiplied by it self in square.

Again for the fourthe figure. 21. maketh. 441. and 28. doeth yelde. 784. whiche bothe beyng added, doe amounte vnto. 1225. And so moche doeth there arise by. 35. multiplied into it self.

The fift figure hath. 21. also, and his square is 441. and the seconde side beyng. 72. maketh in square 5184. So that bothe those squares doe make. 5625. And the like number is made by. 75. multiplied in square forme.

Now in the sixt figure 27 beyng multiplied square bryngeth forth. 729. And. 36. like waies multiplied doeth make. 1296. and that with the other will make by addition. 2025. whiche somme (as is well seen) doeth come of the multiplication of. 45. by it self.

In the seuenth figure. 27. multiplied square, doeth giue. 729: and the other side (whiche is. 120.) doeth bryng forth. 14400. These bothe ioyned together doe make. 15129. And the like somme is gathered by the multiplication of. 123. squarely.

So that all those figures doe appere true.

But how thei maie agree with your former rule, I can not see.

Walter. That rule did I make for nōbers vncompounde. For numbers compounding haue not onely in their owne name, the vse of that rule, but also thei followe the forme of those numbers, of whiche thei bee compounding.

So. 9. beyng compounding of. 3. foloweth the forme  
of



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of. 3. And therefore as. 3. hath. 4. so to make the second side with hym, so. 9. (beeyng thriſe. 3.) ſhall haue. 12. (whiche is thriſe. 4.) ſo a matche ſide with hym.

Likewaiſes. 15. beeyng compounde of. 5. and. 3. ſhall haue their formes in the making of the *diametralle numbers*. For as. 3. hath. 4. ſo. 15. (beeyng five tymes. 3.) ſhall haue. 20. (whiche is five tymes. 4.) ſo the ſeconde ſide.

Again, as. 5. hath. 12. ſo ſhall. 15. beeyng three tymes. 5. haue. 36. (that is three tymes. 12.) ſo his ſeconde ſide.

Likewaiſes. 21. beeyng compounde of. 3. and. 7. ſhall haue bothe their formes.

And. 27. whiche is compounde of. 3. and. 9. ſhall haue all the varieties of their formes.

Scholar. I ſee it is euen ſo, and that in the *diameter*, as well as in the ſeconde ſide. But the *diametralle number* doeth varie moche in them.

Maſter. Yet doe thoſe numbers agree in a marueilouſe good proportion. For if you doe conſider the proportion of bothe the ſides in one figure, to bothe the ſides in an other figure; and adde thoſe two proportions together, the addition of them doeth make the number that representeth the proportion betwene their two *diametralle numbers*. Whiche thynge I will now onely touche, as briefly as maie bee, to giue you occaſion to marke it better hereafter: With this place doeth not fully ſerue for it. As. 3. and. 4. beeyng the two ſides of a *Diametralle number*, doe make. 12. So if 9. and 12 be the ſides of a *diametralle number*, that number muſt be. 9. tymes. 12. that is. 108. For. 9. is triple to. 3. and. 12. is triple to. 4. And becauſe the addition of proportions, is like the multiplication of fractions, I muſt multiplie. 3. by. 3. or els  $\frac{3}{1}$  by  $\frac{3}{1}$ , whiche is all one, and that will maie. 9.

Likewaiſes, if 3. and. 4. be taken for the ſides of the

## The seconde parte

lesser number *diametralle*, and. 15. and. 36. for the sides of the greater number: As the lesser number shall bee 12. so the greater must be. 540. that is. 45. tymes. 12.

For. 15. vnto. 3. is in a *quintuple* proportion, and is written thus.  $\frac{15}{3}$ : and. 36. vnto 4 is a *nonuple* proportion, and is written thus  $\frac{36}{4}$ . Now if you multiplie these numbers together, thei will make 45: whiche declarcth the proportions of the twoo *diametralle* numbers. And so of all the reste, as you maie easily consider.

Scholar. I praye you, let me examine one of twoo of the, in comparison to that first *diametralle* number. 12.

I see that 15 beynge the lesser side, and 20. the greater side, doe make. 300. as their *diametralle* number: and that. 300. is. 25. tymes so moche as. 12. is. Therefore by your sayng the proportion of 15. to. 3. and of. 20. to 4. must make. 25. And so it doeth. For eche of them is a *quintuple* proportion. And it is quickly gessed, that 5. multiplied by. 5. doeth make. 25.

For farther prooffe, I take the *diametralle* number 1680. whose sides are. 15. and. 112. First I see, that. 15. to. 3. beareth a *quintuple* proportion: and. 112. to. 4. is as. 28. to. 1. Therefore I multiplie. 28. by. 5. and it maketh. 140. Then if I multiplie that number by. 12. it will make. 1680.

This is a sufficiente trialle for these numbers.

*Of euen sides* But of soche *diametralle* numbers, as haue euen numbers for their lesser side, you haue giuen no rule, neither examples, saue onely of. 8. wherfore I praye you tell me, how shall I finde out the *diametralle* number, with his other side, and the *diameter* in soche euen numbers.

Master. You shall make it square, as you did in the other numbers, that wer odde: And of that square you shall take twoo quarters, whiche you shall alter in soche sorte, that you shall abate. 1. fro the one quarter, and put it to the other quarter. And so haue you twoo

## of Arithmetike.

twoo numbers, differing onely by .2. and bothe being odd. The lesser of them twoo, is the greater side of the *diametralle number*: and the other is the *diameter* to it. As .8. being your lesser side, the square of it is 64. whose quarter is .16. from whiche I abate .1. and there resteth .15. and that is the seconde side. Also I adde .1. to 16. and it maketh .17: whiche is the *diameter*.

Scholar. This is no thyng harde. As by example I will proue. If .12. bee the lesser side: his square is 144. and the quarter of it is .36. Then abatynge .1. I see there will bee .35. for the other side of the *diametralle number*. And addynge .1. to .36. it maketh .37. to be the *diameter*. And if I multiplie .35. by .12. it byngeth forth .420. whiche is the *diametralle number*.

Now for prooffe of these numbers, I multiplie .12. by it self, and it maketh .144. Then I multiplie the other side, that is .35. by it self, and it yeldeth .1225. Those bothe together doe make .1369. And seynge 37 multiplied by it selfe, doeth make the same number. Therefore are thei all true numbers.

An other example. 10. being set for the lesser side, I doe multiplie it squarely: and there riseth .100. whose quarter is .25. for whiche I take (as you taught me). 24. and .26. And so the whole *diametralle number* is .240. For prooffe of the other numbers, I take .100. whiche commeth of .10. multiplied square, and to it I adde .576. whiche is the square to .24. and thei bothe doe make .676. And so much amounteth by the multiplication of .26. squarely.

Master. This maie suffice for this presente: if you marke that the euē numbers haue not onely one generalle forme, whiche I did expresse in the former rule, but also soche as be compounde of any other numbers, euē or odd: haue the like numbers in proportion, for the greater side, and for their *diameter* as the numbers haue, of whiche thei bee compounde. And because

## *The seconde parte*

bicause I will not staie to long on this matter, I will here set forth the diuerse varieties of *diametrall numbers*, whereby you maie gather not onely the true vnderstanding of the former rules : But also in them you maie see other notable cōclusions: and straunge workes of the natures of numbers.

Marke well this table forme, with the titles ouer it: whiche declare the true meanyng of it.

And where you see one number in the firste columpne against two, three, or fower in the other columpnes, you shall vnderstande that that number is the side to so many seuerall numbers *diametrall*.

*The table of diametrall numbers.*

The lesser side.	The greater side.	The diameter. meter.	The number diametral
3.	4.	5.	12.
5.	12.	13.	60.
6.	8.	10	48.
7.	24	25	168
8.	15	17	120
9	12	15	108
	40	41	360
10	24	25	240
11	60	61	660
12.	16	20	192
	35	37	420
13	84	85	1092
14	48	50	672
15	20	25	300
	36	39	540
	112	113	1680
16	30	34	480
	63	65	1008
17	144	145	2448
18	24	30	432
	80	82	1440
19	180	181	3420
20	48	52	960
	99	101	1980
21	28	35	588
	72	75	1512
	220	221	4620
22	120	122	2640
23	264	265	6072
24	32	40	768
	45	51	1080
	70	74	1680
	143	145	3432
25	60	65	1500
	312	313	7800

The lesser side.	The greater side.	The diameter. meter.	The number diametral
26	168	170	4368
27.	36	45	972
	120	123	3240
28.	364	365	5928
	96	100	2688
29	195	197	5460
	420	421	12180
30	40	50	1200
	72	78	2160
31	480	481	14880
32	60	68	1920
	126	130	4032
33	255	257	8160
	44	55	1452
34	180	183	5940
	544	545	17652
35	288	290	9792
36	84	91	2940
	120	125	4200
37	612	613	21420
	48	60	1728
38	105	111	3780
	160	164	5760
39	223	225	11628
	684	685	25308
40	360	362	13680
41	52	65	2028
	262	255	9828
42	760	761	29640
	75	85	3000
43	96	104	3840
	198	202	7920
44	399	401	15960
45			

## The seconde parte

This table maie you extende infinitely. And these thinges maie you se, as thinges of greate admiratiō.

1. Where is no *diametrall* number, but it maie be diuided by. 12. Wherefore thei be all euen numbers euenly and oddely.
2. Again, there is no *diametrall* number, but it endeth in. 0. iii. 2. or in. 8.
3. Thirdely, there is no *diametrall* number, that can haue any moze *diameters* then one.
4. Yet maie one number bee the *diameter* to diuerse other.  
As you se 25. is the *diameter* to. 168. and also to. 300.  
50. 65. is the *diameter* to. 1008. and also to. 1500.  
Likeliwaies. 145. is the *diameter* to. 2448. and to 3432.
5. Fiftely: No square number can bee a *diametrall* number.

Scholar. These properties be notable.

To knowe a *diametrall* number. But how shall I knowe, when a number is proposed, whether it be a *diametrall* number, or not?

Maister. In that thyng I finde a tediousse trauell, by any rules, in those that write of it. But I wil ease you of moche paine therein.

Firste remember the properties of those numbers.

And if you haue any other figure in the first place, then. 0. 2. or 8. it is no *diametrall* number.

Secondarily, if it maie not bee diuided by. 12. although it ende in one of those. 3. figures, it is no *diametrall* number.

Wherefore if it haue bothe those twoo properties (whiche an infinite multitude of numbers doe want) and be no square number (as none be that ende in. 2. or 8. or with odde cyphers) then sette out all the partes of it, in soche sorte, that the lesser parte doe stande directly ouer those greater partes, which beyng multiplied together, will make the whole number.

And

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And then examine those partes, whiche seme to haue any likelihod: accoꝝdyng to the foꝝmer doctrine.

As foꝝ example: if. 72. be pꝝoponed to be examined in that sorte, I sette his partes in oꝝder thus.

2.     3.     4.     6.     8.

36.   24.   18.   12.   9.

Howbeit I neded not to set doune. 2. nother. 4. foꝝ lesser partes, nother those other greater partes, that aunswere to them: Foꝝ, as I said befoꝝe, thei can not bee the lesser side in any *diametralle number*. Wherfoꝝe thei nede no examination.

Farthermoꝝe, foꝝ them that you shall nede to examine, if the lesser number bee an odde number, the square of it must contain double to that greater number (that is coupled with it) and one moꝝe.

And if the lesser be an euen number (of them twoo that you would examine) then must the square of it containe the greater number (that standeth by it). 4. tymes, and. 4. moꝝe. And this is not onely a shoꝝter waie, then I see to be taughte by other artes menne: but it is also moꝝe certaine, foꝝ all numbers not compounded of other *diametralle numbers*.

Scholar. By this doctrine it appeareth quickly, that. 72. is no *diametralle number*.

Foꝝ although it doeth ende in. 2. and maie be diuided by. 12. yet no couple of numbers here haue those properties that is required.

Foꝝ vnder. 3. is. 24. whiche is to greate: and vnder 6. there is. 12. whiche is to greate also.

But vnder. 8. standeth. 9: whiche is to litle, by a greate deale.

Passer. Then pꝝoue in this other number. 132.

Scholar. His partes will stande thus.

f. ij.     3.

## The seconde parte

3.	6.	11.
44.	22.	12.

Where I see quickly that it can not bee a *diametral* number. For the numbers vnder. 3. and. 6. be to greate: sith no number that should bee sette vnder. 3. maie be aboue. 4.

Neither vnder. 6. maie any number bee set greater then. 8. As it doeth sufficiently appeare by that that is taughte before.

And vnder. 11. there can bee no lesse number placed then. 60: and therfore. 12. is to smalle.

And herein I perceiue greate helpe by this table, whiche you haue set forth.

Master. It is well marked of you. But yet trie this other ex ample. 6072.

Scholar. I set doune his partes in order, thus.

3.	6.	8.	11.	12.	22.	23.	24.
2024.	1012.	759.	552.	506.	276.	264.	253.
33.	44.	46.	66.	69.			
184.	138.	152.	92.	88.			

And here I see a greate sorte of numbers, whiche can not serue to my purpose, because those that bee euen, and are lesse then. 44. make to litle a square, to be 4. times so moche as the number vnder any of the.

And. 44. maketh to greate a square: wherfore it can be none of the euen numbers.

Again, those that be odde vnder. 23. doe make to litle a square, to bee double to the greater number vnder it. And those that bee odde aboue. 23. doe make to greate a square. So that. 23. doeth remain to bee the true nōber for the lesser side: and 264 the greater side.

Master. Because exercise is the beste instrument  
in



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in learning : therfore will I propounde to you one example more.

What saie you of. 5460? Is it a *diametralle number* or no?

Scholar. I will trie it, by settynge doune his partes thus.

3.	5.	6.	7.	10.	12.	13.	14.	15.
1820.	1092.	910.	780.	546.	455.	420.	390.	364.

20.	21.	28.	30.	35.	42.	52.	60.	70.
273.	260.	195.	182.	156.	130.	105.	91.	78.

And here I see diuerse and many numbers, whiche at the firste sighte, appere nothyng mete for this purpose. For. 20. is to smalle a number, as I maie sone iudge : and therfore all other numbers vnder it, must nedes be to smalle, of force.

Againe, I see that. 30. is to greate a number, and therfore, of necessitie, all other numbers aboute it, must nedes be to greate. So that, 21. other. 28. must be the true number, or els none.

Wherfore I examine first. 21. whose square is 441 whiche should bee one more then double, to the number vnder it, that is to saie, it should bee. 521. And so it is not: Wherfore I refuse it, and examine. 28. whose square is . 784. And that should bee fower tymes so moche as. 195. (whiche is the number vnder it) and 4. more. Wherfore I doe *quadruple*. 195. and it maketh. 780. And then I see that it wanteth, but fower of the other square: wherfore I take those twoo numbers, I meane. 28. and. 195. for the true sides of. 5460. whiche I finde to be a *diametralle number*.

Waster. By the waie, remember that you could easily perceiue, that all nōbers vnder. 20. were to small for your purpose: and contrary waies, all about. 50.

ff. iij. to

## The seconde parte

*A shorte  
meane in  
working.*

to be to greate. So that you neded not to sette doune so many partes of your firste number.

Wherfore if your number bee soche a one, as hath many partes, you maie chose one by gesse, which you thinke will go nigh to serue your purpose: and if you finde it to small, then set theim doune onely that bee greater then it, til you finde one other iuste: and then haue you your purpose. Or if you finde any to great, after that whiche was to small, and betwene theim none iuste. then is not your number a *diametrall nōber*.

But and if the parte whiche you tooke by gesse, be to great, you shall refuse all partes aboue it, and take onely lesser partes, til you finde a iuste parte for your purpose: or els one that is to litle.

And if in descendynge orderly, you finde no iuste parte, befoze you come to one that is to litle, then is your number no *diametrall number*.

Scholar. This is a greate ease in shorTENynge of woꝝke: whiche I will pꝛoue in this number. 9786.

Master. If you remembꝛed well your foꝛmer rules, you would not admitte this to be examined for a *diametrall number*: bicause it endeth in none of the thze peculiere terminations: that is. 0. 2. 02. 8.

Scholar. I cōfesse my faulte. And therfore I take this number. 9780. whose. 20. parte is. 489. But seynng. 20. doeth make in square but. 400. therfore is it very moche to litle.

Then I take the. 30. parte of it, whiche is. 326. and finde it also to litle.

Thirdey, I take the. 40. parte of it, whiche is 244½: and seynng. 40. maketh in square. 1600. I see that it is almoste. 7. tymes so moche as. 244½: and therfore is it to greate.

So must the true number be betwene. 30. and. 40: or els there is none at all.

Therfore firste I take. 35. whiche is the middelle number,

## of Arithmetike.

number (as the moſte apte for a coniecture) and it yeldeth.  $279\frac{3}{4}$ . And the ſquare of.  $35$ . is.  $1225$ . whiche is farre moze then the double of.  $279\frac{3}{4}$ .

Wherefore, againe I proue with.  $32$ . whiche giueth  $305\frac{5}{8}$ . And ſeyng the ſquare of.  $32$ . is.  $1024$ . it is not 4. tymes ſo moche as.  $305\frac{5}{8}$ . for that is.  $1222\frac{1}{2}$ .

Wherefore I take a greater number, betwene it and.  $35$ . And firſt I take.  $33$ . whiche bringeth forth the  $296\frac{1}{4}$ . wherby I maie ſee that.  $33$ . is to greate. And ſeyng there is no number leſſe betwene.  $32$ . and.  $33$ . therefore I iudge that firſte number.  $9780$ . to bee no *diametralle number*.

Maſter. Examine this number.  $43200$ .

Scholar. Bicauſe I ſee it to be a greate number, I will begin with a greate parte of it. And therfore, I take.  $100$ . whiche yeldeth.  $432$ . And conſidering that the ſquare of.  $100$ . is.  $10000$ . whiche is farre to greate, I muſt ſeke a leſſer number.

Maſter. I will eaſe you of your paines in that. For bicauſe here is moze to bee conſidered. You remember that I tolde you beſore, in making of *diametralle numbers*, how that ſome numbers doe followe the rules of other, of whiche thei be compounde. And farthermore, that ſoche compounde *diametralle numbers*, did beare proportion to the leſſer, as the proportion was of bothe their ſides added together.

Scholar. That is true.

Maſter. Of like reaſon all ſoche *diametralle numbers*, muſt bee excluded from theſe rules, whiche bee made peculiarly for numbers that haue their owne proper formes, and depende not of other.

And yet ſome common rule muſt bee giuen, that maie extende as well to them, as to any other.

Wherefore let this be it.

That the two ſides of all *diametralle numbers*, haue ſoche a proportion together, as here you ſee expreſſed in

## *The seconde parte*

in some one of these formes : if thei bee continued as  
here thei be begon .

### ¶ The firste order.

$$\frac{1}{4} : \frac{5}{12} : \frac{7}{24} : \frac{9}{40} : \frac{11}{60} : \frac{13}{84} : \frac{15}{112} : \frac{17}{144} : \frac{19}{180} : \frac{21}{224} :$$

$$\frac{23}{264} : \frac{25}{312} : \frac{27}{360} : \frac{29}{420} : \frac{31}{480} : \frac{33}{544} : \frac{35}{612} : \frac{37}{684} : \frac{39}{760} \text{ ¶}$$

### ¶ The seconde order.

$$\frac{8}{15} : \frac{12}{35} : \frac{16}{63} : \frac{20}{99} : \frac{24}{143} : \frac{28}{195} : \frac{32}{255} : \frac{36}{323} : \frac{40}{399} : \frac{44}{483} :$$

$$\frac{48}{575} : \frac{52}{674} \text{ ¶}$$

Here haue I sette the lesser side as the numerator,  
and the greater side as the denominator . Wherby  
you maie perceiue the cause of their distinction.

For the first order is, when the lesser side, or num-  
ber, is odde.

The seconde order is , when that lesser side is an  
euen number.

*Strifelinus* doeth set them so, that the numerator stand-  
eth for the seconde , or greater side: and the denomi-  
nator for the firste number, or lesser side. And for the  
more delectable contemplation, to behold their forme  
of progression, he setteth doune as many whole num-  
bers, as the fraction will giue.

And this is his forme.

### ¶ The firste order.

$$1\frac{1}{3} : 2\frac{2}{5} : 3\frac{3}{7} : 4\frac{4}{9} : 5\frac{5}{11} : 6\frac{6}{13} : 7\frac{7}{15} \text{ ¶}$$

### ¶ The seconde order.

$$1\frac{7}{8} : 2\frac{11}{12} : 3\frac{15}{16} : 4\frac{19}{20} : 5\frac{23}{24} : 6\frac{27}{28} : 7\frac{31}{32} \text{ ¶}$$

¶ Where

## of Arithmetike.

Where in the first order, you se bothe in the whole numbers, and also in the numerators of the fraction, the naturall order of numbers. And in the denominators, the naturall progression of odde numbers.

But in the seconde order, you see that the whole numbers go in their naturall order, and the numerators and denominators, kepe an *Arithmeticalle* progression, by equall distaunce of .4. saue that in the numerators, all the numbers bee odde: and in the denominators, they be all euen.

Now by this generall rule, if you finde any two partes of any number, in one of these former proportions, you maie bee sure that it is a *diametralle number*. But for the more apte conference of the partes, you shall doe best to reduce them to their least numbers: as you haue learned in the first parte of *Arithmetike*.

So in your last number, whiche was  $43200$ . you shall finde his .180. parte, to bee .240. whiche beynge reduced to their smallest numbers, will bee  $\frac{4}{3}$ . wherefore I am assured, that it is a *diametralle number*.

Let one thyng more shall you marke.

If any number ende in Ciphers, abate euery Cipher, as often as you can (I meane .2. 4. 6. 8. and if the reste be a *diametralle number*, so was the first. And therfore in this last example.  $432$ . is a *diametralle number*, as well as.  $43200$ .

Also if any number beynge diuided by any square number, doe make a *diametralle number* in the quotient, then was the first number a *diametralle number* also.

And this, for this tyme, shall suffice for *diametralle numbers*.

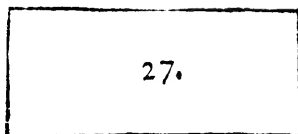
Now will I speake somewhat briefly of *like flattes*: Of like  
and then procede to other *figuralle numbers*. flattes.

Scholar. I remember you defined them before, to bee suche flatte numbers, as had one forme of proportion betwene their sides.

## The seconde parte

As here 27. and 12. be  
like flattes: bicause their  
sides be in one proporti-  
on. For as. 9. is to. 3. so 6 3.  
is to. 2. both beeyng in  
triple proportion.

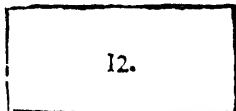
9.



Walter You saie well.

6.

And that is the cause why thei  
be called like: for the likenesse 2  
in the proportiō of their sides.  
Although some menne delite



*Squarelike  
figures.*

more to call them *squarelike figures*: bicause thei haue  
some properties agreable with square numbers ( for  
as *Euclide* saith in his. 8. booke, and. 18. proposition:  
*Euery twoo numbers, beeyng like flattes, haue  
one meane number betwene theim in proporti-  
on. And the one flatte number beareth vnto  
the other flatte double that proportion, that  
their sides doe.*

For declaration of whiche proposition, marke the  
twoo flatte numbers before: I meane. 27. and. 12.  
whose sides are in proportion *Sesquialter*: And the flat  
numbers themselves be as  $\frac{3}{2}$ . or. 9. to. 4: that is double  
*Sesquiquarte*. Now doe you double the proportion *Ses-  
quialter*, and it will make double *Sesquiquarte*.

Scholar. Thus doe I sette them in order.  $\frac{3}{2}$ :  $\frac{3}{2}$ .  
And I multiplie the numeratozs together, and the  
denominatozs also. (For I remember, you tolde me  
before, that proportions are added, as fractions are  
multiplied) and then will it be.  $\frac{9}{4}$ : euen as you saied.

Walter. Again *Euclide* saith in the twentieth pro-  
position of thesame booke.

*If any number stande as a middle number in  
proportion,*

## of Arithmetike.

*proportion, betwene other twoo numbers, those twoo are like flattes.*

That is to saie : if any twoo numbers, beyng multiplied together, doe make a square number (for none but soche can haue a middle number betwene them) then are thei *like flattes*.

As. 3. and. 12. multiplied together doe make. 36. whiche is a square number : and. 6. therby appeareth to bee the middell number betwene them. And therfore are. 3. and. 12. *like flattes*

Likewales. 3. and. 27. for thei make. 81. whiche is a square: and their nuddle number is. 9.

And so are. 2. and. 8: 2. and. 18: 2. and. 50. 2. 4. 72  
3. and. 48: 3. and. 75: 4. and. 9. 4. and. 16: 4. and  
25. 5. and. 20. 5. and. 45: 6. and. 24: 6. and. 54.

And so of infinite other.

This exposition is confirmed by the firste and seconde proposition of the ninth booke of *Euclide*, where he saith thus.

*If twoo numbers beyng like flattes, bee multiplied together, the number that thei make, shall be a square number.*

*And if. 2. numbers beyng multiplied together, do make a square nōber, then are thei like flattes.*

By whiche rules it doeth appere, that you cā haue no *progreſſiō Geometricalle*, but it must be made either of square numbers, or els of *like flattes*, wherby there appeareth a greate agreableness, betwene *like flattes*, and square numbers. And therfore saith *Euclide* also in the. 26. proposition of the eight booke.

*Numbers that bee like flattes, haue soche proportion together, as one square number bea-*

G.ij. reth

## The seconde parte

reth to an other.

This maie you proue by any of the former exam-  
ples. For 12. to. 3. is in like proportion, as. 16. to. 4.  
or. 36. to. 9.

Also. 27. to. 3. hath like proportion as. 36. to. 4: or  
144. to. 16. other. 81. to. 9.

And farther, if you deuide the one of theim by the  
other, the *quotiente* will be a square number.

Scholar. What doeth appeare euidentely at the  
firste betwe.

For 12. diuided by. 3. doeth make. 4. And. 75. di-  
ded by. 3. giueth. 25.

So. 54. by. 6. maketh. 9. And. 72. by. 2. yeldeth. 36.  
And so I see in the reste, that all the *quotientes* will be  
square numbers.

how like flat-  
tes be made.

But I desire moche to knowe, how those numbers  
be produced. For that I knowe not yet.

Master. Take any twoo square numbers, what  
so euer thei bee, and multiplie them by any one num-  
ber, that you list: and thei will make. 2. like *flattes*.

So. 4. and. 9. multiplied by. 2. doe make. 8. and. 18:  
whiche bee like *flattes*.

Again, if you multiplie them by. 5. thei make. 20.  
and. 45. whiche be also like *flattes*.

Scholar. I am perfect mough in this, if that be al.

Master. An other waie you maie make them al-  
so: If you take any twoo square numbers, that will  
admitte one diuisor, and diuide them bothe by it.

As for example. Seyng 9. and. 36. will be bothe di-  
uided by. 3. I doe so diuide them: and their *quotientes*  
are, 3. and. 12. whiche are *diametralle numbers*.

So in like maner, if I diuide 196 and 49 (whiche  
bothe are square numbers) by. 7. the *quotientes* will be  
28. and. 7.

Again, 16. and. 100. beyng bothe square numbers  
and



## of Arithmetike.

and diuided by. 4. doe make. 4. and. 25. as their *quotiente*, and thei be *like flattes*.

Scholar. And in these I see an other straunge worke: that if those twoo *like flattes* bee multiplied together: thei will make the greater square, of whiche thei canre.

For 3. tymes. 12. maketh. 36: and. 7. tymes. 28. giueth. 196: And so. 4. tymes. 25. byngeth forth. 100.

Master. It doeth so happen often times: but it is not allwaies so.

For if you diuide. 16. and. 100. by. 2. the *quotientes* will be. 8. and. 50. whiche twoo numbers multiplied together, doe make. 400. farre differvyng from. 100. So. 36. and. 196. bevyng bothe square numbers, and diuided by. 2. doe make. 18. and. 98. whiche be *like flattes*: and those *like flattes* multiplied together, doe yelde 1764. whiche is a square number, but it is. 9. tymes so greate as is. 196.

Scholar. Yet one doubte I haue: whether all square numbers be *like flattes*, and so bee not distincte from them?

For although in the diuision of figuralle numbers you did distincte them, yet in the examples of *like flattes*, you put certain square numbers emongest other.

Master. All square numbers are *like flattes*, bevyng compared together: and els not. For as any. 2. square numbers maie be compared together: so maie thei be referred to their rootes, without comparison together. Or els thei maie be compared to other numbers that bee not square.

Therefore marke these two rules well. that no one number can bee called a *like flatte*: but in comparison to some other. For. 2. by hymself is not called a *like flatte*, excepte he bee compared to. 8. or to. 18. other to 32. or. 50. or some other soche.

So likewaies. 4. whiche by nature is a square number,  
C. iij. ber,

## The seconde parte

ber, and allwaies shall bee so: yet is it not accepted as a *like flatte*, onles it bee referred to some other square number.

Scholar. What if it be compared with. 12. which you named befoze to be a *like flatte*?

Maister. You remember: one of *Euclide* his rules (whiche I repeated befoze) is, that *like flattes* beeyng multiplied together, will make a square nōber. And sodoeth not. 12. beeyng multiplied by. 4.

Scholar. Now I doe vnderstande your wordes better. So. 3. and. 8. compared together, bee not *like flattes*: yet eche of them compared to other numbers, maie be *like flattes*. As. 3. compared to. 12. or to. 27: and 8. compared to. 18. or to. 50.

*Of rooted  
numbers.*

Maister. Now will we lette these *like flattes* alone for a tyme: And intreate moze of rooted nōbers. And first I will tell you somewhat of the names and natures of soche numbers as haue rootes: Then secondarily I will teache you the order to extract their rootes: And afterwarde will I shewe some parte of the vse of them.

*A roote.*

Wherfoze to begin, where we lefte a litle befoze, the explicatiō of rootes: I saie, that the roote of number, is a number also: and is of soche sorte, that by sondrie multiplications of it, by it self, or by the number resultyng thereof, it doeth produce that nōber, whose rooe it is. And accor dyng to the number of times that it is multiplied, the number that resulteth thereof, taketh his name.

So that one multiplication maketh a *square number* And two multiplications doe make a *Cubike number*.

Likewaies. 3. multiplications, doe giue a *square of squares*. And. 4. multiplications doe yelde a *surfolide*.

And so infinitely.

For as multiplication hath no ende, so the numbers amountyng of them be innumerable, and their  
rootes

## of Arithmetike.

rootes as infinite. But their names thei take certainly, of the numbers that thei doe make.

So the roote of a square number, is called a *Square roote*: and the roote of Cubike number, is named a *Cubike roote*: In like sorte that roote is called a *Squared square roote*, whiche maketh a square of squares in number: And that roote is a *Surfolide roote*, that yeldeth a *Surfolide number*: in whiche sorte of multiplication, you maie procede infinitely, as I saied.

*A square roote.*

*A cubike roote.*

*A squared square roote.*

*A surfolide roote.*

Notwithstanding for your ease, I haue set forth here in a table, certain of the most notable kindes of rooted numbers.

And to the intente you maie partly conceiue the reason of their names, I will after the table, set forth a brief explication of their names, with the portraiture of the figures, that thei doe resemble in multiplications *Geometrically*: where pointes, lines, platte formes, or soundformes be multiplied: and bynge forth the other formes agreeable to suche multiplications.

But first marke the table well: And it will giue you greate lighte, and aptnesse to vnderstande all that foloweth, moche the better.

For examples are the  
lighte of teaching.

The

*The vulgar  
names.*

*The table of rooted numbers.*

*The authors  
names.*

1. Rootes.	2	3	4	5	6	7	8	9	10	Rootes.
2. Squares.	4	9	16	25	36	49	64	81	100	Squares.
3. Cubikes.	8	27	64	125	216	343	512	729	1000	Cubikes.
4. Squares of Squares.	16	81	256	625	1296	2401	4096	6561	10000	Longe Cubes.
5. Sunfolides.	32	243	1024	3125	7776	16807	31768	59049	100000	Squares of cubes
6. Squares of cubes	64	729	4096	15025	46656	117649	262144	531441	1000000	Cubike Cubes.
7. Seconde Sunfolides.	128	1187	16384	78125	279936	823143	2897152	4782969	10000000	Longe Cubike Cubes.
8. Squares of square red squares.	256	6561	65536	390625	1679616	5764801	16277219	43946721	1000000000	Squares of Cubike Cubes.
9. Cubes of Cubes.	512	19683	163144	1973725	10077696	40353607	134217728	387410489	10000000000	Cubus of Cubike Cubes.
10. Squares of Sunfolides.	1024	59049	163876	9765625	60466176	281475249	107374824	348678401	100000000000	Longe Cubes of Cubike Cubes.

## of Arithmetike.

Here you see diuerſe rowes of numbers , and againſt euery rowe twoo names witten : one on the right hande, and the other on the leſte hande, whiche ſerue for all the numbers in that rowe.

The names on the leſte hande bee thoſe names, whiche bee commonly vſed , and attributed to thoſe numbers.

The names on the righte hande, are names of my addition , whiche doe aptly expreſſe the very natures of the numbers, vnto whiche thei bee aſſigned : as a none I will declare.

And now concerning the numbers , you ſee firſt in the hedde of the table, a rowe of numbers ſet in order, as thei followe in common nombryng, from one forward. And thei bee called rootes, for that the multiplication of eche of them, by theiſelves, or by that, that thereof amounteth, byngeth forth the all thother, that bee ſet vnder them. Of the whiche, the ſeconde rowe is called *Square numbers*: becauſe that their length *Square* and their breadth (whiche I vnderſtand by the .2. numbers of their multiplication) is equalle.

As .2. tymes .2. doeth make .4. whiche is a ſquare number, and maie bee figured thus.  $\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$

Likewaies .3. tymes .3. maketh .9. whiche is a ſquare number, and is repreſented thus.  $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$

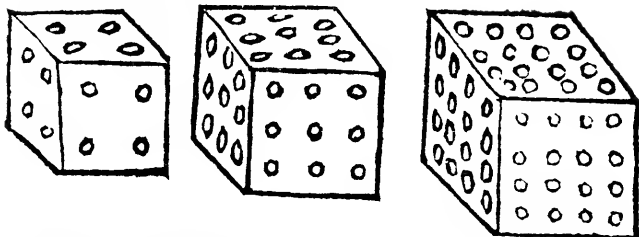
And here you ſe, that if you diuide the *Square number* by his roote, the *quotiente* will be the ſame nōber alſo.

Scholar. That muſt nedes be ſo.

Maſter. Then in the thirde rowe are placed *Cu Cubike bike numbers* : whiche are produced by triple multiplication. As .2. tymes .2. twiſe, maketh .8. And .3. tymes .3. thriſe, yeldeth .27. So .4. tymes .4. ſower tymes, giueth .64. Theſe numbers can not be expreſſed aptly in flatte, but proſpectiuelly, as Dice maie be made in portraiture.

## The seconde parte

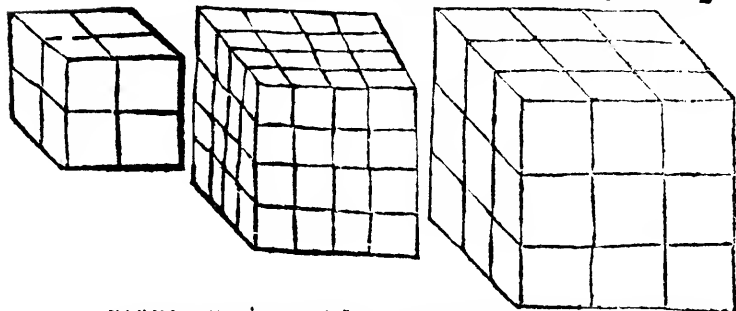
And these are their foymes.



In the firste figure you see . 2 . expresse in lengthe bredthe, and depth. And in the second foyme, 3. is represented in all those, 3. dimensions. In the, 4. figure 4. is the roote, and is drawen agreeably to that foyme.

Scholar. This is manifeste inough to sighte.

Master. Yet reason ought to waigh it more exactly, then sight can comprehend it. For as their triple multiplication doeth resemble the nature of sounde bodies, so it might appeare more iuste expresseing of their figures, agreeably as sounde bodies ought: in whiche euery parte can not appeare to sighte, sith diuerse of them loke inwardly. As by these, 3. laste fig-



res you maie partely coniecture. Of whiche at this tyme and in this place, some men will thinke it an oversighte to speake, and moche more oversighte to write of them any thyng largely. Saue, that we maie vse theim for the apter explication of that triple multiplication,

## of Arithmetike.

ttiplication, wherby thei be made.

So that as it is multiplied thise, so the number that doeth amounte thereof, hath gotten. 3. dimens-  
ones, whiche properly belongeth to a bodie, or sound  
forme. And therefore is it called a *Cube*, or *Cubike number*  
Whiche number if you diuide by the roote, the *quoti-*  
*ent* wil be the square of the same roote. As I said afore.

But to procede, if you doe multiplie that *Cubike*  
number by his roote, the number that riseth of it, is  
called a *Square of squares* commonly: bicause that not *Squares of*  
onely it is a *Square number*, but the roote of it also is *Squares*.  
a *Square number*. As you maie perceiue by examina-  
tion, of all those numbers that be in the fourth rewe,  
whiche numbers I doe call *longe Cubes*: bicause thei *Long Cubes*.  
make a line of Cubes. And hath in lengthe so many  
Cubes, as the firste roote doeth containe vnties.

This line of Cubes, although it haue for his  
bredthe, and depthe also, the thickenesse of one Cube,  
yet bicause it hath no number of Cubes, in bredthe,  
nor in depthe (or generally no number of that thyng,  
whereof it is called a line) therfore maie it tollerably  
beare the similitude and name of a line. And so doe  
we commonly call lines, those smalle cordes, whiche  
are onely long, and haue litle bredthe to their length.  
But yet are thei not without all bredthe.

Scholar. And thereof (I thinke men call a line of  
Bricks, and a line of Asphclers stones, when many  
bee laied in a rowe, in lengthe: and but one (or fewe)  
in bredthe.

Master. You saie truthe. And that name doeth  
continue still, emongest all our countrie menne: saue  
that mosse menne doe not call it sharply a line, but  
more broder (after tholde Englishe language) a *laine*  
And so men vse to saie, a *laine* of wine buttes, and a  
*laine* of brode clothes: and soche other like.

And vse hath so largely applied this name, that it  
U. ij. maie

## The seconde parte

make seme no greate absurditie, to name any thyng  
a line or laine, that hath moche moze lengthe then  
bryethe: and is made by often addition, or multipli-  
cation of any one quantitie. But yet for auoidyng of  
erroure, it ought to bee limited, whereof that line is  
named. As in our mater to saie, a line of vnities: a line of  
Cubes: a line of Cubike Cubes: and a line of Cubike Cubes Cu-  
bikely: and so forth.

In likewates must we iudge of platte formes, that  
they haue no depthe or thickenesse. When one nom-  
ber is multiplied by an other, onely twise: that is to  
saie, in bryethe and lengthe onely: and is not multi-  
plied the thirde time by any number, to make it beare  
depthe.

And this must be considered generally, though the  
number so multiplied bee a Cube, or any other sounde  
nōber. For in soche case, that Cube, or sounde number,  
what so euer it be, standeth but as an unitie.

Scholar. Sir, I doe very well vnderstande the  
meanynge, and reasonablenesse of those names, line,  
and square, in any thing. But I knowe not those ter-  
mes, Cubike Cubes, and Cubike Cubes Cubikely. Although  
I se them set in the table, whiche you haue giuen me.

Master. No more then doe you vnderstande di-  
uerse other names there, whiche I will therfore de-  
clare vnto you.

If you agree to the vse of the name, of a line and a  
square, in that sorte that you haue consented vnto:  
then if I multiplie a Cubike number by his roote.

As to saie. 8. by. 2. or. 27. by. 3. other. 64. by. 4. then  
shall I haue a line of Cubes. whiche I doe therfore  
call longe Cubes: but commonly they bee called Squared  
Squares, or Squares of Squares: and of some men they are  
named Zenzizenzikes, as square numbers are called  
Zenzikes. Whiche name although in sounde bodies,  
it hath no vse, yet in practice of sounde numbers, it

maie

Squares of  
Squares.



## of Arithmetike.

maie and doeth expresse some properties aptly. As namely that all those numbers, whiche rise of 4 multiplications, maie be as well made by twoo multiplications. But then the roote of that multiplication shal be a square number also.

Scholar. So I vnderstande that. 16. is a number of that sorte, which here is called *Square of squares*. And yet maie it bee called a square number: and is so in deede, in comparison to. 4. And therfore, I perceiue, it is set twise in the table: ones emongest square numbers, vnder 4 whiche then is his square roote: And again it is set emongest *squares of squares*, vnder 2 which in that place standeth as his squared square roote.

Likewates. 64. is twise set in thesame table, ones emongest *squares*, vnder 8. whiche is his square roote: And again emongest *Cubike numbers*, vnder. 4. whiche is his Cubike roote.

Master. You saie truthe. Although the lasse example be not to your purpose, concerning *Squared squares* or *Zenzizenzikes*. And if you did note it onely, for this cause it is twise set in the table: then maie you see it thrise sette in thesame table, for it is in the sixte rewe vnder. 2.

Scholar. So I see, wherfore I might rather haue take. 81. whiche is a *Zenzizenzike number*, and so hath for his roote. 3: And also it is a square number, and hath. 9. for his roote.

Master. Farther to procede, if I multiplie those *squares of squares* by their roote, thei will make *Surso: Surfolides*, *lide numbers*.

Scholar. I perceiue by the numbers in the table, that you meane the leaste roote of the twoo: bicause vnder. 16. I see. 32. in the rewe of *Surfolides*.

Master. Reason maie driue you to thinke so. For the number and his roote, muste beare alwaies one name. So that if I name. 16. as a square number, I

must

## The seconde parte

must referre it to his square roote. And if I name it as a *Zenzizenzike number*: it muste bee referred to his *Zenzizenzike roote*. And in like sort of al other names.

As when I call. 64. a square number, & demaunde what is his roote: you muste nedes aunswere by his *Square roote*, whiche is. 8. But if I name. 64. as a *Cube*, and doe then seke for his roote: you must vnderstande his *Cubike roote*, and that is. 4. But if I name it to bee a *Square of Cubes*, or *Zenzicube*: then is. 2. his roote. As you maie by the table perceiue. And also by the orderly multiplication of euery rewe, or order of numbers by their roote. For therby amounteth the nexte rewe.

And so maie you increase the numbers of those rewes, or orders, accordyng to the tymes of your multiplication, as moche as you list. And euery order shall beare soche names, as agreeth to the nature of their rootes.

Wherefore thei appeare to bee ouersene, that call those formers numbers *Surdesolides*, seing thei are not any waies *Surde numbers*, but haue their rootes. And yet, to confesse the truthe, I cannot well tell you the true *etymologie* of their name: except thei be so named, as it were *solide* vpon *solide*. And that interpretation were to straightly racked. But the name beyng receiued and well knowen, wee maie moze easily with libertie vse it, then with scrupulositie, curiously scā it.

These numbers are simple numbers in their kind. For thei rise of. 5. multiplications. And if their roote bee a digite number, then is it the same number, that standeth in their firste place. And if their roote be an article, then hath that *Surfolide*. 5. tymes so many *Cyphers* together in the firste places, as his roote hath: and the nexte figure after those *Cyphers*, is the firste figure signifiatiue of his roote.

Scholar. I see it so in all these numbers, that bee  
in

## of Arithmetike.

in the table.

Maſter. And ſo ſhall you finde it in all others.

And farther if the roote bee a number mixte, then the firſte number of the *ſurſolide*, is the firſt number of the roote. And this I doe tell you for ſome helpe, in geſſyng at their rootes.

This name therfore of theſe, I meane *Surſolides*, in *Arithmetike*, maie ſerue to admoniſhe you of their roote. But in *Geometrie*, or in compoſition of ſounde bodieſ, it ſerueth to no uſe: and therfore I doe call the agreeable to their figure, *Squares of Cubes*: becauſe thei make a ſquare forme: but ſo that euery unitie of that ſquare, is in it ſelf a *Cube*: As by the figures that fol-  
*Squares of Cubes.*  
lowe, you maie well coniecture.

And alſo thei are made by multiplication of a *Cubike number*, and a *Square number* together, bothe hauyng one roote: and the *Surſolide* hauyng the ſame roote. Wherefore reaſon with the nature of their ſounde figure, inſoꝛceth me to call the *ſquares of cubes*.

Yet other menne attendyng more to the nature of their rootes, then to their owne formes and nature, doe giue that name to the next reſe of numbers, becauſe thei maie be made of multiplication, of any *Cubike number* by it ſelf, that is to ſaie ſquarely.

Scholar. It is ſo. For 8. whiche is a *Cubike number* multiplied ſquarely maketh. 64. And that. 64. is ſet amongeſte the *Squares of Cubes*.

Maſter. And this commoditie commeth by that name: that it putteth menne in remembraunce of the ſpedie and eaſie extraction of their roote: As you ſhall learne hereafter.

But I conſideryng their owne nature and mixtynge, as ſounde numbers or bodie�: doe call theiſe *Cubes of Cubes*, or *Cubike Cubes*.

After theſe numbers in the ſeuenth reſe, there do followe thoſe numbers, whiche commonly are called  
*Surſolides*,

## The seconde parte

*Seconde  
surfolides.*

*bsurfolides, or bisurfolides, that is, seconde surfolides, or double surfolides. But I maie call them seconde squares of cubes, alludying at the same name. Howbeit if I looke to their forme and nature, I shall be enforced to call the, longe cubes of cubes, or longe cubike cubes.*

*Squares of  
squared  
squares.  
Cubes of  
Cubes.*

*And so by like reason, doe I cal the nexte numbers square cubes of cubes, or square cubike cubes: whiche other men doe cal *zenzizenzikes*, that is squares of squared squares.*

*The ninth rewe of numbers, is commonly called Cubike Cubes, or Cubes of Cubes: bicause the Cubike rootes of those numbers are Cubike numbers also. But I after their true nature, doe call them Cubes of Cubes Cubikely: or Cubes of Cubike Cubes.*

*Squares of  
Surfolides.*

*The tenth rewe of numbers is named vulgarly, Squares of surfolides, bicause thei haue a Square roote, whiche is of it self a surfolide number. And for their figure Grometricalle, I name the longe cubes of cubike cubes.*

*So that I considering their nature, that thei be figuralle numbers, am constrained to name them, according to their figure, I meane in this place, where I doe make explication of their natures and names.*

*But other men for aide of woork, in extraction of rootes, haue giuen them soche names, as maie beste put minne in remembraunce of redy woork therein. Whiche names I will vse also hercafter, in my wrytynges, bicause I will not bee an authoꝝ of vnnecessary singularitie. And yet bicause truthe in nature is as well to be regarded, as ease in woorkyng, and rather moze, I could not omitte in this place, the declaration of their true nature and very formes.*

*And so bothe of vs hauyng good reasons, for those names, neither maie contempne other, neither contende together.*

*A generall  
reason for na*

*And although the names that I doe giue, maie seme to some minne ( whiche are scarce apte iudges) moze*

## of Arithmetike.

more obiousse, for the newe inuention (as thei maie <sup>mes of these</sup> think) then needfull to the practise of tharte: yet shal <sup>numbers.</sup> you see in theim a naturall sequele, and orderly progression.

For all those numbers are considered, in one of 2. soymes firste. That is to saie, other thei bee taken as numbers absolute, without any consideration of multiplication: And so thei maie be named numbers onely, without name of relation. Or els thei bee considered as numbers multiplied, and that can be but in 3. varieties.

If thei be multiplied but ones, then doe thei make a line of numbers, or a linitarie number. And that number hath onely lengthe, without bredthe, or depth: And therfore maie be the roote to a *Square*, or a *Cube*. But is of it self, in that consideration, nother *Square* nor *Cube*.

Secondarily, it maie bee multiplied twise, the one number stadyng for the lengthe, and the other for the bredthe: and so is it a *Square number*, and therfore a *flat number*.

Thirdly, it maie bee multiplied thrise, and therby gette lengthe, bredthe, and depth: wherby it is made a *sounde number*. And because the sides bee equalle, it is specially a *Cube* or *Cubike number*.

Now can there be no sowerth waie, that any multiplication maie increase: for there are no more dimensions in nature.

But if any manne doe multiple the fourth time, then must he accompte that he maketh a *line of Cubes*: and the fifth multiplication maketh a *Square*, in which euerie unitie is a *Cube*: So the sixte multiplication maketh a *Cube of Cubes*, accomptyng euerie lesser *Cube* for an unitie. And there is a staie again.

Wherfore if any man multiplie the seuenth time, he retourneth againe to the firste nature of numbers

3.). multiplied,

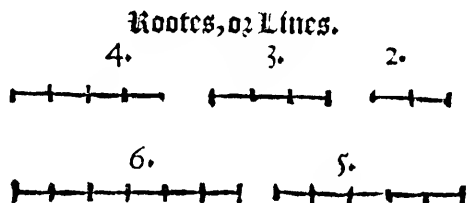
## *The seconde parte*

multiplied, whiche are *liniarie numbers*: And the 8. multiplication, woorketh as the seconde did, and maketh *flatte numbers*. The ninth multiplication agreeably with the thirde, doeth make *Cubes*.

And so infinitely these. 3. woorkes maie bee reiterate, but a fourthe forme can neuer be deuised.

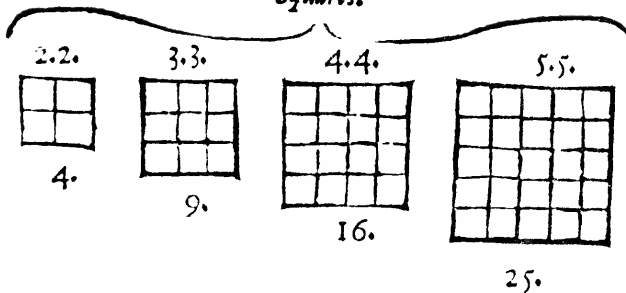
And therefore doe I, as reason doeth compell me, reduce all numbers to those. 3. formes, as their verie originall springes and fountaines.

But to the intente that you maie the more aptly iudge of theim, and their natures, I haue here sette foorth the formes, whiche thei make in figures *Geometricalle*, or *sounde quantities*. Admonishyng you to remember this well. That after any number is become a sounde number, it is against reason, to reduce him to an absolute flatte number again, and mosse of all by multiplication. But now marke these figures.

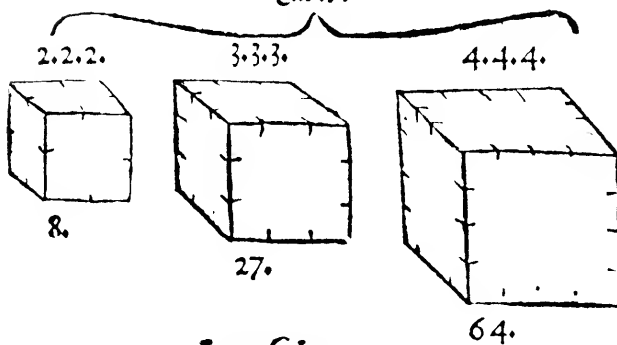


# of Arithmetike.

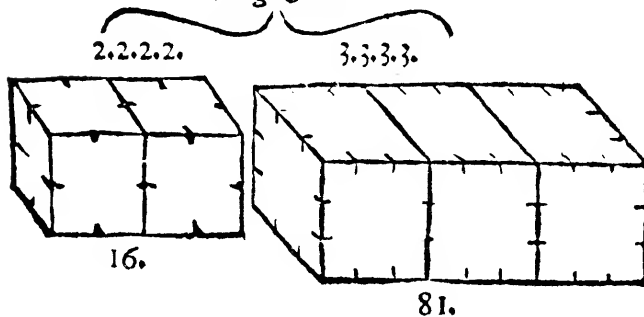
## Squares.



## Cubes.



## Longe Cubes.

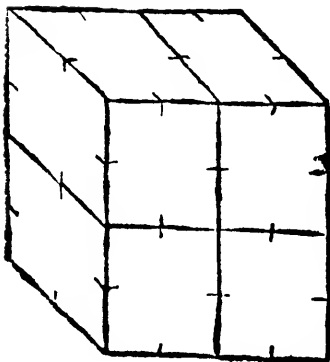


3.4.

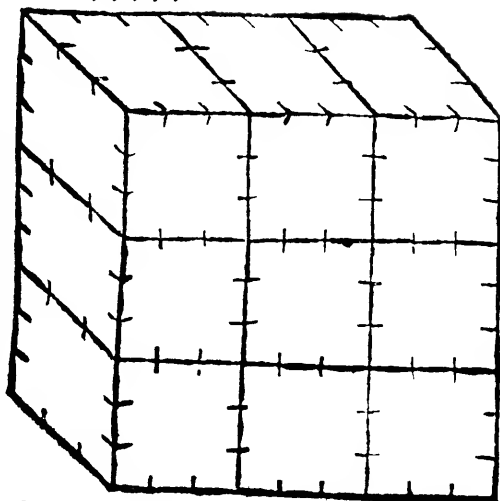
Squares

*Squares of Cubes.*

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

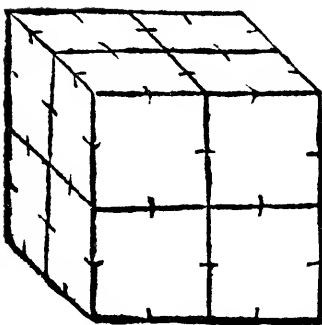


$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

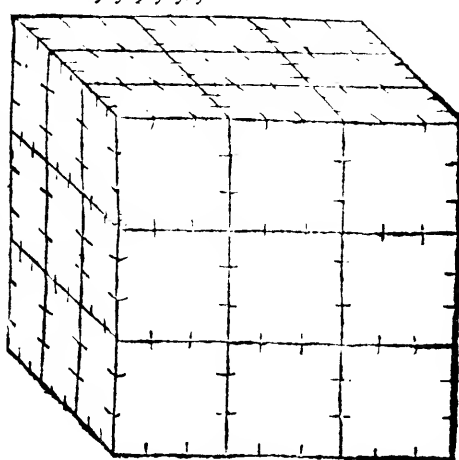


*Cubike Cubes.*

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$



$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$



Here



## of Arithmetike.

Here, as you see, I haue set first certaine lines, containing soche partes as thei bee made of by multiplication: that is to saie, 2. 3. 4. 02. 5. And these bee produced by the first multiplication, where an vnitie of any thyng is multiplied by a number.

And so an inche multiplied by 3. maketh. 3. inches: And a foote multiplied by 6. maketh. 6. foote: and so of other measures and quantities, in like sorte. All whiche multiplications, doe make onely longe lines, or measures in lengthe onely, without bredth or thickenesse.

And in this multiplication, nother the number, nother yet the vnitie, is accounted or called a roote. But the line that is made thereby: maie bee a roote to any of all the other kinde of numbers before rehearsed, and sette forth in the table. For if you multiplie the same line, by the number that his lengthe doeth include, then there will be made thereof, by this seconde multiplication, a square figure, containing a square number in it: As you see amongst those figures, the firste folow to be, whiche are marked with these numbers. 4. 9. 16. and. 25.

Scholar. I perceiue well in eche of the, that their lengthe is agreable with their bredthe, and so thei make square figures, but I knowe not what those numbers doe meane, that be set ouer their heddes.

Master. The quantitie of the number, doeth betoken the value of their roote. And the multitude of the same number repeted, doeth declare the number of multiplications, for eche figure.

And therefore the lines, whiche are made by one multiplication, haue eche of them their number simply set, ones onely.

The squares haue their numbers double: in token that thei haue. 2. multiplications. That is, one in lengthe, and an other in bredthe.

## The seconde parte

The third formes, whiche be *Cubes*, and are made of. 3. multiplications, haue their roote repeted thriſe.

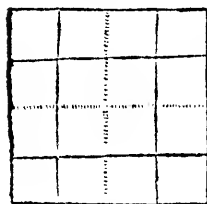
And the like numbers did I ſette, in the ſide of the former table, againſt the like quãtities. Whiche ſhall helpe you ſomewhat in the extraction of rootes.

Scholar. Now doe I perceiue not onely their names, and multiplications, moche better then I did before: but alſo I vnderſtande better the difference of your names, and their reaſons. For by thoſe figures, whiche you haue ſet in the ſowerth place, and doe call them *longe Cubes*, I ſee their forme doeth agree to that name. For thei are longer, then thei are other brode or depe. And ſaue for their depth, I might liken theim to *longe Squares* in *Geometrie*. Howbeit, other men neglecyng their forme, and looking onely to their rootes, doe call them, *Squared Squares*.

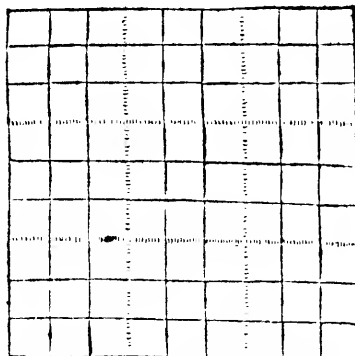
But if you will permitte me, to ſpeake in the defence of theim, as a ſimple ſcholar maie ſpeake for aſſeſſion, in the defence of his maſter, it appereth to me, that thei maie well bee called *Squared Squares*; and might be figured thus.

2.2.2.2.

3.3.3.3.



16.



81.

Where the ſmalleſt ſquares, whiche be contained within the prickled lines, beyng taken as rootes, and multiplied

## of Arithmetike.

multiplied by the same number again, whiche thei do containe ( other els twice by their rootes ) will make the whole greater Squares.

And by this figuring of theim, there doeth appere no inconuenience nor absurditie, in their vulgar names : but rather a iuste expresseing of their naturall formes.

For in the first figure. 2. standyng as the side of the lesser square, and multiplied by it self, doeth make. 4. whiche is the quantitie of the lesser square. Then if I multiply that lesser square. 4. by his owne number, it maketh 16. whiche is the greate and whole square: and is a *Square of squares*.

So in the seconde figure. 3. standeth for the roote of the lesser square, contained within the pricked lines, and if it bee multiplied by it self, it maketh. 9. whiche is the quantitie of the same lesser square. Then if I multiplye that. 9. by it self, it will make. 81. whiche is the quantitie of the greate Square, and is a *Square of squares*.

Master. I commende you well : not onely for so diligente excusyng of theim, whiche for their honeste trauell, deserue moche thankes, but also for that you seke to byyng manifest reason, and some the we, at the least, of linearie demonstration for your purpose. So that you will not seme to speake, without some good grounde.

But as in deece, your figure doeth truely expresse a square of squares, so it doeth suppose the other number, whiche by order of multiplication, doeth go next before it, to be a flatte number also. For it is not possible that a sounde number (as a *Cube* is alwaies) being multiplied by any other number, maie lese the nature of a sounde number : But shall continue a sounde number still. And therfore seeyng the nerte number, before a *Square of squares* was a *Cube*, it is not possible

## The seconde parte

possible that a *Square of squares* can be a mere *flatte number*, as you haue drawen it.

¶ therfore if thei had intended, that a *flatte number* should occupie the .4. place, then should thei haue set some *plat forme* in the third place also. ¶ hiche might haue been made in this sorte.

And then will it be a *longe Square*, and not a *Cube*.

3.3.3.


But in as moche as thei doe not admitte this *longe Square* (whiche by that name hath no roote) therfore maie not the number that soloweth it, bee any other then a *founde number*. For euery *Cubike forme*, beeyng multiplied by his roote, doeth make a *Square pillar*. Whose length beareth vnto his bredth the same proportion, that his roote doeth vnto an unitie.

Scholar. I am very well satisfied now: concerning the names and formes of those numbers. And by this that you haue saied, I doe farther perceiue, that .5. multiplications doeth make the *square of Cubes*, whiche be set in the fiftie place, emongeste the former figures. And also I vnderstande by the former table, that thei be called *Sur-solides*.

Likewaies I see in the sixte place of the foresaied figures, *Cubike Cubes*, made by .6. multiplications. But commonly the numbers of those quantities, be named *Squares of Cubes*. So that for their names, thus farre I am perfecte inough.

The

# The extraction of Rootes. Pastter.



**N**owe will I shewe you, *The extraction of rootes* how you shal extract the roote out of any soche number.

And first I must admonithe you, that you shal alwaies vnderstande, soche a roote, as the number doeth admit. So that in a square number, you shall seke a *Square roote* onely, and

no *Cubike roote*, nother any other kinde.

Likewaises a *Cubike number* hath no other roote, but a *Cubike roote*. Excepte the namebee compounde, as *zenzicubike*, or *Squared Cube*. For in soche there are 2. sortes of rootes, accordyng to the 2. names that they beare. That is bothe *Square* and *Cubike roote*: as I will anon shewe you. But firste I will beginne with *Square numbers*, and their rootes.

And this generall order muste you obserue, befoze all other: That you shall haue by harte, in readie memorie all soche numbers, whose rootes are digites. For as it is superfluous to seke rules for them, so must they helpe in all greater numbers, whose rootes are aboue 9. And for youre ease in remembraunce, I haue here sette forth a table for square numbers.

Where in the firste columpne, you se the rootes set, and in the seconde pillar, right against eche roote, there is set his square. Touchyng whiche I nede to saie no more, but that you be not in any vncertaintie of them, whē

*The table of  
Square rootes vn-  
compounde.*

Rootes.	Squares.
1.	1.
2.	4.
3.	9.
4.	16.
5.	25.
6.	36.
7.	49.
8.	64.
9.	81.

## The extraction

you shall neede their aied, whiche shall be continually in vse of searching for other greater rootes.

Now for greater numbers, this is the order.

1. First set doune the number as it is. Then sette a prick under euery odde place, I meane the firste, the thirde, the fift, the seuenth, and so forth: and so shall euery prick haue .2. numbers, excepte the laste, whiche some tymes hath but one.

2. Secondly, marke the numbers that belong vnto the laste prick, toward the lefte hande: And whether he haue belonging to it one number, or two, looke what the roote maie be of that number, if it bee square. And that roote sette by a crooked line, as you place the *quotiente* in diuision: & cancell all that square number, belonging to that prick.

3. But and if the number belonging to that prick, bee not a square number, then take the roote of the greatest square, whiche is contained in it, and place the roote as I saied before. And the square of it shall you abate from the number, that belongeth to that laste prick, and let the rest be set ouer those numbers cancelled, as you doe in diuision. And so haue you ended your worke for that prick.

Scholar. This moche is easie inough, if I vnderstande you rightly.

Master. Then proue it in a number, or two. And first worke with this number. 5152900.

Scholar. I muste marke euery odde place with a prick, thus.

And here I perceiue that vnto the first prick, there belongeth 2 Cyphers only, and to eche of the other .2. prickes folowynge, there are appointed. 2. figures. But the fourth prick hath but one number, and that is .5.

Now according to the second rule, I seke the roote of 5. (for bicause there belongeth no more numbers to that

## The extraction

that pycke) and I see, it is no square number. Wherefore according to the thirde rule, I take the greatest square in it, whiche is . 4. and the roote of . 4. is. 2. Therefore I doe subtracte . 4. out of . 5. | 1  
and cancell that . 5. and the . 1. that remaineth, I set ouer . 5. as here you see. | 5 152900 (2.  
And the roote. 2. I sette behinde the *quotiente* line, as you taught me, and then the nōbers stand, as you se.

Master. You haue doen wel. Proue again in this number. 18766224.

Scholar. First I set them doune | 18766224.  
and pycke them, as here doeth appear. And now I see, that the laste pycke hath twoo numbers belongyng to it, that is. 18. with whiche I must begin. And seying it is no square number, I find 16. to be the greatest square in it: wherefore I subtract 16. out of 18. and set. 2. ouer the . 8. 2  
And the roote of. 16. whiche is. 4. | 4 18766224 (4.  
I sette behinde the *quotiente* line, as here is seen.

Master. This maie suffice for the first wooyke.

Now to procede, you shall double your roote, and put that double vnder the nexte space, towarde the right hand, that is behinde the nexte pycke. Alwaies forseyng, that if the double doe contain moze figures then one, that the first shall be sette vnder that place, and the seconde vnder the nexte figure, towarde the left hande. 4.

Then seke a *quotiente*, as you doe in diuision, whiche shall shewe how often that double number maie be found in that, that is ouer it, appertainyng to that place: whiche *quotiente*, you shall set before the firste roote, within the *quotiente* line. 5.

But this regarde muste you haue here specially, that you maie leaue ouer the nexte pycke, toward the right hande, as moche as the square of that *quotiente*,

h. v. with

## The extraction

With which you worke, for out of that rest, the square of that *quotiente* muste bee abated. And then make bothe subtractiones, and note the remainder, if any be, and place your *quotient*, and then haue you doen with that prick also.

For the more plaines, I will giue you an example in your firste number, whiche stood thus, after your worke was ended.

<p>Here I ce over the laste prick saue one. 1 1 5. vnder the middell fi- gure of whiche I must set the dou- ble of the former roote. 2. that is. 4. And then I seke how often. 4. is to bee founde in. 1 1. And I finde that I maie haue it two tymes, and. 3. remainyng. Whi- che. 3. with. 5. over the next prick, doe make. 35. and that is more then the square of my <i>quotiente</i>. 2. Ther- fore am I bolde to sette doune that <i>quotiente</i>: And acco:dyng to it, to a- bate twise. 4. (whiche is. 8.) out of 1 1. and there resteth. 3. Therfore I cancell. 1 1. and sette. 3. over it. Then doe I multiplie the laste <i>quotiente</i> squarely: and it maketh. 4. whiche 4. I subtracte out of the number over the prick, that is. 35. where. 5. maie suffice for this number. Ther- fore I abate. 4. out of. 5. and cancell that. 5. and set. 1. whiche remaineth, over the. 5: And then will the whole number stande thus.</p>	<table style="border-collapse: collapse; margin: 0 auto;"> <tr><td style="text-align: right;">1</td></tr> <tr><td style="text-align: right;">5 152900 (2.</td></tr> <tr><td style="text-align: right;">4</td></tr> <tr><td style="text-align: right;">13</td></tr> <tr><td style="text-align: right;">5 152900 (22.</td></tr> <tr><td style="text-align: right;">4</td></tr> </table>	1	5 152900 (2.	4	13	5 152900 (22.	4
1							
5 152900 (2.							
4							
13							
5 152900 (22.							
4							

This worke, whiche I haue  
wrought now, must be repeted as often as there bee  
any prickes, or pricked numbers remainyng. Wher-  
by you maie easily gesse, that it must bee twise more  
repeated in this example, bicause there resteth yet. 2.  
prickes vntouched.

Scholar. Although I thinke, I could doe, as I  
haue marked you to doe, yet for more certaintie I  
pate



## of Rootes.

praie you worke out this example.

Master. Then marke it well.

I shall begin againe with doubling of all, that is within the *quotiente* line. And that double is 44. whiche I must set vnder. 312. that remaineth of the laste worke. And then will the nombers stande, as here you see.

<p>Then I loke how often tymes maie I finde. 44. in. 312. And I see it will be abated 7 times, and 4 remain: whiche 4 with the. 9. ouer the next prick doeth make. 49. And that will suffice to extracte the square of my <i>quotiente</i>. 7. For. 7. tymes. 7. maketh iuste. 49. Thus seying I maie take. 7. for my <i>quotiente</i>, I worke with it, as the rule teacheth: abating first. 7. times. 44. (that is. 308) out of. 312. and there resteth. 4. ouer the space before the next prick. Whiche. 4. with. 9. ouer the prick doe make. 49. out of whiche I abate the square of my <i>quotiente</i>. 7. (that is. 49.) and so resteth nothing, but. 2. Cyphers. And the number standeth thus.</p>	<table border="0" style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">131</td> <td style="padding-left: 5px;">152900 (22.</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">44</td> <td style="padding-left: 5px;">44</td> </tr> </table>	131	152900 (22.	44	44
131	152900 (22.				
44	44				

And seying there remaineth one prick vntouched, I should repeat the same order of worke againe, by doubling all the *quotiente*, whiche would bee. 454. and setting it so that. 4. whiche is in the first place, should be sette vnder the Cypher, that is without the prick, and the other figures in order, toward the left hand. But all this worke were in vaine, seying there is nothing lefte, to serue for the subtraction.

Yet because there is lefte one picked place vntouched, I must set for it a Cypher in the *quotiente*.

For this rule is generall: that how many prickes so euer your square number doeth containe, your *quotiente*, or roote shall haue so many numbers. Wherefore this roote must be made by thus. 2270.

R. 14.

And

## The extraction

*The prooffe.*

And so it appeareth that your number. 5152900. is a iuste square number. Whiche you maie proue by the orderly prooffe of extraction of rootes. That is to multiplie that *quotiente*, or roote (whiche you haue founde) by it self. And if it doe make the first number exactly, then haue you wrought well.

Scholar. That prooffe is as certaine, as can be. And therfore I will proue, whether it will agree with this worke. Wherfore multipliyng 2270. by it self, I see that it yeldeth the firste somme. As here it doeth appeare. So is this worke approued good.

$$\begin{array}{r}
 2270. \\
 2270. \\
 \hline
 158900. \\
 454 \\
 \hline
 454 \\
 \hline
 5152900
 \end{array}$$

And now will I attempte the like worke in the seconde example. Whiche was. 18766224.

But after the firste worke was ended, and the greatest square subtracted out of 18. it did remain in this forme.

$$\begin{array}{r}
 2 \\
 18766224 \text{ (4.}
 \end{array}$$

Now to continue the worke as you did, and as the rule doeth teache, I must double. 4. which is the roote, and standeth by the *quotiente* line: and must set it vnder. 7. that standeth in the space, betwene the laste prick (whose worke is ended) and the nexte prick towarde the right hande. And then will it stande thus as you see.

$$\begin{array}{r}
 2 \\
 18766224 \text{ (4.} \\
 8
 \end{array}$$

That doen, I must seke a *quotiente*, that maie declare how often

8. maie bee subtracted out of. 27. and that *quotiente* I finde to be. 3: because that after I haue taken. 3. tymes 8. (that is. 24. out of. 27. there will remain. 3. which is 3. with. 6. that standeth ouer the prick, doe make. 36. And I see that number to be greate enough, for the abatemente of the square of my *quotiente*: which is but. 3. tymes. 3. that is, 9.

Wherfore

## of Rootes.

Therefore I sette downe. 3.  
for my *quotiente*, before . 4. in  
the *quotiente* line. And multi-  
plyng 8. by that 3. there riseth  
24. whiche I doe subtract out  
of. 27. that is ouer. 8. and there will remain. 3. That  
3. with. 6. ouer the pücke, maketh. 36. out of whiche  
I must abate. 9: whiche is the square of my *quotient*. 3.  
and so will there reste. 27. ouer that pücke.

$$\begin{array}{r} 2 \\ 2 \ 3 \ 7 \\ 18 \ 7 \ 6 \ 2 \ 2 \ 4 \ (43. \\ 8 \end{array}$$

And thus haue I ended. 2. pückes, and yet. 2. more  
doe remain: in whiche bothe I must repeate the same  
forme of worke.

Therefore I double the whole *quotiente*, and it ma-  
keth. 86: whiche I set vnder. 276.

And then I seke the *quotiente*, declaring how many  
tymes. 86. may be abated out of. 276. whiche may  
be. 3. tymes. And for that cause I set. 3. in the *quotiente*  
before the. 43.

Then doe I firste multi-  
plic. 86. by that. 3. sayng. 3.  
tymes. 8. maketh. 24. which  
I abate out of. 27. and there  
resteth. 3. And again I saie,  
3. tymes. 6. is. 18. whiche I  
abate out of. 36. and there doeth remain. 18.

$$\begin{array}{r} 1 \\ 2 \ 3 \ 7 \\ 2 \ 3 \ 7 \ 8 \ 3 \\ 18 \ 7 \ 6 \ 2 \ 2 \ 4 \ (433. \\ 8 \ 6 \end{array}$$

That doen, I take the square of my *quotiente*, that  
is. 9. whiche I doe subtract out of. 12. (for the. 2. ouer  
the pücke must borrowe. 1. of. 8.) and then will there  
remain ouer that pücke. 173.

And thus is that pücke ended.

Now, for the laste pücke in worke, though he be  
firste in place. The double of my *quotiente* is. 866.  
whiche I muste sette vnder  
1732. As here is doen, where  
I leaue out many cancelled  
figures, as superfluous in

$$\begin{array}{r} 1 \ 7 \ 3 \\ 1 \ 7 \ 3 \\ 1 \ 7 \ 3 \ 6 \ 2 \ 2 \ 4 \ (433. \\ 8 \ 6 \ 6 \end{array}$$

## The extraction

this place.

And then seeking for a new *quotiente*, I finde it to be. 2. whiche I set with the other numbers in the *quotiente*. And by it I multiplye and subtract the 866. say-  
 yng; 2. tymes. 8. is. 16. whiche I abate out of. 17. and  
 there resteth. 1. Again. 2. ti-  
 mes 6 is. 12 that I subtract  
 out of. 13. and there remai-  
 neth. 1. Thirdly, I saie. 2.  
 times. 6. giueth. 12. whiche  
 I abate from. 12. and there  
 is left nothing. Saue that ouer the prycke there stan-  
 deth 4 whiche is equall with the square of my *quotient*.

$$\begin{array}{r}
 18 \\
 173 \\
 186624(4332. \\
 \underline{866} \\
 \end{array}$$

Wherefore abatynge the square of my *quotiente* out  
 of it, there resteth nothing at all.

And therby I see that. 18766224. is a iuste square  
 number. And his roote is. 4332.

*The prooffe.*

Master. Although I knowe it to bee so, yet for  
 your better exercise, and full perswasion: I would  
 haue you trie it, by square multiplication.

Scholar. What maie I sone doe.

And so I finde it to be true.

For. 4332. multiplied by it self,  
 doeth make. 18766224. As this  
 woork here set, doeth shewe.

$$\begin{array}{r}
 4332, \\
 4332. \\
 \hline
 8664. \\
 12996. \\
 12996. \\
 17328. \\
 \hline
 18766224.
 \end{array}$$

Master. Yet bicause some other  
 small doubtles, maie happen in wo-  
 king, that maie trouble a yong prac-  
 tiser, I will propounde to you one or  
 twoo examples more. Wherein you shall finde some  
 varietie, as well in the number propounded. as also  
 in the *quotiente*.

And firste to begin, I will you to extract the roote  
 of this number. 22071204.

Scholar. I must set doune the number, and note  
 it with pryckes in euery odde place: For that rule I  
 perceiue

## The extraction

perceiue neuer faileth.

Master. No more doeth any of the other, although the woork be maie varie in some smalle pointes: whiche yet maie be greate mough to trouble a young learner.

Scholar. Then accoꝝdyng to the firste rule, I seke out the greatest square in. 22. (so I see it is no square number it self) and it appereth to be 16. And his roote 4. wherfoze I doe sette downe. 4. in the *quotiente*, and then I doe abate. 16. out of. 22. and the remainer is. 6. whiche I sette ouer the pickke, and cancell the. 22. as here is seen.

$$22 \div 07 \cdot 12 \div 04 ($$

$$\begin{array}{r} 6 \\ 22 \div 07 \cdot 12 \div 04 (4. \end{array}$$

Now goyng on with the nexte pickke, I shall double the former roote in the *quotiente*, and sette it vnder the Cypher, betwene the. 2. pickkes.

Then do I seke how ofte that 8 (whiche is the double of the *quotiente*) maie be found in 60 and I finde it to be 7 times, and 4 remainyng to be set ouer the Cypher. So that foꝝ the pickke there remaineth. 47. out of whiche I should abate the square of my *quotient*. But seing that. 49 (whiche is the square of 7) can not be taken out of. 47. there is a newe *quotiente* to be sought.

Wherfoze I take 6. And see that it will serue. So I set. 6. in the *quotiente*: and by it I multiplie 8 whercof commeth 48 That. 48. abated out of. 60. leaueth. 12. Therefore I cancell the 60. and set. 12. ouer it.

$$\begin{array}{r} 19 \\ 621 \\ 22 \div 07 \cdot 12 \div 04 (46. \\ \quad 8 \end{array}$$

Then doe I multiplie the *quotiente*. 6. by it selfe: whercof riseth. 36. And that abated out of. 127. leaueth. 91. And so haue I ended the seconde woork.

Now foꝝ the thirde woork, I double. 46. and it doeth yelde. 92. to bee sette vnder. 911. as I haue put it here.

And then seeking foꝝ a *quotient*: I se that I maie take

## The extraction

9. Wherefore I set that 9 in the  
quotiente with. 46. and by it I  
multiply 92 and subtract that,  
that riseth, in this forme.

Nine tymes. 9. maketh .81. 22071204 (469.  
whiche I abate out of. 91. and 92  
there resteth 10. Then 9 tymes 2 giueth 18. whiche I  
must abate out of. 10. and there will remain. 83.

And now muste I multiplie that laste quotiente. 9.  
squarely, wherby will amounte. 81. that shall I sub-  
tract out of. 832. and there will remain. 751. and so  
that picke with his woork is ended.

Wherefore procedyng to the fourth picke, I dou-  
ble all the quotiente, whiche will  
be 938. And I set it vnder 7510. 751

Then doe I seke a newe quoti- 22071204 (469  
ente, whiche I finde to bee. 8. For 938

8. times. 9. giueth. 72. whiche I abate out of. 75. and  
there remaineth. 3. Again. 8. tymes. 3. is. 24. and that  
I deduce out of. 31. and so resteth. 7. Then saie I. 8. ti-  
mes. 8. is. 64. whiche beeyng subtracted from. 70.  
doeth leaue. 6. And that. 6. with the 4. ouer the picke  
maketh. 64. out of whiche I muste withdraue the  
square of. 8. that is my quotient, and it beeyng 64. there  
resteth nothing. And the whole woork standeth thus.

$$\begin{array}{r} 37 \\ 7518 \\ 22071204 \overline{) 4698} \\ 938 \end{array}$$

Wherefore I saie that the first nōber  
22071204. is a square nōber: and  
hath for his roote. 4698 As I maie  
proue also, by square multiplicati-  
on. For, as in this example you see:  
4698. multiplied by it self, doeth  
bryng forth, 22071204.

$$\begin{array}{r} 4698 \\ 4698 \\ \hline 37584 \\ 42282 \\ 28188 \\ 18792 \\ \hline 22071204. \\ \text{Master.} \end{array}$$

*The prooffe.*

## of Rootes.

Maſter. Yet one example more ſhall you proue: *Another example.*  
and that is this.  $9 \circ 174 \circ 841$ .

Scholar. I ſet it downe, and picke it accoꝛdyng to the rule: And then I ſee ouer the laſt picke, one onely number, that is, 9. whiche hath, 3. for his ſquare roote. That, 3. I ſet within the *quotiente* line, and therfore I cancell. 9.

$$9 \circ 174 \circ 841 \overset{6}{\circ}$$

After this I ſhould proceade with doublynge the roote, 3. and that double ſhould I ſet in the next ſpace, ouer whiche remaineth no number, for, 9. beynge cancelled, the Cypher is nothyng. And ſo am I at a ſtaie.

Maſter. Seeyng that you can not ſet the double of your *quotiente* downe there, where no number is (or if it ſo chaunce, as ſome times it doeth, that the number ouer it, is leſſer then the double) then ſet a Cypher in the *quotiente*, and ſo haue you doen with that picke. For in ſoche caſe there nedeth no multiplication, nor ſubtraction.

Scholar. Then am I inſtructed fully for that point: The worke is ſo eaſie. I muſt therfore ſet my numbers thus.

$$9 \circ 174 \circ 841 \overset{6}{\circ} \overset{30}{\circ}$$

Maſter. And doe you not ſee, that the double of the *quotiente*, is greater then the number ouer it?

Scholar. I was ſo mindfull of the one halfe of the rule, that I forgot the other halfe.

But now I ſee, I muſt ſet an other Cypher yet in the *quotient*. And then ſhall I ſet the double of all that, in the thirde ſpace, after this ſorte.

And now we proceadynge to ſearche for anſwe *quotiente*, I ſee that, 2. ſhall ſerue me.

$$9 \circ 174 \circ 841 \overset{6}{\circ} \overset{30}{\circ} \overset{600}{\circ}$$

Wherfore I ſette, 2. in the *quotiente* line, with, 300. And by it ſhall I multiplie the double aforeſaid: ſaiyng, 2. tymes, 6. maketh, 12.

L. ij.      to

## The extraction

to bee abated out of. 17. and the remainder will bee. 5.

Then shall I ouerpasse the twoo Cyphers, bicause thei make nothing by multiplication: and so comyng to the picke, I bate the square of my *quotiente*: whiche is 4 out of. 8. and there resteth. 4. Therfore I cancell. 8. and set doune. 4. and so haue I ended that picke. And haue but one worke moze behinde.

$$\begin{array}{r} 5 \quad 4 \\ 9 \overline{) 1740841} \quad (3002. \\ \underline{600} \end{array}$$

Therfore I set doune the numbers, with the double of al the *quotiente*, thus.

And then I loke for a new *quotiente*, whiche I finde to be. 9. by it therfore I multiplie, first 6 and it maketh

$$\begin{array}{r} 5 \quad 4 \\ 9 \overline{) 1740841} \quad (3002. \\ \underline{6004} \end{array}$$

54. that doeth abate the 54. ouer it. Then omit I the 2 Cyphers, and multiplie 4. by 9 whereof there cometh. 36. whiche I abate out of. 44. beyng ouer it, and there remaineth. 8. That. 8. with. 1. ouer the picke maketh. 81. out of whiche I muste abate the square of. 9. beyng also. 81. And so is nothyng lefte, wherby it appeareth, that. 901740841. is a square number, and his roote is. 30029. The ppoofe of it doeth confirme the same. For 30029 multiplied by it self, doeth byynge for the. 901740841.

$$\begin{array}{r} 30029. \\ \underline{30029.} \\ 270261. \\ 60058. \\ 90087. \\ \underline{901740841.} \end{array}$$

*The nigheste  
roote of vn-  
square nom-  
bers.*

Master. This shall suffice for soche numbers as bee fully square. Other numbers there bee infinite, whiche be not square, and therfore haue thei no square rootes. Yet of ten tymes it happeneth, that we shall bee occasioned to searche for the nigheste number, that maie resemble their rootes.

Therfore in soche case, this shall you doe. Firste extracte



## of Rootes.

extract the roote, as if it wer a square nōber. And that roote wil serue for the greatest square, that is in your former number: and there will be a remainder beside. Of whiche remainder with the *quotient*, you shal make a fraction, in this sorte.

Set the remainder ouer the line, for the numerator, and the double of the roote (that you haue founde) set vnder the line, for the denominator. And this shal be a sufficiente precisenesse in greate numbers, for any common woork.

Scholar. I will by an example, taken by chaunce, proue this rule. For it semeth to haue no difficultie. Therefore I take. 296882.

And this, I am assured, can be no square number. For, I remember you told me before, that no soche number might be a square, which had 2 for his first figure.

Then to searche his highest roote, I place it, and picke it thus.

<p>And vnder .29. I finde the greatestte roote to bee .5. whiche I set in the <i>quotiente</i> line, and cancell 29 settyn 4</p>	$\begin{array}{r} 4 \\ 29\overline{)6882}5. \end{array}$
<p>ouer it. After that I double it, and there cometh 10. that double I set in the nexte space vnder 46. Then finde I a newe <i>quotiente</i>, whiche is 4 and by it I multiplie. 10. whereof amounteth 40. to be abated out of 46. And so remaineth .6. Again I multiplie. 4. by it self squarely, and there riseth. 16. whiche I abate from 18. (seyn 8. is to small) and the remainder will be. 2. So standeth the whole number, as you se. Therefore I double the <i>quotiente</i>, whiche is. 54. And it yeldeth. 108. that must be set vnder 528 as I haue here doen.</p>	$\begin{array}{r} 452 \\ 29\overline{)6882}54 \\ \underline{108} \end{array}$

<p>Then I looke for a <i>quotiente</i>, how often I maye abate. 108. out of. 528. And I see it will be but. 4.</p>	$\begin{array}{r} 452 \\ 29\overline{)6882}54 \\ \underline{108} \end{array}$
--	---

L. iij.

tymes

## The extraction

tymes. Wherfore I set. 4. in the *quotiente*, with the other numbers, and then doe I woork with it: Firſt multiplying. 4. and. 1. together, whereof cometh but 4. whiche I abate out of. 5. And there remaineth. 1.

Again I multiplie. 4. by. 4. whereof cometh. 32. that doe I ſubtract out of. 128. and there will remain 96. When ſhall I take the ſquare of my *quotiente*. 4. whiche is 16. And that muſt I abate out of 96. And ſo remaineth. 946. of whiche number ſet as the numerator, with the double of the roote, ſet for the denominator, I ſhall make a fraction in this ſorte.

$$\begin{array}{r} 494 \\ 8266 \\ 2968882(544. \\ 188 \end{array}$$

$\frac{946}{1018}$ . whiche is almoſt.  $\frac{2}{3}$ .

Maſter. You haue doen wel. And ſo you perceiue that the higheſt roote of your former number is 544  $\frac{473}{1018}$ . For thoſe fractions are all one.

And hereby alſo you maie underſtande, that if the remainder ouer your number bee euen, you maie take halfe of it for the numerator, and the whole *quotiente* for the denominator.

So maie you take the quarter of the remainder (if it will ſo bee parted) for the numerator, and the halfe of the roote for the denominator.

And in like maner generally, if the remainder and the roote in the *quotiente*, bee numbers *communicants*, diuide them ſo, that the diuiſor of the remainder, be euen double to the diuiſor of the *quotiente* roote. And ſo maie you eaſily reduce that fraction, to his leaſt termes.

But now for prooſe of this woork, there be two waies: the one is certain, and the other but in a neceſſe. For as the roote of ſoche numbers, is not a precise roote: So if you multiplie that roote by it ſelf, it will make a number, very nigh to that former number, but not exactly the ſame.

Whiche faulte ſome men thinke to redreſſe, by ad-  
dyng

*The firſt  
prooſe.*

## of Rootes.

dyng of. 1. to the denominatoꝝ: and yet that amende-  
mente sometymes increaseth the erreure.

But bicause you shall not wante a sure prooffe, doe  
thus: Multiplie the *quotiente*, or *Roote* of whole nomi- *The seconde*  
bers by it self, and vnto the number that amounteth *prooffe.*  
thereof, adde the whole remainer. And if then it make  
your firste number, your worke was well doen: els  
haue you misse.

Scholar. That maie I proue here quickly. The  
*quotiente* in whole numbers was. 544. whiche beyng  
multiplied squarely, doeth yelde. 295936. vnto whi-  
che number, if I doe adde. 946. that did  
remain, it will amounte to. 296882.  
and that was the number proponed to  
me: wherfore it appereth that the worke  
was well doen.

Master. You shall neede no more  
examples, for this forme of worke.

But one other waie wil I shew you,

how you shall gesse verie nigh vnto the roote. And *An other*  
you shall go as nigh as you will desire, in any prac- *waie to finde*  
tise worke. If you desire to gesse within lesse then  $\frac{1}{2}$ . *the nigheste*  
of one, then set before your number. 2. Cyphers. And *roote.*  
if you would not erre  $\frac{1}{2}$ . then set downe 4. Cyphers:  
But and if you liste to sette downe. 6. Cyphers before  
your number, you shall not misse  $\frac{1}{1000}$  of an unitie fro  
the true roote. And if you list to go any higher in pre-  
cisenesse of partes, adde still euen Cyphers.

Scholar. I would faine proue this forme, in the  
same example, whiche I wroughte laste: Bicause I  
would se the agreemente betwene the bothe workes.

Master. Go to. Your consideration is reasonable  
And bicause the partes maie the better agree, sette  
downe. 6. Cyphers. And then shall your roote expresse  
thousand partes of the whole number.

Scholar. I sette downe the number, and picke it  
thus,

$$\begin{array}{r}
 544. \\
 544. \\
 \hline
 2176. \\
 2176. \\
 \hline
 2720. \\
 \hline
 295936.
 \end{array}$$

## The extraction

thus. Whereby I perceiue that I shall haue the same order of woorkes, and the selfe same numbers that I had before, till that I come to the Cyphers and their prickes.

Master. Truthe it is. And therfore maie you in soche a case sette downe onely the remainer, with the Cyphers. Or els cancell all the numbers, saue the remainer, and the Cyphers: and set the former whole roote, without the fraction, in the *quotiente*.

Scholar. Then will it stande thus.

Now accordyng to the rule I will proceede: as if this whole nōber wer the first number proponed vnto me. And therfore I doe double al the *quotiente*, whiche maketh. 1088. and that doe I set vnder. 9460. And then shall I seke a *quotiente*, that maie declare how often tymes, that double is cōtained in the number ouer it. And I see it will bee. 8. wherfore I set downe. 8. in the *quotiente*,

and by it I multiplie the double, and subtrakte it, in this sorte: sayng 8. tymes. 1. out of 9. leaue. 1. remainyng. Again. 8. times. 8. (that is. 64.) out of 146 will leaue. 82. Then farther I abate. 8. tymes. 8. out of. 820. and there resteth. 756. And last of all, I take the square of the *quotiente*, whiche is also. 64. out of 7560. and there will remain. 7496. And so haue I doen with the firste prick of the Cyphers.

*A notable  
consideratiō.*

Master. Consider now that by those. 2. Cyphers you haue gotten 8 into the *quotient* more then you had before. And all your former number of the roote, removed by it into one place higher, then it was before

So

## of Rootes.

So that, where by the first woꝝke, your roote was 544. and almoste  $\frac{2}{3}$ : by this woꝝke you haue founde it to bee  $\frac{548}{3}$ , and  $\frac{1748}{3}$  of  $\frac{1}{3}$ : whiche is verie nigh the same number, that you had before.

Scholar. In dede, if I reduce the fractions, it wil bee .544.  $\frac{8}{15}$  and  $\frac{917}{135}$  of  $\frac{1}{15}$ : whiche is in one fraction,  $\frac{11833}{1563}$  about. 544.

Master. Marke this triall. And vse the like after euery twoo Cyphers are ended: And you shall see a goodly agremente of the woꝝkes together.

Scholar. In the meane tyme, to procede with the former woꝝke, I set  
 doune the number  
 with the remainer,  
 and the double of the  
*quotiente*, as here appeareth.

7496	
2968	82000000 (5448.
10896	

And searchyng for a newe *quotiente*, I finde that it will be. 6.

Wherefore I sette doune. 6. in the *quotiente* with the other numbers. And by that .6. I doe multiplie the double of the whole *quotiente*, and subtract it orderly, sayng: 6. times. 1. be-  
 yng abated out of. 7.  
 leueth. 1.

5	
968	
12120	
749644	
296882000000 (54486	
12996	

Likewises. 6. ty-  
 mes. 8. maketh. 48,  
 whiche I shall abate  
 out of. 49. and so re-  
 steth. 1. Then 6. times. 9. (whiche is. 54.) must be sub-  
 tracted out of. 1016. and there will remaine. 962.  
 Again I shall abate. 6. tymes. 6. (that is. 36.) out of  
 9620. and there is left. 9584. Then take I the  
 square of my *quotiente*, whiche is also 6 times 6, 0236.  
 and that I must abate out of. 40. and there resteth. 4.  
 And thus is the seconde pꝛicke of the Cyphers ended.

And now I finde in the *quotiente* not.  $\frac{2}{3}$  as I did in  
 M. 1. the

## The extraction

the lasse woork before this. But I finde  $\frac{86}{100}$ : whiche goeth more nigh to  $\frac{9}{10}$ . For  $\frac{90}{100}$  would be  $\frac{9}{10}$ : and  $\frac{86}{100}$  is equalle with  $\frac{8}{10}$ . And I make easily se, that  $\frac{86}{100}$  is more nigher to  $\frac{90}{100}$  then to  $\frac{80}{100}$ : beside the remainder, whiche will make  $\frac{47902}{54486}$  of  $\frac{1}{100}$ . or els  $\frac{47902}{54486}$  of one.

Master. I see, a well willing mynde can marke diligently, and learne speedily: wherfoze go forwarde with your woork.

Scholar. I muste sette doune the double of all my *quotiente*, whiche will be. 108972. And it will stande thus.

Wherfoze I doe seke for a newe <i>quo-</i> <i>tiende</i> , and I finde it to be. 8. whiche. 8. I	<div style="margin-bottom: 5px;">9 5 8 0 2</div> <div style="margin-bottom: 5px;">29688200000054486</div> <div style="margin-bottom: 5px;">108972</div>
---	---

set in the *quotiente*, with the other numbers, and by it I woork after my rule, sayng: 8. tyme. 1. is. 8. whiche I abate from. 9. and there resteth. 1. Then take 8. tymes. 8. (that is. 64.) out of. 158. and the remainder will be. 94.

Again I subtract. 8. tymes. 9. (becyng. 72.) from. 940. and there is left. 868. Farthermoze I take. 8.

times. 7. (whiche is. 56) out of. 82. and there re- steth. 26. Then doe I withdralve. 8. tymes. 2. 02. 16. out of. 60. And there remaineth. 44.	<div style="margin-bottom: 5px;">8624</div> <div style="margin-bottom: 5px;">19486</div> <div style="margin-bottom: 5px;">958024</div> <div style="margin-bottom: 5px;">296882000000544868</div> <div style="margin-bottom: 5px;">108972</div>
--	--

Last of al I take 8 times 80264 (whiche is the square of my last *quotiente*) out of 862440 and the remainder will be. 862376. And so have I ended all my woork.

And now I have for the roote  $\frac{544868}{1000}$  that is. 544. and  $\frac{868}{1000}$  beside  $\frac{431188}{544868}$  of  $\frac{1}{1000}$  or in lesser termes  $\frac{107797}{136217000}$  of one: whiche beynge reduced into one fraction with the  $\frac{868}{1000}$  will make  $\frac{118344153}{136217000}$ .

Master. You have doen well,

And

## of Rootes.

And here you see, that you drawe nigher & nigher still, to the very roote, if it might haue any. For  $8$  is a nigher number to  $\frac{2}{10}$ , then is  $\frac{80}{100}$  as that was nigher then  $\frac{8}{10}$ .

And if you would worke with more Cyphers, you should perceiue still, that it would drawe nigher and nigher. But this maie suffice for examples sake.

Scholar. When I praye you tell me, what is the chief vse of this rule: and for what maters it serueth.

Master. One yere will not suffice, to expresse the commodities of it. It serueth so many waies, in building: in proiection of plattes, for measuring of ground Timber, or stone: And also in warre, for framing of battailes, for making of diuerse engines, and generally for all woorkes of *Geometrie* and *Astronomie*. But for to satisfie you partly, I will sette forth the two or three questions, that depende of this worke of extraction of square rootes.

And firste of a battaile: because it semeth to serue leaste for that purpose.

*A question  
of an armie.*

A capitaine generall hauyng three greate armies, would caste them into three square battailes, but he knoweth not how many men, he shall set in the fronte of eche battaile.

The numbers of the three armies, are for the firste 5625: For the second 9216: And for the third 15129

Scholar. I dooe perceiue easily, that for eche of these numbers, I muste searche out the square roote, and then haue I the fronte, or flanke. With bothe are equalle in a square battaile.

Wherefore I set doune the first number thus, with his prickles. And then vnder the first pricke towarde the lefte hande, I finde the greatestte roote to bee . 7. seeyng the greatestte square is. 49. What roote doe I set within the quote line: and his square doe I abate from. 56. and so

P. ij.

remaineth

## The extraction

remaineth. 7.

Then doe I double that roote, and sette the double vnder. 72. and see that the newe *quotient* will bee. 5. And there will remaine. 25. whiche is the iuste square of the last *quotiente*.

$$\begin{array}{r} 7 \\ 5625 \text{ (75} \\ \underline{14} \end{array}$$

Wherby it is euident, that his first armie contained a square number, and the roote, or side of it is 75. And so many menne shall be in the fronte of the firste battaile, and as many in the flanke.

Now for the seconde battaile, I seke the square of 9216. and finde it to bee. 96. As in this example I haue wrought it.

$$\begin{array}{r} 8 \\ 118 \\ 9216 \text{ (96} \\ \underline{18} \end{array}$$

For the firste number is. 9. seying it is the greatestte square roote, that can bee founde in. 92. And so is the double of it. 18. and the *quotiente* for it. 6. as it appeareth manifestly inough.

Wherfore I saie that the second battaile shall haue in euery ranke. 96. men.

And now for the thirde battaile, I sette downe the number, accordyng to this rule: and I finde the firste roote to be. 1. bicause. 1. tymes. 1. maketh. 1. And his double is. 2. whiche I abate twice from the number ouer it: and after double those bothe numbers, whiche make. 24. And finde that to be abated. 3. tymes.

$$\begin{array}{r} 1 \\ 17 \\ 15129 \text{ (123} \\ \underline{24} \\ 2 \end{array}$$

And so haue I gathered that the number is square and the roote 123. Accordyng to whiche number, that thirde battaile must be marshalled.

Master. Seyng you are so redy in this pointe so sone. Tell me how many menne, shall be sette in the fronte, if all these. 3. armies be ioined into one square battaile.

Scholar. Firste I must adde all. 3. numbers together.



## of Rootes.

ther. And so will thei make. 29960. as  
here by example doeth appere.

But this number can bee no square  
number, because it hath one odde Cypher  
in the firste place: for I remember your  
sayng, that square numbers can not be-  
gin with odde Cyphers. Wherefore this number will  
not make a square battaile.

Yet wil I proue, what male be the frōt of the grea-  
teste square battaile, that male be made of that nōber.

And for that purpose I picke the numbers, and  
finde the greatestte roote in. 2. to be. 1  
and the same nōber to bee the square  
also. Then double I that roote, and  
place his double vnder. 9. that is vn-  
picked: and serchyng for a *quotiente*,  
I finde it to be. 7. with whiche I woork by the rule,  
and so doeth remain for the nexte picke. 10.

Then doe I double that. 17. whereby commeth 34  
whiche I set vnder. 106. And for it I finde. 3. to be the  
nexte *quotiente*: with whiche if I woork accordyn-  
gly, there will remaine. 31. as the excesse about the  
greatestte square.

Whereby it appeareth that. 29929. is a square nō-  
ber: and hath. 173. for his roote. And that should bee  
the fronte of this greate battaile.

After. Now will I proue you with an other  
question of like sorte.

A Prince hath an armie verie greate. With whi: *The seconde*  
che he passeth in a Vallie, so that in marchynge the *question of*  
fronte can be but. 18. menne. And by that meanes the *an armie.*  
flanke containeth. 449352.

After that the armie is passed that valie, the kyng  
myndyng to occupie all the beste grounde, willethe the  
battaile to be set square. How would you doe it?

Scholar. first I multiple the flanke, by the front.

M. iij. And

## The extraction

And so I finde the whole number to be.  $8088336$ .

<p>That number doe I picke as my rule teacheth me, and I finde the first roote to be. 2. and his square. 4. whiche first I subtrakte out of. 8. and so re- steth. 4. Then doe I double that <i>quotiente</i>, and finde that double. 8. tymes in the somme ouer it.</p>	$\begin{array}{r} 2223 \\ 484471 \\ 8088336(2844. \\ 48668 \\ 8 \end{array}$
---	--

And so doe I procede till I haue founde out all the  
4. figures, accordyng to the. 4. pickes vnder that nō-  
ber. And then the roote appeareth to be. 2844.

*The thirde  
question of  
an armie.*

Master. Yet one question more, for to exercise  
your penne, will I propounde of a like mater.

A generalle hath thre armies, to the number of  
28289. men: and none of those thre armies is apte  
to make a square battaile, yet he is appointed by his  
soueraigne, to sette theim in thre square battailes.

These be the. 3. numbers of the. 3. armies. In the  
firste there are. 10296. men: In the seconde. 9493:  
and in the third. 8500. Now let me see how you can  
cast them into thre square battailes.

Scholar. I thinke it reasonable, to take the grea-  
testte squares of the first and second numbers, and the  
excesse of them bothe, to put to the thirde number.

Master. So are you not sure that the third nom-  
ber, will be a true square.

Scholar. Then knowe I not how to doe it.

Master. Take the greatestte square in the thirde  
number also. And note those thre excesses, and their  
rootes also.

Then put one to euery roote, and marke the squa-  
res that will rise of them.

Thirdly, subtrakt the firste 3. numbers, out of those  
3. newe squares, and note the difference of eche of the  
firste numbers, from those squares: and so haue you. 3  
numbers

## of Rootes.

numbers of ercesse, and, 3. other of wante.

Now compare those ercesse and wantes well together: and you shall easily see from whiche you shall take any number, and to whiche you shall adde any.

Scholar. In the firste nōber the greatest square is 10201. and therby the ercesse is. 95. and the roote 101.

In the second number the greatest square is. 9409 and his roote 97. So is the ercesse. 84.

And in the thirde number, the greatest square is 8464: and the roote of it. 92. Wherefore the ercesse appeareth to be. 36.

And thus haue I founde the. 3. ercesse.

Now for to finde the 3 defaultes or wantes, I adde one to eche roote, and multiplie them square: and so of. 102. I finde the square to bee. 10404. and if I subtrate the firste number, whiche is. 10296. out of it, there will remain. 108. for the firste wante.

Then for the seconde roote. 97. I take. 98. whose square will bee. 9604. out of whiche I abate the seconde number, whiche is. 9493. and there is left 111 as the wante of the seconde number.

Thirldy, I take 93 for the newe roote, next aboue 92. and I finde his square to bee. 8649. from whiche when the thirde number. 8500. is abated, the defaulte appeareth to bee. 149. And thus haue I the. 3. defaultes or wantes, and also the. 3. ercesse. Whiche for ease of comparng, I set in order thus.

	A.	B.	C.	A. B. and C. beto-
Excesse.	95.	84.	36.	ken the order of
Wantes.	108.	111.	149.	the 3 first nōbers.

And here I compare the ercesse with the wantes, to see if any. 2. ercesse will make up the others want And I see by a lighte pproofe, it will not serue.

As for the wantes, I doe not compare theim to the ercesse,

## *The extraction*

excesses, for I see that every one want, is greater then any one excess. And therefore. 2. wantes are farre to greate aboue any one excess. And so am I at a state.

Master. Therfore although that rule bee generalle, yet where it faileth, this shall you doe.

Take the. 2. wantes, of any. 2. numbers, and adde theim firste together, and then abate theim from the thirde number: and if the remainder be a square number, then haue you gotten your purpose.

Scholar. What will I proue here. And first I take the wantes, of the. 2. firste numbers, whiche make 219. And that doe I abate from the thirde number 8500. and there remaineth. 8281. whiche as I see, maie be a square number. And therfore I proue it, in my tables, and I finde it so to bee. And. 91. to see the roote of it.

Therfore I saie to the question, that these shall be the numbers of the 3 battailes, as here I haue set the.

The firste battaile. 10404. and his fronte. 102.

The second battaile. 9604. and his fronte. 98.

The third battaile. 8281. and his fronte. 91.

The somme of all  
the. 3. battailes. } 28289.

And bicause these nōbers are not onely square, but also their whole somme doeth agree, with the somme of the 3 seuerall armies, you maie be sure that thei are well parted, accordyng to the intente of the question.

But bicause soche questions, haue more difficultie then commoditie, to them that are not mete, to be trauelled in soche marshall affaires, I wil leaue that matter to marshall men, and will come to lower matters in warre.

*A question  
of scalyng.*

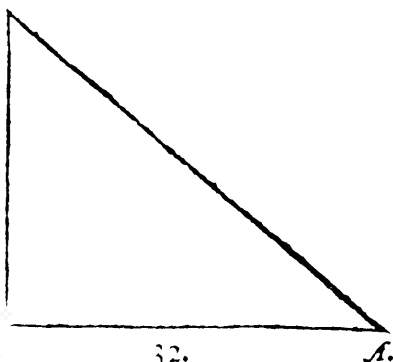
A citie should bee scaled, beyng double ditched. And the inner dicke. 32. foote broade. And the walle. 21. foote high. The capitaine commaundeth ladders to be made

## of Rootes.

made of that iuste lengthe, that maie reche from the viter brow of the inner dicke, to the toppe of the wal- as in this figure C.

is partly cryp-  
sed.

where the line  
*A B.* standeth for  
the breadth of the  
dicke. And the  
line *B. C.* for the  
heighte of the  
walke. Nowe I  
demaunde, what  
shall be the length  
of the line *A. C.* whi-  
che here doeth represente the ladder?



Scholar. This figure doth occasiō me to remēber the 33. theozeme of the pathewate, whiche saith thus.

*In all righte angled triangles, the square of that side, whiche lieth against the righte angle, is equalle to the twoo squares of bothe the other sides.*

Wherby I vnderstand, that I must multiply those twoo sides squarely, that is, eche of them by it selfe. And then adding those. 2. squares together, I muste extract the roote of that whole number: whiche roote shall be the true lengthe of the slope line.

<div style="text-align: right;">                 21 21 — 42 441             </div>	Therefore, firste I multiplie. 32. by it self, and there riseth of it 1024. Again, I multiplie. 21. by it self, and it yeldeth. 441. These bothe sommes, beyng added together, doe make. 1465. whiche number maie bee square, bicause it beginneth
--	---

32 — 32 64 96 — 1024
--

R. J. neth

## The extraction

meth with. 5.

Maſter. It is no ſquare number, as it appeareth at the firſte ſighte. For although the firſte number be 5. yet in ſoche numbers it is requiſite. that the ſeconde figure ſhould be. 2. els can it not be ſquare: and here, you ſee, that the ſeconde figure is. 6. ſo that it can not be a ſquare number.

¶ herefoze you ſhall ſeke the nightheſte roote, that you can finde in it, and take that for your purpoſe.

Scholar. Here is my woork ſet forth.

And ſo it appeareth well that the nightheſte roote is.  $38\frac{1}{4}$ , whiche is leſſe then a quarter of a foote, aboue 38. foote and that muſt be the lengthe of the ladder.

Maſter. Yet one queſtion moze will I propound agreeable to the firſte forme.

*A queſtiō of  
encampyng.*

A capitaine generalle hauynge three armies, in three ſeueralle battailes, in the firſt. 4900. menne, in the ſeconde. 2401. And in the thirde. 2500. (ſo that the greateſte armie, is as moche as bothe the other, excepte one manne) is inſoyced to ioine all three battailes in one. But is in doubtte, whether he maie haue good and conueniente grounde to encampe the, in battaile forme. ¶ herefoze conſideryng, that all. 3. battailes together, are but double to the greateſte of the. 3. alone. The capitaine deſiryng a mete grounde for his armie, ſo ioined in one ſquare battaile, is in doubtte, what ſquare of grounde will ſerue his purpoſe. But ſure he is, that it muſte bee double to the grounde, that the greateſte armie of the. 3. did occupie and that was ſquare euery waies. 210. foote. ¶ herefoze his demaunde is, how many foote ſquare, ſhall the ſide of that grounde bee, that is double to the former ſquare platte, whoſe ſide was. 210. foote euery waie?

Scholar.

## of Rootes.

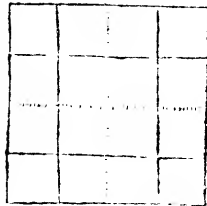
Scholar. Firſte I muſt multiplie. 210. by it ſelf, and ſo haue I the iuſt platte of grounde, of. 44100. foote, that muſt I double, and it will be. 88200. And out of this number, ſhall I ſeke the nightheſte ſquare roote. For a iuſte ſquare, I ſe, it is not: by reaſon that after the euen Cyphers, there foloweth. 2, whiche is one of thoſe figures, that can not beginne any ſquare number.

Therefore, ſekyng for the nightheſte roote, I finde it to bee 296. <sup>13</sup>/<sub>16</sub> that is almoſte. 297. foote euery waies ſquare. And ſo moche muſte the ſquare ſide of that grounde bee, whiche ſhould ſerue for that whole ar-  
mie.

$$\begin{array}{r} 14 \\ 14 \ 58 \\ 42 \ 124 \\ 88 \ 200 \ (296 \frac{13}{16}) \\ 4 \\ 58 \end{array}$$

And hereby I doe perceiue, the ouerſight of many men: whiche being required to double a ſquare platte do double the ſide of it, thinking the matter eaſily doen

But if thei marke it well, thei maie perceiue, that thei doe make, by that meanes, a ſquare ſolwer times ſo bigge as their firſte ſquare was. As by this figure, any man maie ſee.



For if 2. be the ſide of the ſquare then is the ſquare 4. But if I double the ſide, and make it. 4. the ſquare thereof will be 16. whiche is. 4. tymes. 4. and not onely double.

So that the roote of the double platte, ſhould bee the roote of. 8. whiche is ſomewhat leſſe then. 3. and therefore moche leſſe then. 4.

Maſter. You maie perceiue theſame, with the reaſon of it, by the 18. propoſition of the. 8. booke of *Euclide*, as it is before alleged.

But now for to ſhewe the larger uſe of this rule,

Q. ii.      I

## The extraction

*A question  
geographical*

I Demaunde this question.

Where be. 2. townes, as *Chichester* and *Yorke* whiche  
lye Southe and Northe, and betwene them. 220. mi-  
les. A thirde towne as *Excester*, lieth plaine Weste fro  
*Chichester*. 120. miles. I desire to knowe the iuste di-  
stance of *Yorke* from *Excester*.

Scholar. I must set those. 3. townes, in forme of a

Triangle, with *A*

their distaunces:

As here is repre-

sented. Where

*A*. standeth for *Ex-*

*cester*, *B*. for *Chiche-*

*ster*, & *C*. for *Yorke*.

And then accor-

dyng to the rule,

I multiplie. 120. *B*, 220. *C*.

squarely: and it maketh. 14400. Likewises I dooe  
multiplie. 220. and it yeldeth. 48400.

These bothe numbers I shall ioine in one, and so  
haue 3.62800. whose roote is very nigh. 250. miles  
and  $\frac{1}{2}$  of a mile.

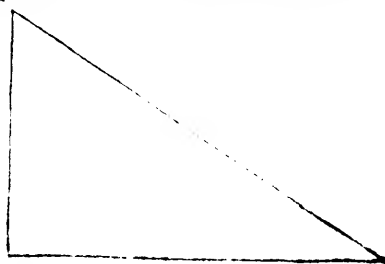
And that is the true distaunce  
of *Yorke* and *Excester*.

By this example I gather,  
that this rule doeth helpe to Geo.

graphic, for to dralve the true platte of any countrie.

Master. If I should stande in propoundyng ex-  
amples of this rule unto you, bying but one for euery  
arte and science. and for euery different kinde of com-  
modious practise: it would make a greate booke.

And therefore omittyn that, till occasion serue o-  
therwaies, I will proccade to the extraction of *Cubike*  
rootes.



$$\begin{array}{r} 250 \\ 62800 \div 250 = 252 \\ \hline 252 \end{array}$$



# of Rootes.

## Of Cubike rootes.



When any *Cubike number* is propoun-  
 ded, whose roote you should extract  
 After the number is written doune  
 orderly: you shall set a prick under  
 the firste figure: and under the. 4.  
 and so under euery third figure, o-  
 mitting till. 2. figures vnpricked.

And looke how many prickes, your number hath,  
 so many figures shall the roote of your nōber contain

Then to begin the searche, for the firste figure of  
 the roote, in this order) you shall looke what maie be  
 the roote of the number, belonging to the last prick  
 toward the lefte hande. And that roote shall you sette  
 by a *quotiente* line, as you did in square rootes. 1.

And if the whole number ouer that prick, be a *Cu-  
 bike number*, you shall cancell it all. But if it bee no *Cu-  
 bike number*, then subtracte out of it, the greatestte *Cube*  
 in it, and cancell the whole number, and set the reste  
 ouer it: as you did in square rootes.

But considering, that you ought to haue in ready  
 remembraunce, all those *Cubike rootes*, whiche be digi-  
 tes, with the *Cubes* that they make: for without them  
 you can not procede in this worke. I thinke it good  
 to set forth here in a table, all those rootes with their  
*Cubes*, that ther by you maie be the more  
 adured in tyme of your worke. For els  
 a litle mistakynge, might be the occasion  
 of a greate erreure.

And now for this first rule I saie, as  
 I saied of *Square rootes*, this shall be eu-  
 more the firste worke, and shall not be  
 repeted in any one *Cubike nōber*. All here  
 as all the other rules folowynge, shall be  
 so often repeated, as there are prickes in

1.	1.
2.	8.
3.	27.
4.	64.
5.	125.
6.	216.
7.	343.
8.	512.
9.	729.

say.

your

## The extraction

your number.

2. And of theim this is the firste: that you shall triple the firste roote. And that triple shall you set vnder the nexte number, toward the righte hande, before that picke, whiche you did laste ende.

3. Then multiplie that triple, by thesame *quotiente*. And set it doune vnder the first triple: and that number shall be called your diuisor.

4. Thirdly, loke out a *quotient*, that maie declare how often the diuisor is in the number ouer it.

In whiche doyng, you must haue this regard, that betwene that picke that is ended, and the nexte that standeth toward the right hande, you must subtrate 2. other numbers. That is to saie, the square of the laste *quotiente*, multiplied by the former triple. 10. tymes: and the *Cube* of thesame *quotiente*.

Scholar. This rule is very obscure in wordes.

Maister. Then will I terme it thus.

2. 4. 3. Take the square of your whole *quotiente*, 300. tymes: and that shall be your diuisor. Then seke a newe *quotiente*, declaring how often that diuisor, maie be founde in the number, that doeth belong to the nexte picke. But so that the square of that newe *quotiente*, multiplied by the last *quotiente*, 30. tymes: and also the *Cube* of that newe *quotiente*, ioyned all in one somme, maie be taken out of thesame number. And if you vnderstande this, there resteth no more difficultie.
- 4.

Scholar. I trust by exāple, to vnderstand it better

Maister. Then take you this exāple, 26463592 whiche I shall set doune and picke, as I taught you before: and as you maie here see. ¶ here the .3. pickes declare vnto me, that the roote will haue .3. figures,

And then vnder the picke that is nexte the leste hande, whose number is. 26. I finde the greatestte *Cubike* number to bee. 8. and his roote. 2.

26463592(

## of Rootes.

For. 27. whiche is the nexte *Cube*, is to greate.

Therefore I set. 2. in the *quotiente*, and his *Cube*, being. 8. I doe abate out of 26. and so remaineth. 18.

That. 18. I doe sette ouer. 26.

whiche I muste cancell: and then standeth the number, as here you doe see.

18

26 46 3592(2

This is that firste worke, whiche is not repeted.

Then to procede forward, I doe triple the *quotiente* 2, and so haue 3.6. whiche I shall set vnder. 4. being the nexte number, on the righte hande of the prick that is ended.

And that triple must I multiplie, by the first *quotiente*, wherby amounteth that number, that must be the diuisor: and it is in this worke 12. whiche must be set vnder the same triple: as here I haue placed it.

18

26 46 3592(2.

6

Then shall I take for a newe *quotiente*, declaring how often times.

12

12. maie be founde in the number ouer it, that is 184. And I see it maie be in appearaunce. 15. tymes, but more then. 9. you shall neuer take for a *quotiente*: wherefore it appeareth, that I maie boldly take. 9. whiche I shall sette in the *quotiente* with the firste. 2. And then shall I multiplie. 12. whiche is the diuisor, by. 9. and thereof cometh. 108. to bee sette vnder 184. benethe the line, whiche shall euermore be drawen vnder the diuisor.

2

18 07 4

26 46 3592(29

6

12

Now muste I take the square of my laste *quotiente*. 9. (whiche is 81.) and multiplie it by the triple of the former *quotiente* (that is by 6.) and so haue 3.486. to be sette on a place more toward the right hande.

108

486

729

16 389

Last

## The extraction

Lasſe of all, I ſhall multiplie the laſte *quotiente* *Cu-  
bikely*: and that maketh. 729. whiche muſt be ſet, yet  
one place more toward the right hand, that is to ſaie,  
vnder the nexte prick. And then ſhall I adde thoſe 3  
ſommes into one: wherby will riſe. 16389. to be ſub-  
tracted out of. 18463. and ſo will remaine ouer that  
pricke. 2074.

And the woork of that prick is doen.

This order of woork, if you marke well, you haue  
learned the whole arte of extraction of *Cubike rootes*.

For how greate ſo euer your number be: you ſhall  
not haue any newe kinde of woork.

But yet becauſe I did teache you befoze, theſame  
woork in other woordes, I will woork theſame ex-  
ample again, accoꝝdyng to theſe woordes.

And firſte, after that the number is ſet doune, and  
the firſt *Cubike roote* taken, and the *Cube* abated. Then  
take the ſquare of that roote. 300. tymes, that is in  
this example. 4. tymes. 300, whiche maketh. 1200.  
and that ſhall be your diuiſor. This number, and all  
other in this woork, ſhall you ſet doune ſo, that the  
firſte number, ſhall be vnder the nexte prick, toward  
the righte hande.

Then ſeke your *quotiente*, with the former cautele,  
and it will be. 9. Wherefoze  
multipliyng. 1200. by. 9. there  
will amounte 10800. to be ſet  
vnder the line.

After this, I ſhall take the  
ſquare of. 9. (whiche is the new  
*quotiente*) and multiplie it by. 2.  
(whiche was the laſte *quotiente*  
befoze) 30. tymes. So muſte I  
multiplie 81. by. 60. and it will make. 4860. whiche  
I place orderly.

18	
26463592	(29
1200	
<hr style="border: none; border-top: 1px solid black;"/>	
10800	
4860	
729	
<hr style="border: none; border-top: 1px solid black;"/>	
16389	

Then ſet I doune the *Cube* of the *quetiente*, whiche  
maketh

## of Rootes.

maketh. 729. And so are the .3. numbers placed, and agree with the former worke, in all thinges, saue in 2. pointes. For here the triple of the *quotiente*, is not set doune, but kepte in memorie. And again, here are diuerse cyphers, whiche are not in the former worke.

Scholar. Sir, I perceiue, that the Cyphers dooe nothing els, but set the numbers in their due places. And the triple of the *quotiente*, is supplied in worke by 2. multiplications. First by. 300. and then by. 30. So that it is all one in effecte.

And by the one worke, I vnderstande the other the better: when I compare them bothe together. But yet I praye you, ende the worke that you began.

Master. To continue that worke, firste I must set doune the numbers, as thei should remaine, after 16389. is abated out of. 18463. and then will thei stande thus.

Then shall I repeate the former worke, by setting doune the triple of all the *quotiente*, whiche will be. 87. and that must be placed vnder. 45.

Perce that I shall multiplie that. 87. by. 29. and there will come. 2523. whiche must be the diuisor.

Wherefore I seke for a new *quotiente*, that maie shewe me how often. 2523. is contained in. 20745. And it will be. 8. That 8 doe I set in the *quotient* and by it I multiplie. 2523. and it giueth. 20184 whiche I sette doune, as here you see.

Then doe I multiplie that *quotient* squarely, and that will be 64. Whiche I shall multiplie by the triple, that is 87, and there will amounte. 5568. to be set one place more toward the righte hande.

2074	
26463592	(298.
87	
2523	
20184	
5568	
512	
2074592.	

## The extraction

Last of all, I must take the *Cube* of. 8. that is. 512, and it shall bee sette yet one place more towarde the righte hande.

And then by additiō, I shall bying the all into one number: and it will bee. 2074592. whiche is equall with the whole number aboue, that is vncancelled. And therfore if I abate the one out of the other, there will remain nothyng.

Wherefore I see, that the firste number, is a iuste *Cubike* number. And his roote is. 298.

Scholar. I haue marked you so well, that I trust to doe the like, without erreure.

But I praye you wooke this laste parte also, by your seconde rule, as you did wooke the other: that I maie see the due agremente of thein bothe: and also perceiue the righte vse of this wooke, the better by that other forme.

*The seconde  
wooke.*

Master. I must in that case sette doune the numbers, as thei were set in the other wooke. And then I shall multiplie al the *quotient*, whiche is. 29. by it self squarely, and it will make. 841. whiche must be multiplied by. 300. And so there amounteth. 252300. to be sette doune, as here you see.

Then I shall seke out a *quotiente*, declarynge how often 252300. maie bee founde in 2074592. And that *quotiente* will bee. 8: whiche I set in the *quotient* roome, with the other numbers.

And then I doe multiplie the diuisor by the *quotiente*, and thereof riseth 2018400 whiche I set vnder a line, as you maie see.

Perce that, I doe multiplie the newe *quotient*, by it self,

$$\begin{array}{r} 2074 \\ 28483592 \end{array} (29.$$

$$\begin{array}{r} 2074 \\ 28483592 \\ \hline 252300 \\ \hline 2018400 \\ 55680 \\ \hline 512 \\ \hline 2074592 \end{array}$$

# of Rootes.

self squarely, whereof commeth. 64. and that square of the last *quotient*, I shall multiply by. 870. whiche is. 10. times the triple of the former *quotiente*. 29: and thereof commeth. 55680. whiche I set downe also orderly.

8
8
64
870
4480

Laske of all, I multiply. 8. (that is the laske *quotiente*) Cubikely, and it maketh. 512. whiche also I set downe in cōueniēt order.

512
55680

And then shall I adde them all together. And so haue I thesame somme, that I had befoze in the other former woork, and it is. 2074592.

Scholar. I neede no more instruction for this: I thinke my self so cunning, by occasion of your exam- ples, whiche you haue wroughte so in double forme.

Master. That maie you proue, by this number 47832147.

Scholar. Firste I shall pricke it, as you taughte me, omitting still. 2. numbers.

*An other example.*

And then out of the number ouer the laske pricke, I shall seke out the *Cubike roote*, and abate the *Cube* thereof, out of thesame number, and set the remainer ouer it, cancellng the reste.

And so in this number, I finde in. 47. the greatest *Cube* to bee. 27. and the roote of it 3. Wherefoze I abate. 27. out of. 47. and finde the reste to be. 20. therfoze I cancell. 47. and set. 20. ouer it. And the. 3. whiche is the roote, I set in the *quotient*. And so is the first woork ended.

20
47832147 (3

Then doe I triple that *quotiente*, and it maketh. 9. whiche I set downe vnder. 8.

Again I multiply that. 9. by. 3. and it yeldeth. 27. whiche I set vnder the triple, and take it for my diuisor.

Wherefoze I shall now seke a *quotiente*, that maie  
D. II. declare

## The extraction

declare how often. 27. is in. 208  
and I see, it will bee. 7. tymes.  
Wherefore I sette doune. 7. in the  
*quotiente*: and by it I multiplie 27  
and it maketh. 189. whiche I set  
vnder the line: and then I dooe  
multiplie. 7. by it self, whiche  
maketh. 49. & that square doe I  
multiplie by the triple of the for-  
mer *quotiente*, that is, by. 9. and it yeldeth. 441. whi-  
che I set one place moze toward the righte hande.

$$\begin{array}{r}
 20 \\
 47 \overline{) 832147} (37. \\
 \underline{9} \\
 27 \\
 \hline
 189 \\
 \underline{441} \\
 343 \\
 \hline
 23653
 \end{array}$$

Last of all, I take the *Cube* of. 7. whiche is. 343. and  
that doe I sette doune, yet one place moze toward the  
righte hande.

These. 3. sommes beyng added together, doe make  
23653.

Master. That will be hardely abated out of a les-  
ser somme.

Scholar. I see now my errour. I must take a lesse  
*quotient*: whiche thyng I might haue perceiued by the  
seconde number. For thei twoo wer to greate, befoze  
the thirde was added.

So that I should haue taken but. 6. for the *quotiente*  
And then would the firste number haue been but 162  
and the seconde. 324. and the  
thirde. 216. but that their pla-  
cyng would make them to be of  
other values, saue the last of the.

$$\begin{array}{r}
 1 \\
 2 \overline{) 176} \\
 47 \overline{) 832147} (36. \\
 \underline{9} \\
 27 \\
 \hline
 162 \\
 \underline{324} \\
 216 \\
 \hline
 19656
 \end{array}$$

Wherefore, I set euery one in  
his due roome: and adde them  
together, and there amounteth  
19656. to bee subtracted out of  
20832. and the remainer will  
be 1176. And thus is that pricke  
with his woozke ended.

Then for the nexte pricke, I repeate the same very  
fozme



## of Rootes.

forme of worke again. First setting dounce the triple of the whole *quotiente*, whiche is. 108. so that it shall stande vnder. 11761.02 vnder. 761. accompting figure for figure.

That triple must I multiplie againe by the whole *quotiente*. 36. and it will make. 3888. whiche number I muste take for my diuisor.

Wherefore I seke how many times, I maye finde that diuisor in. 11761. and I see, it will bee. 3. tymes. Wherefore I set. 3. as my *quotiente*, in his due place: and by that *quotient* I do multiplie. 3888. and so haue I for my firste number. 11664.

$$\begin{array}{r}
 1176 \\
 47 \overline{) 32147(363} \\
 \underline{108} \\
 3888 \\
 \underline{11664} \\
 972 \\
 \underline{27} \\
 1176147
 \end{array}$$

Againe I doe multiplie the laste *quotiente*. 3. squarely, and so haue I. 9. whiche I shall multiplie by the triple of the former *quotient*, and it yeldeth. 972. that shall be set more nigher the right hande, by one place.

Thirdly, I take the *Cube* of. 3. whiche is. 27. and that doe I set yet one place more towarde the righte hande.

Then doe I adde those 3 sommes into one, and they make. 1176147. whiche is equalle somme, with all the numbers ouer it, that be vncancelled.

Wherefore I saie that. 47832147. is a *Cubike number*, and the *Cubike roote* of it is. 363.

Master. Now doeth the order of teachynge re: *The nigheste* quire, that I should instructe you, how to extracte the *roote in a nōs* nigheste *Cube roote*, out of any number, that is not a *ber not Cube*. As this number for example maye serue. *bike*. 694582951.

Where firste I muste extracte the nigheste roote, as I tanghte you, for the nigheste *Square rootes*, in *nōs* bers that are not square: and then shall I note the re:

D. iij. manner:

## The extraction

manner: whiche I shall set for the numerator. And his denominator shall be founde, as I will tell you anon. But firste doe you worke the example, to his nigheste roote in whole numbers.

Scholar. I set it doune, and picke it, and finde the greatest Cube ouer the laste picke to bee 512. and the roote of it is. 8.

$$\begin{array}{r} 182 \\ 8 \overline{) 694} \end{array}$$

Wherefore I set doune. 8. in the *quotiente*. And I abate. 512. out of. 694. and so resteth 182. and the former. 694. cancelled.

Then to procede, I must triple that roote. 8. and it maketh. 24. whiche. 24. I set vnder. 1825. And then I doe multiplie that again, by the *quotiente* or roote. 8. and it maketh 192. to be set vnder the said triple. 24: as the diuisor. For whiche I seke a new *quotient*, and it will be 8. That. 8. I set in the *quotiente* place, and by it I multiplie the diuisor. 192. and there riseth. 1536. to be set vnder the line, in conueniente order.

Nexte I multiplie the *quotiente* squarely: whiche yeldeth. 46. and that square I multiplie again by the triple, and so haue I. 1536. also. But this must stand more forwardly by one place.

Last of all I take the Cube of the *quotient*. 8. and that is. 512. whiche I set vnder the other two sommes, and that by one place more forwardly.

$$\begin{array}{r} 13 \\ 1825110 \\ 8 \overline{) 69472} \end{array}$$

Now gatherng all these. 3. somes into one, thei will make 169472 whiche I shall abate out of. 182582. and so remaineth there. 13110. And that picke with his worke canceled.

$$\begin{array}{r} 1536 \\ 1536 \\ 512 \\ \hline 169472 \end{array}$$

Wherefore hauing one other space to worke, I must repeate the same order of worke again: by tripling the whole *quotiente*

*of Rootes.*

quoyente, 88. and that will bee. 264. And againe I must multiplie that triplede number, by thesaide quoyente, and it will make . 23232. whiche shall bee the diuisor.

Wherefore I seke a newe  
quotiente, whiche is easily per  
ceiued to be. 5. That. 5. doe I  
set in the *quotiente*, and by it  
I dooe multiplie the deuisor  
23232. and there amounteth  
116160. as the firste nomi  
ber, to bee set vnder the line.

Again I shall multiplie the *quotient* squarely, whiche giueth . 25. and that square shall I multiplie by the triple rise . 6600. to bee sette, as der the line : and one place in the righte hande.

Last of all, I shall sette vnder them bothe, and one place more towarde the righte hande, the *Cube* of. 5. whiche is. 125.

And then shall I adde all those. 3. sommes together of whiche commeth. 11682125. to bee abated out of 13110951. and so the remainder will bee. 1428826. Whereby I see, that the firste number that was proposed, I meane 694582951 is no Cubike number, but the greatestte Cube in it is. 693154125. and his roote is. 885.

And so, I see, all other numbers of like kinde must  
bee wroughte.

But now for the remainder, how shall I dooe to  
byunge it vnto a fraction, that maie aptly expresse the  
r. p. theste roote in that sorte?

**Maister.** There bee as many waies, as there bee  
wyters almofte, for euery manne deuifeth, how to  
byng

$$\begin{array}{r}
 1428 \\
 \cancel{1311} \cancel{826} \\
 694582981(885. \\
 \quad 264 \\
 \quad 23232 \\
 \hline
 116160 \\
 \quad 6600 \\
 \quad \quad 125 \\
 \hline
 11682125
 \end{array}$$

## The extraction

*Cardane,*

brynge it mosse nigheste to a true roote, if any soche were: whereof *Cardane* his rule is this.

*Multiply the roote squarely, and againe by 3. and that number shall be the diuisor vnto the remainer,*

¶ Here he might haue vsed moze plainesse in wordes, if he had saied: and that number shall be the denominator, to the remainer. ¶ Herefoze as here your roote is. 885 so is the square of it 783225 and the triple of that is. 2349675. So would that fraction bee

$\frac{1428826}{2349675}$

But how nigh this doeth go to the truthe, I leaue it till an other tyme.

*Scheubell.*

*Scheubelius* doeth allege an other reason, and inferreth an other order, diuerse frō this, and soche as impugneth this, sayng:

*Triple the roote, and the square of it also, and adde bothe those numbers together, and. 1. more: And so haue you a denominator for your numerator.*

The numerator euermoze is vnderstād to be the remainer. By whiche meanes the fractiō in this worke would bee  $\frac{1428826}{2349675}$ : whiche is a lesser fraction by a good deale, then is the former fractiō, after *Cardanes* forme.

But bicause at this presente, I maie not spende so moche time, to scan their seueralle opinions, where in eche of theim, pleaseth hymself well: the one alleging demonstration (whiche scarcely serueth) and the other namynge it a secrete, as it is worthe to bee: I will procede to a thirde waie, moze certain then ether of these bothe. And that is by addition of certain Cyphers, to the remainer, in soche sorte, that thei muste all waies bee ternaries, as. 3. 6. 9. 12. &c. And then searche

## of Rootes.

searche forward with the like order of worke, as you used before.

In this manner of practise, looke how many pickes your cyphers hath (or els how many ternaries of Cyphers, there bee set to your number) so many figures shall the numerator of your fraction contain. And the denominator shall euermore, contain 1. more. Althet of the laste onely shall bee an unitie, and all the other shall bee Cyphers.

That is to saie, that if I adde but 3. Cyphers to the number, the fraction shall contain certain. 10. partes And if I adde. 6. Cyphers, it shall expresse. 100. partes. So. 9. Cyphers maketh the denominator to bee 1000. partes: And 12. Cyphers geueth 10000 partes.

For example. I will adde to our laste number that remained. 12. Cyphers. And then will the number be 1428826.000.000.000.000. vnto whiche I set no more pickes, then serueth for the cyphers, because I haue paied all the other pickes, in my former worke.

And now to continue my worke, I shall triple all the former *quotiente*, and it will be 2655. whiche number I shall place, as here you see it set. And then shall I multiplie that triple, by the former *quotiente*. 885. whiche will yelde. 2349675. to be set vnder the said triple: as I haue sette it here also. And this number shall be the diuisor.

Then shall I seeke for a *quotiente*, whiche can bee none other then. 6: wherefore I sette. 6. in a *quotiente* line, and by that. 6. I dooe multiplie the said diuisor 2349675. and it giueth. 14098050. to be the firste number vnder the line.

After that, I take the square of the said *quotiente*, whiche is. 36. and by it I multiplie the triple. 2655:

p. j.      whereby



*of Rootes.*

it. And so is the woozke of that pꝛicke canded, without any moze trauell.

Wherfore to go forward, I triple all that *quotiente*  
and set it dounc, as the rule would, & as here is seen.

[illegible]

Then dooe 3 multiplie that triple, by the whole  
quotiente, whercof cometh. 23528620800. and that  
shall bee the diuifor. And the quotiente for it will be. 7.

So then if 3 multiplie that diuisor by .7. there will amounte. 164700345600. so2 the first number to be set vnder the line.

And for the nexte woorkke, I shall multiplie. 49.  
(whiche is the square of the newe *quotiente*) With the  
triple of the former *quotiente*, and it  
will bring forth. 13018320. whi-  
che shall bee the seconde number, to  
bee set vnder the line.

265680
49
2391120

The thirde number shall bee the  
Cube of. 7. whiche is. 343.

And those . 3. sommes added together , will make 16 47 0 16 47 4 35 43. whiche is to bee abated out of 18 06 4 98 4 0 0 0 0 0 0. and then shall there remain 15 9 4 8 1 9 2 5 6 4 5 7. And so haue I ended . 3. pickes of the Cyphers. And thereby make saie, that the fraction is  $\frac{697}{159}$  and somewhat more : What is somewhat more then  $\frac{1}{2}$ .

Scholar. I see by the fraction, that it is  $\frac{1}{2}$  and  $\frac{7}{1000}$ ,  
p. y. beside

## The extraction

beside the quantitie of the remainer. But I praye you  
cande the woork of that other prycke, whiche dooeth  
remaine.

Waster. I muste triple all the *quotiente*: whereby  
will rise. 2656821. whiche muste bee multiplied by  
the said *quotiente*: and thereof  
will procede the diuisor, beyng  
2352899275347. And his  
*quotiente* will bee. 6.

Wherefore firste I set. 6. in  
in *quotiente* line, with the other  
numbers: and then doe I mul-  
tiplic the diuisor by that *quoti-  
ente*, and it byngeth forth the  
14117395652082. For the  
firste number to be sette vnder the line.

183078734793024	
<del>1894819286457000</del>	(8856076.
2656821	
2352899275347	
14117395652082	
95645556	
216	
1411740521663976	

And again the square of 6. beyng multiplied by the  
triple, will yelde. 95645556: whi-  
che shall bee the seconde number vn-  
der the line.

The thirde number shall be. 216. because it is the *Cube* of. 6. And those  
3. numbers beeyng added together, doe make. 1411740521663976. to be abated out  
of. 1594819256457000. And so doeth there re-  
maine. 183078734793024.

Wherefore



## of Rootes.

Therefore it dooth appeare, that beside the first, 3 numbers of the roote, that is. 885. the rest (that is 6076.) standeth for the numerator of a fraction, and the denominator vnto it is. 10000.

So that the higheste roote is.  $885\frac{6076}{10000}$ . beside the fraction that doeth remaine: whiche would make but  $\frac{1}{10000}$  of an unit.

Scholar. This is a sufficiente precisenes. And so I iudge it sufficiently taughte.

Therefore I praye you propounde some questions, that doe require this arte, for their solution.

Master. I am contente. And let this be the firste.

The Grecians giuen to idle banketting, and sothe like wantonnesse, did procure thereby soche mortalle sicknesses: that the quicke were scarce hable to burie the dedde. Therefore consultynge with their Goddes, for redresse thereof, they receiued answer, that when they would double the Altare, whiche was of Cubike forme, they should bee deliuered from that plague. Meanyng that learning is a due meane, to deliuer realmes from plagues and enuities. But to the question, what saie you? If the side of a Cube be. 3. foote (as that altare might bee) how many foote shall the side be of that Cube, whiche must be double vnto it.

Scholar. This I consider. That firste I must finde the quantitie of the Cube, that is proponed. And then shall I double that quantitie. Thirdly, I must extracte the Cubike roote, of that double number.

So in this question, the side of the knowne Cube is 3. and therefore the whole Cube is. 27. whose double is 54. And the Cubike roote is. 3. and  $\frac{27}{8}$  by Cardanes rule: That is. 4. whiche is plainly false, for. 4. is the roote of. 64. and not of. 54. But by Scheubelius rule, it wil be.  $3\frac{27}{8}$  that is.  $3\frac{3}{4}$  almoste: whiche is moche nigher the truth. For.  $3\frac{3}{4}$  multiplied Cubikely, doeth make. 52.  $\frac{27}{8}$ . whiche is to litle by a good deale, that is by.  $1\frac{1}{8}$ .

W. lii. Whereas

## The extraction

whereas  $3\frac{27}{27}$  doeth make a lesser somme: that is to say but  $51\frac{2592}{2592}$  and so wanteth.  $2\frac{4694}{2592}$ . And although bothe these sommes goe nigher to the truth, then Cardanes rule, whiche misleth. 10. Wholy: Yet maie it be easily seen, that Scheubelius rule is not so good, as he would it were. And the worse here, for the addynge of that one more.

**Master.** You are lepte berie sodenly from a scholar, to a cōptroller. And yet I can not but praise your diligente obscurynge of soche thynges.

Where now by the Cyphers, how it will frame.

**Scholar.** I sette doune the number with .6. Cyphers, and picke them thus.

Then dooe I take the greatestte Cubike number in. 54. whiche is. 27 and that I doe abate from 54. and so resteth. 27. the roote of the Cube is. 3. whiche I sette in the *quotiente* line.

And then I triple. 3. whiche maketh. 9. that muste be multiplied by the *quotiente* againe, and so commeth 27. to be the diuisor. And his *quotiente* semeth to be. 9.

Wherefoze woorkyng with it, the firste number is. 243. and the seconde is. 729. that is. 81. multiplied by 9. whiche is the triple.

Again, the Cube of. 9. is. 729. And all thei together, dooe make 32319 whiche seme is to greate, and therfoze I must take a lesser *quotiente*. As I mighte haue per-

27
84000000(38.
9
27
216
576

ceiued well inough by the second nōber, if I had marked it in time.

But now amendyng my ouer sighte, I take. 8. for the *quotiente*. And woorkyng with it I see, the firste number vnder the line, will

be

## of Rootes.

bec. 216. and the seconde. 576. And here all ready I  
espie my ouersight again.

¶ Therfore I take .7. to be the *quotiente*. And by it I  
multiplie the diuisor, and so haue  
3.189. for the firste number.

And for the seconde number, I  
doe worke with .49. whiche is the  
square of the *quotiente*, multiplied  
by .9. that is the triple: and it yel-  
deth. 441.

¶ Thirdly, I take the *Cube* of .7.  
whiche is. 343. And then addynge  
al. 3. numbers together, I finde the  
somme to bee. 23653. whiche is to bee abated out of  
27000. and so resteth 3347. ¶ Therby I see, that. 37.  
with somewhat more is the roote that I should finde.

But for farther triall, I triple all the *quotiente*, and  
finde thereby. 111. whiche I mul-  
tiplie by the same *quotiente* again,  
and so commeth 4107. to bee the  
diuisor. And his *quotiente* will bee-  
8. as it semeth: and so the first nū-  
ber will bee. 32856. And the se-  
conde shall bee. 7104. but those. 2. are to greate, as it  
is manifeste all readie.

¶ Therfore I take 7 for the *quotiente*. And by it mul-  
tipling the diuisor, there riseth  
28749.

And for the seconde somme,  
there is founde. 5439.

And for the thirde some. 343.

All whiche. 3. sommes ioined  
in one, dooe make. 2929633.  
And that beeyng abated out of  
the higher somme. 3347000.  
doeth leaue. 417367.

$$\begin{array}{r} 3 \\ 27 \overline{) 347} \\ 84 \overline{) 347} \end{array}$$

$$\begin{array}{r} 9 \\ 27 \overline{) 347} \end{array}$$

$$\begin{array}{r} 189 \\ 441 \overline{) 23653} \end{array}$$

$$\begin{array}{r} 343 \\ 343 \overline{) 3347} \end{array}$$

$$\begin{array}{r} 23653 \\ 3347 \overline{) 27000} \end{array}$$

$$\begin{array}{r} 3347 \\ 111 \overline{) 3347} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

$$\begin{array}{r} 3347000 \\ 111 \overline{) 3347000} \end{array}$$

¶ Therfore

## The extraction

Wherefore I maie boldly saie, that the fraction is  $\frac{77}{100}$  and more, by the portion of the remainer, whiche is nighbe  $\frac{1}{700}$ .

And it is sone seen that  $\frac{77}{100}$  are equalle to  $\frac{1}{4}$ : wherefore  $\frac{77}{100}$  shall be more then  $\frac{1}{4}$ .

And so dooeth *Scheubelius* rule erre more, then I thought befoze.

So is your question aunswerd, that the side of the double *Cube*, shall be. 3. foote and  $\frac{77}{100}$  and  $\frac{1}{7}$  of  $\frac{1}{100}$ .

*Of the rootes  
of fractions.*

Master. For the rootes of fractions, I shall nedde to saie no more but this: that if the numerator and denominator bothe be *Squares*, or *Cubes*, &c. then maie you finde in that fractiō the like roote. But if any of bothe doe swarue from that name, then hath that fraction no soche roote.

As  $\frac{16}{25}$  is nother *Cubike* nor *Square*, bicause his partes dooe not agree in *Square* name, nor in *Cubike* name: although the numerator bee a *Square*, and the denominator a *Cube*.

Scholar. That doeth appeare reasonable, at the firste sighte.

Master. Then seeing you are so readie in learning: aunswere me to this question.

*A question of  
a Gonne.*

A Gonne of sixe inches diameter in the mouth, doeth shotte a bollet of twentiepound weight: what weighte shall that bollette haue, that serueth for a gonne of. 14. inches in the mouth?

But to helpe you in this question, and in all soche like, you shall marke well *Euclide* his saying, in the 18 proposition of his. 12. booke, whiche is this.

*All Globes bise together triple that proportion, that their diameters doe*

So in this example, the proportion of the diameters beyng as. 14. to. 6. Or as. 7. to. 3. I shall triple it, and then haue I the proportion of their Globes.

Wherefore

## of Rootes.

Wherefoze I sette the 3. fractions thus.  $\frac{7}{4} \frac{7}{4} \frac{7}{4}$  and thei make  $\frac{343}{64}$ . that is. 5  $\frac{19}{64}$ . And so is the proportion of the Globes, as well in weighte, as in bignesse.

Wherefoze I must multiplie. 20. that is the weight of the lesser bollette, by the numerato<sup>r</sup> of the proportion, and diuide it by the denominato<sup>r</sup>.

And so shall I haue. 254  $\frac{1}{2}$  for the weighte of the greater bollete.

Now p<sup>r</sup>ooue you the like  
 woork. Rememb<sup>r</sup>ing that  
 Cubes also, as well as Glo-  
 bes, doe beare triple p<sup>r</sup>opo<sup>r</sup>-  
 tion, in comparison of their  
 sides. As you learned befoze by the. 19. p<sup>r</sup>oposition,  
 of the. 8. booke of *Euclide*.

$\begin{array}{r} 43 \\ 20 \overline{) 860} \\ 860 \\ \hline 0 \end{array}$	$\begin{array}{r} 113 \\ 2412 \\ 6860 (254\frac{1}{2} \\ 2777 \\ \hline 22 \end{array}$
---	---

A Cube of Masse of. 4. inches square, doeth weighe 7. pounce weighte, what shall a Cube of Masse of. 9. inches square, waie? *A question of. 2. Cubes.*

Scholar. The proportion of the sides is as  $\frac{9}{4}$  wh<sup>ic</sup>he I must set downe th<sup>is</sup>e, and multiplie them together, as fractions should bee. And so will it bee thus.  $\frac{9}{4} \frac{9}{4} \frac{9}{4}$ . that maketh.  $\frac{729}{64}$ .

Wherefoze I multiplie the weighte of the lesser Cube, beyng. 7. by. 729. and it maketh. 5103. and that doe I diuide by. 64. and so finde 79  $\frac{32}{64}$ , whereby I maie knowe, that the weighte of the greater Cube, is 79. pounce weighte, and very nigh  $\frac{1}{2}$ .

Master. These. 2. questions dooe teache you, rather the proportion of Cubes, then the vse of the rule: wherefoze to make the questiōs moze agreable to this rule, I p<sup>r</sup>opounde them thus, in backer order.

A bollette of yron of. 7. inches diameter, doeth waie 27. pounce weighte: what shall be the diameter to that bollette that shall waie. 125. pounce weighte?

Scholar. I p<sup>r</sup>aie you aunswer to it your self, that I maie see the apte forme of applyng soche questions

## The extraction

to this rule.

**Maſter.** As the *Cubes* are in triple proportion to the ſides, ſo are the proportions of the ſides, to bee founde by triple diuiſion: that is to ſaie, by ſeking the *Cubike rootes*, of the 2. termes of the proportion.

**¶** Herefore I doe firſte ſet doune the termes of the proportion of the bollettes, thus:  $\frac{125}{27}$ . And I ſee, that the *Cubike roote* of. 125. is. 5. and the like roote of. 27. is. 3. whiche numbers I ſhall ſet in the roome of the 2. others, thus:  $\frac{5}{3}$  And thei declare the proportion, betwene the *diameters* of the. 2. bollettes. **¶** Hereof one that is the leſſer, is knowne to be. 7. Therefore I multiplye that. 7. by. 5. whereof commeth. 35. and that. 35. doe I diuide by. 3. whiche giueth.  $11\frac{2}{3}$ .

**¶** Herefore I ſaie, that if. 7. inches bee the *diameter* to a bollette of. 27. pounce weighte, then. 11. inches and  $\frac{2}{3}$  ſhall be the *diameter* to the bollete of. 125. pounce weighte.

**Scholar.** The prooſe of this had neede bee certain, ſeeing the woork is obſcure, to the common iudgemente.

*The prooſe.*

**Maſter.** You ſaie well. And this is the very order of prooſe for it. Multiplye bothe theſe rootes *Cubikely*. And if their *Cubes* be in ſoche proportion as their weightes bee (that is to ſaie in this exaple as  $\frac{125}{27}$ ) then is the woork good: els not.

**Scholar.** What muſt needes bee ſo. And therefore will I proue it ſo in theſe numbers.

And for that eande, firſte I multiplye. 7. *Cubikely*, and it giueth. 343. Then I multiplye.  $11\frac{2}{3}$ . *Cubikely*, and it maketh  $41\frac{4}{27}$ . But now ſeing the one number is a fraction, I will for eaſe tourne the other into a fraction of theſame denomination: and it will bee  $\frac{2101}{27}$  in whiche. 2. fractions, the proportion muſte conſiſt betwene the numeratours. So that thei bothe beeing diuided by one common number, muſte come to this fraction

## of Rootes.

fraction  $\frac{115}{37}$ .

And so I see it will be: for the lesser being diuided by. 343. will yelde 27. And the greater diuided by the same. 343. will giue. 125. So that by triall, that worke is approued good.

Maſter. I will now proue your cunnynge, in a newe question, whiche Maſters often tymes, haue occasion to vse: as thus.

I haue a dice of Masse of. 64. vneces of Troye weighte, whose side is. 3. inches and  $\frac{1}{2}$  and would haue an other dice of the same mettall of. 18. pounce weighte. *A question of weightes.*

My demaunde is: what shall be the side of the dice?

Scholar. This question must firste bee reduced to one kinde of denomination in the weightes, and then will it be moze apte to be answered.

Wherefore I shall tourne. 18. pounce into vneces, multiplying it by. 12. and it will be. 216.

And then I consider the proportion, that is betwene those. 2. numbers of weighte. 64. and. 216. and it is certainly.  $3\frac{3}{4}$ , or  $2\frac{7}{4}$  out of whiche proportion, I must extracte the Cubike roote, as I maie easily dooe, seying bothe the numerator and the denominator, are Cubike numbers.

And so is their roote  $\frac{1}{2}$ : whiche is the proportion of the sides of the two dice.

And seying the side of the lesser die, is knowne to be 3. inches and  $\frac{1}{2}$ , the other his side must be in *Sesquialter* proportion to it, that is.  $5\frac{1}{4}$ : whiche is wrought also thus. I multiplie.  $3\frac{1}{2}$  by. 3. and it maketh.  $10\frac{1}{2}$  whiche I shall diuide by. 2. and there cometh.  $5\frac{1}{4}$ .

Maſter. Yet one question moze I will propounde to giue you occasion, to vnderstande the apte conference of masses, of diuerse stuffe.

And for that purpose, I suppose this proportion in weighte, to be betwene masses of one biggenesse.

Q. y.

That

## The extraction

Examples of  
rates for  
weightes.

That if I compare  
Wodde and Stone of one  
quantitie together, the  
stone shall weighe moze  
then the wodde by  $\frac{2}{3}$ .

Like wates yron to be  
heuer then stone by  $\frac{1}{2}$ .

And Brasse to bee he-  
uier then yron by  $\frac{1}{3}$ .

Ledde to be heuer then Brasse by  $\frac{2}{3}$ .

All whiche rates, although thei be taken for exam-  
ples, and not of truthe, yet thereby make you learne,  
how to woozke with truc rates, set in a like table.

And now for the vse of this table, take this questiō.

A questiō  
of weighte.

I would haue .5. weightes of Cubike forme, made  
of these .5. stufes.

The weighte of the wodde shall be. 28. pounce.

The stone. 56. pounce.

The yron. 112. pounce.

The Brasse. 224. pounce.

And the Ledde. 448. pounce.

Of all these I haue but the yron weighte: whose  
side, or Cubike roote is. 12. inches  $\frac{2}{3}$ .

And my desire is to knowe, of what quantitie the  
sides of all the other weightes shall bee.

Scholar. The questiō is pleasaunt: and yet some  
what harder then the other.

Master. The table will helpe you fully, so that  
you cōferre it well, with that you haue learned before.

But bicause I haue litle leiser, to spende moche  
tyme with you (saue that zeale to your furtheraunce  
doeth make me partly to forgette my owne businesse)  
therefore will I leaue this questiō to your self, to be  
answered at your laisure.

And so in all the rest, I must passe it ouer: and giue  
an eye to suche maters, that touche me moze nigh:  
and

Stoffe.	Weighte.			
Wodde.	60.	1		
Stone.	100	$\frac{2}{3}$	1	
Yron.	150	$\frac{2}{3}$	$\frac{2}{3}$	1
Brasse.	200	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Ledde.	280	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$



## of Rootes.

and weighe more heuily, then all soche weightes, by  
20. folde.

¶ herfore, touchyng all the rootes of compounde  
numbers, you shall at my hand now, haue no priuate  
declaration. But soche as you haue learned all reddie.

### Of compounde rootes.



If the number bee com-  
pounde, other of *Square num-*  
*bers*, or of *Cubike numbers*, then  
acco2dyngly as the cōposition  
is, so shal you draw the roote:  
and without one of these two  
there can bee no composition.

¶ herfore to begin with  
the smallest compounde num-  
ber in that sorte, whiche is a *Square of squares*, you *Squares of*  
shall firste extracte the square roote, as you haue lear- *Squares.*  
ned before. And out of that roote (whiche must nedes  
bee a *Square number*) you shall extracte his square roote  
also: and that roote is the *zenzizenzike* roote, of the  
firste *Square of squares* or *zenzizenzike* number.

For example take .14641. whose *Square roote* is  
121. and that same roote is it self,  
a *Square number*: and hath for his  
roote. 11.

¶ herfore I make saie, that. 11.  
is the *Squared square roote*, or the *zenzizenzike* roote of  
14641.

Again 8503056. is a *Square*  
*of squares*, and therfore a *Square*  
*number*. And his *Square roote* is  
2916. whiche is a  
*Square number* also,  
and hath .54. for

$$\begin{array}{r} 2 \\ 14641 \end{array} \begin{array}{l} 2916 \\ 224 \end{array}$$

$$\begin{array}{r} 341 \\ 499893 \\ 8883856 \end{array} \begin{array}{l} 2916 \\ 54 \\ 8 \end{array}$$

¶. 11.

his

# The extraction

his roote.

So that. 54. maie well bee called the *zenzenzenlike* roote of. 8503056.

And so shall you woork, with all of that name.

*Zenzenzenzenlike* But and if the number be compounde, of. 3. *zenzenzenlike* (whiche some men for shortheite, call *zenzenzenzenlike*). Then shall you drawe firste the Square roote, and then the Square roote of that roote, and thirdly the Square roote of that laste roote.

As for example. 6561. is a Square of squared squares. And his firste roote is. 81. whiche is also a Square number, and hath 9. for his roote. That. 9. likewaies is a Square number, and hath. 3. for his roote.

1
6561 (81.
16

So that the *zenzenzenzenlike* roote of. 6561. is. 3.

And for these formes of numbers, I shall not neede to state for any more explication, or examples: seeing the mater is plaine.

Now for compounde Cubike numbers, you shall vnderstande the like forme.

*Cubes of cubes.*

If the number be a Cube of Cubes, you shall firste extracte the Cubike roote. And because that roote is a Cubike number also, therfore shall you seke the Cubike roote of it. And that seconde roote shall bee the Cubicubike roote of the firste number.

As for example. 512. is a Cubike number, or a Cube of Cubes. And his Cubike roote is. 8. whiche. 8. againe is a Cubike number and hath. 2. for his roote.

So that. 2. is the Cubicubike roote of. 512.

Likewaies. 10077696. is a Cubicubike number, and his firste Cubike roote is. 216. as you may easily perceiue by these woordes: where I haue sette forth the order of extraction of his Cubike roote, whiche is. 216. And that. 216. is a Cubike number, you neede not to doubt,

*of Rootes.*

$  \begin{array}{r}  816 \\  77696(216. \\  \underline{63} \\  1323 \\  \hline  7938 \\  2268 \\  \underline{216} \\  81696  \end{array}  $	<p>doubt, for that it is one of          the, which you          have, I dare          say, in perfect          memory: Be-          cause his root          is a digit, and          that is, 6.</p>	$  \begin{array}{r}  2816 \\  177696(21 \\  \underline{6} \\  12 \\  \hline  1261 \\  \hline  1261  \end{array}  $
---	---	--

816696 By this you may iudge of Cubicubikes Cubikely, or Cubes of Cubicubes, that in them Cubikely. you shall firste finde their Cubike roote: And then the Cubike roote of that roote. And thirdly the Cubike roote of that roote againe. And so haue you the Cubicubientike roote of that firste number.

The third way of composition is, when Squares and Cubes be compounded together: as *Zenzicubes*, *Zenzicubicubes*, *Zenzicubicubicubes*, or soche like, as it happeneth diversely.

In all these you shall as often abate the *Zengke* root, as that name is in the composition, and so takes waies of the *Cubike* root.

So that in a *Zenzicubike*, you shall extract first the *Zenzicubike*.  
Square roots: and out of that Square roots, you shall ex-  
tract the *Cubike* roots.

As. 64. is a Zenzicubike number, whose Square roote is 8. and that . 8. is a Cubike number, and hath. 2. for his roote.

So, 531441. is a Zenzizencube: whose first Square  
 root is. 729. whiche number is  
 a Zencicube, & hath for his Square  
 root. 27. And that num-  
 ber is a Cube, and hath  
 for his root. 3. where-  
 fore I make lustily saie,  
 that. 3. is the Zenzizencubike root of. 531441.

Zenzizencu-  
bicube.

444
430
531441 (729
4444
4

But as I said before, that I might not stale long  
at

## *The extraction*

at this presente, so the vse of these greate numbers is rare in practise; and therefore I will ouerpasse them, for this tyme.

And yet for your aied in the meane season, I haue here drawen a table, whiche maie bee called the table of ease: in whiche you haue greate plentie of these numbers, with their rootes in diuerse kindes.

The table it self is so manifeste, that it needeth no declaration: if you haue not forgotten, what you learned before.

And if you liste to enlarge this table, you maie easily doe it, multipliyng the numbers still by their rootes, whiche bee set ouer theim, in the hedde of the table. And so maie you make it to extende infinitely: whiche shall ease you wonderfully, in the extraction of any kinde of rootes. For which at some other time if my leasure serue me better, with quietnesse, I will giue you more specialle rules.

And also I counsell you, well to examine this table, and trust not to my castinge. For haste and other troubles, maie often times cause erreure in supputation.

**T**he

*The frutefull table, whiche maie be called the table of ease.*

1)Rootes.	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
2)Squares.	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400	441	484	529	576
3)Cubes.	8	27	64	125	216	343	512	729	1000	1331	1728	2197	2744	3375	4096	4913	5832	6859	8000	9261	10648	12167	13824
4)Squares of Squares.	16	81	256	625	1296	2401	4096	6561	10000	14641	20736	28561	38416	50625	65536	83521	104976	130321	160000	194481	234256	279841	331776
5)Surfolides.	32	243	1024	3125	7776	16807	32768	59049	100000	161051	248832	371293	537824	759375	1048556	1419857	1889568	2476099	3600000	4084101	5153632	6436343	7962624
6)Squares of Cubes.	64	729	4096	15625	46656	117649	202144	351441	1000000	1771561	2985984	2816809	7529536	11390625	16777216	24137569	34012224	47045881	64000000	85766121	113379904	148035889	191102976
7)Seconde Surfolides.	128	2187	16384	78125	279936	823543	2097152	4781969	10000000	19497171	35831808	62748517	105413504	170859375	268435456	410338673	612220032	893871739	1280000000	1801088541	2494357888	3404825447	4586471424
8)Squares of squared /qres	256	6561	65536	390625	1679616	5764801	16777216	43046721	100000000	214468881	429981696	815730721	1475789056	2542820625	4294967256	695587441	11019960576	16983563041					
9)Cubes of Cubes.	512	19683	262144	1953125	10077696	40353607	134217728	387420489	1000000000	2359157691	5159780352	10604499373											
10)Squares of Surfolides.	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401															
11)C.Surfolides.	2048	177147	4194304	48828125	362777056	1977326743	8589934592																
12)Squares of Zenx cubes.	4096	531441	16777216	244140625	217666236																		
13)D.Surfolides.	8192	1594323	67108864	1220703125	13059974016																		
14)Squares of Bsurfolides.	16384	4782969	268435456	6103515625																			
15)Cubes of Surfolides.	32768	14348907	1073741824																				
16)ZenxZenxZenxZenxikes	65536	43046721	4294967296																				
17)Bsurfolides.	131072	129240163																					
18)Squares of Cubicubes.	262144	387420489			1.	Rootes.	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
19)Bsurfolides.	524288	1162261467			2.	Squares.	625	676	729	784	841	900	961	1024	1089	1156	1225	1296	1369	1444	1521	1600	
20)ZenxZenxSurfolides.	1048576	3586784401			3.	Cubes.	15625	17576	19683	21952	24389	27000	29791	32768	35937	39304	42875	46656	50625	54872	59319	64000	
21)Cubes of Bsurfolides.	2097152				4.	Squares of Squares.	390625	456976	531441	614656	707281	800000	903521	1008576	1128521	1253636	1390625	1679616	1874161	2085136	2313441	2560000	
22)Squares of Csurfolides.	4194304				5.	Surfolides	9765625	11881376	14448907	17210368	20211149	23300000	26629151	30154432	33935395	45435424	52521875	60466176	69343957	79235168	90224199	102400000	
23)Gsurfolides.	8388608				6.	Zenx cubes.	244140625	308915776	336120489	481890304	594823321	729000000	887503681	1073741824	1291467969	1544804416	1838165725	226782336	2565726009	3010936384	3518743761	4096000000	
24)ZenxZenxZenxZenx cubes	16777216				7.	Bsurfolides.	6103515625	8031810176	9075253103	13492928512	17249876309	21870090000	27512614111										

of Cossike numbers.  
Of numbers denominate.



Now haue I lightly ouer run the moſte *Numbers* common kindes of numbers *Abſtraſte*, *contraſte*. And now reſteth the treatiſſe of numbers *Contraſte*, or *Denominate*. Of whiche kinde there bee ſome called numbers *denominate* *vulgarely*: and other bee called numbers *denominate* *Cosſikely*. And a thirde ſorte there is of numbers *radicalle*, whiche commonly bee called numbers *irrationalle*: becauſe many of theim are ſoche, as can not bee expreſſed, by common numbers *Abſtraſte*, nother by any certain *rationalle* number. Other men call them more aptly *Surde numbers*.

And although many menne would not accountpe them, with numbers *denominate*, yet I maie iuſtly doe it, for that thei require a reduction to one denomination, if thei haue ſeueralle ſignes of quantities, as you ſhall heare hereafter. And thoſe numbers neuer goe alone, without ſome other ſigne, and name of rooted quantitie, annexed to them.

Of the firſt kinde of numbers *denominate*, whiche are *vulgarely* *denominate*, as. 10. ſhillinges. 10. men 20. ſhippes, 100. ſhepe. 1000. yerres, and ſoche like, I will ſpeake nothyng in this treatiſſe. But of the other twoo kindes I will ſomewhat write, for youre learning and contentation.

Scholar. Sir, I am moche bounde vnto you: And therefore remit all to your owne diſcretion and good will. Truſtyng ſo to applie my ſtudie, and employe my knowlege, that it ſhall neuer repente you, of your eurteliſſe in this behalfe.

Maſter. Then marke well my wordes, and you ſhall perceiue, that I will uſe as moche plainneſſe, as I maie, in teachyng: And therefore will beginne with *Cosſike numbers* firſt.

# The Arte Of Cossike numbers.



**N**umbers Cossike, are soche as bee contrate vnto a denomination of some Cossike signe as 1. number. 1. roote. 1. square 1. Cube. &c.

But as for cōpendiousnesse in the vse of them, there bee certain figures set for to signifie them: so I thinke it good to expresse vnto you those figures, before wee enter any farther, to thintente we maie procede allwaies in certentie, and knowe the thynges that wee intermedle withall: for thei are the signes of all the arte, that followeth here to be taught.

And although there be many kindes of irrational numbers, yet those figures that serue in Cossike nōbers, bee the figures also of all irrtrionalle numbers, and therfore being ones well knowen, thei serue in bothe places commodiously.

These therfore be their signes, and significations briefly touched: for their nature is partly declared before.

- q. Betokeneth number absolute: as if it had no signe.
- ℞. Signifieth the roote of any number.
- ☐. Representeth a square number.
- ℥. Expresseth a Cubike number.
- ☐☐. Is the signe of a square of squares, or Zenzicubike.
- ℞☐. Standeth for a Surfolide.
- ☐℥. Doeth signifie a Zenzicubike, or a square of Cubes.
- ℞☐☐. Doeth betoken a seconde Surfolide.
- ☐☐☐. Doeth represent a square of squares squared

## of Coslike numbers.

	ly, 02 a zenxixenzixenzike.
$\text{C} \cdot \text{C} \cdot$	Signifieth a Cube of Cubes.
$\tilde{\text{S}} \cdot \tilde{\text{S}} \cdot$	Expresleth a Square of Surfolides.
$\text{C} \cdot \tilde{\text{S}} \cdot$	Betokeneth a thirde Surfolide.
$\tilde{\text{S}} \cdot \tilde{\text{S}} \cdot \text{C} \cdot$	Representeth a Square of Squared Cubes : 02 a Zenxixenzixenxibike.
$\text{D} \cdot \tilde{\text{S}} \cdot$	Standeth for a fourthe Surfolide.
$\tilde{\text{S}} \cdot \text{S} \cdot \tilde{\text{S}} \cdot$	Is the signe of a square of seconde Surfolides
$\text{C} \cdot \tilde{\text{S}} \cdot$	Signifieth a Cube of Surfolides.
$\tilde{\text{S}} \cdot \tilde{\text{S}} \cdot \tilde{\text{S}} \cdot \tilde{\text{S}} \cdot$	Betokeneth a Square of Squares, squaredly squared.
$\text{E} \cdot \tilde{\text{S}} \cdot$	Is the firste Surfolide.
$\tilde{\text{S}} \cdot \text{C} \cdot \text{C} \cdot$	Expresleth a square of Cubike Cubes.
$\text{F} \cdot \tilde{\text{S}} \cdot$	Is the sixte Surfolide.
$\tilde{\text{S}} \cdot \tilde{\text{S}} \cdot \tilde{\text{S}} \cdot$	Doeth represente a square of squared surfolides.
$\text{C} \cdot \text{S} \cdot \tilde{\text{S}} \cdot$	Standeth for a Cube of seconde Surfolides.
$\tilde{\text{S}} \cdot \text{C} \cdot \tilde{\text{S}} \cdot$	Is a square of thirde Surfolides.
$\text{G} \cdot \tilde{\text{S}} \cdot$	Doeth betoken the seuenthe Surfolide.
$\tilde{\text{S}} \cdot \tilde{\text{S}} \cdot \tilde{\text{S}} \cdot \text{C} \cdot$	Signifieth a square of squares, of squared Cubes.

And though I maie proceade infinitely in this sorte, yet I thinke it shall be a rare chaunce, that you shall nede this moche : and therefore this maie suffice. Notwithstandinge, I will anon tell you, how you maie continue these numbers, by progression, as farre as you liste.

And farther you shal vnderstande, that many men doe euer more call square numbers *zenxikes*, as a shorter and apter name, other men call those squares the *firste quantities*, and the cubes thei call *seconde quantities*: squares of squares thei call *thirde quantities*, and surfolides *fourthe quantities*. And so namyng them all quantities (excepte numbers and rootes) thei dooe adde to them for a difference, an ordinall name of number, as thei doe goe in order successiuelly.



## The Arte

As here foloweth in example.

Ʒ.	Firſte.	}	<i>Quantities.</i>
℥.	Seconde.		
Ʒ. Ʒ.	Thirde.		
Ʒ. Ʒ.	Fourth.		
Ʒ. ℥.	Fifte.		
℥. Ʒ.	Sixte.		
Ʒ. Ʒ. Ʒ.	Seuenthe.		
℥. ℥.	Eighthe.		
Ʒ. Ʒ.	Nineth.		
Ʒ. Ʒ.	Tenth.		
Ʒ. Ʒ. ℥.	Eleuenthe.		
℥. Ʒ.	Twelfth.		

And so forth, of as many as maie bee reckened.

But althoughe ſomemen accompte this the moze eaſie waie: bicauſe the other names be comberouſe, yet thoſe other names beſore, do expreſſe the qualitye of the number,

better then theſe later names doe.

Scholar. I thanke you double, ſith you are contente to teache me double names: ſo ſo ſhall I be acquainted with bothe formes, as I ſhall chaunce on them in other mennes bookes.

Therefore now you maie procede to numeration: whiche I thinke it nerte.

Maſter. There be other. 2. ſignes in oſten uſe, of whiche the firſte is made thus —+— and betokeneth moze: the other is thus made — and betokeneth leſſe.

And where theſe come in any number *Coſike*, or other, that number is called a compoſunde number, bicauſe it conſiſteth of. 2. numbers. And where neither of theſe is, the number is called vncompoſunde, althoughe the ſigne be compoſunde. For the compoſunde ſigne, maketh not a compoſunde number. And now I will procede to numeration.

of *Coslike* numbers.  
Of Numeration in numbers  
*Coslike*, vncompounde.

Master.



Numbers *Coslike* vncompounde, haue no *Numeration* difficultie in their numeration: for euer more the nōber representeth, so many of that *Coslike* denominatiō, be the nōbers, rootes, squares, Cubes, squares of squares, or any other like as ther be vnities in that nōber  
So. 6. 9. is. 6. numbers: And. 6. 2. is. 6. rootes:  
20. 5. is. 20. squares: 30. 2. betokeneth. 30. Cubes.  
Scholar. I see it well. For by this nōber. 20. 5. is not appointed any nōber absolute, of one certaintie, but onely so many *quantities* of that kinde: whiche maie bee. 80. if. 4. be one square. And if. 9. be one square, then 20. squares make 180. And if. 25. be one of those squares thereby represented, then. 20. squares make 500. And as for the signes, you taught me the before.

Of Addition.

Master.



This numeration is so plaine, that wee *Addition of* maie passe from it vnto addition: whiche *like signes.* is as easie also, if the quantities be of one denomination. For then nedeth no more, but to adde the numbers together, and to put that same common *Coslike* denomination, to the totall thereof.

Scholar. I take it thus, 20. 2. added to. 30. 2. will make. 50. 2. And. 12. 3. added to. 16. 3. bryngeth forth. 28 3.

Master. As you doe easily see al the mater of this addition, so maie you as easily conceine, all the worke *Subtraction* of subtraction. For it is wrought as in vulgare nōbers *of like signes*  
Scholar.

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Scholar. When if I abate. 6.  $\text{℥}$ . out of. 10.  $\text{℥}$ . there will reste 4.  $\text{℥}$ . And so. 9.  $\text{℥}$ . out of. 25.  $\text{℥}$ . doeth leaue. 16.  $\text{℥}$ .

Master. This is all for numbers of like signes *Cosike*.

Scholar. What then if I would adde. 10.  $\text{℥}$ . to 6.  $\text{℥}$ . where the signes bee unlike: maie it be doth: se: yng thei be not of one denominatiō, nor signe *Cosike*.

*Addition of vnlike signes* Master. As well as shillinges maie bee added with poudes, or penies: and in like forme.

For thei shall stand still as thei wer, with the signe of addition, whiche is this.  $—+—$ . & betokeneth more.

So that. 10.  $\text{℥}$ . put to. 6.  $\text{℥}$ . maketh. 6.  $\text{℥}$ .  $—+—$  10.  $\text{℥}$ . that is. 6.  $\text{℥}$ . more. 10.  $\text{℥}$ . or. 6.  $\text{℥}$ . and. 10.  $\text{℥}$

Scholar. And why not. 10.  $\text{℥}$   $—+—$  6.  $\text{℥}$ ?

Master. Bicause it is moſte orderly, to sette the greateſte ſigne *Cosike*, for moſte in order.

As you ſaie. 20. ſhillinges, and. 6. pennies: rather then. 6. pennies and. 20. ſhillinges.

Scholar. When I ſe. if. 15.  $\text{℥}$ . be added to. 18.  $\text{℥}$ . it will make. 18.  $\text{℥}$ .  $—+—$ . 15.  $\text{℥}$ . An ſo. 12.  $\text{℥}$ . toynd with. 20.  $\text{℥}$ . dooe make. 20.  $\text{℥}$ .  $—+—$  12.  $\text{℥}$ .

## Of Subtraction.

Master.

*Subtraction of vnlike signes*



*Subtraction* is as easie: for it doeth depend onely of the ſigne of abatemente, which is this.  $—$ , and ſignifieth leſſe, or abat yng. And therefore if I would abate 6.  $\text{℥}$ . out of. 10.  $\text{℥}$ . I muſt ſette it thus 10.  $\text{℥}$ .  $—$ . 6.  $\text{℥}$ : that is to ſaie. 10.  $\text{℥}$ . leſſe. 6.  $\text{℥}$ . or abat yng. 6.  $\text{℥}$ .

Scholar. When if I haue 30.  $\text{℥}$ . and would abate out of the. 12.  $\text{℥}$ . I muſt ſet it thus. 30.  $\text{℥}$ .  $—$ . 12.  $\text{℥}$ . that is. 30. cubes ſaue. 12. numbers. And if multiplication

## of Coslike numbers.

tion and diuision, bee as easie, thei shall neade no greate studie.

## Of Multiplication.

Maister.



Some what more labour is there *Multiplication* in multiplication and diuision, to finde out the newe signes as I will tell you anon. But for finding of the numbers, the common multiplication and diuision doeth serue.

So that when.  $12. \text{z}$ . is multiplied by.  $6. \text{z}$ . it maketh.  $72. \text{z}$ . And if.  $24. \text{z}$ . bee multiplied by.  $5. \text{z}$ . there riseth.  $120. \text{z}$ .

Scholar. This passeth my cunningge, for the finding of the newe signe: although the multiplication of the numbers, be as easie as can be.

Maister. If you did well remeiber, what you haue learned before: the mater would not seme so harde.

Doe not you knowe, that a roote multiplied by a roote, doeth make a square: And a square multiplied by his roote, doeth bring forth a cube?

Scholar. That I knowe right well: and therefore a *Square of Squares* multiplied by his roote, will yelde a *Surfolide*.

Maister. Then by like reason, a *Cube* multiplied by a *Square*, shall make a *Surfolide*.

Scholar. In deece it is all one, to multiplie a *Cube* by a *Square*, and a *Square of Squares* by a roote.

Maister. Then for a generall rule, I will sette forth here a presidente for you: where by you maie knowe the newe signe, in all multiplication or diuision: not onely by sight very speedily, but that you maie also commit it aptly to memorie.

Wherefore marke wel this table folowing: where you see in the higher rowe, a line of numbers, set in naturall

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naturall progression : and vnder them you see the signes of *coslike* numbers.

## The table of *Coslike* signes, and their peculiernumbers.

0.	1.	2.	3.	4.	5.	6.
9.	℥.	℥.	℥.	℥.℥.	℥.℥.	℥.℥.
7.	8.	9.	10.	11.	12.	13.
b/℥.	℥.℥.℥.℥.℥.	℥.℥.℥.	℥.℥.℥.	℥.℥.℥.	℥.℥.℥.℥.℥.	℥.℥.℥.

This table is largely set forth, in the title of progression, whereunto you maie haue recourse, if your number be to greate for this table.

By this table maie you easily knowe, the signe that shall serue for your newe somme, in multiplication.

As for example, if I dooe multiple squares by rootes : I looke in the table, what numbers stande ouer them bothe, and puttyng those .2. numbers together, I seeke the totall in the same line, and vnder it I finde the newe denomination *coslike*, whiche I should haue

Scholar. I perceiue ouer ℥. the number of 1. and ouer ℥. the number .2. whiche bothe added together make .3. And because vnder .3. I find the figure or signe of. ℥. I muste take that for the newe denomination.

Master. You saie truthe.

Scholar. Then if I multiplie. 12. ℥. by. 8. ℥. the somme will be. 96. ℥. For ouer. ℥. I finde 3. and ouer. ℥. standeth. 6. whiche bothe together doe make. 9. and vnder. 9. I see. ℥.℥. whiche I take for the denominator.

And if the same rule bee generall, I am cunnynge  
inoughe

## of Cossike numbers.

trouge in it.

Master. It is generall, for multiplication in this kinde.

## Of Diuision.

**D**ivision. It for diuision, you muste abate the one number out of the other, to finde a new denomination.

Wherefore if you would diuide 96. by 8. the *quotiente* will be 12. because that ouer the signe of your diuident, standeth 9. And ouer the diuisors signe is set 3. Wherefore abating 3. from 9. there resteth 6. vnder whiche is the signe 3. that I must take, to put to my *quotiente*.

Scholar. Then for an other triall, if I would diuide 260. by 5. the *quotient* will be 52. For because that ouer 5. I finde 17. and ouer 5. standeth 5. then subtracting 5. from 17. there resteth 12 vnder whiche in the table I finde 2. 2. 0. 0.

So diuiding 20. by 4. the *quotiente* will be 5. and so of other.

Master. But and if you would diuide 12. by 5. that must be set in forme of fraction, thus.

So 12. by 5. maketh  $\frac{12}{5}$  and 6. by 2. yeldeth  $\frac{6}{2}$ . of whiche fractions, wee will speake amongst the fractions of Cossikes compoūde. For they degenerate out of this kinde.

Wherefore this maie suffice briefly, for the customeable woorkes of whole Cossike numbers.

## Of Fractions in Cossike numbers,



As for fractions, the woorking is like in euery point, vnto the woork of numbers *Abstracte*: remembryng onely that as those broken numbers, haue a Cossike denomination annexed with them, so must

E. i. that

# The Arte

that denomination followe the rules , now laste declared.

And therefore I shall not neede to doe any more, but  
to set forth the onely certain examples, of euery kinde of  
woorde in them.

### Examples of Numeration.

$\frac{2}{3}$   $\sqrt[3]{\phantom{x}}$ . Signifieth  $\frac{2}{3}$  of a Roote.

$\frac{8}{9}$  Z. Betokeneth  $\frac{8}{9}$  of a Square.

$\frac{12}{17} \mathcal{L}$ . Representeth  $\frac{12}{17}$  of a *Cube*.

And so of all other formes of *Cosike* signes: where by is intended, that the *Cosike quantitie*, is divided into so many partes, as the denominator containeth, and there is here represented onely so many of them, as the numerator doeth importe.

Scholar. Hereby I dooe perceiue, that a fraction  
*Coſike*, maie ſignific a number, and not onely a parte  
of an vnitie, as it did in numbers *Abſtraſte*.

For when I saie  $\frac{2}{3} \delta^2$ . if that Square be. 9. then that fraction signifieth. 6. But if the Square be. 4. then that fraction doeth represente.  $2\frac{2}{3}$ .

Likeliwaies  $\frac{3}{4}C$ . if the *Cube* be. 8. then that fraction doeth signifie. 6. But if the *Cube* be. 27. then that fraction is equalle to. 2 o.  $\frac{1}{4}$ .

Master. You doe consider it well.

### *Of Addition.*

*Additions.*

Now for addition, take these examples.

$\frac{2}{3}Z$ , added to  $\frac{3}{4}Z$ , doe make  $\frac{17}{12}Z$ , or,  $1Z\frac{5}{12}$ .

$\frac{1}{7}C$  joined with  $\frac{7}{8}C$ , doe make  $\frac{19}{56}C$ , or 1.  $C \frac{11}{16}$ .

And in vnlike signes,

$\frac{3}{4}$  of 8, added to  $\frac{4}{5}$  of 10, doe make  $\frac{4}{5}$  of 20. —  $\frac{1}{4}$  of 20, or els  
thus by one common denomi- 16. — 15. —  
nato 2.

**Def**

## of Cossike numbers.

Of whiche I will speake more in the *Binomialles*, and therefore will omitte it, till we come to them.

Scholar. As for the reste, I see it well: For the woork is all one with fractions *Abstracte*.

And here the denominatiō of *Cossike* signe is not varied, although here be vsed diuerse multiplications.

Maister. And good reason: for the whole *quotiente* whiche is represented by that *Cossike* signe, is not multiplied, but certaine partes of it: and therefore oughte that *Cossike* signe, to stand vnaltered, as the quantitie represented by it, is not multiplied nor altered.

## Examples of Subtraction.

$\frac{1}{2} \text{℥}$ . abated out of  $\frac{3}{4} \text{℥}$ . doe leaue  $\frac{1}{4} \text{℥}$ .

$\frac{1}{2} \text{℥}$ . out of  $\frac{7}{8} \text{℥}$ . there resteth  $\frac{3}{8} \text{℥}$ .

$\frac{1}{2} \text{℥}$ . subtracted frō  $\frac{4}{8} \text{℥}$  doe leaue  $\frac{3}{8} \text{℥}$ . or  $\frac{1}{2} \text{℥}$ .

And in unlike signes.

$\frac{1}{2} \text{℥}$  abated frō  $\frac{1}{2} \text{℥}$  doe leue  $\frac{1}{2} \text{℥}$  —  $\frac{1}{2} \text{℥}$ .

$\frac{1}{2} \text{℥}$  taken out of  $\frac{3}{4} \text{℥}$ . the reste is  $\frac{1}{4} \text{℥}$  —  $\frac{1}{2} \text{℥}$ .

Likewaies as in additiō, so in this sorte of subtraction, there maie be an other kinde of woork, whiche I will remit to the treatise of *Binomialles*.

## Examples of Multiplication.

$\frac{1}{2} \text{℥}$  multiplied by  $\frac{1}{2} \text{℥}$ . doe make  $\frac{1}{4} \text{℥}$ .

$\frac{1}{2} \text{℥}$  multiplied by  $\frac{3}{4} \text{℥}$ . bringeth forth  $\frac{3}{8} \text{℥}$ .

$\frac{1}{2} \text{℥}$  multiplied by  $\frac{1}{2} \text{℥}$ . doe yelde  $\frac{1}{4} \text{℥}$ , or  $\frac{1}{2} \text{℥}$ .

Where the signes doe alter, as in the multiplication of whole *Cossike* numbers.

℥. ij.

Scholar.



## The Arte

Scholar. This doeth somewhat trouble me: that the *Coslike* signes should chaunge here, rather then in addition, or subtraction: Seyng there was as moche multiplication, in any of them bothe, as there is here.

Master. Marke the mater well, and you shall bee soon satisfied.

For in addition and subtraction, the multiplicatio serueth onely for the reduction, of the .2. fractions, vnto one denomination: And therefore in them, you neuer multiplie the numeratoꝝ together: but you multiplie crosse waies, the numeratoꝝ of the one, by the denominatoꝝ of the other, where as in multiplicatio, you vse no reduction, but doe make a plaine multiplication.

And so likewaies in diuision, there is vsed no meane of reduction: and therefore in it the signes must alter, as befoze is declared.

## Examples of Diuision.

$\frac{6}{7} \div \frac{8}{11}$ . diuided by  $\frac{6}{11}$ . doe make in the *quotiente*  $\frac{44}{7}$ . or  $\frac{11}{7}$ .  
 $\frac{2}{9} \div \mathcal{C}$ . diuided by  $\frac{8}{15} \mathcal{C}$ . doeth yelde  $\frac{105}{72} \mathcal{Q}$ . or els  $\frac{15}{8}$ .

For seying I shall diuide.  $\mathcal{C}$ . by.  $\mathcal{C}$ . I must therefore abate .3. from .3. and so resteth nothing, whiche is signified by this Cipher. 0. and that standeth ouer the signe of number: therefore the fraction, that is as the *quotiente*, must be taken as a number *Abstracte*.

Likewaies  $\frac{8}{7} \div \frac{8}{9}$ . diuided by  $\frac{8}{9}$ . doeth make  $\frac{24}{7} \mathcal{Q}$ . that is to saie .3. And so  $\frac{8}{11} \div \mathcal{C}$ . diuided by  $\frac{2}{11} \mathcal{C}$ . doeth bying forth  $\frac{40}{11} \mathcal{Q}$ . or  $\frac{16}{5} \mathcal{Q}$ .

Scholar. This is sufficient for diuision. Now if you thinke good to speake of progression, I can not but remember you of your promise.

## of Cossike numbers. Of Reduction.

Master.



Although Reduction should go in order before Progression, yet seeing this Reduction, consisteth in the onely numbers, and not in the signes: and therefore agreeth with vulgare reduction of fractions (as here you maie see before in diuerse examples) therefore will we omitte it, and go in hande with Progression: whiche is more straunge.

Scholar. I praie you so: For I see this reduction, is but to reduce the greater fraction, to a lesser in number: as I learned long agoe by your other booke.

## Of Progression in Cossike signes.

Master.



Regression is thus wroughte: Firste sette doune as many vulgare numbers, in their naturall progression, as you liste to haue Cossike signes, that by them you maie the better know, the true places of the Cossike signes: so that you set in the firste place a Cipher, and vnder it. 9. And then vnder. 1. set. 2. vnder. 2. put. 3. and vnder. 3. write. 4. As you see in the table folow- yng. And by these shall you set, as many as you liste.

For all the vulgare numbers, whiche you haue set in the higher rewe, be other compounde numbers, or els vncompounde: and if the place, where you would set any Cossike signe, be noted with a number vncompounde, then must there be set one of the Surfolides.

For vnder the first number vncompounde, you must set the firste Surfolide, and the second vnder the second number vncompounde: and the thirde vnder the thirde,

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and so forth.

The numbers vnccompounde, are these in this progression.

5. 7. 11. 13. 17. 19. 23. 29. 31. 37. 41.  
43. 47. 53. 59. 61. 67. &c.

Under nethe .5. must you set . $\frac{1}{5}$ . and vnder .7.  $\frac{1}{7}$ .  
vnder .11.  $\frac{1}{11}$ . and vnder .13.  $\frac{1}{13}$ . and so forth, til  
you come to .67. vnder whiche you must set . $\frac{1}{67}$ . and  
vnder 71 you must set  $\frac{1}{71}$ . and so as farre as you list.

But for any other place, because the vulgare number is compounde, that is set (as the pecuhare number, in the higher rewe) therefore the *Cosike* signe must nedes be compounde, other of .2. or of .3. or els of bothe. And if it be compounde of .2. then set downe . $\frac{1}{2}$ . so often tymes, as .2. is in the composition of that number.

As for example: 16. is compounde of .2. so twer tymes (not by addition, but by multiplication, as in sayng, twise .2. twoo tymes, twise.

Scholar. I perceiue twise .2. to bee .4. and twise that to be .8. and twise that to make .16.

Master. So make you worke backward, in sayng .16. diuided by .2. maketh .8. that is ones: then .8. by .2. yeldeth .4. that is twise. Again .4. by .2. maketh .2. that is thise: and .2. for himself, is the fourth: wherefore vnder .16. I must set downe . $\frac{1}{2}$  . $\frac{1}{2}$  . $\frac{1}{2}$  . $\frac{1}{2}$ .

And so vnder .32. I muste sette . $\frac{1}{2}$ . in one thus.  
 $\frac{1}{2}$  . $\frac{1}{2}$  . $\frac{1}{2}$  . $\frac{1}{2}$  . $\frac{1}{2}$  .

And vnder .64. I shall sette it .6. tymes, thus.  
 $\frac{1}{2}$  . $\frac{1}{2}$  . $\frac{1}{2}$  . $\frac{1}{2}$  . $\frac{1}{2}$  . $\frac{1}{2}$  . Bicause .64. is made of .6. multiplications by .2.

Scholar. Were by I see, that vnder .8. I muste put 3. tymes that signe: and vnder .4. twise the same.

Master. So must you in deede.

And now for other places, if their numbers bee compounde

## *of Cossike numbers.*

pounde of. 3. onely, then must you set downe the signe of *Cube*, as oftentimes as . 3. is multiplied, to make that number.

As for example. 27. is compounde onely of. 3. and not of. 2. (for of all other compounde numbers here in then of soche as be cōpounde of. 2. or. 3. we take no regard.) And. 3. multiplied thise, doeth make. 27. in sayng. 3. tymes. 3. thise. And therefore vnder. 27. I shall set this signe of.  $\mathcal{C}$ . three times, thus.  $\mathcal{C}\mathcal{C}\mathcal{C}$ . whiche betokeneth a *Cube of Cubes Cubikely*.

But and if the number bee compounde bothe of. 2. and. 3. then for euery tyme that. 2. is multiplied, so that composition, I shall sette.  $\mathcal{J}$ . and for euery tyme that. 3. is multiplied, I shall set.  $\mathcal{C}$ . remembryng full to set.  $\mathcal{J}$ . before.  $\mathcal{C}$ . and not after hym.

As for example. Vnder. 24. I shall set.  $\mathcal{J}\mathcal{J}\mathcal{C}$ . bicause that. 2. 2. 2. 3. that is to saie. 2. tymes. 2. twise thise, doeth make. 24. Or by resolution, thus. 24. diuided by. 2. giueth. 12. For that firste. 2. set.  $\mathcal{J}$ . Again 12. diuided by. 2. yeldeth. 6. for this seconde. 2. set.  $\mathcal{J}$ . also. Then diuide. 6. by. 2. and it maketh. 3. For the. 2. I must set.  $\mathcal{J}$ . and for. 3. I must put.  $\mathcal{C}$ . and so all together maketh.  $\mathcal{J}\mathcal{J}\mathcal{C}$ . in the. 24. place.

Likewaies vnder. 36. I must sette.  $\mathcal{J}\mathcal{C}\mathcal{C}$ . bicause that. 2. 2. 3. 3. doeth make it, that is. 2. tymes. 2. thise, thise. And by resolution, thus. 36. diuided by 2. giueth. 18. For that. 2. I set.  $\mathcal{J}$ . Again. 18. diuided by. 2. maketh. 9. For that. 2. I sette doune againe.  $\mathcal{J}$ . Thiroly, for bicause. 9. can not bee diuided by. 2. but by. 3. 3. tymes: therefore I muste sette doune. for those twoo. 3. twise.  $\mathcal{C}$ . so the whole signe is.  $\mathcal{J}\mathcal{C}\mathcal{C}$ .

Now if the number of the place, or peculiare number, bee compounde of one of theim twoo, with some other number vncōpounde, then must we toyne their signes together.

As. 10. is compounde of. 2. and. 5. therefore must I  
61

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set vnder. 10. the signe that is in the fifth place, which is  $\text{fz}$ , and before it I muste set the signe of.  $\text{z}$ . so 2. So must that signe be.  $\text{zfz}$ .

Likewaies, because. 15. is compounde of. 3. and. 5. I shall ioine together the signe of  $\text{c}$ . and of  $\text{fz}$ . and make it.  $\text{cfz}$ .

Scholar. So I vnderstande it now, that I cannot misse it. Haue that for lacke of vse, and throughte forgetfulnessse, when I heare the name of composition in numbers, I doe mistake it sometimes for addition, els here can be no erroure. For when I doe consider, that. 20. is compounde of. 2. 2. 5. that is twise. 2. and. 5 (fith. 2. tymes. 2. maketh. 4. and. 5. tymes. 4. maketh 20.) I maie sone consider, to set.  $\text{z}$ . twise before.  $\text{fz}$ . and then it will be.  $\text{zfzfz}$ . to be put in the. 20. place.

Likewaies in the. 21. place, I set.  $\text{cfz}$ . seying 21 is compounde of. 3. and. 7. and.  $\text{c}$ . is the signe to the thirde place, as  $\text{bfz}$ . serueth for the. 7. place.

Master. What shall you set in the. 84. place?

Scholar. 84. is compounde of. 2. 2. 3. 7. therefore his signe must be.  $\text{zfzcfzfz}$ .

Master. Now I see, you are cunnyng inough in this, and therefore take here this table, for a patron: and then will we procede to the worke of *Cossike numbers* compounde,

*The table for progression Cossike,  
whiche maie increase it self infinitely,  
without any difficultie.*

|    |    |    |    |    |     |     |     |     |     |     |     |
|----|----|----|----|----|-----|-----|-----|-----|-----|-----|-----|
| 0. | 1. | 2. | 3. | 4. | 5.  | 6.  | 7.  | 8.  | 9.  | 10. | 11. |
| ၅. | ၆. | ၇. | ၈. | ၉. | ၁၀. | ၁၁. | ၁၂. | ၁၃. | ၁၄. | ၁၅. | ၁၆. |

|     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 12. | 13. | 14. | 15. | 16. | 17. | 18. | 19. | 20. |
| ၁၇. | ၁၈. | ၁၉. | ၂၀. | ၂၁. | ၂၂. | ၂၃. | ၂၄. | ၂၅. |

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 21. | 22. | 23. | 24. | 25. | 26. | 27. | 28. |
| ၂၆. | ၂၇. | ၂၈. | ၂၉. | ၃၀. | ၃၁. | ၃၂. | ၃၃. |

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 29. | 30. | 31. | 32. | 33. | 34. | 35. | 36. |
| ၃၄. | ၃၅. | ၃၆. | ၃၇. | ၃၈. | ၃၉. | ၄၀. | ၄၁. |

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 37. | 38. | 39. | 40. | 41. | 42. | 43. | 44. |
| ၄၂. | ၄၃. | ၄၄. | ၄၅. | ၄၆. | ၄၇. | ၄၈. | ၄၉. |

|     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|
| 45. | 46. | 47. | 48. | 49. | 50. | 51. |
| ၅၀. | ၅၁. | ၅၂. | ၅၃. | ၅၄. | ၅၅. | ၅၆. |

|     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|
| 52. | 53. | 54. | 55. | 56. | 57. | 58. |
| ၅၇. | ၅၈. | ၅၉. | ၆၀. | ၆၁. | ၆၂. | ၆၃. |

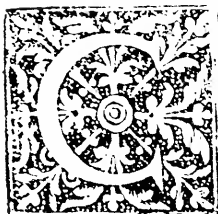
|     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|
| 59. | 60. | 61. | 62. | 63. | 64. | 65. |
| ၆၄. | ၆၅. | ၆၆. | ၆၇. | ၆၈. | ၆၉. | ၇၀. |

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 66. | 67. | 68. | 69. | 70. | 71. | 72. | 73. |
| ၇၁. | ၇၂. | ၇၃. | ၇၄. | ၇၅. | ၇၆. | ၇၇. | ၇၈. |

|     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|
| 74. | 75. | 76. | 77. | 78. | 79. | 80. |
| ၇၉. | ၈၀. | ၈၁. | ၈၂. | ၈၃. | ၈၄. | ၈၅. |

In this table, ခ, င, and ခ, are the groundes:  
of all the reste above them. For of these  
three, all those other bee made.

## The Arte Of Coslike numbers compounde.



Coslike numbers compounde, are made by addition of. 2. or more simple Coslike numbers together :

As. 6.  $\text{ze}$ . — — — — — 5.  $\text{z}$ . 02.

12.  $\text{C}$ . — — — — — 4.  $\text{ze}$ . — — — — — 3. 9. and

so for the in diuerse formes, whiche be infinite. Wherbeit for brieftnesse, we maie comprehend, vnder thesame name (because of the like worke) all other *residualles* Coslike, whiche be made by subtraction : as. 3.  $\text{C}$ . — — — — — 4.  $\text{z}$ . And also those that bee made by addition and subtraction, bothe together: As. 9.  $\text{z}$ .  $\text{z}$ . — — — — — 4.  $\text{ze}$ . — — — — — 6.  $\text{z}$ . In whose numeration is no hardnesse.

Scholar. When your rules maie be the shorter.

## Of Numeration.

Master.



His Numeration is easily vnderstande by addition of simple Coslikes. For this is the forme. 6.  $\text{z}$ . — — — — — 10. 9. that is. 6. *Squares*, more. 10. *nöbers*. Likewises. 8.  $\text{C}$ . — — — — — 11.  $\text{ze}$ . is 8. *Cubes* and. 11.  $\text{ze}$ .

Now for *residualles*, take these examples. 9.  $\text{z}$ .  $\text{z}$ . — — — — — 12.  $\text{C}$ . whiche is. 9. *Squares* of *Squares*, saue. 12. *Cubes*. Also. 4.  $\text{z}$ . — — — — — 15.  $\text{z}$ . that is. 4. *surfolides*, abatynge. 15. *Squares*.

And for bothe together, this is the forme.

10.  $\text{z}$ .  $\text{z}$ . — — — — — 6.  $\text{C}$ . — — — — — 30.  $\text{ze}$ . whiche signifieth 10. *Squares* of *Squares*, and. 6. *Cubes*, abatynge. 30. *rootes*.

Scholar. This is plaine. For so maie I vnderstande of all other As. 9.  $\text{z}$ .  $\text{C}$ . — — — — — 3.  $\text{z}$ . — — — — — 8. 9 that is. 9. *Squares* of *Cubes*, lesse 3. *Squarss*, more. 8. *nöbers*.

Master.

## of Cossike numbers.

Master. It were moze orderly, to kepe the signes of moze and lesse in order, then to followe the order of the *cossike* signes: because that addition, is orderly placed before subtraction. So were it better to set them thus 9.  $\text{Z}$ . — | — 8.  $\text{Y}$ . — | — 3.  $\text{Z}$ . Hobert in deede all is one in these kinde of numbers, but not so in other *Surde numbers*, where the order foloweth of necessity, as shall be declared in their place moze largely.

## Of Addition.



In addition, you must haue consideration of the *Cossike* signes: for noe other number, maye bee added into one, then soche as appertain to one signe *Cossike*.

As in vulgare denominations, you doe not adde the numbers of shyllinges, to the numbers of pennies: but you some shyllinges to shillinges, and pennies to pennies: & poun- des to poun- des, so in *Cossike* numbers, *Cubes* muste bee ioined to *Cubes*, *Squares* to *Squares*, and generally, like to like.

Scholar. If this be al, I cā marke it well inough.

Master. There is somewhat moze to be considered, that if there bee any signe in the one number, whiche is not in the other, that seuerall signe with his number, muste bee sette doun: with his figure of — | — .02. — | — . as it standeth there.

And farther, touchyng those twoo signes. — | — . whiche bee the figures of moze and lesse, you must giue regarde, whether thei bee like or vnlke, in those numbers that must be added: For if thei be like in numbers, of one denomination, then muste thei so remain as thei be. But if thei be vnlke, euermoze abate the smaller number of theim, that followe those

A. y. vnlke



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with the signes, out of the greater: and sette downe the reste, with the signe of the greater number.

Scholar. By examples, I shall better conceiue those rules.

Master. Take these examples.

|  |   |
|--|---|
| $\begin{array}{r} 10. \text{ſ}. - + -. 12. \text{ſ}. \\ 4. \text{ſ}. - + -. 8. \text{ſ}. \\ \hline 14. \text{ſ}. - + -. 20. \text{ſ}. \end{array}$ | $\begin{array}{r} 10. \text{ſ}. ——. 12. \text{ſ}. \\ 4. \text{ſ}. ——. 8. \text{ſ}. \\ \hline 14. \text{ſ}. ——. 20. \text{ſ}. \end{array}$ |
|--|---|

|   |  |
|---|--|
| $\begin{array}{r} 10. \text{ſ}. ——. 8. \text{ſ}. \\ 4. \text{ſ}. ——. 12. \text{ſ}. \\ \hline 14. \text{ſ}. ——. 20. \text{ſ}. \end{array}$ | $\begin{array}{r} 10. \text{ſ}. - + -. 8. \text{ſ}. \\ 4. \text{ſ}. - + -. 12. \text{ſ}. \\ \hline 14. \text{ſ}. - + -. 20. \text{ſ}. \end{array}$ |
|---|--|

|  |   |
|--|---|
| $\begin{array}{r} 10. \text{ſ}. - + -. 12. \text{ſ}. \\ 4. \text{ſ}. ——. 8. \text{ſ}. \\ \hline 14. \text{ſ}. - + -. 4. \text{ſ}. \end{array}$ | $\begin{array}{r} 10. \text{ſ}. ——. 12. \text{ſ}. \\ 4. \text{ſ}. - + -. 8. \text{ſ}. \\ \hline 14. \text{ſ}. ——. 4. \text{ſ}. \end{array}$ |
|--|---|

|   |  |
|---|--|
| $\begin{array}{r} 10. \text{ſ}. - + -. 8. \text{ſ}. \\ 4. \text{ſ}. ——. 12. \text{ſ}. \\ \hline 14. \text{ſ}. ——. 4. \text{ſ}. \end{array}$ | $\begin{array}{r} 10. \text{ſ}. ——. 8. \text{ſ}. \\ 4. \text{ſ}. - + -. 12. \text{ſ}. \\ \hline 14. \text{ſ}. - + -. 4. \text{ſ}. \end{array}$ |
|---|--|

Here haue I varied one example diuersly, to the u-  
 sente you make marke the vse of your rules in theim.  
 And for the reason of those rules, you shall marke  
 those

## *fo Cossike numbers.*

those examples well.

For where in the first example, bothe signes are —|—, it must nedes be, that after the addition of the first numbers, the seconde muste bee added with the signe. —|—.

In the seconde example, where bothe the signes be ———. because there wanteth. 21. 9. of the first. 10. 3. Therfore is it reason, that bothe those wantes should be sette doune with the signe of. ———; and so in the thirde and fourthe examples.

In the fifth example, the seconde somme is not fully. 4. 3. but there wanteth of it. 8. 9. and therefore if you put donne the. 4. 3. fully, you must abate. 8. out of the. 12. 9. in the higher somme: and so of the other examples.

But for more practise, and better declaration of the vse of them, here are other exâples, of more varietie.

$$20. \text{ 3. } \text{℥} \text{ —|— } 9. \text{ 3. } \text{ ——— } 120. \text{ 3. } \text{℥}.$$

$$15. \text{ 3. } \text{℥} \text{ —|— } 5. \text{ 3. } \text{ —|— } 16. \text{ 3. } \text{℥}.$$

$$35. \text{ 3. } \text{℥} \text{ —|— } 14. \text{ 3. } \text{ ——— } 104. \text{ 3. } \text{℥}.$$

$$16. \text{ 3. } \text{ 3. } \text{ —|— } 28. \text{ 9. } \text{ ——— } 16. \text{ 3. } \text{℥}.$$

$$12. \text{ 3. } \text{ 3. } \text{ —|— } 12. \text{ 3. } \text{ ——— } 19. \text{ 9. } \text{℥}.$$

$$28. \text{ 3. } \text{ 3. } \text{ —|— } 9. \text{ 9. } \text{ ——— } 4. \text{ 3. } \text{℥}.$$

In the first example of these. 2. you see. 120. 3. ℥. with the signe of lesse, to bee added with. 16. 3. ℥. with the signe of more: and therefore, seeing the signes of one *Cossike* denomination disagree, I dooe subtracte the lesser, out of the greater: and that. 104. whiche remaineth, I doe set doune with the signe of

℥. iij. lesse

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lette, bicause the remainer is of that uoimber, that bare that signe.

And in the seconde exāple, the placynge of the signe  
 —+— befoze ——— maketh numbers to bee sette be-  
 foze squares: and so the like denominations, dooe not  
 stande one ouer an other. Yet is the wooke dooen as  
 if thei did stande eche ouer his like.

Scholar. I praie you lette me trie my cunnynge,  
 with an example oꝝ twoo.

$$\begin{array}{r}
 17. \text{ 3 } \text{ 3 } \cdot \text{ —+— } 10. \text{ C } \cdot \text{ ——— } 2. \text{ 7 } \cdot \\
 16. \text{ 3 } \cdot \text{ C } \cdot \text{ —+— } 12. \text{ 3 } \cdot \text{ ——— } 6. \text{ 7 } \cdot \\
 \hline
 16. \text{ 3 } \cdot \text{ C } \cdot \text{ —+— } 17. \text{ 3 } \text{ 3 } \cdot \text{ —+— } 10. \text{ C } \cdot \\
 \text{ —+— } 12. \text{ 3 } \cdot \text{ ——— } 8. \text{ 7 } \cdot
 \end{array}$$

I set the example, as numbers came to my mynde:  
 but I had almoste set my self on ground: saue that I  
 called to remembꝛaunce, the comparison that you  
 made, to vulgare denominations of poundes, shillin-  
 ges, and pennies: and so was instructed to place eu-  
 ry seueralle denomination seuerally. And to sette the  
 greatestte denominatiō first, & eche other in his order.

Now will I pꝛoue an other example, oꝝ twoo.

$$\begin{array}{r}
 3. \text{ 3 } \cdot \text{ —+— } 4. \text{ C } \cdot \text{ ——— } 20. \text{ 3 } \cdot \\
 20. \text{ C } \cdot \text{ ——— } 8. \text{ 3 } \cdot \text{ ——— } 16. \text{ 3 } \cdot \\
 \hline
 3. \text{ 3 } \cdot \text{ —+— } 24. \text{ C } \cdot \text{ ——— } 8. \text{ 3 } \cdot \text{ ——— } 36. \text{ 3 } \cdot \\
 \\
 13. \text{ 3 } \cdot \text{ C } \cdot \text{ —+— } 8. \text{ C } \cdot \text{ ——— } 4. \text{ 7 } \cdot \\
 7. \text{ 3 } \cdot \text{ C } \cdot \text{ ——— } 6. \text{ C } \cdot \text{ ——— } 7. \text{ 3 } \cdot \\
 \hline
 20. \text{ 3 } \cdot \text{ C } \cdot \text{ —+— } 2. \text{ C } \cdot \text{ ——— } 4. \text{ 7 } \cdot \text{ ——— } 7. \text{ 3 } \cdot \\
 \text{ 6. 3 } \cdot
 \end{array}$$

## of Cossike numbers.

$$\begin{array}{r}
 6. \text{g} \cdot - + - . 10. \text{ze} \cdot - - - . 8. \text{g} \cdot \\
 4. \text{g} \cdot - + - . 17. \text{g} \cdot - - - . 7. \text{ze} \cdot \\
 \hline
 10. - + - . 3. \text{ze} \cdot - + - . 9. \text{g} \cdot
 \end{array}$$

$$\begin{array}{r}
 4. \text{g} \cdot \text{cl} \cdot - + - . 5. \text{g} \cdot - + - . 6. \text{ze} \cdot \\
 8. \text{cl} \cdot - - - . 8. \text{g} \cdot - - - . 10. \text{ze} \cdot \\
 \hline
 4. \text{g} \cdot \text{cl} \cdot - + - . 8. \text{cl} \cdot - - - . 3. \text{g} \cdot - - - . 4. \text{ze} \cdot
 \end{array}$$

Maister. You haue doene well : And for prooffe of your worke, you maie in this arte not onely proue it, by the contrary kynde, as you did in nōbers *Abstraite*, but also by the resolution of all those *Cossike* numbers into nōbers *Abstraite*, takyng any number for a roote and then the *Squares* and *Cubes*. &c. accordingly. As here in this table, you maie hysely see, but more largely in the table at the ende of numbers figurallie.

### *A table for trialle by resolution, of any worke in this arte.*

| ze | g  | cl | g·g | sg   | g·cl | g/g   |
|----|----|----|-----|------|------|-------|
| 2  | 4  | 8  | 16  | 32   | 64   | 128   |
| 3  | 9  | 27 | 81  | 243  | 729  | 2187  |
| 4  | 16 | 64 | 256 | 1024 | 4096 | 46384 |

| g·g·g | cl·cl  | g/g     |  |
|-------|--------|---------|--|
| 256   | 121    | 1024    |  |
| 6561  | 19683  | 59049   |  |
| 65536 | 262144 | 1048576 |  |

And if this table in any parte, seme to shoyte or to little;

## The Arte

title: you maie haue recourse to the table, at the ende of figurall numbers, whiche therfore is made large and generall: so that it maie well be called the true: full table, or table of ease.

But now for triall of the laste example: firste there is. 4.  $\sqrt[4]{\phantom{x}}$ : for whose roote I take 2. and therfore those. 4.  $\sqrt[4]{\phantom{x}}$ . make. 256. 256.  
 whiche I sette doune in number *Abstracte*. 20.  
 Nexte is. 5. squares, whiche accordyng to that roote, must nedes be. 20. and that. 20. I sette doune also: and then. 6. rootes, whiche make 12. And all thei yelde. 288. and that is all the firste somme.

Then for the seconde somme, I see firste. 8. Cubes, whiche make. 64. to bee added. 32.  
 Then foloweth. 8. squares lesse, that is. 32. to bee abated, and also. 6. rootes lesse, that is. 20. also to bee abated: So must I abate. 52. (for theim bothe) out of. 64. and then there resteth but 64.  
 12. whiche added vnto 288. of the first somme doe yelde. 300. 52.  
 12.

Now if the totall agree with this, then is the woork good. 288  
 12.

For triall whereof, I resolve. 4.  $\sqrt[4]{\phantom{x}}$ . in to number *Abstracte*, and thei will make. 256. 300  
 then. 8.  $\sqrt[4]{\phantom{x}}$ . maketh. 64: whiche bothe yelde 256  
 320. Then foloweth in the same somme. 3.  $\sqrt[3]{\phantom{x}}$  and. 4.  $\sqrt[4]{\phantom{x}}$ . to be abated. The. 3.  $\sqrt[3]{\phantom{x}}$ . make. 12. 64  
 and the 4. rootes yelde. 8. whiche together do amounte to. 20. and that must bee abated fro the said somme of 320 and then there remaineth onely 300. agreeable to the former somme above the line.

Scholar. This prooffe I like well: And I perceiue that if I would woork the like, takyng for the roote 3, or any other number, the prooffe will succede a like.

After. Now to make an cande of Addition, bicause

## *of Coflike numbers.*

cause you shall the better remember the rules of it, I  
will giue you them in this brief forme.

*In greatnesse like and signes also,  
Adde like to like there nedes no mo:  
And where the greatnesse disagree,  
Place eche by other seuerally.*

*with signe of eche, as doeth require,  
But if the signes unlike appere  
Then from the more abate the lesse,  
The greater his signe with the excesse.*

*will make the somme,  
Of that addition.*

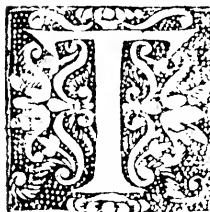
*The prooffe is by resoluyng,  
Eche number into his rekenyng.*

This lesson doeth containe the former rules onely  
in brief, and therefore needeth no declaration: but the  
greatnesse doeth betoken the *Coflike* denomination,  
and signes betoken specially, — + and . — — . the  
signes of more and lesse, and no other signes.

Scholar. This brief lesson will helpe memorie  
moche: and shall suffice for the rules of Addition.

## *Of Subtraction.*

Pastor.



Then for subtraction, this shall you  
marke in especiall: that when your  
numbers are sette downe, after the  
common maner, firste the totall, and then the deduction: you shall con-  
sider well, whether the signes bee  
— + or — — . For in the de-  
duction, if you haue — + then must that be subtrac-  
ted from the like above.

And if that somme in the deduction, that hath the 2. Rule.  
I. J.                      signe

## *The Arte*

signe  $—+—$ , bee greater then the number of the like quantitie ouer hym, with the like signe  $—+—$ . then abate the higher out of the lower, and write the reste with this signe  $—$ .

3. Rule. But if the like quantitie in the totall, haue the signe  $—$ , then adde bothe numbers together and set them vnder the line with that signe  $—$ .

4. Rule. And if the seconde somme (that is the deduction or abatement) with any number, haue this signe of lesse  $—$ , it must be accounted for more, and must be added to the like number ouer it, excepte the ouer number haue the signe of lesse also: For then must you abate the lesser, out of the greater, and sette downe the reste, with the signe of the greater number: whiche thei haue at this conferre: I meane to regarde what the signe of the seconde somme is by estimation, and not by writing, for thei are contrary.

Scholar. I see good reason in this: For in any abatemente, the more is abated, the lesse by so moche shall remain: and the lesse is abated, the more doeth remain by so moche.

5. Rule. Master. Yet one thyng more is to bee marked, that if there be some denominations, in the one some that are not in the other, you shall marke in whiche somme thei bee. For if thei bee in the firste, then shall thei kepe still their owne signe. And if thei bee in the seconde somme, whiche is the deduction, then shall thei chaunge their signe to the contrary: But where soeuer thei be, thei must be set in the remainer.

Scholar. I can better vnderstande you, then remembre those rules.

Master. Then take this byrsef lesson, apter to bee remembred, then to bee vnderstande, but by the letter befoze, and by the examples folowynge. But me more liketh well soche aide.

# of Cōsikeuomers.

## A brief rule of Subtraction.

1. When signes and greatenesse bothe agree,  
Your woorkes procedeth forthe commonly.
2. But if thabatemente greater bee,  
The excessse shall chaunge his signe therby.
3. And where the signes doe dissagree,  
The higher signe must rest duely:  
And though the batement be the greater,  
The reste still ioyneth bothe sommes together.
4. If quantities doe dissagree,  
Place them with signes all severallie:  
The totall kepeth the signe he had,  
The batementes fill, to chaunge is glad.

Scholar. Now some examples, will lighten these rules well.

Master. I will propounde the like, as I did in addition, to the intete you maie iudge the likenesse, and diuersities in bothe woorkes.

|                   |  |                  |
|-------------------|--|------------------|
| 10. 8. —+— 12. 9. |  | 10. 8. —+— 8. 9. |
| 4. 8. —+— 8. 9.   |  | 4. 8. —+— 12. 9. |
| 6. 8. —+— 4. 9.   |  | 6. 8. ——— 4. 9.  |

|                   |  |  |
|-------------------|--|--|
| 10. 8. ——— 12. 9. |  | 10. 8. ——— 8. 9.   |
| 4. 8. ——— 8. 9.   |  | 4. 8. ——— 12. 9.   |
| 6. 8. ——— 4.      |  | 6. 8. —+— 4.   |
|                   |  | <div style="display: inline-block; width: 150px;"></div> 3. 9.      10. 8. |



# The Arte

|   |   |
|---|---|
| $\begin{array}{r} 10. \text{z} . - + . 12. \text{q} . \\ 4. \text{z} . \text{---} . 8. \text{q} . \\ \hline 6. \text{z} . - + . 20. \text{q} . \end{array}$ | $\begin{array}{r} 10. \text{z} . - + . 8. \text{q} . \\ 4. \text{z} . \text{---} . 12. \text{q} . \\ \hline 6. \text{z} . - + . 20. \text{q} . \end{array}$ |
|---|---|

|  |  |
|--|--|
| $\begin{array}{r} 10. \text{z} . \text{---} . 12. \text{q} . \\ 4. \text{z} . - + . 8. \text{q} . \\ \hline 6. \text{z} . \text{---} . 20. \text{q} . \end{array}$ | $\begin{array}{r} 10. \text{z} . \text{---} . 8. \text{q} . \\ 4. \text{z} . - + . 12. \text{q} . \\ \hline 6. \text{z} . \text{---} . 20. \text{q} . \end{array}$ |
|--|--|

The firste and thirde examples be very plaine: and in the seconde where, 12. should bee abated out of 8. there is, 4. to fewe: and therefore I abate the higher, out of the lougher, and I set doune, 4. with the signe of wantyng, or abatements.

In the fourth example: because the higher number is the lesser, I doe subtrade him out of the nether, and sette doune the reste. 4. with a contrary signe of  $- +$ .

But in the, 4. later examples, where the signes do disagree, the numbers that followe the signes, are not subtraced one from an other, but are added together: and thei take still the higher signe. Because in value, the signe of abatements is contrary, to that it appeareth to bee.

And for your exercise, to make you full prompt in this arte, I haue set forth the more examples.

|  |  |
|--|--|
| $\begin{array}{r} 6. \text{c} . - + . 120. \text{q} . \\ 9. \text{c} . \text{---} . 40. \text{q} . \\ \hline 160. \text{q} . \text{---} . 3. \text{c} . \end{array}$ | $\begin{array}{r} 8. \text{z} . \text{c} . \\ 9. \text{z} . \text{c} . \text{---} 89. \text{q} . \\ \hline 89. \text{q} . \text{---} . 1. \text{z} . \text{c} . \end{array}$ |
|--|--|

*of Cosſike numbers.*

|   |  |
|---|--|
| $3. \text{ſ}. - + - 18. \text{ze}.$<br>$12. \text{ze} - - - 3. \text{ſ}.$ | $18. \text{ze} . - + - 3. \text{ſ}.$<br>$12. \text{ze} . - - - 3. \text{ſ}.$ |
| $6. \text{ſ}. - + - 6. \text{ze}.$  | $6. \text{ze} . - + - 6. \text{ſ}.$  |

|  |
|--|
| $3. \text{ſ}. - + - 18. \text{ze} . - - - 10. \text{ſ}.$<br>$12. \text{ze} . - + - 8. \text{ſ}.$ |
| $3. \text{ſ}. - + - 6. \text{ze} . - - - 18. \text{ſ}.$  |

|  |
|--|
| $4. \text{ſ}. - + - 10. \text{ce} . - - - 6. \text{ſ}.$<br>$5. \text{ſ} \text{ſ} - + - 12. \text{ſ}. - - + - 3. \text{ſ}.$ |
| $4. \text{ſ} - + - 10. \text{ce} . - - 5. \text{ſ} \text{ſ}. - - 18. \text{ſ}. - - 3. \text{ſ}.$                           |

Here in the first example, where I would abate 9 ce. out of 6. ce. I may easily perceive, that there are 3. ce. to seve. And therefore doe I sette downe. 3. ce. with this signe —, whiche signifieth wante or abatements: and the. 2. numbers that followe the vnlike signes, I set downe bothe added into one: and put therto the signe of the totall or ouermosse somme.

In the seconde example, there is the like woork: For in abatynge. 9. out of 8. I finde. 1. to seve: that. 1. doe I set downe with his denomination of. 3. ce.: and the signe. —.

And the number 89 that soloweth the signe — in the seconde somme, standeth in force as — + —, for the lesse is abated, the more must remain: therefore in the remainer, I set not the signe of more, before that number of. 89. but I put it in the first place of the somme: whiche place of it self, signifieth still more.

## The Arte

And bicause ouer that number 89, there are no numbers in the totall, therefore I muste putte downe that somme as it is, without adding to it, or abating fro it, in it self.

Scholar. Those .2. examples might be set thus, as I thinke, bicause the places doe so require.

$$\begin{array}{r}
 6.\text{C}.\text{---}+-.120.\text{q} \\
 9.\text{C}.\text{---}.40.\text{q} \\
 \hline
 \text{---}.3.\text{C}.\text{---}+-.160.\text{q} \\
 \\
 8.\text{z}.\text{C} \\
 9.\text{z}.\text{C}.\text{---}.8.\text{q} \\
 \hline
 \text{---}.1.\text{z}.\text{C}.\text{---}+-.8.\text{q}
 \end{array}$$

Master. Remember your self well, and marke the remainder how it is written.

Scholar. I see my owne oversight: for no number maie begin, with signe of lesse: and therefore must their places be altered of necessitie, and set in order as they were before.

Master. Then for all the reste of the examples, or any other like, I shall not neede to glue you any farther instruction: sith that by these former, you maie iudge of all other.

*Prooffe.*

And for the examination of your worke, the trialle by resolution doeth serue here, as well as els where: remembryng onely (as the order of subtraction maie admonithe you) that the somme of the totalle, whiche is the firste somme, must counteruaile the other bothe sommes: that is of the deduction, and of the remainder.

So to trie the firste example, takyng .3. for a roote: 6.C. make .162. whiche I put to .120. and it yeldeth 282. When in the seconde somme .9.C. are .243. whereof .40. must bee abated for the signe —, so

is

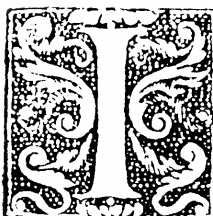
## of Cossike numbers.

is that somme. 203. Again in the remainer. 3. cē. are 81. whiche must bee abated out of. 160. and so resteth 79. whiche with. 203. doe make. 282. agreable with the firste somme.

Scholar. This doe I well vnderstande, and prae you to procede to multiplication.

## Of Multiplication.

Master.



In multiplication, there is no difficultie, so that you doe well marke the signes  $+$  and  $-$ , whiche beyng bothe like, will haue the signe  $+$  sette in the totalle. and beyng vnlike, thei will haue in the totalle the signe  $-$ .

And likewise in diuision  $+$  diuided by  $-$  or contrary waies  $-$  by  $+$  will alwaies haue in the totalle  $-$ : but  $-$  diuided by  $-$ , or  $+$  by  $+$ , will make alwaies  $+$ .

Whiche rule for ready remembraunce, I haue giuen you here in meter.

*Who that will multiplie,  
Or yet diuide trulie:  
Shall like still to haue more,  
And mislike lesse in store.  
Their quantities doe kepe soche rate,  
That M. doeth adde: and D. abate.*

Scholar. So meane you, that like signes multiplied together, doe make more, or  $+$ : And vnlike signes multiplied together, doe yelde lesse, or  $-$ .

Master. So is the rule. But to go forward now: of the nexte difficultie, as touchyng Cossike quantities that chaunge their denomination, here is no more to be

## The Arte

bee saied, then was taught in multiplication of numbers *Coslike* vncompounde, and in the table set foꝛ the foꝛ the chaunge of their names.

Scholar. I vnderstande, that in multiplication (that is. *M.*) their figures must bee added. And in. *D.* (oꝛ diuision) thei muste bee abated. Wherefoꝛe a fewe examples shall suffice foꝛ the reste.

Master. Take these foꝛ a presidente, of all that woꝛke: by whiche you maie iudge of all other like.

$$\begin{array}{r}
 10. \text{℥} . \text{—} | \text{—} . 9. \text{ʒ} . \text{—} | \text{—} . 20. \text{ʒ} . \\
 5. \text{ʒ} . \text{—} | \text{—} . 7. \text{ʒ} . \text{—} | \text{—} . 8. \text{ʒ} . \\
 \hline
 80. \text{℥} . \text{—} | \text{—} . 72. \text{ʒ} . \text{—} | \text{—} . 160. \text{ʒ} . \\
 70. \text{ʒ} \text{ʒ} . \text{—} | \text{—} . 63. \text{℥} . \text{—} | \text{—} . 140. \text{ʒ} . \\
 5 \text{ʒ} \text{ʒ} \text{—} | \text{—} . 45 \text{ʒ} \text{ʒ} \text{—} | \text{—} 100 \text{℥} . \\
 5 \text{ʒ} \text{ʒ} \text{—} | \text{—} 115 \text{ʒ} \text{ʒ} \text{—} | \text{—} 83 \text{℥} \text{—} | \text{—} 68 \text{ʒ} \text{—} | \text{—} 160 \text{℥}
 \end{array}$$

$$\begin{array}{r}
 15. \text{ʒ} \text{℥} . \text{—} | \text{—} . 12. \text{ʒ} . \\
 14. \text{ʒ} . \text{—} | \text{—} 2. \text{ʒ} . \text{—} | \text{—} . 5. \text{ʒ} . \\
 \hline
 \text{—} | \text{—} . 75. \text{ʒ} \text{℥} . \text{—} | \text{—} 60. \text{ʒ} . \\
 30. \text{ʒ} \text{ʒ} \text{—} | \text{—} 24. \text{℥} . \\
 210. \text{ʒ} \text{ʒ} \text{ʒ} \text{—} | \text{—} 168. \text{ʒ} \text{ʒ} . \\
 210. \text{ʒ} \text{ʒ} \text{ʒ} \text{—} | \text{—} 30. \text{ʒ} \text{ʒ} \text{—} | \text{—} 75. \text{ʒ} \text{℥} \text{—} | \text{—} 168. \text{ʒ} \text{ʒ} . \\
 \text{—} | \text{—} 24 \text{℥} . \text{—} | \text{—} 60. \text{ʒ} .
 \end{array}$$

Scholar. I perceiue, that these woꝛkes doe appere moze hard, then thei bee in deede, and that because of their straunge formes: but by vse I truste to bee acquainted with them well inough: and therfoꝛe I will begin with moze easie examples. As these bee, that folowe

# of Cossike numbers.

followe here.

$$\begin{array}{r}
 18. \text{z.} \text{---} | \text{---} 20. \text{q.} \\
 15. \text{ze.} \text{---} | \text{---} 4. \text{q.} \\
 \hline
 \text{---} 72. \text{z.} \text{---} | \text{---} 80. \text{q.} \\
 270. \text{ce.} \text{---} | \text{---} 300. \text{ze.} \\
 \hline
 270. \text{ce.} \text{---} | \text{---} 300. \text{ze.} \text{---} 72. \text{z.} \text{---} | \text{---} 80. \text{q.}
 \end{array}$$

$$\begin{array}{r}
 16. \text{z.} \text{---} | \text{---} 14. \text{ze.} \\
 8. \text{ce.} \text{---} | \text{---} 7. \text{q.} \\
 \hline
 \text{---} 112. \text{z.} \text{---} | \text{---} 98. \text{ze.} \\
 128. \text{fz.} \text{---} | \text{---} 112. \text{z.} \text{z.} \\
 \hline
 128. \text{fz.} \text{---} | \text{---} 112. \text{z.} \text{z.} \text{---} 112. \text{z.} \text{---} | \text{---} 98. \text{ze.}
 \end{array}$$

And this I see farther now, that these woordes seme moze difficulte to looke on, then thei be in practice, if a manne giue good hede to the signes, and the quantitties.

Master. Beseze we go any farther, I will shewe you somewhat of the reason, why the signes ought to chaunge. And that by twoo plaine woordes, in numbers *Abstraite*. As here foloweth.

Where you see, that when I had multiplied. 16 — | — 12 by 20 it made. 320 — | — 240 that is in all. 560.

But bicause the multipliers are ought not to be so moche by 4 therfore it is reason, that I shall multiplie the higher somme by . 4. and abate that out of the former totall.

|                |
|----------------|
| 16. —   — 12.  |
| 20 —   — 4.    |
| — 64 —   — 48. |
| 320 —   — 240  |
| 560 —   — 121  |
| that is. 448.  |

## The Arte

Whiche thyng you see here doen by, ———. 64.  
———. 48. whiche bothe make. 112. to bee deducted  
out of 560. and so remaineth 448. The iuste somme  
that commeth of that multiplication.

Scholar. This I vnderstande well: and  
maie proue it in this sorte. 16. ———. 12.  
maketh. 28: and. 20. ———. 4. is. 16.  
Then if I multiplie. 28. by. 16. it will yelde  
448. as the woozke here declareth.

|      |
|------|
| 28.  |
| 16.  |
| 168. |
| 28.  |
| 442  |

And hereby maie I iudge, of *Coslike* nom-  
bers like waies.

Master. Yet one example moze will I propound  
bicause I would put you out of all doubt. Therfore  
marke this forme of woozke.

Here you maie see, that if  
the firste somme of 24 ——— 3  
wer multiplied by 15 it would ——— 15. ———. 2.  
make. 360. ———. 45. that is ——— 48. ———. 6.  
315. But it ought not to bee so ——— 360 ———. 45.  
moche, but lesse by. 2. tymes ——— 366 ———. 93.  
24 ——— 3. that is. 48 ——— 6: that is. 273.  
bicause the multiplier doeth wante. 2. of. 15.

And so abatynge. 42. of. 48. ———. 6. out of. 315.  
there resteth. 273. whiche is the iuste totall, when. 21  
is multiplied by. 13. wherby the multiplication is de-  
clared to bee good.

And soz bicause that ——— multiplied with ———  
doeth make ———: marke here, that you maie not a-  
bate fully. 48. but 48. ——— 6.

Then seeyng in abatements, the signes in figure  
are contrary to their owne estimation and force: ther-  
fore that. 48. must be made ———. and the ——— be-  
fore. 6. tourned into ———.

Scholar. I see it well, it must nedes be so.

For if thei were set, to bee subtraced, then should  
thei stande so. 48. ———. 6: whiche declareth that 42  
should

## of Cossike numbers.

should bee abated.

But when the same numbers, are set amongst themselves to be added: as it is here in working of multiplication, then must they be written thus. — 48 — + — 6  
declaryng that if you abate. 48. you muste adde. 6. again, because you abated. 6. more then you ought.

Master. You vnderstand it well. Wherefore here will wee make an cande of multiplication: sith there resteth nothing but the prooffe of it: whiche maie bee wrought by resolution, of all the *Cossike* numbers, into numbers *Abstrakte*, as in other kindes before. *The prooffe of multiplication.* Once ly considering that the resolutions of the first and seconde sommes, must be added together.

And therfore if you liste to proue the firste example takyng. 2. for the roote, you shall finde the firste idine 80. — + — 36 — + — 40. that is. 156. And the seconde somme is, 20. — + — 14. — .8. that is. 26. The thirde somme is. 1600. — + — 1840. — .664. — + — 272. — .320. whiche maketh. 4056. And so voeth. 156. multiplied by. 26.

Scholar. This maie I proue at any tyme: so that you shall not neede to staie aboute it.

## Of Diuision.

Master.



**D**iuision is nexte in order, and agreeable in the generall rules: and hath not more speciall, then the very nature of the woork dooeth require. For as concernyng the signes of — + — and — — —. the same order is here, as is in multiplication. And touchyng the *Cossike* signes, it is all one with that I saied in diuision of numbers *Cossike* vncompounde.

Scholar. When a fewe examples maie supplie the  
B. y. declaration



# The Arte

Declaration of the vse of the rules, with the practike  
wooke.

Master. Take these for your purpose.

An example of the firste wooke.

$$\begin{array}{r} 60. \\ 12. \cancel{8} \cancel{8} \text{---} | \text{---} 76. \text{C.} \text{---} | \text{---} 80. \cancel{8} \cdot (2. \cancel{8} \cdot \\ 6. \cancel{8} \cdot \text{---} | \text{---} 8. 20. \end{array}$$

The remouyng of the diuisor,  
for the seconde wooke.

$$\begin{array}{r} 60. \\ 12. \cancel{8} \cancel{8} \text{---} | \text{---} 76. \text{C.} \text{---} | \text{---} 80. \cancel{8} \cdot (2 \cancel{8} \text{---} | \text{---} 10 20 \\ 6. \cancel{8} \cdot \text{---} | \text{---} 8. 20. \end{array}$$

The prooofe in numbers *Abstrakte*,  
accountyng. 2. for roote.

$$\begin{array}{r} 3 \qquad 480. \\ 192. \text{---} | \text{---} 608. \text{---} | \text{---} 320. (8. \\ 24. \text{---} | \text{---} 16. \end{array}$$

$$\begin{array}{r} 480 \\ 192 \text{---} | \text{---} 608 \text{---} | \text{---} 320. (8 \text{---} | \text{---} 20. \\ 24. \text{---} | \text{---} 16. \end{array}$$

The same wooke in  
vulgare forme.

$$\begin{array}{r} 3 \\ 1920 (28. \\ 440 \end{array}$$

Here I haue not onely parted  
the wooke, for your ease in vn-  
derstanding: but I haue also put  
against it, the declaration of the  
same, by resoluyng the *Cosike*  
nōbers, into numbers *Abstrakte*.

And finally, I haue putte one example of the same  
numbers,

## of Coslike numbers.

numbers, after the vulgare forme: all whiche. 3 agree together: and haue one an other.

Scholar. Yet I praye you worke, one example more.

Master. Here is an other.

¶ The firste extraction  
of the diuifor.

$$\begin{array}{r} 48.3.c. \text{---} | 48.3.3 \text{---} | 20.c. \text{---} | 24.ze. \quad (8.3.3. \\ 3.3. \text{---} | \text{---} 6.9. \end{array}$$

¶ The remouynge for-  
ward of the diuifor.

$$\begin{array}{r} 48.3.c. \text{---} | 48.3.3 \text{---} | 20.c. \text{---} | 24.c. \quad (8.3.3. \text{---} | 4.ze \\ 5.3. \text{---} | \text{---} 9. \end{array}$$

¶ The comprobation of the same by resolu-  
tion, accomptyng still. 2. for a roote.

$$\begin{array}{r} 2868 \text{---} 768 \text{---} | 160 \text{---} | 48. \\ 28 \text{---} | 8. \end{array} \quad (128.$$

¶ The setting forward of the diuifor.

$$\begin{array}{r} 2868 \text{---} | 768 \text{---} | 168 \text{---} | 48. \quad (128 \text{---} | 8. \\ 28 \text{---} | \text{---} 8. \end{array}$$

Scholar. Yet ones again, I praye you worke the like.

For although I perswade my self, that I perceiue the worke: yet would I see more confirmation of it, before I would be to constante in my perswasion.

Master. Good aduifemete is euer sure: but if you doubt, your counselloure is not farre absente.

Scholar. I maie iustly reioice thereof: But for euery mater to require aied, and neuer to trauell my owne witte, it might seme mere dastardlineffe. And



## of Coslike numbers.

And. 8. rootes yelde. 16. that is. 72. From whiche I must abate. 3. 8. that is. 12. And then it is iuste. 60.

Master. This is well doen.

Scholar. Yea sure, I am perfecte inough, in this feate of diuision, I trowe.

Master. You doe well to doubt.

Scholar. I thinke my self sure without doubte: As by one or twoo examples, I will declare.

And first I take this nōber 322  $\frac{b}{8}$ . — + — 115  $\frac{8}{\ell}$ .  
 — 42.  $\ell$ . — + — 69.  $\frac{8}{\ell}$ . — + — 30.  $\frac{7}{\ell}$ . to be diuided  
 by. 14.  $\frac{8}{\ell}$ . — + — 5.  $\frac{7}{\ell}$ . wherefore I sette them doune  
 thus.

$$\begin{array}{r} 322 \frac{b}{8} - + - 115 \frac{8}{\ell} - 42 \ell - + - 69 \frac{8}{\ell} - + - 30 \frac{7}{\ell} \quad (23 \frac{8}{\ell} - + - 3 \frac{7}{\ell} \\ 14 \frac{8}{\ell} - + - 5 \frac{7}{\ell} . \quad | \quad 14 \frac{8}{\ell} - + - 5 \frac{7}{\ell} . \\ \hline 32 \frac{b}{8} - + - 115 \frac{8}{\ell} . \quad | \quad 42 \ell - + - 15 \frac{8}{\ell} . \end{array}$$

And finde the firste *quotiente* to bee. 23.  $\frac{8}{\ell}$ . by whiche I multiplie the diuisor, and it taketh awaie all the numbers ouer it: Wherefore I set the diuisor forward, & finde 37. for the *quotiente*, whiche I multiplie into the diuisor, & it maketh 42  $\ell$ . — + — 15  $\frac{8}{\ell}$ . wherby I am at a stale. For although I see in the diuidende, the like numbers, yet the signe of — de- clareth, that it is not possible, to abate this newe nōber thens: seying — 42.  $\ell$ . is lesse then naughte.

Master. Wherefore consider it, in chosyng your *quotiente*: and giue your *quotiente* the like signe.

Scholar. But then riseth an other doubte. For there will be — 15.  $\frac{8}{\ell}$ . whiche disagreeth in signe from the number ouer it.

Master. Yet maie you subtract it well inough, if you haue not forgotten, your rules of subtraction.

Scholar. Now I dooe better remember my self: that by good reason, I must leaue as a remainer, not onely the whole number ouer it, whiche is. 69.  $\frac{8}{\ell}$ .

but

## The Arte

but I must adde therto. 15. 3. more.

So shall I cancell the. 69. and set ouer it. 84. And then doe I remoue the diuisor forward, setting 4 3 vnder. 84. 3. and the reste in order, wherebp I perceue, that the newe *quotiente* will be. —. 6. 9.

$$\begin{array}{r}
 84. \\
 322.6/3. - | - 1153. \text{C} - | - 42 \text{C} - | - 683. - | - 30. \text{C} (23/3. - 7e + 69 \\
 14. 3. - | - .5. 7e | - | - 143. - | - .5. 7e. \\
 \hline
 322.6/3. - | - 1153. \text{C} | - | - 42 \text{C} - | - 153. \\
 84. 3. - | - 30. 7e. \\
 \hline
 14. 3. - | - .5. 7e.
 \end{array}$$

Whiche *quotiente* I doe multiplie into the diuisor, and it doeth make. 84. 3. —. 30. 7e. agreeable to the somme ouer it. And so there remaineth nothyng.

Master. You haue dooen well. But in choysege your diuidende, and the diuisor, your lucke was better then your cunnyng.

Scholar. What shall I proue againe, by an other example, takyn also at all aduentures.

I would diuide this somme.

16. 3. C. —. 20. 3. —. 12. 7e. —. 8. 9. by  
4. 3. —. 2. 7e. And therfore I set theim doune in order thus.

$$\begin{array}{r}
 16. 3. \text{C} - | - 203. - | - 127e - | - 89. (4. 3. 3. \\
 4. 3. - | - 2. 7e.
 \end{array}$$

And firste I see, that. 4. is contained in. 16. fouer tymes: and so maie I finde. 2. in any other numbers there. 4. tymes. Wherfore I set. 4. in the *quotiente*.

And bicause the. 4. in the diuisor are. 3. and the 16 to bee diuided, are. 3. C. accordyng to the former rules, I finde the newe denomination *Cosike* to be. 3. 3. whiche

## of *Cosike* numbers.

whiche I set in the *quotient* with 4 and so is it. 4.  $\text{ſ}\cdot\text{ſ}\cdot$

Then ſaie I. 4.  $\text{ſ}\cdot\text{ſ}\cdot$ . multiplied by. 4.  $\text{ſ}\cdot$ . do make 16  $\text{ſ}\cdot\text{C}\cdot$ . and therfore cleareth and conſumeth al that ſome ouer it. Then farther ſaie I 4.  $\text{ſ}\cdot\text{ſ}\cdot$ . multiplied by. 2.  $\text{ſ}\cdot$ . doe yelde. 8.  $\text{ſ}\cdot\text{ſ}\cdot$ : But I ſet noe ſorche deno-  
mination in the diuidende.

Maſter. Then maie you perceiue, that you haue miſſed.

Scholar. Why ſir, I thinke I ought to doe as you did: that is to multiplie the *quotiente* into euery parte of the diuiſor.

Maſter. That is true: but I wil detecte the faute vnto you. And that is this.

That all numbers *Cosikes* compounde, can not bee diuided orderly, by diuiſors compounde. And thoſe that can bee diuided, will not receiue any other diuiſor, of theſame kinde, but one of. 2. numbers, by multiplication of whiche, it was made: and ſo the other of thoſe. 2. ſhall be the *quotiente*: As it came to paſſe in all thoſe. 3. examples, which I ſet forth. And therfore it is loſſe labour, to goe aboute to diuide theim in that ſorte.

Scholar. Then are there but ſelwe numbers of *Cosikes* compounde, that maie be diuided.

Maſter. So many men ſaie. But I ſaie thereto, that though many of them can not be diuided, by like numbers *Cosikes* compounde, yet are there many thouſandes, that maie be ſo diuided.

And again I ſaie, that all ſortes of theim, maie bee diuided, by an *Abſtraſt* number. And alſo any of them maie be diuided, by conuerſion into a fraction: And ſo maie your example be ſet thus.

$$\begin{array}{r}
 16.\text{ſ}\cdot\text{C}\cdot \text{---} | \text{---} 20.\text{ſ}\cdot \text{---} | \text{---} 12.\text{ſ}\cdot\text{C}\cdot \text{---} | \text{---} 8.\text{ſ}\cdot \\
 \hline
 4.\text{ſ}\cdot \text{---} | \text{---} 2.\text{ſ}\cdot\text{C}\cdot \\
 \text{Z. j.}
 \end{array}$$

And

## The Arte

And in all other cases like, sette the diuidende ouer a line, and the diuisor vnder the same line, and so is your diuision randed: and this is the reddicste waie, and the moſte indifferēce, in all ſoche numbers.

Scholar. What is ſone learn'd. And therfore needeth no moare examiples.

It is like in numbers *Abſtraſte*, when the greater number, doeth diuoe the leſſer. As. 6. diuided by. 11. maketh  $\frac{5}{11}$ .

Maſter. Somewhat like it is. Howbeit here is a woozke moare like therevnto, as when we ſhould diuide the leſſer *Coſike* number, by the greater, for then we muſt ſet them in that ſoyme. So. 6.  $\text{z}$ . diuided by 7.  $\text{c}$ . ſhall be ſet thus:  $\frac{6}{7\text{c}}$ . And. 20.  $\text{c}$ . diuided by 5.  $\text{z}$ . muſt ſtande in this maner:  $\frac{20\text{c}}{5\text{z}}$ .

Scholar. Why 20. maie be diuided by. 5.

Maſter. But.  $\text{c}$ . can not be diuided by  $\text{z}$ . And in *Coſike* numbers, the chieſ regard is to be had, to the *Coſike* ſignes.

Scholar. Then, as ſoꝛ any other ſoyme, of regular diuiſion, here is none.

Maſter. Doe, excepte your diuiſoꝛ, bee a number *Abſtraſte*: Or at the leaſte, if it haue one onely *Coſike* ſigne, and be vncompounde, that ſigne muſt be other equalle, oꝛ leſſer then the leaſte *Coſike* ſigne, in the diuidende.

For ſo. 60.  $\text{z}$ .  $\text{c}$ . — + — 48  $\text{c}$ . — + — 18.  $\text{z}$ . maie bee diuided by any number, hauyng one of theſe. 3. ſignes *Coſike*.  $\text{z}$ .  $\text{c}$ .  $\text{z}$ .

Scholar. I vnderſtand it well. For.  $\text{z}$ . is the laſte ſigne in the diuidende: And.  $\text{c}$ . and.  $\text{z}$ . are not onely leſſe then it, but alſo.  $\text{z}$ . leaueth the number, as if it were a number *Abſtraſte*.

So if I would diuide your number, aſſigned by 40.  $\text{z}$ . the *quotiente* would bee thus.

60.  $\text{z}$ .

## of *Coslike* numbers.

$$\begin{array}{r} 60.8. \text{C} - | - 48. \text{C} - | - 18.8. (1\frac{1}{2}.8.8 - | - 1\frac{1}{2}. \text{C} - | - 3.9. \\ 40.8. \qquad 40.8. \qquad 40.8. \end{array}$$

Master. Before we can do this worke of diuision, I will admonish you, of one easie aied, in the diuision of diuerse numbers. And that is, to consider, whether your diuidende, doe omit any *Coslike* denominations, betwene them, whiche it hath. For if it doe, you must yet supplie their roomes, with signes and Ciphers. As by example, you shall vnderstande.

I require to haue this number.  $8. \text{C} - | - 64.9.$   
diuided by.  $2. \text{C} - | - 4.9.$

Scholar. What will I doe quickly. For I see. 4. will be the first *quotiente* and his denomination will be.  $8. \text{C}.$  diuided by.  $2. \text{C}.$  doe make.  $8. \text{C}.$

But firste I sette downe the numbers orderly. And then I multiplie the diu:  
for by the *quotiente*, & there  
riseth.  $8. \text{C} - | - 16.8.$

Master. Stande you  
now amazed, for all your  
greate confidence: You see that you can not finde any  
 $8.$  in the diuidende. Therefore set downe the number  
as I told you before, in this sorte.

$$\begin{array}{r} \text{---} 16.8. \\ 8. \text{C} - | - 0.8. - | - 0. \text{C} - | - 46.9. (4.8. \\ 2. \text{C} - | - 4.9. \\ \hline 8. \text{C} - | - 16.9. \end{array}$$

And then I take the same *quotient* that you did, and  
I finde the remainer to be.  $\text{---} 16.8.$  And therefore  
I doe againe sette forward the diuisor: And finde the  
*quotiente* to be  $\text{---} 8. \text{C}.$  by whiche I multiplie the  
 $2.9.$  diuisor,



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diuisor, and it maketh.  $16. \text{z}$ . ———.  $32. \text{ze}$  so that abatynge the.  $16. \text{z}$ . the reste, that is, ———.  $32. \text{ze}$ . shall be the remainer with the signe ——— by the rule of subtraction.

$$\begin{array}{r}
 \text{—————} 16\text{z} \text{ ———} + 32\text{ze} \\
 8\text{℥} \text{ ———} | \text{—————} 8\text{z} \text{ ———} | \text{—————} 8\text{ze} \text{ ———} | \text{—————} 64\text{q} (4\text{z} \text{ ———} 8\text{ze} \text{ ———} | \text{—————} 16\text{q} \\
 2\text{ze} \text{ ———} | \text{—————} 4\text{q} \\
 2\text{ze} \text{ ———} | \text{—————} 4\text{q}
 \end{array}$$

Then vnder that remainer, I remoue the diuisor, and finde the newe *quotiente* to bee ———  $16. \text{q}$ . And so is the number clerely consumed.

Scholar. If I forgette any parte of this, I am deuised to foule.

Master. Then haue you learned this parte, well enough, for this tyme. And therfore will we go forth vnto fractions, whiche partly were omitted befoze, and partly are compounde of them self.

### Of fractions, and their numeration.



Fractions of this kinde appere simple: and yet are scante so to bee iudged: as  $\frac{4}{3}\text{z}$  betokeneth  $4. \text{z}$ . to bee diuided by.  $3. \text{℥}$ . Likewales this fractio  $\frac{12}{5}\text{z}$  doeth import that  $12. \text{z}$  muste bee diuided by.  $5. \text{z}$ . But  $\frac{10}{9}\text{z}$  betokeneth.  $10. \text{z}$ . to bee parted into.  $9.$  portions.

And here shall you note, the doubtfull soyme, that many menne in this arte vse, whiche write that laste fraction thus.  $\frac{10}{9}\text{z}$ . where as this fractio doeth represent  $\frac{10}{9}$  of a square: and not  $10. \text{z}$ . to be diuided by.  $9.$

Scholar. Because you saie, that some doe so vse it,  
and

## of Cossike numbers.

and I would gladly excuse all good wryters : I maie saie for them that as in bulgare numbers, when. 10. should be diuided by 19. And is set thus  $\frac{10}{19}$  it doeth im-  
porte bothe that. 10. is diuided into. 19. and also that euery portion of those. 19. is  $\frac{10}{19}$  of an vnitie : so that if 10. l. should be parted amongest. 19. men, euery man should haue  $\frac{10}{19}$  of. 1. l.

Master. Your wordes haue so moche apperance that thei maie persuaide hym, that is not very pccise in termes, especially seying there is no other *quotiente* there, but thesame number. But as the somme of 10. l. being diuided by. 19. is farre more then  $\frac{10}{19}$  of an vnitie: So. 10. s. to bee diuided by. 19. differ moche from  $\frac{10}{19}$  of a square. For the one is 19. tymes so moche as the other. And therfore oughte to haue a distincte forme in wrytyng.

Scholar. When you would haue me to wryte the so, that  $\frac{10}{19}$  of a square, should haue the signe against the line, as here is set  $\frac{10}{19}\text{Z}$  : and when I would represent. 10. s. diuided by. 19. I shall wryte it thus.  $\frac{10}{19}\text{Z}$ . with the signe aboue the liue.

Master. You maie see their agremente, and their difference by resolution, in this maner  $\frac{10}{19}\text{Z}$  will make  $\frac{10}{19}$  accountpynge. 2. for a roote, and  $\frac{10}{19}\text{Z}$ . maketh  $\frac{10}{19}$  of 4.  $\frac{10}{19}$  of. 1.

Again, accountpynge. 3. for the roote, then  $\frac{10}{19}\text{Z}$  yeldeth  $\frac{30}{19}$  : and  $\frac{10}{19}\text{Z}$  maketh  $\frac{20}{19}$  of an vnitie: so thei appere to bee equall in valewe by reduction.

But now maie you see, that the one doeth betoken the firste nōber, whiche is to be diuided: and the other doeth signifie the *quotiente* of the diuision : and so are thei distincte in office and nature. But bicause by resolution, the one tourneth into the other, therfore many men account them as one. Wholbeit, we stand to longe aboute this, consideryg the erreure, is not alwaies daungerous.

Z. iij.      But

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But their oversighte is more dangerous, whiche misplace the signe, when it should bee sette vnder the line: as a greate clerke doeth (except I shall for his excuse, impute the faulte to the printer) for he meaning to diuide.  $3$ . by  $7$ .  $\frac{3}{7}$ . writeth it thus.  $\frac{3}{7}$ . where he should write it thus.  $\frac{3}{7}$ : and againe, myndyng to diuide.  $7$ . by  $3$ .  $\frac{7}{3}$ . he writeth it thus  $\frac{7}{3}$ . where he should write.  $\frac{7}{3}$ .

Scholar. This faulte is manifeste, and detecteth the firste negligence: for  $\frac{7}{3}$ . doeth make in number, after the former resolution.  $2\frac{1}{3}$  and.  $\frac{7}{3}$ . dooeth make.  $\frac{7}{3}$ .

Master. Well, sayng you perceiue the faulte, we will stande no longer aboute it. Wherefore to procede distinctly and certainly, whether that fraction be compounde, or simple, where the numerator is a *Cosike* number, and the denominator, a number absolute, yet maie you boldly thinke, that fraction to bee compounde, whose numerator is a number *Cosike* and the denominator an other *Cosike* of unlike signe: as.  $\frac{3}{-2}$  and  $\frac{-3}{2}$ .

Yet as in numbers Abstracte, it maie seme moſte aptly to bee called a fraction, when the numerator, is lesser, then the denominator, so in numbers *Cosike*, moſte aptly the signe of the denominator, should bee the greater. Yet bothe formes come in vse.

And forbicause easinesse in working, doeth oftentimes bying certaintie with it beſore we take in haue the addition of fractions, I thinke it good to ſpeake ſomewhat of Reduction, to an other denomination. So that you forgette not, that any. 2. numbers *Cosike* compounde, with a line betwene them, maie be called a fraction. As thus.  $\frac{36}{38} + \frac{32}{39} = 69$  that is,  $36. - + . 8. 20. = . 6. 9$ . to bee diuided by  $38$ .

## Examples of Numeration.

33. — — — 12. 9. and so of other like.

## Of Reduction of fractions.



Fractions *Cosike*, not onely in their numbers, but also in their signes maye be reduced to other valuations, and namely to their leaste termes, and yet continue still in one proportion, betwene the numerator, and denominator.

So  $\frac{17}{12}$  maye bee reduced to  $\frac{34}{24}$ : for so high as. 24 is above. 9. that is in the thirde place from it: So is 3. 3. in the thirde place above. 24.

Againe.  $\frac{17}{12}$ . by reduction doeth make  $\frac{34}{24}$ : And so  $\frac{17}{12}$  will bee by reduction.

And so in all other fractions, where the numbers bee commensurable.

But if any one number, bee incōmensurable with the other, then can there be made no reduction in the numbers. Yet in the signes *Cosike*, there maye be a reduction, other to greater, or to smaller signes: For those signes be ever commensurable.

And there is no exception, but they maye bee reduced to smaller quantities, excepte any one quantitie of them bee. 9. that is a number. For that can bee no smaller. And therefore none other maye be altered, with every one must be abated alike.

And looke how moche, the smallest quantitie of that fraction, is above a number, so moche maye they all bee abated: for they are neuer reduced to the smallest, till one of them be a number.

Scholar. And why maye not this reduction, serue for whole *Cosike* numbers?

Master. Because the whole number, doeth not co  
fit

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list of a proportion, as the fraction doeth, and so maie bee expressed in diuerse termes: but it importeth one somme certaine, whiche maie nother bee increased, nor decreased, but it will chaunge his valewe, and alter his office.

And if I saie: a foote is  $\frac{1}{2}$  of a yarde, I maie saie as truly, increasynge bothe numbers, in the like proportion, a foote is  $\frac{4}{8}$  of a yarde: or in lesser termes: a foote is  $\frac{1}{2}$  of a yarde.

But when I saie in whole number, a yarde is . 3. foote, or a foote is. 12. ynches, I saie truly: and if I doe increase or abate any of those numbers, my wordes will be false.

So although in this number. 8.  $\frac{1}{2}$ . ———. 6.  $\frac{1}{2}$ . ———. 10.  $\frac{3}{4}$ . by reason of bothe numbers and signes, there might bee a reduction, yet because it is a whole number, it should thereby bee abated moche: as here you maie see. 4.  $\frac{1}{2}$ . ———. 3.  $\frac{2}{3}$ . ———. 5.  $\frac{1}{4}$ . whiche by resolution into vulgare numbers, 2. being sette as the roote, doeth make. 32 ———. 6. ———. 5. that is. 33. and the other number before, doeth yelde by the like resolution. 256 ———. 48. ———. 40. that is. 264. and is 8. tymes so moche as the other.

Scholar. I perceiue now good reason, why reduction serueth for fractions onely. And if there bee noe more difficultie in it, then you haue declared. I can worke it easily.

*Reduction in signes onely* For other the reduction consisteth in the signes *Cosike* onely, as  $\frac{10}{12}$  where the numbers bee uncommensurable, and therefore can not bee altered to any lesser termes. But the signes *Cosike* maie bee abated by . 3. denominations: seying the smalleste of them, is so many in order about. 9. And therefore it maie be reduced to  $\frac{10}{13}$ .

*Reduction in numbers onely* Other els secondarily, the reduction consisteth in the numbers onely, when the numbers be communicante.

## of Coflike numbers.

cante. And the signes *Coflike* bee all redy at the leaſte: as when one of them is.  $\frac{9}{11}$ . So  $\frac{100}{119}$  will bee reduced to  $\frac{100}{119}$ .

¶ Els thirdly, the reduction maie bee wroughte, *Reduction in* bothe in signes, and alſo in numbers. When all the *ſignes and* ſignes be aboute.  $\frac{9}{11}$ . and the numbers be communicant *numberses.*

So  $\frac{100}{119}$  maie be reduced well vnto.  $\frac{100}{119}$ .

Maſter. Yet one forme of reduction moze, I will *An other* ſhewe you, where not onely the like woorkes maie be, *reduction.* but alſo the number maie be broughte from his com- poſition, to a moze ſimplicitie, by abatynge ſome of his partes.

As this number  $\frac{100}{119}$  maie bee reduced, firſt by his numbers to  $\frac{100}{119}$ .

Secondarily, by his ſignes it maie be altered thus.

Thirdely, by abatynge the numbers, that followe ſigne of compoſition (that is  $+$ ) it maie be brought to.  $\frac{100}{119}$ . or.  $\frac{100}{119}$ . whiche fractions, kepe the ſelf ſame proportion, that the firſt fraction did.

Likewaies with the ſigne of  $-$ . numbers reſi- dualles, maie bee reduced. As.  $\frac{100}{119}$  will bee reduced, as the other was to  $\frac{100}{119}$ .

Scholar. This is vnto me a marvellouſe mater, that thoſe. 2. contrary numbers, ſhould be reduced to one fraction.

Maſter. The like happeneth in vulgare num- bers. For.  $\frac{100}{119}$  will bee reduced to  $\frac{100}{119}$ . For firſt it maketh  $\frac{100}{119}$  and then  $\frac{100}{119}$ . So likewaies  $\frac{100}{119}$  will make firſt  $\frac{100}{119}$  and then  $\frac{100}{119}$ .

And the reaſon of it, doeth depende of the. 19. pro- poſition, of the fifth booke of *Euclide*, where it is writ- ten thus.

Aa.j. If

## The Arte

If the proportion of the abatements vnto abatements be, as the whole is in proportion to the whole. Then shall the residue bee in like proportion to the residue, as the whole is to the whole.

That is in the last example. As. 18. is vnto. 24. so is 6 vnto 8. Therefore shall 12 be to 16. as 18. is to 24.

And for to exercise you the better, loe, here are one or two examples more, of the like reduction.

$\frac{7\cancel{2}}{8\cancel{3}} = \frac{14\cancel{8}}{16\cancel{8}}$  maketh  $\frac{7\cancel{2}}{4\cancel{3}}$  or  $\frac{7}{2}$ . Again  $\frac{19\cancel{2}}{28\cancel{3}} = \frac{48\cancel{4}}{7\cancel{2}}$  yeldeth  $\frac{19\cancel{2}}{28\cancel{3}}$  or  $\frac{48\cancel{4}}{7\cancel{2}}$ .

But this muste you farther marke, that in *Cosike* numbers, not onely the numbers, but also the *Cosike* signes must bee, according to *Euclides* proposition.

Scholar. What doe I see.

For in the last example: As. 9. is to. 3. so. 2. is to. 2.

And in the next example before: As. 2. is to. 3. so is. 3. to. 3.

Likewise in the other examples, as 2 is to 3-2 so is. 3. to. 3.

Al this is good and reasonable.

Master. Now doe you see, bothe the manner of reduction, and also some reason for it. Therefore I will proceede, to declare the woork of Addition.

### Of Addition and Subtraction.



**A**d Addition there is nothyng more, then you haue learned before: For as for the multiplications of the denominators together, and then crosse waies with the numerator of thother, is luste agreeable with the reductions of Abstracte fractions, to bying them to one common denominator

## of Cossike numbers.

nominator.

And then the numeratoꝛs added together, doe make the newe numeratoꝛ in addition.

And likewise the lesser numeratoꝛ, subtracted fro the other, doeth make the numeratoꝛ in subtraction: wherfoꝛe a fewe exampls maie suffice.

### Examples of Addition.

|   |   |
|---|---|
| $54.\frac{8}{7}.\text{---}1\text{---}.28.\text{C.}$         | $40.\frac{8}{6}.\text{---}1\text{---}.42.\text{C.}$         |
| $\frac{8}{7}.\frac{8}{7}.\text{ to } \frac{4}{7}.\text{C.}$ | $\frac{1}{6}.\frac{8}{6}.\text{ to } \frac{7}{6}.\text{C.}$ |
| $63.$   | $48.$   |

|                                 |  |
|---------------------------------|--|
| That is in smal-<br>ler termes. | $20.\frac{8}{3}.\text{---}1\text{---}.21.\text{C.}$<br>$24.$ |
|---------------------------------|--|

Here you see how the .2. fractions be sette betwene 2. lines: and vnder the nethermoste line, is sette the newe denominatoꝛ: and ouer the higher line, are set the .2. newe numeratoꝛs ioyned in one.

The firste of them, can not be reduced to any smaller termes, bicause the numbers be not all .3. commensurable: & the denominatoꝛ, also is a number *Abstract*.

The seconde hath also a number *Abstracte* foꝛ his denominatoꝛ, and therfoꝛe there can be noe reduction in signes: but the numbers all .3. beyng commensurable, & diuisible by .2. maie be reduced, as there you see.

### More examples of Addition.

|  |   |
|--|---|
| $16.\frac{8}{3}.\text{---}1\text{---}.4.\text{C.}$ |   |
| $12.\frac{8}{3}.\text{---}1\text{---}.9.\text{C.}$ | $4.\frac{8}{3}.\text{---}1\text{---}.5.\text{C.}$ |
| $20.\frac{8}{3}.\text{C.}$                         | $20.\frac{8}{3}.\text{C.}$                        |
| $20.\frac{8}{3}.\text{C.}$                         |   |

An. y

That



## The Arte

What is in final  
ler termes.

$$\begin{array}{r} 4.\text{z}.\text{—}|\text{—}.\text{I}.\text{q}.\text{—} \\ \hline 5.\text{c}.\text{—} \end{array}$$

Here is noe multiplication wroughte, bicause the denominatozs are like.

### Another Example of Addition.

$$\begin{array}{r} 5.\text{z}.\text{c}.\text{—}|\text{—}.\text{20}.\text{c}.\text{—}|\text{—}.\text{3}.\text{f}.\text{z}.\text{—} \\ \hline 5.\text{z}.\text{c}.\text{—}|\text{—}.\text{3}.\text{f}.\text{z}.\text{—} \quad \text{to} \quad 20.\text{c}.\text{—}|\text{—}.\text{6}.\text{f}.\text{z}.\text{—} \\ \hline 6.\text{c}.\text{c}.\text{—} \quad \quad \quad 6.\text{c}.\text{c}.\text{—} \\ \hline 6.\text{c}.\text{c}.\text{—} \end{array}$$

What is in les  
ser termes.

$$\begin{array}{r} 5.\text{c}.\text{—}|\text{—}.\text{20}.\text{q}.\text{—}|\text{—}.\text{3}.\text{z}.\text{—} \\ \hline 6.\text{z}.\text{c}.\text{—} \end{array}$$

Here is noe multiplication, noz redution to one common denominatoz: sith thei bee one all ready: noz ther can the numbers be reduced, to any other lesser: but the quantities onely be reduced as you see.

Scholar. I praye you let me proue.

### Another Example.

$$\begin{array}{r} 80.\text{b}.\text{f}.\text{z}.\text{—}|\text{—}.\text{90}.\text{z}.\text{c}.\text{—}|\text{—}.\text{60}.\text{z}.\text{c}.\text{—}|\text{—}.\text{30}.\text{f}.\text{z}.\text{—} \\ \hline 8.\text{c}.\text{—}|\text{—}.\text{9}.\text{z}.\text{—} \quad \text{to} \quad 6.\text{c}.\text{—}|\text{—}.\text{3}.\text{z}.\text{—} \\ \hline 10.\text{c}.\text{—} \quad \quad \quad 10.\text{z}.\text{z}.\text{—} \\ \hline 110.\text{b}.\text{f}.\text{z}.\text{—} \end{array}$$

What is

Master. Marke your worke well, befoze you reduce it.

Scholar. I see my faulte: I haue sette. 2. numbers seuerally, with one signe C<sup>o</sup>sike: by reason I did not foresee, that, c. multiplied with, c. doeth make the like

## of Cosike numbers.

like quantitie, as,  $\frac{8}{3}$ , multiplied by,  $\frac{3}{10}$ . Therefore it should be thus.

$$\begin{array}{r} 80.b\frac{8}{3}. - + . 150.\frac{3}{10} \mathcal{C}. \text{---} . 30.f\frac{8}{3}. \\ \hline 110.b\frac{8}{3}. \end{array}$$

Whiche maie bee reduced, by meane of the numbers, to this somme.

$$\begin{array}{r} 8.b\frac{8}{3}. - + . 15.\frac{3}{10} \mathcal{C}. \text{---} . 3.f\frac{8}{3}. \\ \hline 11.b\frac{8}{3}. \end{array}$$

And now considering the *Cosike* signes, and woꝝ kyng as I haue marked you to dooe: What is to abate the leaste signe, out of theim all: bicause,  $f\frac{8}{3}$ . is here the leaste, I abate it out of,  $b\frac{8}{3}$ . and there resteth,  $\frac{3}{10}$ . and so doing with the other signe,  $\frac{3}{10} \mathcal{C}$ . there remaineth,  $\frac{2}{10}$  & then  $f\frac{8}{3}$  out of  $f\frac{8}{3}$  doeth leaue,  $\frac{9}{10}$ . nober: So will the fraction bee thus:  $\frac{8}{3} = \frac{1}{10} \frac{2}{10} \mathcal{C} = \frac{1}{10} \frac{1}{10}$  by reduction in signes and numbers also.

After. Seyng you haue so well marked the reduction of the signes (whiche followeth the forme, taught befoze in diuision) I thinke it not nedefull, to staie any longer aboute this.

Therefore we will goe foꝝward to subtraction, after that I haue admonished you of fractions, in apperaunce simple, whiche in deede by addition, bee come compounde. As this  $\frac{2}{3} \mathcal{C}$ . added to  $\frac{1}{4} \frac{3}{10}$ . maie firste be added by the common signe of addition, thus.

$$\frac{2}{3} \mathcal{C}. - + . \frac{1}{4} \frac{3}{10}. \text{whiche by reduction, vnto one denomination, wil be thus written, } \frac{2}{3} \mathcal{C} = \frac{1}{12} \frac{2}{12} \mathcal{C} = \frac{1}{12} \frac{2}{12}$$

But as this is easie inough to vnderstand, so maie it helpe often times, foꝝ speedie woꝝke, as well in addition, as in subtraction, by the onely adding of the signe.

As if I would subtracte this fraction  $\frac{1}{4} \frac{3}{10}$ . out of  $\frac{2}{3} \mathcal{C}$ . *Subtraction.*

$$\frac{2}{3} \mathcal{C}. - \frac{1}{4} \frac{3}{10} = \frac{2}{3} \mathcal{C} - \frac{1}{4} \frac{3}{10}$$

## The Arte

$\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$ . I maie write it thus.  $\frac{2}{10} \div \frac{1}{3} = \frac{6}{10}$ . And so is the Subtraction wroughte.

Yet maie you reduce theim, to one denomination, if you will, after thesame forme, as you did in addition. And then will it bee.  $\frac{6}{10} - \frac{1}{10} = \frac{5}{10}$ . Whiche can not bee reduced to any smaller termes, bicause the numbers are not commensurable: and one of them (that is to saie, the denominato<sup>r</sup>) is a number *Abstract*

Scholar. I see in this, there is no difference from Addition, but in the signes. — + and. —. wherefore I will p<sup>r</sup>ove an other example, by your leave.

I would subtracte  $\frac{1}{2}$  out of  $\frac{4}{3}$ . and it will bee at the firste  $\frac{4}{3} - \frac{1}{2} = \frac{8}{6} - \frac{3}{6} = \frac{5}{6}$ . And by reduction

Master. Your worke is well doen, acco<sup>r</sup>dyng to your firste meanyng: But as the numerator of this laste reduction doeth declare, it can not bee well, that  $1 \frac{5}{6}$  maie bee abated out of  $1 \frac{1}{6}$ . For the greater absolutely, can not well be abated out of the lesser: and therfore you might rather haue abated  $\frac{1}{2}$  out of  $\frac{1}{3}$ .

Scholar. I see it well now: for the  $\frac{1}{3}$ . is alwaies double or triple, or yet more tymes greater, then the  $\frac{1}{6}$ . Bicause the  $\frac{1}{3}$ . commeth by multiplication of the  $\frac{1}{6}$  by his firste roote.

Master. Yet here in is discretion to be vsed, for in fractions, sometyme the number of the greater signe maie be the lesser. As for example  $\frac{1}{2}$  is lesser then  $\frac{1}{3}$ . as by resolution you maie p<sup>r</sup>ove, accompting 2. for the common roote.

Scholar. 2. beyn<sup>g</sup> the roote. 3 2. is the  $\frac{1}{3}$ . and his  $\frac{1}{2}$  maketh. 6. then.  $\frac{1}{3} \div \frac{1}{2} = \frac{2}{6}$ . beyn<sup>g</sup>. 1 2. dooth appere double to it: and therfore greater by moche.

If I doe by the like resoluti<sup>o</sup>n, p<sup>r</sup>ove the other fractions before,  $\frac{1}{4} \div \frac{1}{3}$ . will bee. 2 4: and  $\frac{1}{2} \div \frac{1}{3}$ . will bee 1 2  $\frac{1}{2}$ : whiche is lesser moche.

## of *Cosike* numbers.

So, I perceiue the greatnesse and smalnesse of the fractions, must be considered, as well in the numbers as in the *Cosike* signes. And farther, if their fractions be nigh of one greatnesse, or the fraction of the lesser signe the greater, then can not the subtraction, appeare reasonable.

Master. That is true, if those .2. fractions stande alone: els beyng partes of other numbers, it maie appeare reasonable inough. As in this example of compounde fractions.  $\frac{1}{10} \mathcal{C} . - + - \frac{1}{4} \mathcal{Z} .$  maie bee abated out  $\frac{1}{2} \mathcal{C} . - + - \frac{1}{2} \mathcal{Z} .$  and yet in the abatemente after  $- + -$  not onely the number  $\frac{1}{4}$  is greater, then  $\frac{1}{2}$  in the other, but also, the *Cosike* signe.  $\mathcal{Z} .$  is greater then the other *Cosike* signe.  $\mathcal{Z} .$

Scholar. I consider it to be so: and yet  $\frac{1}{2} \mathcal{C} .$  doeth so moche exceede  $\frac{1}{10} \mathcal{C} .$  that it supplieth sufficiently the other default: els could it not be well doen.

But for this woozke, I must craue your helpe: because I haue not seen the like.

Master. You maie doe in this, as I saied befoze, generally for all subtractions.

Set doune bothe numbers in due order, so that the abatemente dooe folowe in order: and putte betwene them the signe of subtraction: as thus.

$$\frac{1}{2} \mathcal{C} . - + - \frac{1}{2} \mathcal{Z} . \quad \text{---} \quad \frac{4}{10} \mathcal{C} . - + - \frac{1}{4} \mathcal{Z} .$$

Whoebeit, if you will firste reduce euery compounde fraction, into one fraction, it will seme moze apte. As thus.  $\frac{1}{2} \mathcal{C} . - + - \frac{1}{2} \mathcal{Z} .$  beyng reduced by additiō will make  $\frac{15 \mathcal{C} . - + - 15 \mathcal{Z} .}{30}$ . and by farther reduction of numbers.  $\frac{13 \mathcal{C} . - + - 12 \mathcal{Z} .}{12}$ . Likewates  $\frac{4}{10} \mathcal{C} . - + - \frac{1}{4} \mathcal{Z} .$  will make by the firste addition.  $\frac{16 \mathcal{C} . - + - 15 \mathcal{Z} .}{40}$ . and by farther reduction  $\frac{4 \mathcal{C} . - + - 15 \mathcal{Z} .}{10}$ .

Now toyne them together, with the signe of subtraction, and thei will stande thus.

$$\frac{4 \mathcal{C} . - + - 15 \mathcal{Z} .}{10} \quad \text{---} \quad \frac{4 \mathcal{C} . - + - 15 \mathcal{Z} .}{10}$$

Scholar.

## The Arte

Scholar. This doeth appeare verie straunge vn-  
to me: but by vse I shall finde it moze familiare: See-  
yng I see the reason of this woꝝke, to agree with the  
woꝝke of common fractions.

*The prooffe.*

But foꝝ pꝛoofe of it, I will resolue eche woꝝke, in-  
to numbers absolute, accoumptynge. 2. foꝝ a roote.

Maſter. So ſhall you finde it true: But foꝝ caſie  
woꝝke, take rather. 10. foꝝ the roote.

Scholar. I thanke you foꝝ your aide.

Then if. 10. be the roote, the ſquare will be. 100.  
and the Cube. 1000. Now  $\frac{1}{2}$  C. that is  $\frac{1}{2}$  of. 1000.  
is. 600. And  $\frac{1}{3}$  of. 10. whiche is the roote, will bee. 6.  
whiche bothe put together, doe make. 606. and that  
is the greater number.

Then foꝝ the leſſer  $\frac{4}{10}$  C. arc in this example. 400  
Foꝝ the Cube beeyng. 1000. his  $\frac{1}{10}$  is. 100. Againe  
the ſquare beeyng. 100.  $\frac{3}{4}$  of. muſt nedes bee  
75. whiche beeyng put vnto. 400. dooeth  
make. 475.

Then doe I abate. 475. out of. 606. and  
there will reſte. 131. Now now.

Maſter. I perceiue you ſaie, as beeyng aſtoniſ-  
hed, bicauſe in the former woꝝke, there is not leſſe a  
remainder: But the. 2. firſte ſommes enclly altered by  
reduction, and ioyned together, with the ſigne of ſub-  
traction: Wherein if you had continued your woꝝke,  
you ſhould haue ſounde the ſame numbers.

Foꝝ. 3. C. muſt nedes bee. 3000. ſeyng. 1. C. is a  
1000. And alſo. 375. arc. 300: whiche bothe added to  
gether, make. 3300. Diuide them by. 5. (as the deno-  
minatoꝝ would) and it will be. 666. as the valewe of  
the firſte fraction.

Then come to the later number: and you maie ſome  
thinke that. 8. C. arc. 8000. And. 15. Squares are  
1500. adde them together, and thei will make  
9500. whiche muſt bee diuided by. 20. (as the deno-  
minatoꝝ

## of Cossike numbers.

minatoz (into 24) and there will a mounte. 475. the valewe of the lesser fraction: whiche numbers appeare the same, that were before: and thereby the worke is good.

But if you will bring it to a remainer, doe thus. Reduce these. 2. new fractions, into one denomination.

Scholar. What can I doe, by multiplying the numeratozs together: that is. 20. by. 5. and thereof cometh. 100. whiche shall be the common numerator: then must I multiply in crosse waies, the numerator of the firste, by the denominator of the seconde, and contrarily.

So for the firste numerator 3.  $\text{C}.$  — — — — — 3.  $\text{ze}.$  I worke thus. And thereby dooeth amounte (as you see)  $\frac{60.\text{C} - 1 - 60.\text{ze}}{20}.$  And for the seconde numerator, I multiply. 8.  $\text{C}.$  — — — — — 15  $\text{ze}.$  by. 5. and there doeth rise. 40.  $\text{C}.$  — — — — — 75.  $\text{ze}.$  of them hauing one common numerator. 100.

Wherefore, seying bothe numbers, haue one denominator, I shall abate the lesser numerator, out of the greater, as here in example is set forth: and then the

$$\begin{array}{r} 60.\text{C} - 1 - 60.\text{ze} \\ 40.\text{C} - 1 - 75.\text{ze} \\ \hline 20.\text{C} - 1 - 60.\text{ze} - 75.\text{ze} \end{array}$$

remainder will bee (as you see). 20.  $\text{C}.$  — — — — — 60.  $\text{ze}.$  — — — — — 75.  $\text{ze}.$  into whiche I muste adde the common denominator. 100. and it will be thus.

$$\begin{array}{r} 20.\text{C} - 1 - 60.\text{ze} - 75.\text{ze} \\ \hline 100. \end{array}$$

## The Arte

How proue whether this remainer, doe not agree to thother remainer before, in your trial: which was 131

Scholar. 200℥ do make. 20000. & 60℥. yelde 600: those 2 sommes I must adde together, bicause of the signe. — + —. and it will be. 20600. then. 75. 8. are. 7500. whiche I must abate from the former somme of. 20600. and there will 20600. remaine. 13100. for the numerator, and  $\frac{7500}{100}$ . for the denominator, thus.  $\frac{13100}{100}$ . 13100

Master. And what doe you thinke of it?

Scholar. By that I learned in the vulgare fractions, I knowe that it is iuste. 131. and so doeth it agree precisely, with the former prooffe.

Master. Well yet for moare exactnesse in this worke, I will farther reduce that fractiō, by diuiding the numerator by the denominator: wherefore. 20.℥ diuided by. 100. doeth yelde.  $\frac{1}{5}$ ℥. And. 60.℥. diuided by. 100. doeth make  $\frac{3}{5}$ ℥. And lastly. 75. 8. diuided by. 100. will yelde  $\frac{3}{4}$ 8. so is the same fraction so reduced  $\frac{1}{5}$ ℥ — + —  $\frac{3}{5}$ ℥. —  $\frac{3}{4}$ 8. And now trie what that is, by the former prooffe.

Scholar. I maie sone perceiue, that  $\frac{1}{5}$ ℥. is. 200. when the Cube is. 10000: And so  $\frac{3}{5}$ ℥. is. 6. whiche I must adde together, and it will be. 206. Then  $\frac{3}{4}$ 8. is 75. whiche if I dooe abate from. 206. there will remain. 131. agreeably as before. And so is this worke fully examined.

Master. Yet will I propounde one or two examples more, partly to practise your memorie, and partly to admonish you, if you happen to see any soche misse wroughte, in some other booke (as I haue doen) how you maie amende the erreure, and not staie at it.

Firste take this example. I would subtrakte.

$$\begin{array}{r} 48.9. \\ \hline 12.9. \text{ — } 3.8. \end{array} \quad \text{out of} \quad \begin{array}{r} 489. \\ \hline 7.8. \end{array}$$

Scholar.

*of Coßike numbers.*

Scholar. I must first multiplie the denominato<sup>rs</sup> together, and so it will make, as here is sette soozthe  
 $84. \text{C.} \text{---} 21. \text{8} \text{3}.$   $12. \text{50} \text{---} 1. \text{7}.$

7. 8.

84.၉—21.၆၆.

12. 2 ——— 3 8.

48.9.

96

48

576.ze. ————— 144.8.

The abatemeñte that should

After the signe ——— with

gainer complete, as that

02, so vnto it I must adde

4. ㄷ. ———. 21. ㄱㄱ.

$\frac{1}{2}$  . that is in letter termes

12.  $\frac{9}{2} \cdot \dots \cdot 3 \cdot 8$

63.  $\int \frac{1}{x} dx = \ln|x| + C$

008.0. ———. 504.33

Ab. y.

tozs



## The Arte

toys together. And so wil it be.  $63\sqrt{3} \text{ --- } 1008\text{℥}$   
 $\text{---} 504\sqrt{3} \sqrt{3}$ . as by speciall woorkke, I haue here  
 proued.

Then doe I multiplie the numerator of the totall,  
 by the denominator of the abatemente, as here also I  
 haue perticularly set forth in woorkke, for my owne  
 ease, and auoidyng of erreure: And so I finde it to be  
 $1056\sqrt{3} \text{ --- } 6912\sqrt{2} \text{ --- } 696\text{℥}$ . whiche  
 shall bee the numerator of the totalle.

$$\begin{array}{r}
 232\sqrt{2} \text{ --- } 576\sqrt{3} \\
 12\sqrt{2} \text{ --- } 3\sqrt{3} \\
 \hline
 2784\sqrt{3} \text{ --- } 6912\sqrt{2} \\
 \text{---} 696\text{℥} \text{ --- } 1728\sqrt{3} \\
 \hline
 1056\sqrt{3} \text{ --- } 6912\sqrt{2} \text{ --- } 696\text{℥}
 \end{array}$$

Then doe I multiplie the numerator of the abate-  
 mēte, by the denominator of the totalle (whiche thing  
 is easily dooen, bicause the one number, is a number  
*Abstraſte*) and so haue I for the numerator of the aba-  
 temente.  $4032\sqrt{3} \text{ --- } 1008\text{℥}$ .

And seying these two numbers, haue one common  
 denominator, I shall abate the lesser numerator, out

$$\begin{array}{r}
 1056\sqrt{3} \text{ --- } 6912\sqrt{2} \text{ --- } 696\text{℥} \\
 4032\sqrt{3} \text{ --- } 1008\text{℥} \\
 \hline
 6912\sqrt{2} \text{ --- } 312\text{℥} \text{ --- } 2976\sqrt{3}
 \end{array}$$

of the greater, & so wil there be left for the numerator  
 of the remainer  $6912\sqrt{2} \text{ --- } 312\text{℥} \text{ --- } 2976\sqrt{3}$   
 vnto whiche, I shall adde the common denominator,  
 and then wil it be.

$$\begin{array}{r}
 6912\sqrt{2} \text{ --- } 312\text{℥} \text{ --- } 2976\sqrt{3} \\
 63\sqrt{3} \text{ --- } 1008\text{℥} \text{ --- } 504\sqrt{3} \sqrt{3} \\
 \hline
 \end{array}$$

That

## fo Cosbike numbers.

That is in lesser termes.

$$\begin{array}{r} 2304.9. - + - .104.8. - - - .992.ze. \\ \hline 21.8.8. - + - .336.8. - - - .168.℥. \end{array}$$

Master. You haue wrought it well. And hereby I coniecture, that you are experte enough in subtraction. Wherefore now we will goe in hand, with multiplication and diuision.

## Of Multiplication.



And firste, concernyng multiplie: *Multiplication*, here is no more to bee saied, then hath been taughte before.

For the numbers shall bee multiplied, as common fractions are wonte to bee: that is to saie, numerator, by numerator, and denominator, by denominator.

And for the chaunge of their denominations *Cosbike*, the rules giuen before shall suffice: so that a few examples shall sufficiently instruct you, in the worke of it.

As this for the firste.

$$\begin{array}{r} 20.8. - + - 19.ze. \\ 6.℥ - - - 3.8. \\ \hline 120.8. - + - 114.8.8. \\ - - - 60.8. - + - 57.ze. \\ \hline 120.8. - + - 114.8.8. - - - 60.8. - + - 57ze. \end{array}$$

Where I dooe multiplie.

$$\begin{array}{r} 20.8. - + - 19.ze. \text{ by } 6.℥. - - - .3.8. \\ \hline 31.℥. \qquad \qquad \qquad 4.8. \end{array}$$

And these I shall multiplie, numerator by numerator,  
13b.iiij.                      rator:

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ratio: where.  $20. \text{z}$ . multiplied by.  $6. \text{c}$ . dooe make  $120. \text{fz}$ . as the former table of multiplication, so chaunge of *Cosike* signes doeth declare. And so in all the rest, there is no difficultie, if you remember that, that you haue learned before.

Scholar. I percecue it well. And so the whole newe numerator will bee.  $120. \text{fz}$ . —  $114. \text{z}$ .  $\text{z}$ .  
 $\text{---} 60. \text{z}$ . —  $57. \text{ze}$ . And the denominator will be.  $124. \text{fz}$ .

So will the whole fraction bee.

$$\frac{120. \text{fz} \text{ --- } 114. \text{z} \text{ z} \text{ --- } 60. \text{z} \text{ --- } 57. \text{ze}}{124. \text{fz}}$$

That is not to bee reduced to smaller termes of numbers, because they be vncommensurable, but in *Cosike* signes, it mighte bee broughte to one letter, as.

$$\frac{120. \text{z} \text{ z} \text{ --- } 114. \text{c} \text{ --- } 60. \text{ze} \text{ --- } 57. \text{q}}{124. \text{z} \text{ z}}$$

Now will I proue an other number, as fortune doeth offer it to mynde. That is  $\frac{11 \text{c} \text{ --- } 9 \text{fz}}{21 \text{z} \text{ --- } 5 \text{ze}}$ . to bee multiplied by.  $\frac{12 \text{z} \text{ --- } 1}{36 \text{ze}} = ?$ .

*An Absurde* Matter. It appeareth that you take them, at all number ex- adventures. For your firste number, semeth to be an preisseth lesse *Absurde* number. Seyng his numerator, is lesse then then naught, in apperaunce. And then make it not bee diuided by any number: and moche lesse by so greate a denominator.

Scholar. It is easie to see, now that I am admonished thereof. For it is not possible, that any *Surfolide* number, can bee lesse then sover tymes so moche, as the *Cube* of the same nature. Seeyng every *Surfolide* is made, by multiplying the *Cube* by the *Square* of the like *Roote*, but lesse then. 4. is there no *Square*. And therefore every *Surfolide*, doeth excede his *Cube* sover tymes at the leaste.

## of Cossike numbers.

So that.  $32. \mathcal{C}.$  —  $8. \mathcal{f}\mathcal{z}.$  were nothyng, and so is an *Absurde* nōber. And therfore.  $32. \mathcal{C}.$  —  $28. \mathcal{f}\mathcal{z}.$  is moche lesse then nothyng, and is therby an *Absurde* number also.

Master. Yet maie your example serue, to teache and practise multiplication by, as well as any other.

And farthermore, I will tell you by this occasion, that I spake to you, moze after the opinion of the cōmon number of artes men, then after my owne iudmente.

Scholar. I might thinke so, by termynge of your sentence: but yet was your sayng true.

Master. Yet maie that fraction stand well, if you take a brokē number *Abstrakte* for the roote. Although in whole numbers, it bee an *Absurde* number.

Scholar. That will I proue, by setting.  $\frac{1}{4}$ . for a Roote. When will the Square be  $\frac{1}{16}$ . and the Cube.  $\frac{1}{64}$ . Also the Square of Squares will bee.  $\frac{1}{256}$ . And the Surfolide  $\frac{1}{1024}$ .

$\frac{1}{4}$  The Roote.

$\frac{1}{16}$  The Square.

$\frac{1}{64}$  The Cube.

$\frac{1}{256}$  Thezenzenzike

$\frac{1}{1024}$  The Surfolide.

And now to proue by resolution, how my number will rise, I take.  $32. \mathcal{C}.$  that is.  $\frac{8 \times 4}{64}$ , or  $13. \frac{1}{2}$ . whiche I note as the firste somme. When I take likewais.  $28. \mathcal{f}\mathcal{z}.$  whiche yeldeth  $\frac{68 \times 4}{1728}$ , that is.  $6. \frac{105}{216}$ . And now I see that I maie abate it very well, out of.  $13. \frac{1}{2}$ .

Master. So maie you see, that as in whole numbers, euer moare the greater *Cossike* signes, will haue the greateste numbers: So in fractions resolued by *Cossike* signes, the greatest fraction, aunswerech to the leaste signe: and the leaste fractiō, agreeth to the greateste signe.

The reason of it is this. That the moare any fraction is multiplied by a fraction, the lesser it wareth. For as whole numbers by multiplication, maie increase infinitely: so fractions by multiplication, maie decrease

# The Arte

decrease infinitely.

But before wee passe from multiplication, I will proue you with one example moare. I would haue  $\frac{19\cancel{c}}{75} - \frac{12\cancel{c}}{69}$  multiplied by  $\frac{24\cancel{c}}{43}$  . 2.

Scholar. I am troubled with the multiplier. For I knowe not what to make of it?

Master. You doubt (I thinke) of the numeratio of it, bicause you had not the like example before: for it is a mixte number of a fraction, and a whole number. But seying the signe of abatements is set against the whole fraction, and nother against the numerator, nor denominator, therfore must that  $4\cancel{c}$  be vnderstande, to be abated out of the full fraction.

Scholar. Now I perceiue the mater. For there might be .3. diuerse formes, to place that abatemente. As here I haue set them.  $\frac{24\cancel{c}}{43} - \frac{4\cancel{c}}{8} - \frac{14\cancel{c}}{42}$ .

And as it was set by you,  $\frac{24\cancel{c}}{43} - 4\cancel{c}$ . whiche I will resolue into absolute numbers, to see their difference the better. And so, taking 3. for the roote, these will be their .3. formes.

*The firste.*

For the firste  $\frac{648}{81} - \frac{12}{12}$ . or els  $\frac{616}{81}$  that is  $\frac{112}{17}$ .

For the seconde  $\frac{648}{81} - \frac{48}{12}$  or els  $\frac{648}{69}$  that is  $\frac{216}{17}$ .

And for the thirde number, whiche is our specialee number.  $\frac{648}{81} - 12$ . that is .8. . 12. and is an *Absurde* number. For it betokeneth lesse then naught by .4.

Master. If you would haue it no *Absurde* number, you must increase the proportion of the fraction, by augmenting the numerator, or abatung the denominator, or els thirdly, by abatung the number, after the signe of abatements. As  $\frac{45\cancel{c}}{53}$  . 4. 2. or els secondarily, thus.  $\frac{24\cancel{c}}{43} - \frac{4\cancel{c}}{8}$  . 4. 2. or thirdely  $\frac{24\cancel{c}}{53}$  . 2. 2.

Howbeit for examples sake, you maie worke, as well with *Absurde* numbers, as with any other.

But

## of Cossike numbers.

But for you ease, I will shewe you the woork of this example, in twoo formes.

First, you shall multiplie the firste whole number, by the fraction of the seconde number, that is.

$\frac{100}{78} + \frac{120}{9} = \frac{1900}{39}$  by  $\frac{24}{9}$ . and it will bee.

$$\begin{array}{r} 456.80. - + - 72.88. = 120.00. \\ \hline 63.88. = 54.88. \end{array}$$

As here in woork you maie see it plaine.

$$\begin{array}{r} 19.00. - + - 3.20. = 5.80. \\ 24.00. \\ \hline 456.80. - + - 72.88. = 120.00. \end{array}$$

$$\begin{array}{r} 7.88. = 6.88. \\ 9.88. \\ \hline 63.88. = 54.88. \end{array}$$

That is in lesser termes, bothe of numbers, and of signes Cossike.

$$\begin{array}{r} 152.88. - + - 24.88. = 40.20. \\ \hline 21.88. = 18.88. \end{array}$$

And this is the firste parte of your somme.

Then for the nexte parte, multiplie your firste number, that is  $\frac{100}{78} + \frac{120}{9} = \frac{1900}{39}$  by the abatement of the seconde number, that is by  $\frac{4}{9}$ . and it will be.

$$\begin{array}{r} 20.20. = 76.88. = 12.88. \\ \hline 7.88. = 6.88. \end{array}$$

Et. s.

As

## The Arte

As by this woork you maie see.

$$\begin{array}{r}
 19.\text{℥}.\text{---}\frac{1}{2}\text{---}.3.\text{ʒ}.\text{---}.5.\text{ʒ}.\text{---} \\
 \text{---}.4.\text{ʒ}.\text{---} \\
 \hline
 20.\text{ʒ}.\text{---}.76.\text{ʒ}.\text{ʒ}.\text{---}.12.\text{ʒ}.\text{---}
 \end{array}$$

whiche being reduced to the denomination of the former number, will be tripled (sith that denominator is triple to this) and so will it be  $\frac{60.\text{ʒ}.\text{---}.218.\text{ʒ}.\text{ʒ}.\text{---}.36.\text{ʒ}.\text{---}}{21.\text{ʒ}.\text{---}.18.\text{ʒ}.\text{---}}$   
 Now adde those two numbers together, by puttyng their bothe numeratozs in one, and it will be.

$$\begin{array}{r}
 20.\text{ʒ}.\text{---}.76.\text{ʒ}.\text{ʒ}.\text{---}.12.\text{ʒ}.\text{---} \\
 \hline
 21.\text{ʒ}.\text{---}.18.\text{ʒ}.\text{---}
 \end{array}$$

As here appeareth in woork.

$$\begin{array}{r}
 152.\text{ʒ}.\text{ʒ}.\text{---}\frac{1}{2}\text{---}.24.\text{ʒ}.\text{---}.40.\text{ʒ}.\text{---} \\
 60.\text{ʒ}.\text{---}.228.\text{ʒ}.\text{ʒ}.\text{---}.36.\text{ʒ}.\text{---} \\
 \hline
 20.\text{ʒ}.\text{---}.76.\text{ʒ}.\text{ʒ}.\text{---}.12.\text{ʒ}.\text{---}
 \end{array}$$

whiche will not bee reduced to any smaller fraction, bicause the numbers be incommensurable. and one of the *Coslike* signes is.  $\frac{1}{2}$ . And so is that the somme of the multiplication.

An other waie you maie woork it, and all soche like, by reducyng the multiplier, into one vniforme fraction. As here in.  $\frac{14.\text{℥}.\text{---}}{9.\text{ʒ}.\text{---}}$   $4.\text{ʒ}.$  you shall multiplie  $4.\text{ʒ}.$  by.  $9.\text{ʒ}.$  whiche is the former denominator, and it will be  $36.\text{℥}.$  Then putte that to.  $24.\text{℥}.$  ouer the line, and set the common denominator.  $9.\text{ʒ}.$  vnder the line, and it will bee in one fraction reduced  $\frac{14.\text{℥}.\text{---}.36.\text{℥}.\text{---}}{9.\text{ʒ}.\text{---}}$ .

Scholar. Here I maie see at the firste beue, that this fraction is an *Aburde* number: for the abatement after the signe  $\text{---}$  is greater then the number be-  
 foze

## of Cossike numbers.

foze it.

Master. That was cōfessed befoze. But yet make you worke the example by it.

Scholar. That is true : and so will the numeratozs , beeyng multiplid together , make exactly, 60.  $\text{℥}$ . ——— 228.  $\text{ʒ}$ .  $\text{℥}$ . ———. 36.  $\text{ʒ}$ .  $\text{ʒ}$ . As here in example of woork, I haue set it, fo2 my owne case and certentie.

$$\begin{array}{r}
 19. \text{℥}. \text{---} + \text{---} . 3. \text{ʒ}. \text{---} . 5. \text{ʒ}. \\
 24. \text{℥}. \text{---} . 36. \text{℥}. \\
 \hline
 456. \text{ʒ}. \text{℥}. \text{---} + \text{---} 72. \text{ʒ}. \text{ʒ}. \text{---} . 120. \text{℥}. \\
 \text{---} 684. \text{ʒ}. \text{℥}. \text{---} 108 \text{ʒ}. \text{ʒ}. \text{---} + \text{---} . 180. \text{℥}. \\
 \hline
 60. \text{℥}. \text{---} . 228. \text{ʒ}. \text{℥}. \text{---} . 36 \text{ʒ}. \text{ʒ}.
 \end{array}$$

And that is the newe numeratoz.

And then fo2 the seconde number, if the firste denominator. 7  $\text{ʒ}$ . ——— 6.  $\text{ʒ}$ . be multiplied by the seconde denominator, 9.  $\text{ʒ}$ . it is easily seen , that thei will make. 63  $\text{ʒ}$ .  $\text{ʒ}$ . ——— 54  $\text{ʒ}$ . whiche shall be the newe denominator.

And so the intere fraction shall bee.

$$\begin{array}{r}
 60. \text{℥}. \text{---} . 228. \text{ʒ}. \text{℥}. \text{---} . 36. \text{ʒ}. \text{ʒ}. \\
 \hline
 63. \text{ʒ}. \text{ʒ}. \text{---} . 54. \text{ʒ}.
 \end{array}$$

That is in the smalleste numbers and figures Cossike.  $\frac{1072}{128} = \frac{738}{189} = \frac{135}{35}$  : whiche somme, dooeth in all thynges fully agree , with the former number that you wrought.

Master. Proue theim. bothe by resolution : And then shall you knowe, the reason of their agreement.

Scholar. I see that the woork of the denominatozs, doeth agree. Wherefoze I will take. 3. fo2 a roote to proue how the woork of the numeratozs wil agree

And so fo2. 19.  $\text{℥}$ . I shall haue. 513. And fo2. 3.  $\text{ʒ}$ .

Et. y. I



## The Arte

I shall haue. 9. to be added to. 513. And so haue 3. 522 out of whiche somme I must abate. 5.

|   |        |
|---|--------|
| And then remaineth. 517. to be multiplied by. 24. C. that is by. 648. And the   | 648    |
| totalle will bee (as here in worke appeareth). 335016. whiche somme must be abated to a smaller number, in like rate as   | 517    |
| the other was reduced, firste by partition  | 4536   |
| into. 3. And then will it be. 111672. And   | 648    |
| again, it must bee diuided by. 9. for that is the quantitie of a square, by whiche the former reduction, was wroughte for the Cobike signes: and then will it be. | 3240   |
| 12408. And that is the firste parte of the first worke.   | 335016 |

When for the seconde parte of that worke, I shall multiplie the firste numbers, that is 517 by the abatemente of the fraction, that is by 47, 02 — 12. (sith. 3. is the roote) and thereof will come — 6204. whiche somme I must triple, as I did his equalle (that is. 207. — 76. 3. 3. — 12. 3.) And so will it be — 18612. Now shall I adde this somme, with the firste parte, whiche was. 12408. and it will be. 12408. — 18612. that is. 6204. lesse then nothyng: and is the numerator of the firste worke.

Wherfore I procede to the seconde worke, where the numerator of the fraction, beeyng reduced to the common denominator, is. 24. C. — 36. C. whiche is — 12. C. and in numbers resolute (keeping 3. still as a roote) it is — 324. by whiche if I multiplie. 517. it will yelde. 167508. And that somme beeyng abated, by diuision into. 3. and. 9. as the other was, 02 els diuided by. 27. whiche is all one, it giueth 6204. as the former worke did.

Master. Thus I see, you are experte inoughe in multiplication: Wherfore I will shewe you now, the order and forme of diuision.

*of Coflike numbers.*  
*Of Diuifion.*

**T**here is noe ſpeciall rule to be giuen, for  
the worke of Diuifion, other then ſoche  
as are all ready taughte in other workes  
of diuiſiō before. ~~¶~~ herfore I will by one  
or 2. examples, ſhewe you the worke of it.

*The firſte example of Diuifion.*

$$\begin{array}{r} 14.\text{℥} \text{ — } 9.\text{ſ} \\ 15.\text{ſ} \end{array} \text{ to be diuided by } \begin{array}{r} 5.\text{ſ} \text{ — } 2.\text{℥} \\ 3.\text{℥} \end{array}$$

doeth yelde.  $\begin{array}{r} 42.\text{ſ} \text{ — } 27.\text{℥} \\ 75.\text{ſ} \text{ — } 30.\text{℥} \end{array}$  that is in a leſſer fraction, by bothe reductions of numbers & ſignes.

$$\begin{array}{r} 14.\text{℥} \text{ — } 9.\text{ſ} \\ 25.\text{℥} \text{ — } 10.\text{ſ} \end{array}$$

*An other example.*

$$\begin{array}{r} 12.\text{ſ} \text{ — } 16.\text{ſ} \\ 2.\text{℥} \text{ — } 5.\text{℥} \end{array} \text{ diuided by } \begin{array}{r} 19.\text{ſ} \text{ — } 3.\text{ſ} \\ 4.\text{ſ} \text{ — } 5.\text{ſ} \end{array}$$

doeth make.

$$\begin{array}{r} 48.\text{ſ} \text{ — } 60.\text{ſ} \text{ — } 64.\text{ſ} \text{ — } 80.\text{ſ} \\ 38.\text{ſ} \text{ — } 15.\text{℥} \text{ — } 101.\text{℥} \end{array}$$

whoſe numbers bee incommenſurable, and therefore  
maie not bee reduced, but by abatynge one denomina-  
tion *Coflike*. And ſo will it be.

$$\begin{array}{r} 48.\text{ſ} \text{ — } 60.\text{ſ} \text{ — } 64.\text{℥} \text{ — } 80.\text{℥} \\ 38.\text{ſ} \text{ — } 15.\text{ſ} \text{ — } 101.\text{ſ} \end{array}$$

C.c. iij. Scholar.

## The Arte

Scholar. I see that you multiplie crosse wales (as in vulgare fractions) the numerator of the one number, by the denominator of the other. And so is diuision of noe difficultie, to hym that remembzeth the former rules.

### Of the golden rule.

Master.



The golden rule, that is the rule of proportion, should folowe now, by the commō order. But seying there is no difficultie in it, nother any other forme of woork, then is in vulgare numbers, I will not staie any tyme aboute it. Saue that for your pleasure, I haue set here certaine exampls, as wel in whole numbers *Cosike*, as in broken.

$$\begin{array}{l} 32.8. \quad \text{Z} \quad 4.8. \\ 6.c. \quad \text{Z} \quad \frac{3}{4}.8.c. \end{array} \qquad \begin{array}{l} 250.c. \quad \text{Z} \quad 20.ze. \\ 26.8. \quad \text{Z} \quad 2\frac{2}{3}.c. \end{array}$$

$$\begin{array}{l} 5.8. \quad \text{Z} \quad 3.ze. \\ 15.c. \quad \text{Z} \quad 9.9. \end{array} \qquad \begin{array}{l} 4.c. \quad \text{Z} \quad 5.9. \\ \frac{6c.8.c.}{58} \quad \text{Z} \quad \frac{111.c.}{3ze} \quad \text{Z} \quad 45.9 \end{array}$$

$$\begin{array}{l} \frac{3c.}{12ze} \quad \text{Z} \quad 14.ze. \\ 618-79 \quad \text{Z} \quad 4.9. \end{array} \qquad \begin{array}{l} \frac{1024888}{3c} \quad \text{Z} \quad \frac{2928c}{58} \quad \text{Z} \quad \frac{11768}{336ze} \end{array}$$

Scholar. These selwe exampls, dooe sufficiently teache the forme of the whole rule. So that here needeth noe farther explication.

Wherfoze, if in this arte, there be any forme of extraction of rootes, I pꝛaie you to pꝛocede therto.

¶

of *Cosike* numbers.  
Of extraction of rootes.

Maſter.



In numbers *Abſtraſſe*, every number is not a rooted number, but some certaine onely emongest theim, so in numbers *Cosike*, all numbers haue not rootes: but ſoche onely emongest ſimple *Cosike* numbers are rooted, whose number hath a roote, agreeable to the figure of his denomination.

So that. 16.  $\mathcal{C}$ . is not a Square number, nother hath any roote. For although. 16. bee a square number, and hath. 4. for his roote, yet the denomination (whiche is.  $\mathcal{C}$ .) hath noe square roote: but. 16.  $\mathcal{Z}$ . is a square number: and hath. 4.  $\mathcal{Z}$ , for his roote.

Likewaies. 8.  $\mathcal{C}$ . is a *Cubike* number, and his roote is. 2.  $\mathcal{Z}$ : but. 8.  $\mathcal{Z}$ . hath noe roote. For bicause. 8. hath no square roote, agreeable to the ſigne.  $\mathcal{Z}$ . nother is it a *Cubike* number, although it haue a *Cubike* roote, bicause the roote is diſagreeable from the ſigne.  $\mathcal{Z}$ .

Scholar. I perceiue that in theſe numbers, as well as in all other, the roote beeyng multiplied by it ſelf, will make the number, whose roote it is. And therefore can no number be called square, or *Cubike*, or any waies els a rooted number, excepte the roote of the number agree with his ſigne: Whereby I perceiue well, that. 32.  $\mathcal{Z}$ . is a rooted number, for bicause that 32. hath a *Surſolide* roote, agreeable to the ſigne. So likelwaies. 125.  $\mathcal{C}$ . is a rooted number, ſeyng 5. is the *Cubike* roote of. 125. But. 27.  $\mathcal{Z}$ . is no rooted number.

Maſter. Thus you vnderſtande ſufficiently, the iudgemente of rooted numbers, and their knowlege, in ſimple *Cosike* numbers, that be utterly vncōpounde.

Wherefore, for extraction of their rootes, take this brief order.

Extrade

## The Arte

Extracte the roote of your number, as if it were absolute, and put to it.  $\mathbb{Z}$ . for the denomination.

So.  $27$ . Cubes hath for his roote.  $3$ .  $\mathbb{Z}$ .

And.  $49$ .  $\mathbb{Z}$ . hath.  $7$ .  $\mathbb{Z}$ . for his roote.

Again, the roote of.  $216$ .  $\mathbb{C}$ . is.  $6$ .  $\mathbb{Z}$ .

Scholar. This I perceiue. And by like reason, the roote of.  $243$ .  $\mathbb{Z}$ . is.  $3$ .  $\mathbb{Z}$ . But why dooe you name nōbers *Coslike* utterly vncompounde? For as I vnderstande, that there bee numbers compounde, in their signes, so I see that thei maie haue rootes also.

As.  $16$ .  $\mathbb{Z}$ . hath for his roote.  $2$ .  $\mathbb{Z}$ . And likewise.  $64$ .  $\mathbb{Z}$ . hath.  $2$ .  $\mathbb{Z}$ . for his roote.

Maister. And dooe you not see, that those compounde numbers, maie haue moare rootes then one? With.  $16$ .  $\mathbb{Z}$ . hath for his square roote.  $4$ .  $\mathbb{Z}$ . as well as it hath.  $2$ .  $\mathbb{Z}$ . for his *zenzenzenlike* roote.

So.  $4$ .  $\mathbb{Z}$ . hath for his square roote.  $2$ .  $\mathbb{Z}$ . And hath no *zenzenzenlike*  $\mathbb{Z}$  agreable to his whole signe.

Likewise.  $9$ .  $\mathbb{Z}$ . hath no *zenzicubike* roote, according to his whole signe: but it hath a square roote agreable to parte of the signe, and that is.  $3$ .  $\mathbb{C}$ .

Scholar. I see that also. And so hath.  $8$ .  $\mathbb{Z}$ . noe *zenzicubike* roote, but a *Cubike* roote: whiche is.  $2$ .  $\mathbb{Z}$ .

Maister. Wherefore in cōpōūde signes, if the signe maie haue soche a roote, as the number will yelde, it is a rooted number, els not.

Whereby you maie perceiue, that if any number cōpōūde in signe, haue a roote agreable to his whole signe, then maie it haue also, as many rootes, as ther be partes in that compōūde signe.

So  $4096$ .  $\mathbb{Z}$ . hath not onely a *zenzenzenzicubike* roote, whiche is.  $2$ .  $\mathbb{Z}$ : but it hath a square roote that is.  $64$ .  $\mathbb{Z}$ . And also it hath a *Cubike* roote, that is,  $16$ .  $\mathbb{Z}$ . Farther moze, it hath a *zenzenzenlike* roote, whiche is.  $8$ .  $\mathbb{C}$ . And fourthly, it hath a *zenzicubike* roote, that is.  $4$ .  $\mathbb{Z}$ .

And

## of Cossike numbers.

And so shall you iudge, of all other like.

Scholar. This shall suffice, as I will practise the mater, at moare leiser. But and if the numbers bee compounde, with signes of addition, is there then any speciall order for their rootes? As in this erample.  $81.3.3. - + - 27. \text{C}$ . where I haue made eche parte to be a rooted number.

Master. In deede.  $81.3.3.$  hath bothe a Square roote, and also a *zenzike* roote. But  $27 \text{C}$ . hath none of those twoo rootes, although it haue a *Cubike* roote, whiche the other number wanteth. And therfore is not that whole number, a rooted number.

But to the intente, that you maie be the moze certein of rooted numbers, I will tell you certein notes, how it maie bee knowen, whether your number be a rooted number.

Firste, if the number annexed to the greatestt signe of that compounde *Cossike* number, bee not a rooted number, the whole number can not be a rooted nōber

Secondarily, if the number that is toynd with the leaste *Cossike* signe, be not a rooted number, the whole number can not be a rooted number.

And eche of these bothe rootes (if thei haue any) are partes of the whole roote, for the compounde *Cossike* number.

Thirdly, if the number be a rooted number, euery parte of it, that is not a rooted number, is a meane number, betwene the greatestt and the leaste.

Fourthly, if  $20$  bee any denomination in it, then is  $9$  an other denomination in it also.

Fiftly, and generally, all rooted nōbers, other are specially framed, by orderly multiplication, or els are numbers equalle to some one rooted number *Abstract*.

Now specially framed are soche, as are made by multiplicatio of one number by it self, and litle or nothing altered from that very forme.

Ed. j. Example

# The Arte

**Of square  
rootes.**

Exāple of.  $529\cancel{\text{z}}\text{c} \text{---} 184\cancel{\text{z}}\text{z} \text{---} 16\cancel{\text{z}}$   
 whiche is a Square number, made by multiplication  
 of.  $23\cancel{\text{c}} \text{---} 4\cancel{\text{z}}$ . by it self. This number maie  
 haue his Roote orderly extracted thus.

$$\begin{array}{r} 529\cancel{\text{z}}\text{c} \text{---} 184\cancel{\text{z}}\text{z} \text{---} 16\cancel{\text{z}} (23\cancel{\text{c}} + 4\cancel{\text{z}} \\ 23 \qquad \qquad 46\cancel{\text{c}}. \end{array}$$

In the firste number, I finde the Square roote to bee  
 $23$ . And for his denomination, I take halfe the *Cosike*  
 signe  $\cancel{\text{z}}\text{c}$ , and that is.  $\cancel{\text{c}}$ . For as.  $\cancel{\text{c}}$ . multiplied by  
 $\cancel{\text{c}}$ . doeth make.  $\cancel{\text{z}}\text{c}$ . So in diuision by.  $2$ . and in ex-  
 traction of Square rootes, I shall take the.  $\cancel{\text{c}}$ . for the  
 halfe of  $\cancel{\text{z}}\text{c}$  and the denomination of his roote: and  
 so set it doune in the *quotiente*.

Then I shall double the number *Abstrakte* of that  
*quotiente* (kepyng his *Cosike* signe vnaltered) and that  
 double shall I set euermore vnder the nexte number,  
 toward the righte hande. As here, you see, I haue set  
 $46$  (whiche is the double of  $23$ ) with his signe  $\cancel{\text{c}}$ . vn-  
 der the seconde number. And there I perceiue, I maie  
 haue it.  $4$ . tymes, if I doe diuide (as I ought)  $184$ . by  
 $46$ . And that.  $4$ . I sette in the *quotiente*, with the signe  
 $\text{---}$ , and the denomination.  $\cancel{\text{z}}$ : seying.  $\cancel{\text{z}}\text{z}$ . diui-  
 ded by.  $\cancel{\text{c}}$ . doeth yelde.  $\cancel{\text{z}}$ .

Laste of all, I muste multiplie that parte of the *quo-*  
*tiente*.  $4\cancel{\text{z}}$ . by it self, and it will yelde.  $16\cancel{\text{z}}$ . whiche  
 beyng subtracted also (as it should) leaueth nothyng  
 remainyng of the Square number.

This order must you kepe in all Square numbers,  
 how greate so euer thei be. As in this seconde exāple.

$$\begin{array}{r} \text{---} 90\cancel{\text{z}}\text{z}. \\ 25\cancel{\text{z}}\text{c} \text{---} 80\cancel{\text{z}} \text{---} 26\cancel{\text{z}}\text{z} \text{---} 144\cancel{\text{c}} \text{---} 81\cancel{\text{z}} (5\cancel{\text{c}} + 8\cancel{\text{z}} \text{---} 9\cancel{\text{z}} \\ 5\cancel{\text{c}}. \qquad 10\cancel{\text{c}} \text{---} 64\cancel{\text{z}}\text{z}. \\ \text{---} 10\cancel{\text{c}} \text{---} 16\cancel{\text{z}}. \text{---} 9\cancel{\text{z}}. \end{array}$$

The

## of Coſſike numbers.

The roote of the firſt number is. 5.  $\text{C}$ , whiche I ſet in a *quotiente*.

Then doe I double that. 5. and it maketh. 10. to be ſette vnder. 8. with his denomination, whiche is.  $\text{C}$ . like to the roote.

That. 10.  $\text{C}$ . maie be founde in. 80.  $\text{f}$ . 8. times, & therfore I ſet. 8. in the *quotiente*, with the ſigne — + — and the denomination.  $\text{f}$ . And then dooe I multiplie that. 8.  $\text{f}$ . ſquaredly, whiche giueth. — + — 64.  $\text{f}$ .  $\text{f}$ . to be ſubtracted out of — 26.  $\text{f}$ .  $\text{f}$ . and ſo remaineth — 90.  $\text{f}$ .  $\text{f}$ .

After this I double all the *quotiente* again, whereof commeth — + — 10.  $\text{C}$ . — + — 16.  $\text{f}$ . And bicauſe there is a remainer, ouer the number that I wrought laſte, I muſt ſet. 10.  $\text{C}$ . vnder the remainer, and the other number in order, as you ſee it ſet.

Then ſeke I how often tymes maie. 10.  $\text{C}$ . diuide 90.  $\text{f}$ .  $\text{f}$ , and I finde the *quotiente* to be — 9.  $\text{ze}$ . And likewaies — + — 16.  $\text{f}$ . multiplied by — 9.  $\text{ze}$  doeth make — 144.  $\text{C}$ . equalle to the ſomme ouer it: And ſo ſubtracteth it cleane. ¶ herfore to ende that worke, I multiplie the laſte *quotiente*, by it ſelf ſquare, and it yeldeth. — + — 81.  $\text{f}$ . whiche is to bee ſubtracted out of the like ſomme, in the ſquare number: and ſo reſteth nothing. ¶ herfore I iuſtly affirme, that the firſte number is a ſquare number, and hath for his roote. 5.  $\text{C}$ . — + — 8.  $\text{f}$ . — 9.  $\text{ze}$ .

Scholar. What maie I ſone proue, if I multiplie

$$\begin{array}{r} 5\text{C} \cdot - + - 8\text{f} \cdot - - - 9\text{ze} \cdot \\ 5\text{C} \cdot - + - 8\text{f} \cdot - - - 9\text{ze} \cdot \end{array}$$

---


$$\begin{array}{r} 25\text{f}\text{C} \cdot - + - 40\text{f}\text{f} \cdot - - - 45\text{f}\text{f} \cdot \\ \quad - + - 40\text{f}\text{f} \cdot - + - 64\text{f}\text{f} \cdot \\ 81\text{f} \cdot - - - 72\text{C} \cdot - - - 45\text{f}\text{f} \cdot \\ \quad - 72\text{C} \cdot \end{array}$$

---


$$25\text{f}\text{C} \cdot - + - 80\text{f}\text{f} \cdot - 26\text{f}\text{f} \cdot - 144\text{C} \cdot + 81\text{f} \cdot$$

that

Dd. y.



## The Arte

that roote by it self, as here I haue doen it. Wherby I haue not onely confirmed it to be a square number: but also I haue espied, that you vsed the number not so plainly set doune, as the particulare multiplication did make it: but rather as a reasonable reduction would expresse it. I meane in the.  $3 \cdot 3 \cdot$ . where the particulare multiplication hath  $—+—64 \cdot 3 \cdot 3 \cdot$ . and  $—90 \cdot 3 \cdot 3 \cdot$ . For whiche twoo numbers you sette one, that resulteth of the bothe, that is  $—26 \cdot 3 \cdot 3 \cdot$ .

Master. But if you would take the nōber in that sorte, the woozke would be not onely all one: but also some what plainer to bee perceiued of a learner. And therefore for your pleasure, I will set forth here, the example of that woozke. And loe, here it is.

$$\begin{array}{r}
 25 \cdot 3 \cdot \mathcal{C} + 80 \cdot \mathcal{f} \cdot 3 \cdot + 64 \cdot 3 \cdot 3 \cdot \longrightarrow 90 \cdot 3 \cdot 3 \cdot \longrightarrow 144 \cdot 3 \cdot + 81 \cdot 3 \cdot (\mathcal{C} + 8 \cdot 3 \cdot - 9 \cdot \mathcal{C} \\
 5 \cdot \mathcal{C} \quad 10 \cdot \mathcal{C} + 64 \cdot 3 \cdot 3 \cdot \quad 10 \cdot \mathcal{C} \cdot -+ 16 \cdot
 \end{array}$$

Scholar. By comparynge these bothe formes of woozke together, I dooe better vnderstande, the reason of the firste woozke.

Master. One example moare of this kinde of extraction of rootes, will I set doune, that maie be a generall patrone, for all the varieties, in this sorte of rooted numbers. And if you examine it diligently, and marke it well, you shall heare felue other examples, for this kinde of square numbers.

The Square number, with the  
wooze of extraction  
of his roote so:  
loweth  
here.

The

## The Arte

that roote by it self, as here I haue doen it. Wherby I haue not onely confirmed it to be a square number: but also I haue espied, that you vsed the number not so plainly set doune, as the particulare multiplication did make it: but rather as a reasonable reduction would expresse it. I meane in the.  $3 \cdot 3 \cdot$ . where the particulare multiplication hath  $—+—64 \cdot 3 \cdot 3 \cdot$ . and  $—90 \cdot 3 \cdot 3 \cdot$ . For whiche twoo numbers you sette one, that resulteth of the bothe, that is  $—26 \cdot 3 \cdot 3 \cdot$ .

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 5 \cdot \mathcal{C} \quad 10 \cdot \mathcal{C} + 64 \cdot 3 \cdot 3 \cdot \quad 10 \cdot \mathcal{C} \cdot -+ 16 \cdot
 \end{array}$$

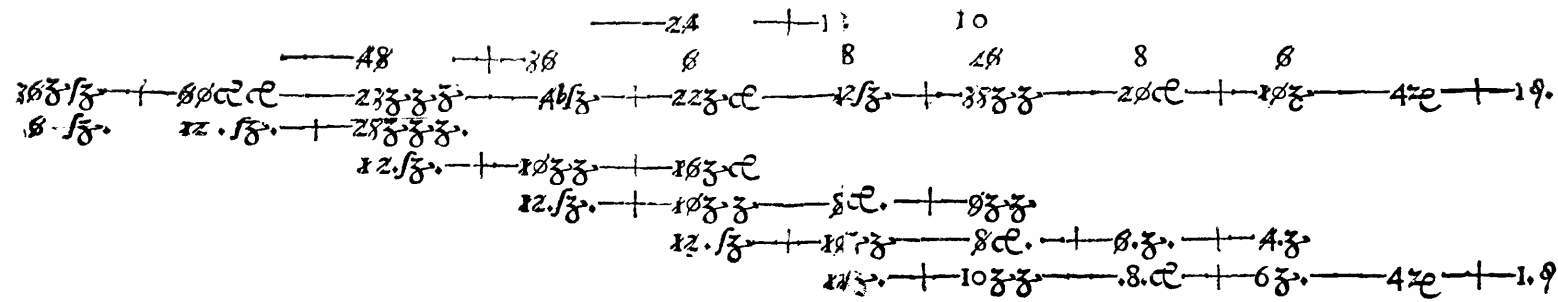
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Master. One example moare of this kinde of extraction of rootes, will I set doune, that maie be a generall patrone, for all the varieties, in this sorte of rooted numbers. And if you examine it diligently, and marke it well, you shall heare felue other examples, for this kinde of square numbers.

The Square number, with the  
wooze of extraction  
of his roote so:  
loweth  
here.

The

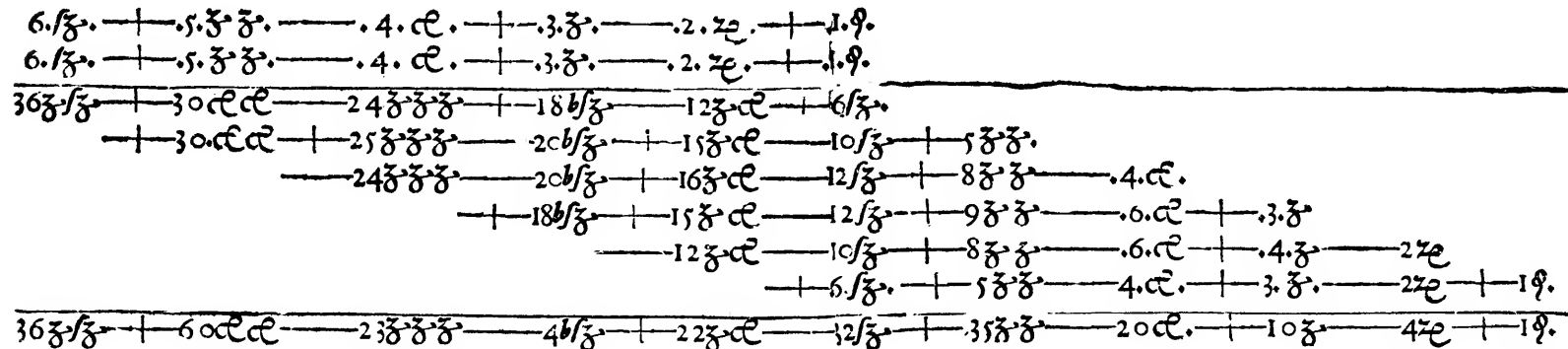
# The square nomber, with the woorke of extraction of his roote.



## The Roote.

6. f. 8. — 5. 3. 3. — 4. c. — 3. 3. — 2. 2. — 1. 9.

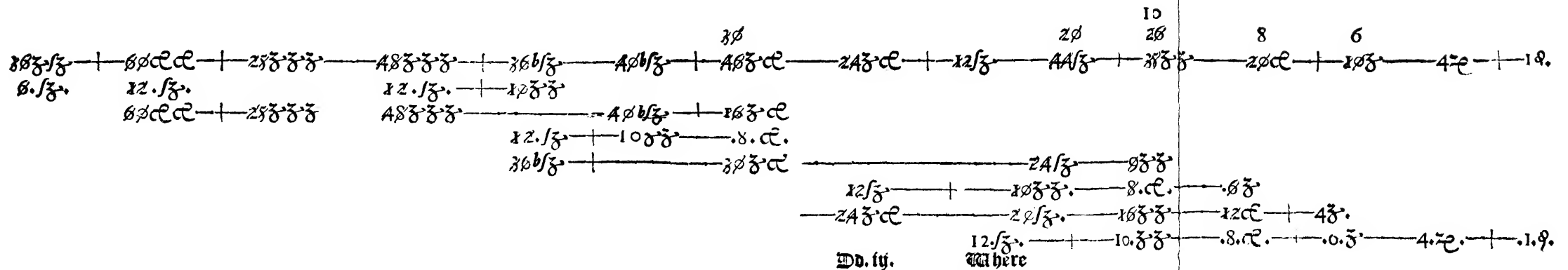
## The prooffe by Multiplication.



Scholar. It maie appeare easily, that this example serueth for many other, it doeth contain so many varieties of signes *Cosike*, multiplied so diuersely.

And in this number also, as well as in the other, I see that many numbers be omitted, by reduction: namely in the thirde, fourthe, fift, and sixthe orders of numbers. For in the 2. first orders, and in the 5. last, there is no varietie of the signes — and —.

Wherfore to see the varietie of woorke, I will sette doune the numbers, as they rise in particulare multiplication, and in it will I make an experimete of my cunningg. As here foloweth.



## of Cosike numbers.

Where for myne owne ease, and aied of memozie, I haue set vnder euery doublyng of the *quotiente*: And the somme that amounteth, by the multiplication of thesame, into the newe *quotiente*, with the Square of thesame newe *quotiente*.

Whereby I perceiue that the numbers, doe not go in soche order, that euery odde place, maketh a newe roote, as it doeth in numbers *Abstraite*. But sometime I must take. 2. places nerte together, and at an other tyme, I shall scippe. 2. or. 3. places.

Maister. You marke it well. And yet that is a good and true rule, that some menne teache: that in these *Cosike* numbers, as well as in other *Abstraite* numbers, you shall marke euery odde place, and vnder eche of them to finde a Square roote. But that is to be vnderstande, when the numbers are sette, in their besteste and exacteste order.

These fewe examles maie suffice, for a declaratiō of extractyng the roote of Square numbers, made by multiplication. And now touchyng those numbers, *The rootes of* that bee equalle to some rooted number, and namely *nōbers equal* to be square, I will teache you how their roote maie be extracted.

But firste you shall marke, that a Square beeyng compared, as equalle to rootes and numbers, the rootes maie bee coupled with the numbers onely, in. 3. formes. That is.  $20 \text{ --- } 1 \text{ --- } 9$  (whiche is all one with  $9 \text{ --- } 1 \text{ --- } 20$ ) or els thus.  $9 \text{ --- } 1 \text{ --- } 20$ . Or thirdly,  $20 \text{ --- } 9$ . And for eche of these. 3. formes, there is some varietie, in the extraction of the roote. And in them all moche agremente.

For the first forme, where  $20 \text{ --- } 1 \text{ --- } 9$  is equalle to *The firste*  $3$ , take these examles  $1 \text{ --- } 3$  is equall to.  $4 \text{ --- } 20 \text{ --- } 1$  *forme.* or.  $1 \text{ --- } 3$  is equalle to  $35 \text{ --- } 9 \text{ --- } 20$ . Likewise  $1 \text{ --- } 3$  is equalle to.  $10 \text{ --- } 20 \text{ --- } 75 \text{ --- } 9$ . or.  $1 \text{ --- } 3$  is equalle to  $10 \text{ --- } 9 \text{ --- } 8 \text{ --- } 20$ .

## of Cosike numbers.

Where for myne owne ease, and aied of memozie, I haue set vnder euery doublyng of the *quotiente*: And the somme that amounteth, by the multiplication of thesame, into the newe *quotiente*, with the Square of thesame newe *quotiente*.

Whereby I perceiue that the numbers, doe not go in soche order, that euery odde place, maketh a newe roote, as it doeth in numbers *Abstraite*. But sometime I must take. 2. places nerte together, and at an other tyme, I shall scippe. 2. or. 3. places.

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These fewe examles maie suffice, for a declaratiō of extractyng the roote of Square numbers, made by multiplication. And now touchyng those numbers, *The rootes of* that bee equalle to some rooted number, and namely *nōbers equal* to be square, I will teache you how their roote maie be extracted.

But firste you shall marke, that a Square beeyng compared, as equalle to rootes and numbers, the rootes maie bee coupled with the numbers onely, in. 3. formes. That is.  $20 \text{ --- } 1 \text{ --- } 9$  (whiche is all one with  $9 \text{ --- } 1 \text{ --- } 20$ ) or els thus.  $9 \text{ --- } 1 \text{ --- } 20$ . Or thirdly,  $20 \text{ --- } 9$ . And for eche of these. 3. formes, there is some varietie, in the extraction of the roote. And in them all moche agremente.

For the first forme, where  $20 \text{ --- } 1 \text{ --- } 9$  is equalle to *The firste*  $3$ , take these exāples  $1 \text{ --- } 3$  is equall to.  $4 \text{ --- } 20 \text{ --- } 21$  *forme*.  
 $02 \text{ --- } 1 \text{ --- } 3$  is equalle to  $35 \text{ --- } 9 \text{ --- } 2 \text{ --- } 20$ . Likewise  $1 \text{ --- } 3$  is equalle to.  $10 \text{ --- } 20 \text{ --- } 75 \text{ --- } 9 \text{ --- } 02 \text{ --- } 1 \text{ --- } 3$  is equalle to  $105 \text{ --- } 9 \text{ --- } 1 \text{ --- } 8 \text{ --- } 20$ .

## The Arte

In all these exāples, and other soche like, you must first consider the number annered with the signe.  $\mathcal{Z}$ . (whiche is the middell quantitie) and the halfe of it shall you note, for with it shall you worke twise. First you shall multiplic halfe of that number by it self, and this is the firste worke, and to it shall you adde the other whole number, that is ioyned with.  $\mathcal{Q}$ . And thei will euer more make a square number: out of whiche square you shall extrate the roote. And to that roote shall you adde halfe the number, that was annered with the signe of.  $\mathcal{Z}$ . (whiche was the number that I bade you to marke). And this is the seconde woozke. The totall that commeth of this addition, is the roote of the compounde Cosike number.

*An example* Example of the firste.  $4.\mathcal{Z}.$  — — —  $2\ 1.\mathcal{Q}.$  halfe the number annered with.  $\mathcal{Z}$ . is. 2. whose square is. 4. that shall I put to. 2 1. and there riseth. 25. beeyng a square number, and haupng. 5. for his roote. To that 5. I loyne halfe the number annered with.  $\mathcal{Z}$ . and it maketh. 7. whiche is the number that I seke for: and is the roote to.  $4.\mathcal{Z}.$  — — —  $2\ 1.\mathcal{Q}.$

*The prooffe.* For triall whereof take. 4. rootes, that is. 28. and putte to it. 2 1. and thereof commeth. 49. whiche is a square number, and hath. 7. for his roote.

*An other example.* Scholar. When can I doe the like with the second exampl.  $35.\mathcal{Q}.$  — — —  $2.\mathcal{Z}.$  And firste the halfe of. 2. is 1. and the square of it is. 1. whiche I put to. 35. and it maketh. 36. a square number: whose roote is. 6. To that. 6. if I adde. 1. that was the halfe before reserved, it will make. 7. whiche is the roote that I doe seke.

*The prooffe.* The prooffe is this: 2. rootes maketh. 14. and. 35. giueth. 49. whose roote is. 7.

*The thirde example.* Like wates for the thirde example  $100\mathcal{Z}$  — — —  $75\mathcal{Q}$  I woozke thus. Halfe. 10. is. 5. and his square is. 25. that dooe I adde to. 75. and there riseth. 100. whose roote is. 10. to whiche roote I add. 5. and there cometh

## of Coslike numbers.

meth. 15. that is the roote whiche I would haue.

And that I maie proue by triall in this sorte. 10. rootes giue. 150. vnto whiche if I adde. 75. there will amounte. 225. whiche is a Square number: and hath 15. for his roote.

The fourthe example is. 105.  $8 - \frac{1}{2}$ . where *The fourthe*  
I take firste the halfe of. 8. that is. 4. and it in Square *example.*

giueth. 16. whiche I adde to. 105. and there amounteth. 121. beyng a Square number, and the roote of it 11. vnto whiche I shall adde. 4. for halfe the number of rootes: and so there riseth. 15. as the roote that I seke for. And to approue it I take. 8. times. 15. whiche *The prooffe.*  
is. 120. and adde it vnto. 105. and so commeth. 225.

For the square, and the roote of it is. 15.

Maister. The like order of worke shall you vse, in *Other for:*  
all numbers Coslike compounde, whē any. 2. numbers *mes in like*  
with immediate Denominatiōs Coslike, are equalle to sorte.  
one of the nexte denomination, in order aboute them.

As. 1.  $\mathcal{C}$ . is equalle to. 3.  $\frac{3}{4}$ .  $-\frac{1}{2}$  10.  $\frac{2}{3}$ .

And again. 1.  $\frac{3}{4}$ . equalle to. 6.  $\frac{3}{4}$ .  $-\frac{1}{2}$  40.  $\mathcal{C}$ .

Likewise. 1.  $\frac{3}{4}$ .  $\mathcal{C}$ . equalle to. 3.  $\frac{3}{4}$ .  $-\frac{1}{2}$  28.  $\frac{3}{4}$ .

But in al these the roote shal beare name of the greater quantie.

Scholar. By the former order of worke, I shall in *The fift*  
the firste of these. 3. examples, take halfe. 3. (because it *example.*  
is the number of the middell quantite). And that is  $\frac{3}{2}$ .

and that shall I multiplie squarely, and so will there rise  $\frac{9}{4}$ , vnto whiche I shall adde 100  $\frac{4}{9}$ . And that maketh  $\frac{409}{9}$ . whiche is a square number, and his roote is  $\frac{20}{3}$ . vnto whiche I must put the firste halfe, that is  $\frac{3}{2}$ , and then will it be  $\frac{17}{3}$ , or els. 5. whiche is the Cubike roote of that number. 3.  $\frac{3}{4}$ .  $-\frac{1}{2}$  10.  $\frac{2}{3}$ . beyng equalle to 1  $\mathcal{C}$

For prooffe whereof, I multiplie. 5. Cubikely, and it *The prooffe.*  
maketh. 125. Then doe I multiplie it squarely, and it will be. 25. Now. 3.  $\frac{3}{4}$ . is. 75. and. 10.  $\frac{2}{3}$ . maketh. 50 whiche bothe added together, giue. 125.

## The Arte

*The seconde  
example.*

In the seconde example, where.  $1.\sqrt[3]{8}.$  is equalle to  $6.\sqrt[3]{8}.\sqrt[3]{8}.$  — + —.  $40.\mathcal{C}.$  I shall take halfe.  $6.$  (whiche is the number of the middell quantitie) and that is.  $3.$  and the square of it is.  $9.$  whiche I must adde vnto  $40$  and thereof commeth.  $49.$  whiche is a square number and hath.  $7.$  for his roote, vnto whiche I adde  $3.$  and so haue I  $10$  for the *Surfolide* roote, of  $6.\sqrt[3]{8}.\sqrt[3]{8}.$  — + —.  $40.\mathcal{C}.$

*The prooffe.*

And for prooffe I saie, if.  $10.$  bee the roote, then is  $100.$  the square, &  $1000.$  the Cube, the  $\sqrt[3]{8}.\sqrt[3]{8}.$  is  $10000.$  And the *Surfolide*.  $100000.$  Wherefore.  $6.\sqrt[3]{8}.\sqrt[3]{8}.$  make  $60000.$  and.  $40.\mathcal{C}.$  yelde.  $40000.$  And bothe thei together doe make.  $100000.$  whiche is the quantitie of the *Surfolide*.

*The thirde  
example.*

In the thirde example.  $1.\sqrt[3]{8}.\mathcal{C}.$  is equalle to.  $3.\sqrt[3]{8}.$  — + —.  $28.\sqrt[3]{8}.\sqrt[3]{8}.$  whose *zenzicubike* roote, I seke in this sorte.

Firste I take halfe.  $3.$  (as the number of the middell quantitie) that is  $\frac{1}{2}$ , & that maketh in square  $\frac{9}{4}$ . whiche I adde vnto  $28$  (that maketh  $\frac{112}{4}$ ) & it yeldeth  $\frac{121}{4}$ . whiche is a square number, and his roote is  $\frac{11}{2}$ . vnto whiche I adde  $\frac{1}{2}$ , and it will be  $\frac{12}{2}$ , or.  $7.$  whiche is the *zenzicubike* roote vnto the foresaid number.  $3.\sqrt[3]{8}.$  — + —.  $28.\sqrt[3]{8}.\sqrt[3]{8}.$

*The prooffe.*

For prooffe whereof I multiplie.  $7.$  *zenzicubikely*, and it maketh  $117649.$  When must the  $\sqrt[3]{8}.$  be  $16807$  and.  $3.\sqrt[3]{8}.$   $50421.$  Again the  $\sqrt[3]{8}.\sqrt[3]{8}.$  is.  $2401.$  and so  $28.\sqrt[3]{8}.\sqrt[3]{8}.$  shall bee.  $67228.$  And those bothe together yelde.  $117649.$

*A thirde  
forme.*

Master. Yet one other forme is there, that in all thinges, saue in one poinde onely: followeth the same rule. And that is whē the denominations doe not go immediatly together, but yet are equally distant. As  $\sqrt[3]{8}.\sqrt[3]{8}.$  &  $\mathcal{C}.$  and.  $9.$  where the distance is one onely quantitie. Likewise.  $\sqrt[3]{8}.\mathcal{C}.\mathcal{C}.$  and.  $9.$  whiche differ by.  $2.$  quantities. And in like sorte.  $\mathcal{C}.\mathcal{C}.\sqrt[3]{8}.$  and  $9\mathcal{C}.$  are distant by.  $3.$  quantities. And so of other, how many so euer bee omitted, so that the difference bee equalle



## of Coſſike numbers.

equalle. In all whiche you ſhall worke, as you did in the former rule, till you haue canded all that worke. But then haue you here, one thing moze to bee conſidered. For the laſte number, whiche you haue ſounde, is not the roote, but a rooted quantitie: And his roote is the roote that you ſeke for.

Scholar. Doe you meane the ſquare roote of that quantitie, or ſome other?

Maſter. It maie be any kinde of roote, in diuerſe numbers, but not in one number. Wherefoze for your certaintie marke this rule.

If the denominations of your numbers, differ one by one, then is it a ſquare nōber, that you doe finde by the praſtiſe of the laſte rule. And therfoze ſhall you take his ſquare roote, for the roote of your number.

But if the denomination differ by . 2 . quantities, then ſhall you extracte a *Cubike* roote, out of your laſte number. And if the diſtaunce bee . 3 . quantities, the roote muſt bee a *zenzizenzike* roote: and for . 4 . quantities diſtante, a *Surſolide* roote, and ſo forth.

As for example. 1.  $\text{Z} \text{Z}$ . is equalle to.  $8 \text{O} \text{Z}$ . *An example*  
 $2000 \text{O} \text{Z}$ . Now for to finde the roote of.  $8 \text{O} \text{Z}$ .  
 $2000 \text{O} \text{Z}$ . I worke thus. Firſt I take the halfe of  $8 \text{O}$ .  
 (becauſe it is the number of the middle quantitie) and that halfe is.  $4 \text{O}$ . whiche I multiplie ſquare, and it maketh.  $1600$ . to it I adde.  $2000$ . and it will bee  $3600$ . whiche is a ſquare number,  $60$ . is his roote: to that.  $60$ . I ſhall adde the ſoꛑſaied.  $40$ . and then will it bee.  $100$ . whiche number in the firſt rule, had been the true roote. But here conſidering the diſtance is of one quantitie, I muſte extracte his ſquare roote, whiche is.  $10$ . And that is the *zenzizenzike* roote, that my number containeth.

An other example. 1.  $\text{Z} \text{C}$ . is equalle to.  $400 \text{C}$ . *The ſeconde example.*  
 $— + — 57344 \text{O} \text{Z}$ . I take  $200$ . for the halfe of the middle quantities number, and multiplying it ſquare, I

Ce. 1.

finde

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finde. 40000. whiche I put to. 57; 44. and then I haue. 97344: whiche is a Square number, and his roote is 312 vnto whiche I shall adde the halfe of 400 and so will it bee. 512. But now must I take the Cubike roote of this number (that is. 8) for my roote, that I desire: Bicause the denominations in the number, differ by. 2. quantities.

Scholar. I see very well the order of this worke: And the prooffe is in like sorte, whiche I maie practise by my self at any tyme. Wherefore I praise you, procede forth to other rules.

*The seconde  
sort of equal  
numbers.*

Master. This is sufficiente for the firste sorte. Now for the seconde sorte, in numbers *diminute* or *residuale* where. 3. is equalle to. 9. — 2. the forme of worke is like vnto the other, in all pointes saue in one. For in steede of the laste addition, you shall vse in these numbers, Subtraction. As here for example, when I saie. 1. 3. is equalle to. 60. 9. — 4. 2. to finde the roote, firste I take the halfe of. 4. (bicause it is the number of the middell signe) and that halfe beyng. 2. doeth make in square. 4. whiche I put to 60 and so is it. 64. a square number, and hath. 8. for his roote. From whiche roote (by the order of this rule) I must abate. 2. that is the halfe of the firste number of rootes. And then will there remaine. 6. for the verie roote of. 60. 9. — 4. 2. beyng equalle to. 1. 3.

*Example.*

*The prooffe.*

Scholar. That is sone proued. For. 6. beyng the roote, then. 4. 2. maketh. 24. whiche beyng abated out of. 60. leaueth 36 and that is the iuste square vnto. 6. as the equation saith.

*The seconde  
example.*

Master. An other example is this. 1. 3. is equall to. 162. 2. — 9. 3. 3.

Scholar. What can I worke, thus: Firste I take the halfe of. 9. (bicause it is the number of the middell signe) and it is  $\frac{9}{2}$ , whiche I multiplie squarely, and it will be  $\frac{81}{4}$ , that must bee added to. 162. 2.  $\frac{81}{4}$ , and then will

## of Coſike numbers.

Will there amounte  $\frac{729}{4}$ . whiche is a Square number, and hath for his roote  $\frac{27}{2}$  out of whiche, by this rule, I muſt abate  $\frac{3}{2}$ , and then riſeth  $\frac{1}{2}$ , that is. 9. whiche is the very roote to 162.  $\overline{162} = 9 \cdot \overline{9} \cdot \overline{9}$ . being equall to. 1.  $\overline{9}$ .

And for the pꝛoofe, I multiplie. 9. Cubikely, and it giueth. 729. ſo that. 162.  $\overline{162}$ . doe make. 118098. out of whiche I muſt abate. 9.  $\overline{9} \cdot \overline{9}$ . that is. 59049. (by the ſame roote, ſith. 1  $\overline{9} \cdot \overline{9}$ . is. 6561). And then will there remaine. 59049. whiche is the iuſte quantitie of. 1.  $\overline{9}$ .

Maſter. Yet one example moꝛe ſhall you haue of The thirde  
a thirde ſorte. example.

When. 1  $\overline{9} \cdot \overline{1}$  is equalle to. 275456.  $\overline{9} = 26 \overline{1}$   
I demaunde of you, what is the valewe of. 1.  $\overline{1}$ ?

Scholar. I ſearche it thus. The number of the middell ſigne is. 26. whose halfe I muſt take, and firſt multiplie it ſquarely, and there will riſe 169. whiche I adde to. 275456. and it will bee. 275625. whiche is a ſquare number, and hath for his roote. 525. from whiche number I muſt abate halfe the number, of the middell ſigne, that is. 13. and ſo there will remaine 512. whose Cubike roote I muſt extract, becauſe the denominations differ by. 2. quantities, and that roote will be. 8. whiche is the Cubike roote to. 512. but to the number pꝛopounded, it is the *zenzicubike* roote.

Maſter. This is enoughe for the worke of the ſeconde ſorte. Now for the thirde ſorte of equation, The thirde  
where.  $\overline{9}$ . is equalle to.  $\overline{1}$ . ——.  $\overline{9}$ . I will giue you ſorte of equal  
a brief admonition enclpy, though it differ from bothe numbers,  
the ether. 2. rules, in forme of worke. For as the equalitie maie be in diuerſe ſortes, ſo ſome tymes you maie uſe the worke of the firſt ſorte, by Addition of halfe the number of the middle ſigne: and ſome times you ſhall worke by ſubtraction. Wherein this is the difference, from the ſeconde rule. That there you doe  
Ec. 9. ſubtrate

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subtracte halfe the number of the middell signe, from the roote whiche you sonde. And in this thirde rule, you shall subtracte the roote from the halfe, and not the halfe from the roote. For because that that roote, is euer lesser then that halfe.

And in this rule, this is specially to bee obserued: that the Square of halfe the number, of the middell signe, will euer moze bee greater, then the number of the lesser signe: And therfore shall the number of the lesser signe, bee abated out of that square. And the remainer will bee a Square number, with whiche you shall woork, as I haue taught you before.

And farther in this rule, it is commonly seen, that euery soche equalle number, hath. 2. valuations for his roote. I meane that any of those. 2. numbers, will bee as the roote in this equation. For otherwises no number can haue. 2. rootes of one denomination.

Scholar. I vnderstande you thus. That no number can haue. 2. square rootes, or. 2. Cubike rootes, and so forth: Els one number maie haue. 3. or. 4. rootes. As. 64. hath. 8. for his Square roote: 4. for his Cubike roote: and. 2. for his *quincubike* roote.

Master. You take it well. And farther for the easie knowledge of those. 2. numbers, or rootes: They must bee soche, as beeyng added together, will make the nōber of the middell signe: and beeyng multiplied together, wil make the number of the least signe. And so mate you finde them without farther multiplication, or extraction of rootes.

*The firste  
example.*

For example, I sette firste. 1. 3. equalle to. 16. 4. ———. 63. 9. where I maie espie quickly, that. 63. can haue no moze partes to his composition, but. 3. 7. 9. 21 And if I take. 3. and. 21. then their addition will bee greater then. 16. but 7. and 9. maketh iuste 16. by addition, and. 63. by multiplication. And therfore they shall be the. 2. rootes.

Scholar.

## *of Cossike numbers.*

Scholar. I will proue that by examination, thus. If 7. be the roote, then is. 49. the square. And. 16.  $\mathcal{Z}$  make. 112. out of whiche I must abate. 63. and there resteth. 49. equalle with the Square: so is that a true roote. When for. 9: his square is. 81. And. 16.  $\mathcal{Z}$ . doe yelde 144 frō whiche I shal abate 63. And the remainer will be. 81. equalle to the square. And so is that al so a true roote.

Master. Now worke it by the other rules, that I taught you.

Scholar. Firste I take. 8. as halfe the number of the middell signe, and that multiplied Square, doeth giue 64 from whiche I shall abate 63 and then doeth there remain but. 1. whiche is counted as a square number, and his roote to be. 1. also, whiche if I adde to. 8. it will make. 9. that is one of the rootes: And if I abate it from. 8. it will leaue. 7. whiche is the other roote. And thus I see one worke confirmeth the other.

Master. Take this for the seconde exāple. 1  $\mathcal{Z}$   $\mathcal{C}$  *The seconde example.*  
is equalle to. 8.  $\sqrt{3}$ . ———. 12.  $\mathcal{Z}$   $\mathcal{Z}$ . what is the roote saie you?

Scholar. To finde it, firste I loke for the partes of 12. And thei be. 2. 3. 4. 6. of whiche. 2. and. 6. doe serue my purpose, for their addition maketh. 8. and so doeth not. 3. and. 4. Therefore I saie, that. 2. maie be the roote, and so maie. 6. But for farther trialle of it: I worke it by the other rule, sayng halfe. 8. is. 4. and his square is. 16. From whiche I abate. 12. and there remaineth. 4. whose roote is. 2. that I maie adde to. 4 and so haue 3. 6. for one roote: or els abatyng it from 4. I shall haue. 2. for the other roote.

The vsoofe is manifeste for. 6. beeyng a roote, the *The prooffe.*  
*zenzicube* is. 46656. The *Surfolide* is. 7776. And the *zenzizenike* is 1296. So that 8.  $\sqrt{3}$ . doe make 62208 And. 12.  $\mathcal{Z}$   $\mathcal{Z}$ . are. 15552. whiche being abated out of 62208 do leaue 46656. the true quantitie of 1  $\mathcal{Z}$   $\mathcal{C}$

Ce. iij. And

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And so is that woꝝke good, 6. beyng a roote.

Now if, 2. be sette for a roote: then is the  $\sqrt[3]{8}$ . 16. the  $\sqrt[3]{8}$ . 32, and the  $\sqrt[3]{8}$ . 64. And so are, 8.  $\sqrt[3]{8}$ . equall to, 256. And, 12.  $\sqrt[3]{8}$ . yelde. 192. Wherfore atating 192. out of, 256. thire resteth, 64, the iuste quantitie of, 1.  $\sqrt[3]{8}$ . And so is that woꝝke also good, and, 2. a true roote.

*The thirde example.*

Maſter. Now pꝛoue this thirde example, where 1.  $\sqrt[3]{8}$  is equalle to, 2000.  $\sqrt[3]{8}$  — 470016  $\sqrt[3]{8}$ .

Scholar. Halfe the number of the middell sign is 1000. And the square of it is, 1000000. From whiche I shall abate . 470016. and there will remaine 529984. whose square roote by trialle of extraction, I finde to be 728. whiche I maie other adde to, 1000 and so there riseth, 1728. whiche I finde to bee (as it ought) a Cubike number. And his roote to be, 12.

But and if I abate 728. from 1000, there will remain, 272. whiche is no Cubike number.

Maſter. So that here semeth to be but one roote. And yet these, 2. numbers, 1728. and, 272. kepe soche a rate, that beeyng multiplied together, thei make 470016. whiche is one of the numbers, and beeyng added together, thei make 2000. whiche is the other number of the same Cosike residuall.

But now pꝛoue in other like nōbers, whiche haue some distaunce, betwene their denominations, whether it will so happen still. As namely in this, where

*The fourth example.*

1.  $\sqrt[3]{8}$  is equalle to, 12.  $\sqrt[3]{8}$  — 32.  $\sqrt[3]{8}$ .

Scholar. Halfe, 12. is, 6. and his Square, 36. from whiche abatying, 32. there is left, 4. whose roote is, 2. And if I adde that 2. to, 6. it maketh, 8. whiche is a Cubike number, and hath, 2. for his roote. But if I abate 2. from, 6. there remaineth, 4. whiche is no Cubike number, and therefore hath no soche roote. And yet these, 2. numbers, 4. and, 8. by addition, make the middell number, and by multiplication, thei make the laste number.

Maſter.

## of *Cossike* numbers.

**Master.** Prove yet ones againe in a number, *The fyste* where one quantitie onely is omitted. As when  $1\sqrt{3}$ , *example.* is equalle to. 2 4.  $\text{C}$ . ——— 1 3 5.  $\text{C}$ .

**Scholar.** 12. maketh in square. 144. from whiche I shall deduce. 135. and then resteth. 9. whose square roote is. 3. whiche if I adde to. 12. it will bee. 15. and hath no square roote, as here is required. But if I abate. 3. from 12. then remaineth 9 whose square roote is. 3. and seructh to the number, as I haue here produced in my tables. And. 9. and. 15. kepe the customa- ble rate. For by addition thei make. 24. And by multiplication, thei yelde. 135.

But in all these examples, where the denominati- ons be are a distaunce, I can finde but one roote, and not. 2. As it was in the other exaples of thesame rule.

And in some of theim, the greater number containeth the roote: but in other, the lesser number hath the roote.

**Master.** Because I can not stae now, about this varietie, I will remitte it till an other tyme. But this by the waie, I must admonishe you, that I doe solowe here, the common forme of writers, in calling these rootes, that rise in equatio, where as thei are not the rootes of those numbers, but are the value of a roote. For of a *Cossike* number, the roote must needs bee a *Cossike* number also. And soche as by multiplication will make the rooted number: But so can not those numbers doe.

And here will I make an eande, of the woorkes of *Cossike* ombers. And now will I applye them to practise in the rule of equation, that is commonly called *Algebers* rule.

# The Arte

## The rule of equation, common- ly called *Algebers Rule*.

*The rule of  
equation.*



Other to haue I taughte you, the common formes of worke, in numbers *Denominate*. Whiche rules are vsed also in numbers *Abstratte*, & likewise in *Surde* numbers. Although the formes of these workes be scuerallye, in eche kinde of number. But now will I teache you that rule, that is the principall in *Coslike* workes: and so; whiche all the other dooe serue.

This Rule is called the Rule of *Algeber*, after the name of the inuentoure, as some men thinke: or by a name of singular excellencie, as other iudge. But of his vse it is rightly called, the rule of *equation*: because that by *equation* of numbers, it doeth dissolue doubtfull questions: And vnfolde intricate riddles. And this is the order of it.

*The somme of the rule of equation:*



Then any question is propounded, apperteyning to this rule, you shall imagine a name for the number, that is to bee soughte, as you remember, that you learned in the rule of *false position*. And with that number shall you procede, accordyng to the question, vntil you finde a *Coslike* number, equalle to that number, that the question expresseth, whiche you shall  
reduce



## of Cossike numbers.

reduce euer more to the leaste numbers. And then diuide the number of the lesser denomination, by the number of the greateste denomination, and the quotient doeth aunswere to the question. Except the greater denominatiō, doe beare the signe of some rooted nōber. For then must you extract the roote of that quotiente, accordyng to that signe of denomination.

Scholar. It seemeth that this rule is all one, with the rule of false position: and therefore mighte so bee called: sayng it taketh a false nōber, to worke with al.

Master. This rule doeth farre excell that other. And dooeth not take a false number, but a true number for his position, as it shall bee declared anon. Wherby it maie bee thoughte, to bee a rule of wonderfull inuention, that teacheth a manne at the firste worde, to name a true number, befoze he knoweth resolutely, what he hath named.

But bicause that name is common to many numbers (although not in one question) and therefore the name is obscure, till the worke doe detect it, I thinke this rule might well bee called, the rule of darke position, or of straunge position: but not of false position.

And for the more easie and apte worke in this arte wee dooe commonly name that darke position. i. *℥*. And with it doe we worke, as the question intendeth, till we come to the equation.

This rule of equation, is diuided by some men, into diuerse partes. As namely *Scheubelius* dooeth make. 3. rules of it. And in the seconde rule, he putteth. 3. senec-  
*The partes of the rule.*  
rallē cannōs. Some other men make a greater nōber of distinctiōs in this rule. But I intende (as I thinke beste for this treatice, whiche maie serue as farre

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as their woordes doe extende ) to distincte it onely into two partes. Whereof the firste is, when one number is equalle vnto one other. And the seconde is, when one number is compared as equalle vnto. 2. other numbers.

Alwaies willyng you to remeber, that you reduce your numbers, to their leaste denominations, and smalleste formes, befoze you procede any farther.

And again, if your equation be soche, that the greatestte denomination Coslike, be ioined to any parte of a compounde number, you shall tourne it so, that the number of the greatestte signe alone, maie stande as equalle to the reste.

And this is all that needeth to be taughte, concerning this woorde.

Howbeit, for easie alteration of equations. I will propounde a fewe examples, bicause the extraction of their rootes, maie the more aptly bee wroughte. And to auoide the tedious repetition of these wordes: is equalle to: I will sette as I doe often in woordes vse, a paire of paralleles, or Gemowe lines of one lengthe, thus: =====, bicause noe. 2. thynges, can be moare equalle. And now marke these numbers.

1.        14.ze.—|—.15.g=====71.g.
2.        20.ze.——.18.g=====102.g.
3.    26.g.—|—10ze=====9.g——10ze——|—213.g.
4.    19.ze——|—192.g=====10g——|—108g——19ze
5.        18.ze——|—24.g.=====8.g.—|—2.ze.
6.    34g——12ze=====40ze——|—480g——9.g.
1.        In the firste there appeareth, 2. numbers, that is  
14.ze.

## of Cossike numbers.

14.  $\mathcal{Z}$ . — + — 15.  $\mathcal{G}$ . equalle to one number, whiche is 71.  $\mathcal{G}$ . But if you marke them well, you maie see one denominatiō, on bothe sides of the *equation*, which neuer ought to stand. Wherfoze abating the lesser, that is 15.  $\mathcal{G}$ . out of bothe the numbers, there will remain. 14.  $\mathcal{Z}$ . — — — — 6.  $\mathcal{G}$ . that is, by reduction, 1  $\mathcal{Z}$ . — — — — 4.  $\mathcal{G}$ .

Scholar. I see, you abate 15.  $\mathcal{G}$ . from them bothe. And then are thei equalle still, seying thei wer equalle before. Accordyng to the thirde common sentence, in the patthelwaie:

*If you abate euen portions, from thynges that bee equalle, the partes that remain shall be equall also.*

Maister. You doe well remeber, the firste groundes of this arte. For all springeth of those principles *Geometricalle*. Wherfoze call to your minde likewises the secende common sentence, in the same booke, and then haue you another reason, whiche will helpe you not onely, in the other formes of woork here, but also very often in the practise of this arte.

Scholar. What is this.

*If you adde equalle portions, to thynges that bee equalle, what so amounteth of them shall be equalle.*

Maister. These twoo sentences doe instructe you that when you see on bothe the sides of the *equation*, any one denominatiō *Cossike*, you shall marke the signe that is annexed to the lesser of them bothe: and if it be the signe of addition. — + —. then shall you abate that lesser number, from bothe the partes of the *equation*. As I did in this firste example. But if the signe be of abatemente — — — —, then shall you adde that lesser number, to bothe partes. And so shall you doe, till there be noe one denomination on bothe partes, but diuerse and distincte.

So the secende number will be. 20.  $\mathcal{Z}$ . — — — — 120.  $\mathcal{G}$  2.  
and in the leasse termes. 1.  $\mathcal{Z}$ . — — — — 6.  $\mathcal{G}$ .

Scholar. I see that you adde. 18.  $\mathcal{G}$ . to bothe partes  
ff. ij. tes

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3. tes of the equation. But by that reason, I doubt in the thirde somme, because.  $10. \frac{ze}{z}$ . is in bothe partes of the equation: in the firste parte with  $+$ , and in the seconde parte with  $-$ , whether I shall adde  $10. \frac{ze}{z}$ , or abate them.

Master. In soche a case, you maie dooe either of bothe, at your libertie: and all will be to one eande.

Scholar. If I adde.  $10. \frac{ze}{z}$ . then will it be.  $26. \frac{z}{z}$ .  
 $10. \frac{ze}{z} = 9 \frac{z}{z} = 213. \frac{z}{z}$ .

Master. And doe you not see.  $z$ . on bothe sides of the equation:

Scholar. I did loke but for one alteration onely.

Master. If there were twentie like denominations, you should alter them all. For that is the principalle and peculiere reduction, that belongeth to equations.

Scholar. Then must I abate.  $9. \frac{z}{z}$ . on bothe partes, and so will there remaine.  $17. \frac{z}{z}$ .  $+$ .  $20. \frac{ze}{z}$ .  
 $= 213. \frac{z}{z}$ .

Master. Now reduce it by abating.  $10. \frac{ze}{z}$ .

Scholar. So it will bee.  $17. \frac{z}{z}$ .  $=$ .  $213. \frac{z}{z}$ .  
 $= 20. \frac{ze}{z}$ .

And now I remeber, that this is the better forme of reduction. Because the greater denomination, that is.  $z$ , is alone with his number on the one side of the equation, and the. 2. lesser denominations, on the other side.

Master. How doe you reduce the other equations, to their smalleste formes?

4. Scholar. In the fourth example, there is noe denomination, before the signe of equation, or in the first parte, but the like is in the seconde parte also, after the signe of equation. Wherefore firste, because I see  $19. \frac{ze}{z}$ . on bothe sides, I will abate it on bothe sides. And then will it be thus.

$192. \frac{z}{z} = 10. \frac{z}{z} = 108. \frac{z}{z} = 38. \frac{ze}{z}$ .  
 But

## of Coslike numbers.

But bicause I see  $\varphi$ . yet remainyng on bothe partes,  
I abate the lesser, that is . 108 $\varphi$ . from bothe partes,  
and it will be. 84. $\varphi$ . ———. 10. $\zeta$ . ———. 38. $\zeta$ .

Master. This equation would bee better, if the  
greater denomination, did stande as one parte of the  
equation alone. Whiche thyng you maie easily doe,  
by addyng. 38.  $\zeta$ . to bothe partes: bicause so moche  
soloweth ———, on the one parte.

And enermore when occasion serueth, to translate *Translations*  
numbers compounde, ——— on the one side is equalle *of numbers.*  
to ——— on the other side.

Scholar. When it will be thus.

$$84.\varphi. - \text{---} 38.\zeta. = 10.\zeta.$$

Master. It were better thus.

$$10.\zeta. = 38.\zeta. - \text{---} 84.\varphi.$$

And in smaller termes.

$$5.\zeta. = 19.\zeta. - \text{---} 42.\varphi.$$

But now procede with the examples.

Scholar. The fiftie is easily reduced, by abatyng 5.  
2. $\zeta$ . on bothe sides: For so will it bee.

$$8.\zeta. = 16.\zeta. - \text{---} 24.\varphi.$$

The furthe equation will be, by addyng. 12. $\zeta$ . on  
bothe sides. 34. $\zeta$ . ——— 52. $\zeta$  ——— 480. $\varphi$  ——— 98. $\varphi$ . 6.

But yet I must reduce it farther, by addyng. 9. $\zeta$ . on  
bothe sides. And then it will stande thus.

$$43.\zeta. = 52.\zeta. - \text{---} 480.\varphi.$$

Master. Now will I shewe you the varieties of *Varities of*  
equations, taught by Scheubelius, bicause you maie per- *equations.*  
ceiue, how thei bee contained in those. 2. formes, nam-  
med by me. As for the manyfolde varieties, that some  
other doe teache, I accoumpte it but an idle bablyng,  
or (to speake moare fauourably of them) an vnnecessary  
F. iij.                      distinc.

# The Arte

*The firste  
equation.*

distinction.

The first equatio after *Scheubelius*, & after my meanyng also, is, when one number is equall to an other: meanyng that thei bothe must be simple numbers *Cosfike*, and vncompounde. As. 6.  $\mathcal{Z}$ . equalle to. 18.  $\mathcal{Q}$ :

$$4.\mathcal{Z}.\text{---}.12.\mathcal{Z}.$$

$$14.\mathcal{C}.\text{---}.70.\mathcal{Z}:$$

$$15.\mathcal{Z}.\text{---}.90.\mathcal{Z}.\mathcal{Z}:$$

$$20.\mathcal{Z}.\mathcal{C}.\text{---}.180.\mathcal{Z}:\mathcal{Z}:$$

$$26.\mathcal{Z}.\mathcal{Z}.\text{---}.117.\mathcal{C}.\mathcal{C}.$$

In all these examples, as you see but one number, compared to an other: so to finde the quantitie of one roote, you shall diuide the number of the lesser Character, by the number of the greater Character, and so shall the *quotiente* byyng forth the quantitie of. 1.  $\mathcal{Z}$ .

Scholar. It semeth at the firste belue, that it is against reason, to diuide the number of the lesser signe, by the number of the greater. But when I consider, that if I compare a number of crounes, or any like denomination, to a number of shillynges in equaltie, the number of crounes, or other soche like, must needs be lesser, then the nōber of shillinges. And so diuiding the nōber of the shillinges (or other lesser name) by the number of crounes (or other greater name) the *quotiente* will shewe, how many shillynges make a croune: and generally, how many of the lesser, dooe make one of the greater.

As if. 20. crounes bee equalle to. 100. shillynges, then. 5. shillynges dooeth make a croune. So when 6.  $\mathcal{Z}$ . bee equall to. 18.  $\mathcal{Q}$ . then. 3.  $\mathcal{Q}$ . doeth make. 1.  $\mathcal{Z}$ . And. 4.  $\mathcal{Z}$ .  $\text{---}$ . 12.  $\mathcal{Z}$ . dooeth cause that. 3.  $\mathcal{Q}$ . must be a roote.

Master. As your cramplarie profe is good, so reduction will be a sufficiente profe in this.

Scholar. I see it manifestly. For if. 14.  $\mathcal{C}$ . bee equalle to. 70.  $\mathcal{Z}$ . then. 1.  $\mathcal{C}$ . is equalle to. 5.  $\mathcal{Z}$ . by that reduction

## of Cossike numbers.

reduction in numbers. And again by reduction in signes.  $1. \text{ze}$ . is equalle to.  $5. \text{q}$ .

Likewais.  $15. \text{fz}$ . beyng equalle to.  $90. \text{z}$ .  $\text{z}$ . reduction by signes and numbers also, will make  $1. \text{ze} = 6. \text{q}$ . So shall.  $20. \text{z}$ .  $\text{cl}$ .  $= 180. \text{fz}$ . be reduced to.  $1. \text{ze} = 9. \text{q}$ . And.  $26. \text{z}$ .  $\text{fz}$ .  $= 104. \text{cl}$ .  $\text{cl}$ . will make.  $1. \text{ze} = 4. \text{q}$ .

Maister. And so generally, when there is no denomination omitted, betwene those. 2. that bee compared in equalitie, still the diuision of the number, of the lesser denomination, by the number of the greater denomination, will bypng forth in the *quotiente*, the quantitie of.  $1. \text{ze}$ .

But if there bee any denominations omitted, be- *The seconde*  
twene those. 2. whiche be compared together in equalitie: loke how many denominations are omitted, and *forme of the*  
so many in order is the rooted quantitie, whose roote you must extract, for the aunswere to the questiō. For *firste equatiō*  
in soche a case, euer moze you shall extracte the roote of your laste number.

As for example, when.  $6. \text{cl}$ . be equalle to.  $24. \text{ze}$ . by the former rule, you shall finde. 4. in the *quotiente*. But here that. 4. is not the quantitie of a roote, but is a rooted number, whose roote I shall extracte. And seying betwene.  $\text{cl}$ . and.  $\text{ze}$ . there is no quantitie omitted, but one, that is.  $\text{z}$ . Therefore I shall accompte. 4. the firste quantitie, that is to saie, a *Square* number, and so take his *Square* roote, beyng. 2. for the quantitie of a roote.

Again if.  $7. \text{fz}$ . be equalle to.  $567. \text{ze}$ . the *quotiente* will be. 81. and declareth a *zenzenzenike* number, because there are omitted betwene.  $\text{fz}$ . and.  $\text{ze}$ . thre numbers: and *zenzenzenike* is the thirde quantitie: as you did learne in the beginning of this treatise, of numbers denominate.

Scholar. I perceiue it. And therefore I must take  
the

## The Arte

the *zenzizenzike* roote of. 81. whiche is. 3. and that is the true roote, where.  $7\sqrt[3]{3}$ . be equalle to. 567.20.

Master. And if those.  $7\sqrt[3]{3}$ . were accopted equalle to. 56.3. the *quotiente* will be. 8. And bicause there are omitted. 2. quantities, that is.  $\mathcal{C}$ . and.  $\mathfrak{z}\mathfrak{z}$ . therefore you shall accompte that. 8. to be 1  $\mathcal{C}$ . or a seconde quantitie. And his roote *Cubike* is. 2. whiche standeth as the valewe of a roote, in the former equation.

And it is not possible that any other number, maie be placed as a roote, in that equation: or in any other forme of this firste kinde. Whobecit in one sorte of equation, of the seconde kinde, there maie be. 2. diuerse rootes, when one number hath. 2. rootes in valewe. As I taught you before in the extraction of rootes.

*The seconde  
kinde of  
equation.*

The second kinde of equatio<sup>n</sup>, after *Scheubelius* minde and myne also, is, when one simple number *Cosike*, is compared as equalle to. 2. other simple numbers *Cosike*, of seueralle denominations, and like distaunce.

And in soche equation, beyng reduced as is taught before, the roote of those. 2. numbers compounded, as in one (or rather the valewe thereof) shal be extracted: As I haue before taughte also. And that roote doeth aunswere to the question.

*The seconde  
forme of the  
second kinde*

Whobecit, here is the like obseruation, as was in the seconde forme of the firste kinde. For if those. 3. denominations be not immediate, but doe omit some other betwene them, then shall you extracte the roote of that laste number, in all pointes, as you did in the firste equation.

Examples of the firste sorte.

$$4\mathfrak{z} \cdot \text{---} \cdot 6\mathfrak{z} \text{---} \text{---} 4\mathfrak{z}$$

whiche beyng reduced, will bee:

$$1\mathfrak{z} \cdot \text{---} \cdot 1\mathfrak{z} \text{---} \text{---} 1\mathfrak{z} \cdot \text{---} \text{---} 1\mathfrak{z} \cdot \text{---} \text{---} 1\mathfrak{z}$$

$$\text{And. } 6\sqrt[3]{3} \cdot \text{---} \cdot 12\sqrt[3]{3} \text{---} \text{---} 18\mathcal{C}$$

That



## of Coslike numbers.

What is by reduction.

$$1.\text{f}\text{z} \equiv 2.\text{f}\text{z}\text{f}\text{z} \equiv 3.\text{c}.02$$

$$1.\text{f}\text{z} \equiv 2.\text{z}\text{c} \equiv 3.\text{f}. \text{ And the roote. } 3.$$

$$5.\text{f}\text{z} \equiv 25.\text{f}\text{z}\text{f}\text{z} \equiv 30.\text{c}. \text{ And by reduction.}$$

$$1.\text{f}\text{z} \equiv 5.\text{f}\text{z}\text{f}\text{z} \equiv 6.\text{c}. \text{ And}$$

$$1.\text{f}\text{z} \equiv 5.\text{z}\text{c} \equiv 6.\text{f}. \text{ whose roote is. } 3.02.2.$$

$$\text{Like waies. } 2.\text{f}\text{z} \equiv 120.\text{f}. \equiv 8.\text{z}\text{c}$$

$$\text{And by reduction. } 1.\text{f}\text{z} \equiv 60.\text{f} \equiv 4.\text{z}\text{c}. \text{ whose roote is. } 6.$$

Examples of the seconde sort.

$$5.\text{f}\text{z}\text{f}\text{z} \equiv 60.\text{f}\text{z} \equiv 320.\text{f}.$$

What maketh by reduction.

$$1.\text{f}\text{z}\text{f}\text{z} \equiv 12.\text{f}\text{z} \equiv 64.\text{f}.$$

And the square roote. 4.

$$\text{Like waies. } 8.\text{f}\text{z}\text{c} \equiv 40.\text{c} \equiv 30208.\text{f}.$$

And by the orderly reduction.

$$1.\text{f}\text{z}\text{c} \equiv 5.\text{c} \equiv 3776.\text{f}. \text{ whose Cubike roote is. } 4.$$

Again in residuales.

$$8.\text{f}\text{z}\text{c} \equiv 864.\text{f}\text{z} \equiv 24.\text{f}\text{z}\text{f}\text{z}.$$

What maketh by reduction.

$$1.\text{f}\text{z}\text{c} \equiv 108.\text{f}\text{z} \equiv 3.\text{f}\text{z}\text{f}\text{z}. \text{ And els.}$$

$$1.\text{f}\text{z}\text{f}\text{z} \equiv 108.\text{f} \equiv 3.\text{f}. \text{ whose roote is. } 3.$$

$$30.9.\text{f}\text{z} \equiv 90.\text{f}\text{z}\text{f}\text{z} \equiv 144.\text{z}\text{c}.$$

$$\text{And by reduction. } 1.\text{f}\text{z}\text{c} \equiv 10.\text{c} \equiv 16.\text{f}. \text{ whose roote is. } 8.02.2.$$

But now because *Seheubelius* dooth make. 2. severall equations of these. 2. formes : And giueth. 3. diuerse rules, or canons for eche of them, I will declare his. 6. canons to be all contained in this seconde kind of equation.

## The Arte

He maketh his diuision thus. When . 1. number is compared as equale to . 2. other, other that one number is of the smalleste denomination. And then is it of the firste Canon. As.  $1.3 \div 8.ze = 65.9.02$  els that one number, is of the greatest denomination: As.  $3.ze \div 4.9 = 1.3$ . And then is it of the seconde Canon: Or els thirdely, the alone number is of the middle denomination: and then is it of the thirde Canon. As.  $1.3 \div 12.9 = 8.ze$ .

The like forme he useth, for the numbers of denominations distaunte.

Wherby you maie perceiue, that in my rule there is noe forme of numbers, like the of the firste Canon, nother yet of the thirde: but onely of the seconde. But then again in my rule, there are . 2. sortes of examples whiche he hath not. And if you compare them well together, you shall perceiue, that thei bee agreeable together.

As for exâple. In his firste canon, this is the forme  $1.3 \div 6.ze = 27.9$ . whiche equation in my rule, by translation, is expressed thus.

$1.3 = 27.9 \div 6.ze$ . bicause I doe still set the greatest denomination alone.

Again in his thirde Canon, this is an example.

$1.3 \div 13.9 = 8.ze$  and that number doe I translate into this forme  $13.9 = 8.ze \div 15.9$ .

Now where as he giueth generall rules, for euery Canon, I saue for them all: extracte the roote of that composide number. For all his rules doe teache nothing els.

Scholar. I doe vnderstande the diuersitie, and agreemente of your rules and his. But for my exercise, I doe couette some apte questions, appertaining to these equations.

*A question  
of ages.*

Master. Take this for the firste question.

Alexander being asked how olde he was, I am. 2. yeres

## of Coslike numbers.

peres elder (quod he) then Ephestio. Wca. saied Ephestio. And my father was as olde as we bothe, and. 4. peres moare. And my father hauyng all those peres, saied Alexander, was. 96. peres of age. I demaunde now of you, how olde was eche of them.

Scholar. I praie you awiswere the question your self, to teache me the forme.

Master. I will begin with the yongeste mannes age, and that will I call  $x$ . whiche is the common supposition in all soche questions. Then is Alexander doctrs age. 2. peres moare, that is.  $x + 2$ . And those bothe together dooe make.  $2x + 2$ . Whereunto if you put. 4. moare, then haue you the age of Ephestio his father, that will be.  $2x + 6$ . And all these put together, that is.  $4x + 8$ . will make 96 whiche is the equation that shall open the question.

Wherefore I set downe the equation thus.

$4x + 8 = 96$ . And because I see on bothe sides, one denomination of. 4. I doe abate. 8. from bothe sides: & then there remaineth.  $4x = 88$ . And by reduction or diuision,  $x = 22$ .

Scholar. Then maie I easily saie, that Ephestio was. 22. peres olde, scyng you did putte. 1. for his age: and now. 1. is founde to be. 22. And therby all the other peres be manifeste. For Alexander being. 2. peres elder, must be. 24. And Ephestio his father had in age. 22. and. 24. and. 4. moare, that is. 50. peres. All whiche make. 96. So is that question fully answered.

Master. An other question is this. I had a somme of money owing vnto me: whereof I did receiue at one tyme  $\frac{1}{4}$  and after ward I receiued  $\frac{1}{2}$  of that residue, whiche remained unpaid. And so remained the reste of the debte 27. l. I would knowe what was the first debte, & what wer the. 2. severall paymentes

C. ij.

Scholar

## The Arte

Scholar. This muste I obserue still, to name the firste doubtfull thyng.  $1. \frac{2}{3}$ . wherefore I saie that the firste debte was  $1. \frac{2}{3}$ . whercof I receiued  $\frac{1}{3}$ . And so did there remain.  $\frac{1}{3}$ .  $\frac{2}{3}$ . of whiche rest, againe I receiued  $\frac{1}{3}$ . that is  $\frac{2}{9}$ . of the whole somme, or  $\frac{2}{9}$ . And that be- yng abated also, then did there remaine  $\frac{2}{9}$ .  $\frac{2}{9}$ . whiche you named to be.  $27. \text{li}$ . Then if  $\frac{2}{9}$ .  $\frac{2}{9}$ . bee equalle to  $27. \text{li}$ . diuide.  $27. \text{li}$ . by  $\frac{2}{9}$ . and the *quotiente* will bee  $81$ . that is.  $60$ . whiche was the whole debte: And then is it plaine, that  $\frac{1}{3}$ . of it is.  $15$ . and  $\frac{2}{3}$ . of the residue is.  $18$ . whiche maketh.  $33$ . and then remaineth,  $27$ .

Master. There is nothyng better then exercise, in attaynynge any kynde of knowlege: And therfore I will proue you with diuerse questions, to make you the moare experie in this rule. And this is one.

*A question of  
pauynge.*

There is a flooze paved with square bricks, the lengthe of that flooze beynge longer then the breadyth, by  $\frac{1}{7}$ . and the whole pavemente containeth.  $3584$ . bricks: I require to knowe the breadyth and lengthe.

Scholar. The lesser quantitie, whiche is the breadyth I doe name.  $1. \frac{2}{3}$ . And then the lengthe will bee, by your proportion.  $1 \frac{1}{7}$ .  $\frac{2}{3}$ . Now must I multiplie the breadyth by the lengthe (for that is the orderly worke in all flatte formes, to finde out the whole platte) that is here.  $1. \frac{2}{3}$ . by  $1 \frac{1}{7}$ .  $\frac{2}{3}$ . and there will amounte the whole platte.  $\frac{2}{3}$ .  $8$ . whiche by your supposition is equalle to.  $3584$ .

Wherefore accordynge to your rule, I diuide.  $3584$ . by  $\frac{2}{3}$ . and the *quotiente* will be.  $3136$ . whiche is a square number, bicause there is one denomination omitted in this equatio. For betwene  $3$  and  $9$ . there is omitted.  $2$ . And therfore must I extracte the square roote of.  $3136$ . and it will bee the quantitie of.  $1. \frac{2}{3}$ . that I looke in my tables, and finde it.  $56$ . whiche must be the breadyth: for that I named.  $1. \frac{2}{3}$ . Then the length must be moare by  $\frac{1}{7}$ . of it: and so shall it be.  $64$ .

Now

## of Coslike numbers.

Now for to confirme my wooke, I multiplie. 56. by. 64 and it will make. 3584. whiche is the number that you old name.

Master. What question is well aunswered: And if you had put. 1. 20. for the lengthe, as you might do, then the bredthe will be 7. 20. and the square 7. 20. and so. 1. 20. would bee. 64. as you maie proue at letter: but in the meane time, what saie you to this questiō?

*An other woorkes of that questiō*

There is a capitaine, whiche hath a greate armie, & would gladly Marshall the, into a square battaile, as large as mighte bee. Wherefore in his firste prooffe of square foume, he had remainyng. 284. to many. And prouyng again by putteng. 1. moare in the fronte, he founde wante of. 25. men. How many souldiars had he, as you gesse:

*A questiō of an armie.*

Scholar. I call the firste fronte. 1. 20. and then multipliyng it squarely: I shall haue for the whole battaile. 1. 7. and so by your sayng, there was leste 284. men, wherefore the whole number of men, was 1. 7. — — — 284. 9.

Now for the seconde prooffe, when the fronte was increased by. 1. man: I shall set the former fronte, and 1. manne moare, that is 1. 20. — — — 1. 9. And multipliyng that number, squarely: there will arise for the whole armie.

$$\begin{array}{r}
 1. 20. - - - 1. 9. \\
 1. 20. - - - 1. 9. \\
 \hline
 1. 7. - - - 1. 20. \\
 1. 7. - - - 2. 20. - - - 1. 9. \\
 \hline
 1. 7. - - - 2. 20. - - - 1. 9.
 \end{array}$$

out of whiche I muste abate 25 that, you saie, did wante, to make hy that square battaile. And then it will be. 1. 7. — — — 2. 20. — — — 24. 9.

Now haue I one number of menne, expressed by. 2 Coslike numbers: Of necessitie therefore must these. 2. numbers be equalle: seying thei represente one armie.

Wherefore I set them thus.

Eg. iij. 1. 7.

# The Arte

$$1. \text{z} \text{---} | \text{---} 284. \text{q} \text{---} | \text{---} 1. \text{z} \text{---} | \text{---} 2. \text{ze} \text{---} | \text{---} 24. \text{q}.$$

And findyng. 1.  $\text{z}$ . on bothe partes of the equation, I doe abate it, & then standeth.  $284 \text{q} \text{---} | \text{---} 2 \text{ze} \text{---} | \text{---} 24 \text{q}.$  Yet again I see.  $\text{q}.$  on bothe sides of the equation, and therfore, seing the lesser of them hath the signe of subtraction, I doe adde. 24. to bothe numbers, and then will there be.  $308 \text{---} | \text{---} 2. \text{ze}.$  that is.  $154 \text{---} | \text{---} 1. \text{ze}$

So that seing  $\text{ze}$  was set for the first fronte: the same front must be. 154. whose square is. 23716. unto whiche I muste adde the. 284. that did abounde, and then will the whole number be. 24000.

|        |  |
|--------|--|
| 154.   |  |
| 154    |  |
| 616.   |  |
| 770    |  |
| 154    |  |
| 23716. |  |
| 284.   |  |
| 24000. |  |

For farther trialle wherof, I take the seconde fronte to be. 155. that is. 1. moare then the firste: and his square will bee 24025. And so is there, 25. moare then the iuste number of the armie, as the question supposed.

*An other  
woorde of  
that questiō.*

Wasser. That question may be wrought also, by namyng the seconde fronte. 1.  $\text{ze}$ . and then will his square bee. 1.  $\text{z}$ . but seing there wanteth. 25. menne, to make that square battaile, the number shall bee 1.  $\text{z}$ . --- 25.  $\text{q}.$

Then for the firste front, you must set. 1. man lesse, as the question importeth, & that will be.  $1. \text{ze} \text{---} | \text{---} 1. \text{q}$  whose square will be  $1. \text{z} \text{---} | \text{---} 1. \text{q} \text{---} | \text{---} 2. \text{ze}.$

$$1. \text{ze} \text{---} | \text{---} 1. \text{q}.$$

$$1. \text{ze} \text{---} | \text{---} 1. \text{q}.$$

---


$$1. \text{z} \text{---} | \text{---} 1. \text{ze}.$$

$$\text{---} | \text{---} 1. \text{ze} \text{---} | \text{---} 1. \text{q}.$$

---


$$1. \text{z} \text{---} | \text{---} 1. \text{q} \text{---} | \text{---} 2. \text{ze}.$$

unto whiche I must adde the. 284. menne that did abounde, whē that battaile was framed, and then will the

## of Coslike numbers.

the number be.  $1. \text{z}.$  —  $285. \text{q}.$  —  $2. \text{ze}.$  And  
it must bee equalle to.  $1. \text{z}.$  —  $25. \text{q}.$  ~~And he. toze to~~  
reduce that equation, strike 3 adde on bothe sides  $25. \text{q}.$   
& then resteth.  $1. \text{z}.$  equalle to.  $1. \text{z}.$  —  $310. \text{q}.$   $2. \text{ze}.$   
When 3 adde.  $2. \text{ze}.$  because 3 will haue noe ~~in~~  
the equation: and it will be,  
 $1. \text{z}.$  —  $2. \text{ze}.$  —  $1. \text{z}.$  —  $310. \text{q}.$  Thirde 3  
abate.  $1. \text{z}.$  on bothe sides of the equation: and then  
remaineth.  $2. \text{ze}.$  —  $310. \text{q}.$  that is.  $1. \text{ze}.$  —  $155. \text{q}.$   
wherby it appeareth that the seconde fronte was.  $155$   
and the firste fronte.  $154$ . & so forth, as you wrought  
it before.

An other question is this.

There is a kyng with a greate armie: And his ad- *A question*  
uerfarie corrupteth one of his heraultes with gistes, *of an armie.*  
and maketh hym swere, that he will tell hym, how  
many Dukes, Erles and other souldiars there are in  
that armie. The heraulte lothe to leafe those gistes,  
and as lothe to bee vntrue to his Prince, diuiseh his  
aunswere, whiche was true, but yet not so plain, that  
the aduerfarie could therby vnderstande that, whiche  
he desired. And that aunswere was this.

Looke how many Dukes there are, and for eche of  
them, there are twise so many Erles. And vnder eue-  
ry Erle, there are fower tymes so many soldiars, as  
there be Dukes in the fiede. And when the muster of  
the soldiars was taken, the.  $200$ . parte of them, was  
9. tymes so many as the number of the Dukes.

This is a true declaratiō of eche number, quod the  
heraute: and 3 haue discharged my othe. Now gesse  
you how many of eche sorte there was.

Scholar. Although the question seme harde, 3 see  
many tymes, that diligence maketh harde thynges  
easie, and therfore 3 will attempte the worke of it.

And firste for the number of Dukes, 3 sette.  $1. \text{ze}.$   
then will the number of Erles bee.  $2. \text{z}.$  that is.  $1. \text{ze}.$   
by

## The Arte

by 1.  $\mathcal{H}$  multiplied twise: And the number of soldiars are. 8.  $\mathcal{C}$ . that is. 2.  $\mathcal{Z}$ . multiplied by 1.  $\mathcal{H}$ . solver tymes, but bicause the. 200. parte of the soldiars is. 9. tymes so moche as the number of the Dukes, therfoze must the. 200. parte of 8.  $\mathcal{C}$  be equalle to. 9.  $\mathcal{Z}$ . And so consequently. 8.  $\mathcal{C}$   $\div$  9.  $\mathcal{Z}$   $\overline{\hspace{1cm}}$  1800.  $\mathcal{Z}$  and 1.  $\mathcal{C}$   $\div$  225.  $\mathcal{Z}$   $\overline{\hspace{1cm}}$  02. 1.  $\mathcal{Z}$   $\overline{\hspace{1cm}}$  225.  $\mathcal{Z}$ .

For if I set  $\frac{1}{100}$  and. 9. as equalle together, & would by the arte of fractions, bynne the same propoztion in whole numbers, I shall haue for. 9. this fraction  $\frac{1 \text{ foot}}{300}$ . And seying the denominato<sup>r</sup>s, be all one in  $\frac{1}{300}$  and in  $\frac{1}{100}$  the propoztion consisteth betwene the numerato<sup>r</sup>s.

When to procede, if. 225. be equalle to. 1.  $\mathcal{Z}$ . I shall take the square roote of. 225. for. 1.  $\mathcal{Z}$ . and that is. 15 whiche must be the number of Dukes.

And so haue I the firste number, wherefoze the seconde number, that is the number of Erles, must bee 15. tymes. 15. twise: that is. 450. And the number of soldiars shall be. 4. tymes. 15. multiplied by. 450. that is. 27000. And for iuste trialle of this worke, I take the. 200. parte of the soldiars that is. 1350. and I

|      |       |
|------|-------|
| 450. |       |
| 60   |       |
|      | 27000 |

finde it to bee. 9. tymes. 15. that is. 9. tymes so moche as the number of the Dukes. And so is that question solved, and tried.

*An other  
question of  
walles.*

Master. This is an other question. There is a grounde inclosed with. 4. walles, beyng like iambes and of one heigthe. The longest. 2. walles are in propoztion to the shorteste, as. 5. to. 3. And vnto the height thei bee double *Se/qualter*. Now multiplying the longest by the shorteste, and that totalle by the heigthe, there will rise. 39936. foote. I demaunde then, what is the lengthe and the heigthe of eche walle?

Scholar. The least quantitie is the heigthe, whiche I call. 1.  $\mathcal{H}$ . and vnto it the longest walle is double *Se/qualter*:



## of Cosike numbers.

*Sesquialter*: that is.  $2\frac{1}{2}$ .  $\Sigma$ . Now that same longeste is in propozition *Superbipartiente quintas*, to the shortestest walle. So must the shorter wall be  $1\frac{1}{2}$ .  $\Sigma$ . Then must I multiplie all those. 3. nöbers together, that is  $1\frac{1}{2}$ . by.  $1\frac{1}{2}$ . whereof doeth come.  $\frac{9}{4}$ . then shall I multiplie that totalle, by  $\frac{1}{2}$ . and it will be  $\frac{9}{8}$ . or  $3\frac{3}{4}$ . whiche must be equalle, by the woordes of the question, to. 39930.

So by reducyng them to one denomination.  $\frac{9}{8}$ . shall be equalle to  $\frac{15720}{8}$  that is. 15.  $\Sigma$ . = 159720.  $\eta$ . and. 1.  $\Sigma$ . = 10648. wherefore I shall extracte the Cubike roote out of. 10648. and that is the quantitie of.  $1\frac{1}{2}$ . or the heighte of the walle.

In my Tables I woode that extraction of Cubike roote, and finde it to be. 22. And therfore must the longeste walle bee double *Sesquialter* to it, that is. 55. And the shortestest walle will be. 33.

For proöfe whereof I dooe multiplie. 22. with. 55. *The proöfe.* and it maketh. 1210. whiche number I shall multiplie by. 33. and it will be. 39930. according to the supposition of the question.

Master. You doe chose still the leasste number, to be equalle to.  $1\frac{1}{2}$ . as the easiest forme. Howbeit you maie put.  $1\frac{1}{2}$ . for the lengthe of any of the walles.

And if you sette it for the longeste walle, then the shortestest walle will be  $\frac{2}{3}$ . and the heighte  $\frac{2}{3}$ . and all those. 3. numbers will make, by multiplication together.  $\frac{8}{27}$ . equalle to. 39930. And so will. 6.  $\Sigma$ . be equalle to. 998250.  $\eta$ . and. 1.  $\Sigma$ . = 166375.  $\eta$ . whereof the Cubike roote is 55. and answereth to the quantitie of.  $1\frac{1}{2}$ . *An other forme of worke.*

But if.  $1\frac{1}{2}$ . be set for the measure of the shortestest walle, then the longeste walle will bee  $\frac{3}{2}$ . and the heighte  $\frac{3}{2}$ . And so all. 3. numbers multiplied together will make  $\frac{27}{8}$ . = 39930. So shall. 10.  $\Sigma$ . be equall to. 359370. And. 1.  $\Sigma$ . = 35937. where

## The Arte

of the *Cubike* roote is. 33. and is the value of. 1.  $\text{ze}$ . in this position.

Scholar. This varietie of woorkes, is not onely pleasaunte, but it maketh the reason of the woorkes to appeare moare plainly. So that I could neuer be we-  
rie to heare soche questions.

Master. Then will I propounde one or 2. moare befoze we passe from this kinde of equation. Where-  
of one shall be somewhat like that last. And this it is.

*A question  
of Bricke.*

A Brickeleiar had a pile of Bricke, whiche he sold by the yarde. The lengthe of it was  $7\frac{1}{2}$  to the bredthe, that is *Triplafesquialters*. And the heighte was five ty-  
mes so moche as the legthe. This pile the owner sold for. 980. crounes. By soche rate that he had for every yarde so many Crounes, as the Pile had yarden in bredthe. Now is the question, what was the lengthe, bredthe, and heighte of this pile?

Scholar. I suppose the bredthe to bee. 1.  $\text{ze}$ . then was the length  $7\frac{1}{2}\text{ze}$ . and the heighte.  $17\frac{1}{2}\text{ze}$ . These 3. numbers dooe I multiplie together, and thei make  $\frac{245}{4}\text{ze}$ . whiche standeth as equalle to all the yarden in the whole pile. But yet what that is, I knowe not.

Wherefoze to proccede farther, I consider that euery yarde coste as many crounes, as the bredthe containe yarden. Now the bredthe being 1.  $\text{ze}$  I must saie, that euery yarde did coste. 1.  $\text{ze}$ . of crounes. And then by the Golden rule: if. 1. yarde coste. 1.  $\text{ze}$ . of Crounes, what shall  $\frac{245}{4}\text{ze}$ . coste?

$$1. \quad \frac{245}{4}\text{ze} \quad \frac{1. \text{ze}}{\frac{245}{4}\text{ze}} = 1. \text{ze}.$$

Woorkyng by the rule, I finde that it shall cost  $\frac{245}{4}\text{ze}$ . And the question doeth suppose that it coste. 980. crounes. Wherefoze I must saie, that. 980. crounes, are equalle to  $\frac{245}{4}\text{ze}$ . And consequently.  $245. \text{ze} = 3920. \text{ze}$ . wherefoze di-  
uidynge the number of the lesser name, by the other, the *quotiente* will be 16. whose *zenzizenzike* roote is 2

And

## of Coslike numbers.

And that therfoze must be the value of a roote, and the bredthe of the pile. So shall the lengthe be .7. yardes, and the heighte. 35. yardes.

For trialle of it, I mutiplic the lengthe, by the bredthe, and that totalle by the heighte, and so haue I *The prooffe.*  
 490. for all the yardes of Wicke. Then considering that euery yarde coste. 2. crownes, bicause. 2. yardes is the bredthe of the pile: the number of crownes must be twice. 490. that is. 980. And so is the woorkie good.

Maister. Now woorkie that question, by settinge *Another forme of woorkie.*  
 1. 20. for the lengthe.

Scholar. If the lengthe be. 1. 20. the bredthe must bee 3. 20. that is *Subtriplesesquialtera* to. 1. 20. And the heighte must bee. 5. 20. All whiche sommes make by multiplication 1. 20.

Then farther, if 1. yarde coste 3. 20. 1. 20. shall coste 3. 20. 3. 20., whiche is equalle to 980. And so is. 20. 3. 20. equal to. 48020. and by diuision 1. 20. 3. 20. = 2401. whose *zenzenzike* roote is. 7. And that is the lengthe of the walle, and is the value of. 1. 20.

The reste of this woorkie, is like as befoze.

Maister. Yet proue the thirde waie.

Scholar. The heighte beeing. 1. 20. the lengthe must be the first part of it, that is 1. 20. And the bredth 3. 20. All these make by multiplication 1. 20. Then for the price, if. 1. yarde coste 3. 20. what shall 1. 20. 3. 20. coste? By the *A thirde forme of woorkie.*  
 Golden Rule there is founde, 1. 20. 3. 20. 3. 20. 3. 20. whiche is equalle to 980. And so shall. 4. 3. 20. be equalle to. 6002500. And. 1. 3. 20. = 1500625. whose *zenzenzike* roote is. 35. And that is the value of. 1. 20. and the heighte of the pile.

Maister. One question more will I propounde,  
 1. h. v. and

## The Arte

and so eande with this equation.  
*A question of* A pooze man died, whiche had solwer childzen, and  
*a Testament.* all his goodes came to. 72. crounes: whiche he would  
 haue parted so, that the seconde & thirde childe should  
 haue. 7. times so moche as the firste. And that the por-  
 tions of the thirde and fourthe childe should bee. 5. ty-  
 mes so moche as the secondes parte: And that the first  
 and the fourthe, should haue twise as moche as the  
 thirde. If you worke the solution wel, you maie seme  
 worthy to be master of those wardes.

Scholar. I trust to obtaine moare benefite by the  
 question, then by that office. Wherefore I will giue  
 good hede vnto it. And for the firste namber, I set. 1.  $\mathcal{Z}$   
 then muste the seconde and thirde portions make to-  
 gether. 7.  $\mathcal{Z}$ . And the fourthe must bee all the reste of  
 the. 72. that is.  $72 - 8 \mathcal{Z}$ . Now the thirde must  
 be halfe the firste & the fourthe, that is.  $36 - 3 \frac{1}{2} \mathcal{Z}$ .  
 And the thirde & fourthe, is. 5. tymes the second. where-  
 fore the seconde shall be the. 5. part of.  $108 - 11 \frac{1}{2} \mathcal{Z}$   
 that is.  $21 \frac{1}{2} - 2 \frac{1}{2} \mathcal{Z}$ , whiche number I shall set  
 in order with Letters, as here I haue dooen for my  
 owne ease, and aide of me-  
 mozy. And then shal I adde  
 them all together. Where-  
 of there commeth.

|  |  |
|--|--|
| A  | 1. $\mathcal{Z}$ .                               |
| B  | $21 \frac{1}{2} - 2 \frac{1}{2} \mathcal{Z}$ .   |
| C  | $36 - 3 \frac{1}{2} \mathcal{Z}$ .               |
| D  | $27 - 8 \mathcal{Z}$ .                           |
| $129 \frac{1}{2} - 12 \frac{1}{2} \mathcal{Z}$ | $129 \frac{1}{2} - 12 \frac{1}{2} \mathcal{Z}$ . |

whiche  
 is equalle to 72. First ther-  
 fore I do adde all that solo weth — to bothe partes of  
 the equatiō. And so haue I  $129 \frac{1}{2} - 12 \frac{1}{2} \mathcal{Z} - 72$ .  
 But bicause there are numbers Absolute on bothe si-  
 des, I shall abate the lesser somme, that is. 72. from  
 bothe partes, and then will there bee left,  $57 \frac{1}{2} - 12 \frac{1}{2} \mathcal{Z}$ .  
 that is.  $288 - 64 \mathcal{Z}$ . And by diuision  
 $4 \frac{1}{2} = 1 \mathcal{Z}$ .

*The prooffe.*

So shall the firste mannes portion bee  $4 \frac{1}{2}$ . And the  
 seconde and thirde mannes portion. 7. times so moche  
 that

## of Coſlike numbers.

that is.  $31\frac{1}{2}$ . Whereby it follo weth,  
that the fourthe manne, ſhall haue  
the reſte of 72. that is. 36.

Then ſeeyng the thirde manne,  
hath halfe ſo moche as the firſt and  
the fourthe, his portiō ſhall be  $20\frac{1}{2}$ .  
And then by diuerſe reaſons, the ſeconde mānes part  
ſhall bee.  $11\frac{1}{2}$ . And all theſe partes added together, doe  
make iuſte 72. Wherefoze the wooke is good.

Maſter. You haue wroughte it well. And yet *Another*  
maie you wooke it thus. Firſte ſette downe.  $1\frac{1}{2}$ . *for forme of*  
the firſte mannes parte. And then for the ſeconde and *wooke.*  
thirde ioynly.  $7\frac{1}{2}$ . ſo ſhall the fourthe manne haue  
 $72\frac{1}{2}$ . —  $8\frac{1}{2}$ . And becauſe the ſeconde mannes  
parte is  $\frac{1}{2}$ . of the thirde and fourthe mannes portion,  
if you ioync all their. 3. partes together, the ſeconde  
mannes portion will be  $\frac{1}{2}$  of that totalle. But therfoze  
 $7\frac{1}{2}$ , whiche is the partes of the ſecond and the thirde  
unto.  $72$  —  $8\frac{1}{2}$ , whiche is the fourthe mannes  
parte, and the totalle will be.  $72\frac{1}{2}$  —  $1\frac{1}{2}$ . whoſe  
firſte parte is  $12\frac{1}{2}$ . —  $\frac{1}{2}$ , for the ſeconde man-  
nes ſhare. Whiche ſomme if you abate out of.  $7\frac{1}{2}$ .  
there wil remain for the thirde  
mannes parte  $7\frac{1}{2}$  —  $12\frac{1}{2}$ .

And ſo haue you euey man-  
nes portiō allotted to hym due-  
ly. As I haue here ſet it for the  
for you. And all thei added to-  
gether, doe make. 72.

Scholar. But here is noe equatiō yet, though the  
partes be diuided iuſtly.

Maſter. Now ſhall you ſee it.

The queſtion ſaileth, that the thirde mannes port-  
ion is halfe the portions of the firſte and fourthe man-  
wherefoze ſeeyng the firſte and fourthe mannes port-  
ions doe make.  $72$  —  $7\frac{1}{2}$ . the thirde mannes por-  
tion

|   |                   |
|---|-------------------|
| A | $4\frac{1}{2}$ .  |
| B | $11\frac{1}{2}$ } |
| C | $20\frac{1}{2}$ } |
| D | $36$ .            |
|   | <hr/>             |
|   | $72$ .            |

|   |                                   |
|---|-----------------------------------|
| A | $1\frac{1}{2}$ .                  |
| B | $12\frac{1}{2}$ . — $\frac{1}{2}$ |
| C | $7\frac{1}{2}$ — $12\frac{1}{2}$  |
| D | $72$ . — $8\frac{1}{2}$ .         |
|   | <hr/>                             |
|   | $72$ .                            |

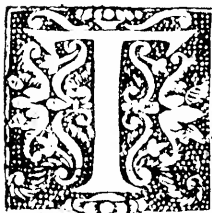
## The Arte

tion beeing doubled, shall make as moche. But the double of the thirde manes parte, is  $14\frac{1}{2}z$  —  $24s$ . and therfore I saie, that those .2. numbers be equalle; that is,  $72s$ . —  $7z$ . —  $14\frac{1}{2}z$ . —  $24s$ . Firste adde.  $7z$ . to eche parte, and it will bee  $72s$ . —  $21\frac{1}{2}z$ . —  $24s$ . Then adde.  $24s$ . on bothe sides, and there will be.  $96s$ . —  $21\frac{1}{2}z$ . that is by reduction.  $288$ . —  $64z$ . as you made it. And then all agreeth.

Like wates for the equation, you maie set the thirde mannes portion, with the halfe of the firste & fourth mannes partes. And so will.  $7\frac{1}{2}z$ . —  $12s$ . be equalle to.  $36s$ . —  $3\frac{1}{2}z$ . And by reduction,  $10\frac{1}{2}z$ . —  $48s$ . That is in other termes of whole number.  $32z$ . —  $144$ . And by diuision it will bee  $1z$ . —  $4$ . And thus will we eande the examles of the firste equation, for this tyme. And will shewe you some questions of the seconde equation.

### Examples of the seconde equation, by questions propounded.

*A question  
of silkes.*



There are two men that haue silke to sell. The one hath.  $40$ . elnes, and the other.  $90$ . And the firste man his silke is not so fine as the seconde man his silke. So that he seileth in euery angell, price more by  $\frac{1}{3}$  of an elne, then the seconde ma doeth. And at the eande, bothe their monies made but  $42$ . angelles. Now I demaunde of you, how moche eche man solde for an angell?

Scholar. I will sololue my olde forme, in putting  $1z$ . for the leasse quantitie, whiche is the seconde mannes somme, and then shall the firste mannes somme be.  $1\frac{1}{3}z$ .

Paster. You are deersued all readie. For you set  $1z$ .

## of Costlike numbers.

1.  $\text{℥}$ . for an elne. Seyng you name  $\frac{1}{2}$  of an elne, to be  $\frac{1}{2}$ .  $\text{℥}$ . And so were the position neadelesse, and likewise all the woorkes.

Scholar. I see my fault: but I knowe not how to amende it. For that 1.  $\text{℥}$ . maie bee a parte or partes of an elne: and so maie it be moare then 1. or 2. elnes so that I ought not to haue set  $\frac{1}{2}$  (whiche is certainly referred, in this question, to an elne) as the parte of a doubtful quantitie, but rather as the parte of a quantitie certaine. Whereas 1.  $\text{℥}$ . is euer put for a number unknowen.

Master. To helpe you herein, I will set the firste numbers, as you began them. The seconde man his numbers of elnes, shall bee 1.  $\text{℥}$ . as you did name it, and then shall the firste mannes portion be as moche, and  $\frac{1}{2}$  of an elne moare: whiche I maie beste call  $\frac{1}{2}$ .  $\text{℥}$ . And so shall it bee disfaunte from 1.  $\text{℥}$ . clerely in all woorkes *Arithmetically*.

But now to procede, I shall diuide eche mannes number of elnes, whiche he had, by the number of elnes, whiche he solde for an angelle, and the *quotiente* will declare how many angelles eche man had receiued. So that the firste mannes number of elnes, being 40. shall bee the numerator, and the somme of measure, whiche he solde for an Angelle, that is 1  $\text{℥}$   $\frac{1}{2}$  shall bee the denominator. And so is the diuision ended. And that fraction is the *quotiente*.

Scholar. Now I perceiue the woorkes. And by like reason: the seconde mannes somme of elnes being 90. shall bee the numerator, and 1.  $\text{℥}$ . being the somme of measure, solde for one Angelle, shall be the denominator, that is in one fraction: accordingly as I haue sette bothe numbers

$$\begin{array}{r} 1. \text{℥} \text{ --- } \frac{1}{2} \text{℥} \\ 13 \end{array}$$

$$\begin{array}{r} 40. \\ 1. \text{℥} \text{ --- } \frac{1}{2} \text{℥} \\ \hline 90. \\ \hline 1. \text{℥} \end{array}$$

here

# The Arte

here distantly.

Master. It were moare ease for you in woorkyng, if you did tourne that fractiō of  $\frac{1}{3}$  into an intere vnitis.

Scholar. That wil easily be doon, by multipling every number, of that whole fraction by. 3. And then will it be  $\frac{110}{300} = \frac{11}{30}$ , whiche is all one in valne with

40. And this I consider farther, that as  $1\frac{2}{3} = 1\frac{1}{3}$  these. 2. fractions, severally dooe expresse the sommes of angelles, that eche of them receyved, so toynaly bothe together, dooe declare the full somme, of all their angelles. Wherefore I shall adde them bothe together. And they will make.

$\frac{130}{35} = \frac{26}{7}$  As here in woork I haue expessed.

$$\begin{array}{r}
 390\frac{2}{3} = 1\frac{1}{3} \\
 \hline
 120. \qquad 90. \\
 3\frac{2}{3} = 1\frac{1}{3} \qquad 1\frac{2}{3} \\
 \hline
 3\frac{2}{3} = 1\frac{2}{3}
 \end{array}$$

And by your supposition, their bothe sommes of Angelles made. 42. So that those. 2. sommes are equalle: and therefore am I come to an equation. In whiche I see a number absolute, equalle to a fraction *Cosike* compounde.

Master. When so euer that, or the like dooeth chaunce, you shall reduce the whole nōber, to the like denomination: and then their numerators will bee equalle.

Scholar. When shall I multiplie. 42. by the denominator 3  $\frac{2}{3} = 1\frac{2}{3}$  & it wil be  $126\frac{2}{3} = 42\frac{2}{3}$  whiche muste bee equalle to.  $390\frac{2}{3} = 1\frac{1}{3}$ . That is in lesser termes.

$21\frac{2}{3} = 7\frac{2}{3} = 65\frac{2}{3} = 15\frac{2}{3}$ .  
 Where firste I dooe abate.  $7\frac{2}{3}$ . on bothe sides: and there remaineth then.  $21\frac{2}{3} = 58\frac{2}{3} = 15\frac{2}{3}$ .  
 But



## of Cossike numbers.

But now I remeber your admonitic, that because the number annexed to the greatestte signe, is moare then. 1. I shall diuide all the numbers by it, and sette their *quotientes* in their stede, with their signes. And so will the number of the greatestte signe, euermoze be 1. And this equation will be  $1. \text{z.} = \frac{1}{2} \text{z.} - \frac{1}{2} \text{z.}$ . Where I must extracte the square root of the latter part, according to your doctrine, and it will be. 3. As it appereth in this worke following, whiche I did frame in my tables.

$\frac{1}{2}$ . in square doeth make  $\frac{1}{4}$ , vnto whiche I muste adde  $\frac{1}{4}$ , whiche is all one with  $\frac{1}{2}$ , by reduction to one denominatio. So is the full additio  $\frac{1}{2}$ . whose square root is  $\frac{1}{2}$ . vnto whiche I shall adde  $\frac{1}{2}$ , and it will bee  $\frac{3}{2}$ , that is. 3.

Master. This is well doen. Now worke the same questiō, as it was proponed, and you shall easily finde all the other numbers to bee true, and agreeable to the questiō.

Scholar. Seyng the seconde manne solde. 3. elnes *The prooffe.* for an angell, the firste manne did sell. 3. elnes and  $\frac{1}{2}$ . So. 4  $\frac{1}{2}$  whiche is the somme of elnes of the first man his silke) diuided by.  $3 \frac{1}{2}$ . doeth yelde. 12. and sheweth how many angelles that man receiued.

Again for the seconde man, whiche had. 90. elnes, diuide that. 90. by. 3. and so shall you finde. 30. for the number of his Angelles. And that. 30. and. 12. dooe make. 42, it needeth not to be proued.

Master. Now againe for your exercise, suppose the firste mannes somme to be. 1.  $\text{z.}$  *An other forme of worke.*

Scholar. Then muste the seconde manne sell for an angelle.  $1. \text{z.} = \frac{1}{2} \text{z.}$ . And their numbers of elnes, diuided by those numbers will make  $\frac{1}{2} \text{z.}$ . and  $\frac{1}{2} \text{z.}$  whiche bothe added together, will bee  $\frac{1}{2} \text{z.}$  equalle to. 42.  $\frac{1}{2} \text{z.}$ . That is by reduction.  $390. \text{z.} = 40 \text{z.} = 126. \text{z.} = 42. \text{z.}$ .  
31.1. And

## The Arte

And by addition of. 42.  $\mathcal{Z}$ . on bothe partes.

432.  $\mathcal{Z}$  ——— 40.  $\mathcal{P}$ . =——. 126.  $\mathcal{Z}$ . And by diuision it will be.  $\frac{24}{7} \mathcal{Z}$  ———  $\frac{10}{63} \mathcal{P}$ . =——. 1.  $\mathcal{Z}$ .

So that now I must extracte the roote of that compounde *Cosike* fraction, thus.  $\frac{12}{7}$  squarely, dooe make  $\frac{144}{49}$  out of whiche I shall abate  $\frac{10}{63}$ . And therfore, firste of all I doe reduce the to one denomination, & theri make  $\frac{9072}{3087}$ . and  $\frac{980}{3087}$ . wherefoze if I abate the lesser out of the greater: there will remaine  $\frac{8092}{3087}$ . that is in lesser termes  $\frac{1616}{617}$  and is a square number, whose roote is.  $\frac{11}{11} \text{ bn}$  to whiche if I adde  $\frac{12}{7}$  that is  $\frac{36}{21}$ . it will make  $\frac{70}{21}$ . or  $2 \frac{10}{3}$ . that is the vale we of. 1.  $\mathcal{Z}$ . And is the firste mannes number of elnes, agreably as I tried it befoze. And so doe bothe woorkes agree.

But now cometh to my remembraunce, that this number, whose roote I did extract, in this last woрке is of that sorte, where.  $\mathcal{Z}$ . ———  $\mathcal{P}$ . is equalle to.  $\mathcal{Z}$ . And therfore hath in it. 2. rootes: thone by addition, as this, whiche I now founde: And the other by subtraction, whiche in this example, by abatynge  $\frac{11}{21}$  out of  $\frac{70}{21}$ , will bee  $\frac{59}{21}$ . But how I maie frame that roote, to agree to this question, I doe not see.

Master. That varietie of rootes dooeth declare, that one equation in number, maie serue for. 2. seueralle questions. But the forme of the question, maie easily instruct you, whiche of those. 2. rootes, you shall take for your purpose. Whobett sometymes you shall take bothe. As for example again, marke this question.

*A question  
of money.*

A gentelman, willing to proue the cunnynge, of a bragging *Aritmetician*, saied thus: I haue in bothe my handes. 8. crownes: But and if I accompte the somme of eche hande by it self seuerally, and put ther to the squares and the *Cubes* of the bothe, it will make in number. 194. Now tell me (quod he) what is in eche hande: and I will giue you all for your labour.

Scholar.

## of Coſike numbers.

Scholar. Soche incoragements, would make me studie harde, and trauell bety willyngly in learned exercises: though learning bee moſte to be loued, for knowledges sake. But for to finde the true aunſwere thus I doe procede.

Fiſt I ſuppoſe the one number in one hand, to be 1.  $\mathcal{Z}$ . And then muſt the other nedes be 8.  $\mathcal{Q}$ . — 1.  $\mathcal{Z}$ . Then doe I make theim bothe Squares. And for the fiſt I haue. 1.  $\mathcal{Z}$ . and for the ſeconde. 1.  $\mathcal{Z}$ . — 64  $\mathcal{Q}$  — 16.  $\mathcal{Z}$ . Thirde I multiplie theim bothe Cubi- kely: and ſo haue I for the fiſt. 1.  $\mathcal{C}$ . and for the other 24.  $\mathcal{Z}$ . — 512.  $\mathcal{Q}$ . — 1.  $\mathcal{C}$ . — 192.  $\mathcal{Z}$ . Then muſt I adde bothe the nōbers, with their Squares, and their Cubes, into one ſomme. As here in worke

$$\begin{array}{r}
 1. \mathcal{Z}. \text{---} | \text{---} . 1. \mathcal{Z}. \text{---} | \text{---} . 1. \mathcal{C}. \\
 \phantom{1. \mathcal{Z}. \text{---} | \text{---} . } 8. \mathcal{Q}. \text{---} | \text{---} . 1. \mathcal{Z}. \\
 1. \mathcal{Z}. \text{---} | \text{---} . 64. \mathcal{Q}. \text{---} | \text{---} . 16. \mathcal{Z}. \\
 24. \mathcal{Z}. \text{---} | \text{---} . 512. \mathcal{Q}. \text{---} | \text{---} . 1. \mathcal{C}. \text{---} | \text{---} . 192. \mathcal{Z}. \\
 \hline
 26. \mathcal{Z}. \text{---} | \text{---} . 584. \mathcal{Q}. \text{---} | \text{---} . 208. \mathcal{Z}.
 \end{array}$$

It is ſet forth. Where for eaſe I haue ſet. 1.  $\mathcal{Z}$ . , 1.  $\mathcal{Z}$ . and. 1.  $\mathcal{C}$ . (whiche is the Roote, the Square and the Cube of one number) all in one line: and the other Roote, Square, and Cube, I haue ſet ſeuerally. And ſo all thei doe make. 26  $\mathcal{Z}$ . — 584  $\mathcal{Q}$ . — 208  $\mathcal{Z}$ . whiche is equalle to. 194. by the intente of the queſtion. Wherefore I adde fiſt. 208.  $\mathcal{Z}$ . to bothe partes, and there remaineth.

26.  $\mathcal{Z}$ . — 584.  $\mathcal{Q}$ . — 208.  $\mathcal{Z}$ . — 194.  $\mathcal{Q}$ . Then I abate. 194. from bothe ſides, and ſo reſteth the equatiō thus. 26.  $\mathcal{Z}$ . — 390.  $\mathcal{Q}$ . — 208.  $\mathcal{Z}$ . That is by diuiſion. 1.  $\mathcal{Z}$ . — 15.  $\mathcal{Q}$ . — 8.  $\mathcal{Z}$ . And by tranſlation of. 15.  $\mathcal{Q}$ . to ſette. 1.  $\mathcal{Z}$ . alone, it wil be. 1.  $\mathcal{Z}$ . — 8  $\mathcal{Z}$ . — 15.  $\mathcal{Q}$ . And now haue I the exacte and complete equation, where I muſt ſeke for

31. y. the

## The Arte

the value of. 1.  $\mathcal{Z}$ . by extractyng the roote. Therefore firste  $\mathcal{I}$  take halfe of. 8. and that is. 4. whose square is. 16. out of whiche  $\mathcal{I}$  abate. 15. and the remainer is 1. whiche  $\mathcal{I}$  maie either adde to. 4. and so haue  $\mathcal{I}$ . 5. or ther,  $\mathcal{I}$  maie abate it from 4. and so haue  $\mathcal{I}$ . 3. Whiche numbers also according to thesame rule, beyng added together dooe make. 8. that is the number of the indell denomination. And beyng multiplied together, thei dooe make. 15. that is the other parte of thesame compounde *Cosike* number.

Master. And if you had marked that firste, you might easly haue found bothe those numbers, by the partes of. 15. whiche can be none other, but. 5. and 3.

And farther, seying thei. 2. doo make. 8. and. 8. is the number (named in the questio) that thei should make, therfoze you shall take them bothe. And name whiche of them you liste to be. 1.  $\mathcal{Z}$ . And the other shall be of necessitie, the reste of. 8.

*The prooffe.*

Scholar. To examine theim, by the order of the question,  $\mathcal{I}$  doe pproceede thus. 3. with his Square. 9. and his *Cube*, 27. dooeth make. 39. And. 5. with his square 25 and his *Cube*. 125. doo yelde 155. And bothe thei together doe byng fo; the. 194. according to the sayng of the question: and therfoze it is certain, that the woork is good.

*An other  
woorke for  
equations.*

Master. Befoze you passe any farther,  $\mathcal{I}$  will admonishe you of one wate, whiche  $\mathcal{I}$  ofte vse in reduction of soche equations, as this is, when there is noe denomination on the one side, but the like is on the other side, with a greater number annexed to it. Then maie you abate all the lesser nōbers, out of their greater, and the reste shall bee accounted equalle to nothing. Whiche chaunce can neuer happen: excepte there bee some numbers on the greater side, with the signe of abatmente. ———.

As here you had.

## of Cossike numbers.

263. — + — 584.9. — — — 208ze. — — — 194.9.  
 Because on the one side, there is noe nōber but 194.9  
 and on the other side, there is. 584.9. beeyng a greater  
 number, and of the same denomination: therefore  
 maie you abate. 194. from bothe sides, and then re-  
 maineth. 263. — + — 390.9. — — — 208ze. — — — 0  
 Wherefore you maie well consider, that the numbers  
 whiche be ioined with — + —. are equalle to the num-  
 bers that bee set with — — —. And therefore the one a-  
 batyng the other iustly, dooe remaine together as e-  
 qualle to nothyng.

Wherefore it is reasonable, that seeyng the num-  
 bers with — — — bee equalle to the numbers with  
 — + — that I maie translate the numbers with — — —  
 from that side of the equation, and set them on the co-  
 trary side, with the signe of — + —. And so in this exā-  
 ple it will bee. 263. — + —. 390.9. — — — 208ze.  
 And this forme shall ease you moche, in reducyng of  
 equations.

Scholar. I thanke you moche. And I will not for-  
 get to vse it, as occasiō shall happen. But I praye you  
 propounde yet some moare questions, that I maie see  
 their diuerse variettes.

Master. There were twoo seueralle men, which *A question*  
 had certaine sommes of angelles, in soche rate, that *of money.*  
 the seconde manne his somme, was triple sesquiquarta  
 to the firste: and if their. 2. sommes were multiplied  
 together, and to that totall the 2 firste sommes added,  
 there would amounte.  $142\frac{1}{2}$ . I demaunde of you,  
 what was eche of their sommes in angelles?

Scholar. The firste mannes somme I call. 1. ze.  
 And the seconde mannes some shall be.  $3\frac{1}{3}$  ze. which  
 2. sommes beeyng multiplied together, dooe make  
 $3\frac{1}{3}$  ze. vnto whiche I must adde bothe the firste nom-  
 bers, that is  $4\frac{1}{3}$  ze. And it will be  $3\frac{1}{3}$  ze. — + —  $4\frac{1}{3}$  ze  
 equalle to.  $142\frac{1}{2}$ . All whiche numbers, I shal bring  
 into

## The Arte

into whole numbers, if I multiplie theſe by. 4. And ſo will it be.  $13\frac{8}{9}$ . —  $17\frac{2}{3}$  —  $570$ . And by reducyng the greateſte denomination *Coſike*, to an vñitie.  $1\frac{8}{9}$ . —  $17\frac{2}{3}$  —  $43\frac{11}{13}$ . And laſte of all, by tranſlatyng the number of.  $2\frac{2}{3}$ . to ſet.  $1\frac{8}{9}$ . alone, on one ſide of the equation, it will be.  $1\frac{8}{9}$  —  $43\frac{11}{13}$   $\frac{9}{9}$  —  $17\frac{2}{3}$ . where I muſt extract the value of the roote thus.  $\frac{17}{13}$ . ſquarely dooe make  $\frac{289}{169}$ . vnto whiche I ſhall adde the.  $43\frac{11}{13}$  (it beeyng firſte multiplied by. 52. to byyng it to the denomination of. 676. And ſo making  $\frac{29640}{676}$ ) And it will be  $\frac{29929}{676}$  whiche is a ſquare number (as I haue proued in my Tables) and his roote is  $\frac{173}{26}$ . from whiche roote I muſt abate  $\frac{17}{13}$ , and then will there remain  $\frac{156}{26}$ , that is. 6.

And that. 6. is the value of.  $1\frac{2}{3}$ , and ſtandeth for the firſte mannes number. So that the ſeconde mannes nōber muſt be as.  $\frac{1}{2}$  to it; that is *triplex ſequiquarto*. And ſo ſhall it be.  $19\frac{1}{2}$ .

**The prooffe.** Maſter. Now proue thoſe numbers, according to the queſtion.

Scholar.  $19\frac{1}{2}$  multiplied by. 6. doeth make. 117. vnto whiche I ſhall adde.  $25\frac{1}{2}$ . amountyng of their. 2 additiōs, and all will be.  $142\frac{1}{2}$ , according to the purpoſe of the queſtion.

**An other** Maſter. So is your woork good. Yet woork it again, by chaungyng the poſition.

**ſame queſtiō.** Scholar. I maie put.  $1\frac{2}{3}$ . to betoken the ſeconde manne his ſomme. And then ſhall the firſte mannes ſomme bee  $\frac{4}{3}$ .  $\frac{2}{3}$ . whiche bothe multiplied together doe make  $\frac{8}{9}$ . And then addyng the. 2. firſte ſommes to it, it will bee  $\frac{4}{3}$   $\frac{8}{9}$  —  $1\frac{4}{3}$   $\frac{2}{3}$ . And that is equalle to.  $142\frac{1}{2}$ . All whiche numbers will bee reduced to whole numbers, by multiplication conueniente. And ſo will it be.  $8\frac{8}{9}$  —  $34\frac{2}{3}$ . equalle to. 3705; that is by reduction,  $1\frac{8}{9}$ . —  $4\frac{1}{2}$   $\frac{2}{3}$  —  $463\frac{1}{3}$ . and by tranſlation of the termes.

## of Coslike numbers.

18. ——— 463  $\frac{1}{2}$ . 9. ——— 4  $\frac{1}{2}$ . 2. out of whiche number I shall extract the value of the roote, in this sorte.

Firste I saie  $\frac{1}{2}$  multiplied Square, doeth make  $\frac{1}{2}$ , vnto whiche number I must adde. 463  $\frac{1}{2}$ , reduced as it ought, and it will bee in all  $\frac{927}{2}$ . whiche is a square number, and hath for his roote  $\frac{30}{2}$ . from whiche I must abate  $\frac{1}{2}$ . And then will there remain  $\frac{29}{2}$ , that is 19  $\frac{1}{2}$ , for the value of. 1. 2. And so consequently for the second mannes nōber: whiche was named in this position, 1. 2. And this maie bee proued as the other was.

Master. What saie you then to this question? *A question of Iorneyng.*  
There is a straunge Iorneye appointed to a manne. The firste daie he must goe 1  $\frac{1}{2}$  mile, and euery daie after the firste, he must still augmente his Iorney, by  $\frac{1}{2}$  of a mile. So that his Iorney shall procede by an *Arithmeticalle* progression. And he hath to trauell for his whole Iorney. 2955. miles. I demaunde in what nōber of daies, shall he cande his Iorney?

Scholar. I knowe not how to procede in this question.

Master. Doe you not heare me name it, an *Arithmeticalle* progression? Whether by you might be adured, that it doeth appertaine to that rule. And accorpyng to the canons of that rule, must you wooke this question. But for your better instruction, I will helpe you in this wooke.

Firste aunswere to the question, by the common position: and saie that the tyme of his Iorney is. 1. 2. of daies. And then shall all the *excesses* (whiche maie also be called the *number of the spaces*) be. 1. 2. ——— 19 The *common excessse* was supposed to bee.  $\frac{1}{2}$  of a mile. And therefore the *somme of all the excesses* muste bee  $\frac{19 \times 1}{2}$  that is to saie, the number of all the *excesses* multiplied by  $\frac{1}{2}$ , that is here, the sixte parte of the number

## The Arte

number of the excesses.

And because that the firste number is  $1\frac{1}{2}$ . I must adde it vnto the somme of the excesses, and so haue I the laste number of that progression. Wherefore adding.  $1\frac{1}{2}$ . (whiche is  $\frac{3}{2}$ , or in like denomination with the other,  $\frac{2}{2}$ ) with  $\frac{12}{6}$  it will make  $\frac{14}{6}$ . And that is the laste number of the progression.

Now you remember, that in progression Arithmetical, if you adde the firste number to the laste: and multiply that totalle, by the number of halfe the places, there doeth amounte the somme totalle of that progression.

And therfore in this exāple, if you adde.  $1\frac{1}{2}$  (whiche is the firste nōber in the progression) vnto  $\frac{14}{6}$  (that is the laste number of the progression) there wil amounte  $\frac{17}{6}$ , whiche being multiplied by the number of halfe the places, that is  $\frac{1}{2} \times 2$ . (For all the number of places is . 1 . 2 ) there will rise,  $\frac{17}{12}$ , whiche is the totalle somme of all the miles: and therfore is equalle to. 2955.

Scholar. All the reste, and this againe can I dooe now. Saying  $\frac{18}{12}$  is equalle to. 2955. I will firste bring the whole number to the like denomination, with the fraction, and it will bee.  $\frac{35400}{12}$ . And then omitting the like denominations.  $1.3$  —  $17.2$ . =  $35460.9$ . That is by translation  $1.3$  =  $35460.9$ . —  $17.2$ . whose roote in value I shall finde out thus. I multiply  $\frac{17}{2}$  squarely, and it will be  $289$ . vnto whiche I shall adde.  $35460.9$  & it will make  $\frac{141129}{4}$ , whiche is a square number, and hath for his roote  $\frac{377}{2}$ , from whiche I shall abate  $\frac{17}{2}$ , and then remaineth  $\frac{360}{2}$ , that is. 180. whiche is the value of. 120. And expresth the number of dayes, whiche the question requireth.

*The prooffe.*

Master. The prooffe in this, and the like questions, is, to set forth the progression with all his termes.



## *of Cossike numbers.*

mes. Excepte you will for shortnesse, sette downe the firste terme, whiche in this example is.  $1\frac{1}{2}$ : and then by the number of the *excesses*, or distaunces (whiche is euer one lesse then the nuber of places) multiplie the quantitie of one *excesse*: and put to it the firste terme: and so haue you the laste terme. When hauyng the firste terme and the laste, with the number of *excesses* you knowe how to finde the totalle.

As in this example, the number of *excesses* beeyng 179. And the quantitie of one *excesse* beeyng  $\frac{1}{2}$ . their multiplication giueth  $89\frac{1}{2}$ . vnto whiche if you adde the firste number, that is  $1\frac{1}{2}$ , it will be  $91$ . And that is the laste number of that progression. Then to trie the totalle somme of the miles, adde the firste number.  $1\frac{1}{2}$  to the laste, and thei will make  $91\frac{1}{2}$ , that you shall multiplie by halfe the number of the places, whiche in our example are. 90 (with the whole number is. 180) and there will amounte. 2955. accorbyng as the question saith.

Scholar. This is sufficient for this question. And at some idle time, I will not sticke to trie it out, by setting the progression sooth the at large. In the meane tyme I praie you for better exercise, giue me some moare questions.

Master. There is a number, whiche I haue for: *Another question*  
gotten: and it is diuided into. 2. partes, whereof the one I haue forgotten also, but the other was. 4. And yet this I remember, that if the parte, whiche I haue forgotten, be multiplid by itself, and then also with 4. those. 2. sommes will make. 117. Now would I knowe what was the whole number, and also what is the parte, whiche I haue forgotten.

Scholar. I suppose the whole number to be.  $1\frac{1}{2}$ . And bicause. 4. is his one parte, the other parte must needs bee.  $1\frac{1}{2}$ . ——— 4. Then doe I accorbyng to the question, multiplie.  $1\frac{1}{2}$ . ——— 4. firste by it self,

lik.), and

## The Arte

and it will make.  $1\ 3\ \text{---}\ 1\ 6\ 9\ \text{---}\ 8\ 2\ 0$ . Secondly, I doe multiplie it, that is.  $1\ 2\ 0\ \text{---}\ 4\ 8\ 0$ . And it giueth.  $4\ 2\ 0\ \text{---}\ 16$ .

Then adde I bothe those numbers together, and it will be.  $1\ 3\ \text{---}\ 4\ 2\ 0$ . whiche by the question shall be equalle to.  $1\ 17$ .

$$1\ 3\ \text{---}\ 1\ 6\ 9\ \text{---}\ 8\ 2\ 0$$

$$4\ 2\ 0\ \text{---}\ 16\ 9$$

---


$$1\ 3\ \text{---}\ 4\ 2\ 0$$

But then must I vse the accustomed translation, to bring the greatestte quantitie in denomination, to stande alone. And so will it bee.

$$1\ 3\ \text{---}\ 4\ 2\ 0\ \text{---}\ 1\ 17\ 9$$

Where I must sicke for the value of a roote. And therfore I multiplie. 2. by it self squarely, and so haue  $4$ . vnto whiche I adde.  $1\ 17$ . and it maketh.  $121$ . whose roote is.  $11$ . vnto whiche I muste adde. 2. and there commeth.  $13$ . as the value of.  $120$  and the quantitie of the whole number.

*The prooffe.*

For prooffe of this worke, I abate. 4. out of.  $13$ . and there resteth. 9. as the other parte. Then doe I multiplie. 9. by it self, and therof riseth.  $81$ . Also I doe multiplie. 9. by. 4. and it maketh.  $36$ . whiche bothe together, doe make.  $117$ . as the question would.

*Another worke.*

Passer. Set.  $120$ . for the unknowen parte, and then worke it, to see the diuersitie of the workes.

Scholar. If. 4. bee one parte, and.  $120$ , the other parte, then will the whole number be.  $120\ \text{---}\ 480$ . Wherefore firste I multiplie.  $120$ . by it self, and it yeldeth.  $14400$ . Then dooe I multiplie.  $120$ . by. 4. and it giueth.  $480$ . whiche bothe sommes together, dooe make.  $14400\ \text{---}\ 480$ . whiche is equalle to.  $117$ .

And by translatiō.  $1\ 3\ \text{---}\ 1\ 17\ 9\ \text{---}\ 4\ 2\ 0$ .

Wherefore I doe multiplie. 2. squarely, and it giueth

## of Coslike numbers.

ueth. 4, whiche added to. 117. maketh. 121. and the roote of that is. 11. from whiche 3 shall abate. 2. and there will rise. 9. as the other parte of the number. This is verie plain, & the pfoe of it as it was before.

Master. Then aunswere to this question.

There are 3 numbers in proportion *Geometricall*. And *A question of proporti*  
one of the extremes is.  $20\frac{1}{4}$ . the other extreme, with the double of the middell terme, doeth make 22. Now would I knowe of you, what those. 2. numbers bee?

Scholar. For trialle, I name the other extreme, 1. 2. And because it, with the double of the middle terme dooeth make. 22. the middell terme shall bee 11. ———.  $\frac{1}{4}$ . 2. for his double is. 22. ——— 12. whiche with. 1. 2. doeth make. 22.

Then to procede, I knowe the propertie of those numbers in proportion *Geometricall* to bee soche, that the multiplication of bothe the extremes is equalle to the square of the middell terme, wherefore I multiplie the. 2. extremes together, and there will rise.  $\frac{1}{4}$  2. Then dooe I multiplie. 11 ———  $\frac{1}{4}$  2. by it self in square, and it will bee. 121. 9. ———  $\frac{1}{4}$  8. ———. 11 2, whiche must bee equalle to  $\frac{1}{4}$  2. or.  $20\frac{1}{4}$  2. When to reduce it, I adde. 11. 2. on bothe sides, and it will be.  $31\frac{1}{4}$  2. ———  $\frac{1}{4}$  8. ———. 121 9. and by translation.  $\frac{1}{4}$  8. ———  $31\frac{1}{4}$  2. ———. 121 9. That is 1. 8. ——— 125. 2. ——— 484. 9.

Now resteth nothing, but to seache the value of 1. 2. Wherefore I take  $\frac{1}{4}$  2, and multiplie it square, and so haue  $\frac{1}{4}$  2. from whiche I must abate. 484. that is  $\frac{1}{4}$  2. And there will remain  $\frac{1}{4}$  2 whose roote is  $\frac{1}{4}$ , whiche I shall abate from  $\frac{1}{4}$ , and there will remain  $\frac{1}{4}$ , that is. 4. for the other extreme.

Then for the middell terme, thus shall I doe. *Mul. The pfoe.*  
multiplie. 4. and.  $20\frac{1}{4}$  together, and there will rise. 81. whose roote is. 9. and is the middell number. That 9 doubled will make. 18. and 4. ioined therto, giueth 22

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So are those. 3. termes in progression *Geometricall*, according to the conditions limited in the question.

**Maſter.** Nowe the worke now, how it wil frame if. 1.  $\infty$ . be ſet for the middell number. For it wer ſollic, to trie whether this question, would admitte addition of the. 2. laſte numbers. Although the rule doe declare that in ſoche ſorte of equations, there is double valuation to eche roote.

**Scholar.** Yet I beſeeke you, let me examine it a little, to ſee the cauſe, why I maie not adde them, and ſo take the roote.

**Maſter.** I muſt beere with you ſo moche. By addition you ſee, there will riſe  $\frac{22}{2}$ , that is 121. And then the middell number will be.  $49\frac{1}{2}$ . And ſo the proportion is  $\frac{22}{2}$ . that is *Dupla ſuperquadripartiens nonas*. And here as in the other. 3. numbers.  $4.9.20\frac{1}{4}$ . the proportion is *Dupla ſeſqui quarta*.

But in the question is one condition, that ſecludeth the roote, that riſeth by additiō. For the double of the middell terme, with the other vnknownen extreme, ſhould make. 22. As in. 4. and. 9. it doeth. But in  $49\frac{1}{2}$  and 121. it would be 220. that is 10. tymes ſo moche.

**Scholar.** And if you had ſaid in the question, that the double of the middell number, with the other extreme, would haue made. 220. then I ſhould haue taken this later roote by additiō, and not the firſt roote by ſubtraaction.

And ſo I perceiue the varietie of conditions in the question dooeth limite, whiche of the. 2. rootes I ſhall of neceſſitie take, and leaue the other.

*An other  
woorke.*

But now to varie that worke, I will ſet. 1.  $\infty$ . for the middell terme. And then the double of it, with the other terme, will make. 22. The double of. 1.  $\infty$ . is. 2.  $\infty$ . So muſt the other terme be  $22\frac{1}{2}$  — 2.  $\infty$ .

Then to ſeke out an equation, I multiplie the. 2. extremes together, that is.  $22\frac{1}{2} \times 2\frac{1}{2}$  —  $2\infty$  by  $20\frac{1}{4}$ .  
And

## of Coſike numbers.

And there riſeth.  $445\frac{1}{2}$ . —  $40\frac{1}{2}$ .  $2$ . And the ſquare of.  $1.2$ . beynge the middell terme, is ſome perceined to be.  $1.3$ . And ſo the firſte equation is,  
 $13 = 445\frac{1}{2} \cdot 9. = 40\frac{1}{2} \cdot 2$ .

Wherefore I take halfe.  $40\frac{1}{2}$ , that is.  $20$ , whoſe ſquare is  $400$ . And vnto it I putte.  $445\frac{1}{2}$ . whereby there commeth  $889$ . whoſe roote is  $29$ . from whiche roote I muſt abate  $20$ , and ſo remaineth  $9$ . that is.  $9$ . As the value of.  $1.2$ . And for the middle number.

Then for the prooffe. if.  $9$ . be the middell number, *The prooffe.* the ſquare of it, whiche is.  $81$ , ſhall bee equalle to the multiplications of the extremes. Wherefore if I diuide.  $81$ . by  $20\frac{1}{2}$ , the *quotiente* beynge.  $4$ . declareth the other extreme.

Maſter. You ſeme experte inough in this forme of woork. Therefore I will procede to other queſtions, that differ ſomewhat from theſe.

There are. 2. menne talking together of their monies, and nother of them willing to expreſſe plainly his ſomme, but in this ſorte. The number of angelles in my purſe, ſaith the firſte manne, maie bee parted into ſoche 2. numbers, whiche beynge multiplied together, will make.  $24$ . And their *Cubes* beynge added together, will make.  $280$ . Then, quoth the other man. And the like maie I ſaie of my money, ſaue that the *Cubes* of the. 2. partes, will make.  $539$ . Now I deſire to knowe, what monie eche of them had. *A double queſtion.*

Scholar. The firſte mannes ſome, I ſet to be  $1.2$  whiche I muſt parte into two ſoche partes, that thei bothe multiplied together, maie make.  $24$ .

Maſter. You erre verie moche. For it is not poſſible, that the partes of any Coſike number multiplied together, can make an absolute number. Wherefore in ſoche caſes, where you perceiue that there is required, after the firſte poſition, any multiplication to make an absolute number, you ſhall call the firſte no-

## The Arte

bers, by some other name of pleasure. As here you maie call the firste mannes somme. *A*. And the second mannes somme. *B*. and then in their partition, vñ the name of. 1.  $\frac{7}{2}$ .

And as thei are twoo questions in one, so shall you make seuerall woorkes for them.

Scholar. Then shall I saie, that the firste mannes somme is. *A*. and it is diuided as he declared. Wherefore for one number of that diuision, I set. 1.  $\frac{7}{2}$ . And then the other shall be  $\frac{13824}{100}$ . for as the one number multiplied by the other, dooth make. 24. So. 24.  $\frac{7}{2}$ . diuided by the one of them, must needs bying for the other.

Master. What is well remembred of you. For as 4. and. 5. by multiplication, doe make. 20. So. 20. diuided by. 5. bringeth for the 4. and diuided by. 4. it yieldeth. 5.

Scholar. So  $\frac{11}{2}$  is but. 4. and  $\frac{20}{4}$  is. 5.

Master. Go forth then with the rest of the worke.

Scholar. The Cube of. 1.  $\frac{7}{2}$ . is. 1.  $\frac{343}{8}$ . and the Cube of  $\frac{13824}{100}$  is  $\frac{13824^3}{100^3}$  whiche. 2. numbers I maie not adde together, vntill I haue reduced them vnto one denomination: whiche thing I shall doe, by setting. 1.  $\frac{343}{8}$ . as a fraction thus  $\frac{13824}{100}$ . And then woorking after the rate of fractions, in the firste reduction thei will stande thus.  $\frac{13824}{100} \times \frac{13824}{100} = \frac{13824^2}{100^2}$ . And by farther addition thus.

And hether to the worke of bothe these. 2. mennes sommes, are indifferent and agreynge. So that this one worke serueth for them bothe. But now thei will differ. For in the firste mannes woordes, and so in the worke for him  $\frac{13824}{100} \times \frac{13824}{100} = \frac{13824^2}{100^2}$  is equalle to 280: but in the seconde mannes worke, it must be accompted equalle to. 539.

But firste to goe forward with the firste man. Seeing  $\frac{13824}{100} \times \frac{13824}{100} = \frac{13824^2}{100^2}$  is equalle to. 280. Therefore by reduction

## of Coslike numbers.

reduction to one d. nomination,  $\frac{13824}{12} = 1152$  is e-  
 qualle to  $\frac{13824}{12}$ . And remouyng the common denomi-  
 nator, the numerators shal kepe thesame proportion:  
 and therefore.  $13824 \cdot 9$ . Shall be equalle  
 to.  $280 \cdot 12$ . And by translation, to leaue the greatestte  
 d. nomination alone,  $13824 \cdot 9 = 280 \cdot 12$   
 Here I shall seke the value of.  $1 \cdot 12$ . whiche shall  
 not be here accounted the square roote, but the zen-  
 zicubike roote, or the Cubike roote of the square roote,  
 accordyng to the greatestte denomination.

Therefore.  $140$ . in square, maketh  $19600$ . from  
 whiche I must abate  $13824$ . And there dooth remain  
 $5776$  whose square roote is.  $76$ . whiche beyng added  
 vnto.  $140$ . dooeth giue.  $216$ . and beyng abated from  
 it, it leaueth.  $64$ . of whiche bothe I must extracte the  
 Cubike roote, bicause in the equation there are.  $2$ . qua-  
 tities omitted. So that of.  $216$ . the Cubike roote is  $6$ .  
 And of.  $64$ . the Cubike roote is.  $4$ . Were I see bothe  
 rootes serue so my purpose, that I shall take the both.

Master, And good reason. For as in setting  $120$   
 for your position, you could not tell whether it were  
 the greater parte, or the lesser, so maie you not now  
 applie it to either of theim bothe, but take bothe roo-  
 tes for the.  $2$ . partes of your number.

Scholar. So doeth the firste mannes number ap- *The prooffe.*  
 peare to be.  $10$ . seying the partes bee.  $4$ . and.  $6$ . whiche  
 I maie examine thus. What thei make.  $24$ . by multi-  
 plication, it is easily seen. And that their Cubes added  
 together, doe make.  $280$ . is sone perceiued: seying the  
 Cube of.  $4$ . is.  $64$ : and the Cube of.  $6$ . is.  $216$ . whiche.  $2$ .  
 numbers by addition, doe make.  $280$ .

Master. Now proue the seconde mannes worke. *The worke*

Scholar. In his worke  $\frac{13824}{12} = 1152$  is equalle *of the second*  
 to  $539$ . And by reduction to one denomination, it is *parse.*  
 equalle to  $\frac{13824}{12}$ . So that.  $1 \cdot 12 = 13824 \cdot 9$ . is  
 equalle to.  $539 \cdot 12$ . and by translation.

$1 \cdot 12$ .

## The Arte

1.  $\text{Z}$ .  $\text{C}$ . ———.  $\text{539}$ .  $\text{C}$ . ———.  $13824$ .  $\text{9}$ . whose *zenzicubike* roote I seke, thus:  $\text{539}$  doth make in square  $\text{290521}$ , from whiche I must abate  $\text{539}$ , and then remaineth  $\text{289982}$ , whose roote is  $\text{539}$  vnto whiche I maie adde  $\text{539}$ . and then will it bee  $\text{290521}$ , that is.  $\text{512}$ . whose *Cubike* roote is.  $8$ . And is one parte of the seconde mannes number. And for the other parte, I shall abate  $\text{29}$  out of  $\text{539}$ , and there remaineth  $\text{510}$ . that is,  $27$ . whose *Cubike* roote is.  $3$ . And is the other parte of the seconde mannes number. As it maie sone be tried thus. For.  $3$ . tymes.  $8$ . maketh.  $24$ . and.  $27$ . whiche is the *Cube* to.  $3$ . added with.  $\text{512}$ . whiche is the *Cube* to.  $8$ . dooeth make  $\text{539}$ , as the question intendeth.

*The prooffe.*

*A question of an armie.*

Master. One other question I will propounde, of.  $2$ . armies beyng bothe square, and of like nombcr. And if you abate.  $4$ . from the one armie, and adde.  $10$ . to the other armie, and then multiplie them bothe together, there will amounte.  $9853272$ . I demaunde of you, what is the fronte of those square battailes.

Scholar. I call the fronte  $1\text{Z}$ . And then must the battaile bee.  $1\text{Z}$ . Now abatynge.  $4$ . from the one, it will bee.  $1\text{Z}$ . ———.  $4\text{9}$ . Then addynge.  $10$ . to the other, it wil make.  $1\text{Z}$ . ———.  $10\text{9}$ . And if you multiplie those.  $2$ . numbers together, there will amounte by it.  $1\text{Z}$   $\text{Z}$  ———.  $6\text{Z}$ . ———.  $40\text{9}$ . whiche somme must be equalle to.  $9853272$ .

$$1\text{Z} \cdot - + - \cdot 10\text{9}.$$

$$1\text{Z} \cdot - - - \cdot 4\text{9}.$$

---


$$1\text{Z} \text{ Z} \cdot - + - \cdot 10\text{Z} \cdot - - - \cdot 4\text{Z}.$$

$$- - - \cdot 4\text{Z} \cdot - - - \cdot 40\text{9}.$$

---


$$1\text{Z} \text{ Z} \cdot - + - \cdot 6\text{Z} \cdot - - - \cdot 40\text{9}.$$

And if you adde.  $40\text{9}$ . to bothe partes of the equation, it will be.  $1\text{Z} \text{ Z} \cdot - + - \cdot 6\text{Z}$ . equalle to.  $9853312$   
And



## of Coſſike numbers.

And by translation.  $1\cancel{8} \cdot \cancel{8} = 9853312$ . ———  $6\cdot\delta$ .  
out of whiche laſte equation, I ſhall ſearche for the  
value of.  $1\cancel{2}$ . by multiplying firſt  $3$ . ſquarely, where  
of comineth.  $9$ . and then adding it to.  $9853312$ . And  
ſo comineth.  $9853321$ . whoſe roote is.  $3139$ . from  
whiche I muſt abate.  $3$ . And then remaineth.  $3136$ .  
whiche is the full number and ſquare of the one ar-  
mie And hath for his roote.  $56$ .

For as here is one onely quantitie omitted, ſo the  
firſt number, whiche in other queſtions of immediate  
equations, was the verie roote, in theſe interrupte e-  
quations is a rooted number, and is here a ſquare num-  
ber: whoſe roote therfore, I haue drawen according-  
ly. And for triall of this woork.  $56$ . in ſquare maketh *The proefe.*  
 $3136$ . from whiche if you abate.  $4$ . there will reſte  
 $3132$ . Again if you adde.  $10$ . there will riſe.  $3146$ .  
And theſe.  $2$ . numbers multiplied together, doe make  
 $9853272$ , as the queſtion intendeth.

Maſter. This you ſee, what uſe is in theſe equa-  
tions, yet are there many other equations, whiche here  
be not ſpoken of: but here after you ſhall haue moare  
largely declared, if you ſhew your ſelf diligente in  
this parte.

And one queſtion I will propounde, & aſſoyle with *A queſtion*  
out woork for breſſenneſſe, that you maie ſee there is *of ſtraunge*  
moare behinde. There is a number whoſe ſquare *equation.*  
abated by.  $16$ . and the firſt number augmented by  
 $8$ . and then bothe thei multiplied together, will bring  
for the.  $2560$ .

Scholar. I will proue the woork of it. And there-  
fore ſuppoſe the firſt number to be.  $1\cancel{2}$ . Then is his  
ſquare  $1\cancel{8}$ . whiche abated by  $16$ . leueth.  $1\cancel{8}$ . ———  $16\delta$ .  
and the nuber augmented by.  $8$ . yeldeth  $1\cancel{2}$ . ———  $8\delta$ .  
Theſe.  $2$ . numbers multiplied together, will make  
 $1\cancel{2}$ . ———  $8\cdot\delta$ . ———  $16\cdot\cancel{2}$ . ———  $128\cdot\delta$ . bring-  
gualle to.  $2560$ .

$11\cdot f.$        $1\cdot\delta$ .

# The Arte

$$1. \text{z} . \text{---} . 16. \text{q} .$$

$$1. \text{z} . \text{---} . 8. \text{q} .$$

$$1. \text{c} . \text{---} . 16. \text{z} .$$

$$8. \text{z} . \text{---} . 128. \text{q} .$$

$$1. \text{c} . \text{---} . 8. \text{z} . \text{---} . 16. \text{z} . \text{---} . 128. \text{q} .$$

And addyng 128. q. on bothe sides of the equation,  
it will be.  $1. \text{c} . \text{---} . 8. \text{z} . \text{---} . 16. \text{z} . \text{---} . 2688. \text{q}$   
Againe addyng. 16. z. on bothe sides, it will bee  
 $1. \text{c} . \text{---} . 8. \text{z} . \text{---} . 16. \text{z} . \text{---} . 2688. \text{q} .$

Master. Where at staie you now?

Scholar. I see no thiste, but other to leaue it, as it  
is, 2. nombers equalle to. 2: other els to make. 1. nom-  
ber equalle to. 3. And all that is aboue my cunningg.  
For hetherto I haue learned noe rule for any of them  
bothe. So that I can not gesse, what the firste number  
might bee.

Master. The number is. 12. And his Square is  
144. from whiche if you abate. 16. it will bee. 128.  
And if you adde. 8. to. 12. it will yelde. 20. Then mul-  
tipliyng. 128. by. 20. the somme will be. 2560. as the  
question declared.

*Of other  
equations.*

But to put you out of doubte, this equation is but  
a trifle, to other that bee vntouched. And yet I will  
tourne this equation a litle, to giue you some light in  
it, and other soche. As here.

$1. \text{c} . \text{---} . 16. \text{z} . \text{---} . 2688. \text{q} . \text{---} . 8. \text{z} .$   
where you see. 1. c. equalle to. 3. other nombers. And  
is it not certaine to you. that this equation is true?

Scholar. Yes, I am adured thereof.

Master. And yet to auoide doubtfulnes the more  
trie it by resolution, accountyng. 12. for. 1. z.

Scholar. Where. 12. is. 1. z. there. 1. z. is. 144.  
and. 1. c. is. 1728. whiche. 1728. must bee equalle to  
16. z.

## of Cossike numbers.

16. 2e (that is. 192) and to. 2688. saue that you must abate. 8. 3. that is 1152. Now if I adde 192. to 2688 it will make. 2880. out of whiche abatinge. 1152. there will remaine. 1728. wherby I see the equation is iuste.

Master. Then you see that the equation is true. And can you doubt, that any number, whiche is equalle to a Cubike number, hath in it a Cubike roote?

Scholar. It must needs be a Cubike number, that is equalle to a Cubike number: and therefore muste needs haue a Cubike roote: although I knowe not how to extrate that roote.

Master. Likewises, when I saie:

8. 3.  $\mathcal{C}$ .  $\equiv$  12. 3.  $\mathcal{S}$ .  $\text{---}$  12. 8. 9. it is certaine, not onely that. 12. 3.  $\mathcal{S}$ .  $\text{---}$  12. 8. 9. containeth in it as moche as. 8. 3.  $\mathcal{C}$ . but that the. 8. parte of it is a 3.  $\mathcal{C}$ . number, and hath a 2enzicubike roote.

Here the  
roote is. 2.

And farther it is manifeste, that as euery. 3.  $\mathcal{C}$ . number, dooeth containe in it certaine. 3. numbers exactly, so if any number be annered with those *Surfolides* (as here in this example are set 128) it is of necessitie, that that. 128. must containe in it certaine *Surfolides* exactly.

So if. 8. 3.  $\mathcal{C}$ . bee equalle to

10. 3.  $\mathcal{S}$ .  $\text{---}$  20. 3. 3.  $\mathcal{S}$ .  $\text{---}$  400.  $\mathcal{C}$ .  $\text{---}$  1250. 9. it must needs be that the. 8. parte of this compounde number shall bee a. 3.  $\mathcal{C}$ . number. And also that the 3. 3. with the other numbers folowyng dooeth containe a certain number of 3. numbers. And the.  $\mathcal{C}$ . in like sorte includeth a number of. 3. 3. numbers. And laste of all. 1250. 9. doeth comprehend certain Cubike numbers exactly.

The roote is 5

In like sorte, when we saie, that. 1. 3.  $\mathcal{S}$ . is equalle to 6.  $\mathcal{C}$ .  $\text{---}$  8. 3.  $\mathcal{S}$ .  $\text{---}$  9. 9. All this compounde number is a *Surfolide*, and hath a. 3.  $\mathcal{S}$ . roote. And 3. 3.  $\text{---}$  9. 9. includeth certaine Cubes. And so

The roote  
here is. 3.

L. y. doeth

## *The Arte*

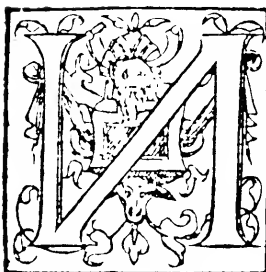
doeth. 9. 9. containe exactly. 1. 8. 02 moare.

But of these and many other verie excellent and  
wonderfull woorkes of equation, at an other tyme I  
will instructe you farther, if I see your diligence ap-  
plied well in this, that I haue taughte you.

And therfore here will I make an  
eande of *Cosike* numbers,  
so this tyme.

# Of Surde numbers, in diuerſe ſortes

And firſte of Surde numbers  
vncpounde.



Now that you haue ſome-  
what learned the arte of *Coſe*  
*ſike* numbers, with the rule of  
equation, it ſemeth good time  
and apte place, to teache you  
the arte of Surde nōbers, whi-  
che are diuerſe in name, accor-  
dyng as there are diuerſe na-  
tures of rootes, whiche maie

giue theim name.

For generally, a Surde number is nothyng els, but  
ſoche a number ſet for a roote, as can not be expreſſed  
by any other number absolute. *A Surde  
number.*

As the *Square* roote of. 1 0, 02 of. S. 02 of any number,  
that is not ſquare. Likewaies the *Cubike* roote of. 4, 02  
of. 5. 02 of any number that is not *Cubike*. So the *zen-  
zenzike* roote of. S. 1 2. 02. 2 0, 02 of any number that  
is no *zenzenzike*, is called a Surde number. And in  
like maner, any other roote of any number, that hath  
noe ſoche roote, doeth cauſe that number to be a Surde  
number.

For if you ſee thoſe ſignes annexed with numbers,  
that hath ſoche rootes, thoſe numbers are not Surde  
numbers properly, but ſette like Surdes. As the *Square*  
roote of. 4. 02 of. 9. 02. 2 5. &c. The *Cubike* roote of. S. 2 7.  
02. 1 2 5. &c. whiche ſometymes is vſed for apte worke,  
as you ſhall ſee here after.

## Of Numeration.

The numeration of the doeth conſiſt, in know-  
lege of their figures, whiche partly be declared  
before. But their common and peculiere ſignes  
are theſe.  $\sqrt{\phantom{x}}$ .  $\sqrt[3]{\phantom{x}}$ .  $\sqrt[n]{\phantom{x}}$ . Although there maie be moare

Ll. 19. varieties

## The Arte

varieties: Yet these for this tyme maie suffice.

The firste, that is.  $\sqrt{\quad}$ . is customably set, to signifie a *Square roote*. As this.  $\sqrt{5}$ . betokeneth the *Square roote* of. 5. And.  $\sqrt{12}$ . is the *Square roote* of. 12. Nowbeit many tymes it hath with it, for the moare certaintie the *Cosike* signe.  $\text{Z}$ . And is written thus.  $\sqrt{\text{Z}} \cdot 20$ . the *Square roote* of. 20. And.  $\sqrt{\text{Z}} \cdot 56$ . the *Square roote* of. 56.

The seconde signe is annexed with *Surde Cubes*, to expresse their rootes. As this.  $\sqrt[3]{16}$ . whiche signifieth the *Cubike roote* of. 16. And.  $\sqrt[3]{20}$ . betokeneth the *Cubike roote* of. 20. And so forth. But many tymes it hath the *Cosike* signe with it also: as  $\sqrt[3]{\text{Z}} \cdot 25$  the *Cubike roote* of. 25. And.  $\sqrt[3]{\text{Z}} \cdot 32$ . the *Cubike roote* of. 32.

The thirde figure doeth represente a *zenzizenzike roote*. As.  $\sqrt[4]{12}$ . is the *zenzizenzike roote* of. 12. And  $\sqrt[4]{35}$ . is the *zenzizenzike roote* of. 35. And like waies if it haue with it the *Cosike* signe.  $\sqrt[4]{\text{Z}} \cdot 24$  the *zenzizenzike roote* of. 24. and so of other.

Scholar. It were againste reason, to seke reason for those signes, whiche be set voluntarily to signifie any thyng: although some tymes there bee a certaine apte conformance in soche thynges. And in these figures, the number of their minomes, seemeth disagreeable to their order.

Maister. In that there is some reason to bee shewed: for as.  $\sqrt{\quad}$ . declareth the multiplication of a number, ones by it self: So.  $\sqrt[3]{\quad}$ . representeth that multiplication *Cubike*, in whiche the roote is represented thise. And.  $\sqrt[4]{\quad}$ . standeth for.  $\sqrt{\sqrt{\quad}}$ . that is. 2. figures of *Square* multiplication: and is not expresse with. 4. minomes. For so should it seme to expresse moare then. 2. *Square* multiplications. But of voluntarie signes, it is inoughe to knowe that this thei doe signifie. And if any manne can devise other, moare easie or apter in vse, thei maie well be receiued.

## of Surde numbers.

But concerning the numeration of *Surde* numbers this shal you marke: that when any compoūde signe is putte before a number, whiche hath any roote, that maie bee expresse by parte of that signe, that number is not absolutely so to bee expresse, onlesse it bee for ease or aptnesse in worke. As.  $\sqrt{3 \cdot 3 \cdot 36}$ . whiche betokeneth the *zenzizenzike* roote of. 36. Seyng it is well knowen, that. 36. hath. 6. for his *Square* roote, it were moare apte expresseynge that number thus.  $\sqrt{3 \cdot 6}$ . that is the square roote of. 6.

Otherwaies, if the nōber that foloweth the signe, haue a roote agreable to that signe: it is noe *Surde* number. As.  $\sqrt{16}$ . is. 4. and is noe *Surde* number. So.  $\sqrt{27}$ . is. 3. and needeth not to bee written in *Surde* forme, excepte it bee for aptnesse of worke. And by this maie you iudge of all other, as thei come in vse

Scholar. If this bee all that is requisite to numeration, I praye you proceed. to addition. For that is nexte in order.

Maister. That is the common order. I dolebet in vulgar fractions, you remember that multiplication and diuision, are set before addition and subtraction: because of the easier formes of worke, in multiplication and diuision. And so in these *Surde* numbers, because the worke of multiplication, and of diuision, be not onely moare easie, then the worke of addition, and of subtraction, but also be requisite to them, therefore will I begin with them, and so come to the other.

## Of Multiplication.



Multiplicatiō in *Surde* numbers encoūpōnde hath noe difficultie, if thei be of one denomination: els must thei be reduced to one denomination: and that by multiplication, accordyng to their signes.

But where noe reduction needeth, you shall multiply

## The Arte

ttplie the numbers together , and sette their common  
figne before the number, that resulteth of that multi-  
plication.

### Examples of square Surdes.



If you will multiplie.  $\sqrt{\cdot 3} \cdot 15$ . by.  $\sqrt{\cdot 3}$ .  
26. it will make.  $\sqrt{\cdot 3} \cdot 390$ .  
So.  $\sqrt{\cdot 3} \cdot 32$ . multiplied by.  $\sqrt{\cdot 3} \cdot 48$ .  
dooth make.  $\sqrt{\cdot 3} \cdot 1536$ .  
And.  $\sqrt{\cdot 3} \cdot 56$ . multiplied by.  $\sqrt{\cdot 3} \cdot 21$ .  
dooth yelde.  $\sqrt{\cdot 3} \cdot 1176$ .

Notwbeit some tymes it happeneth, that the nom-  
ber, whiche is made by that multiplication, is a nom-  
ber absolute, and not a *Surde number*.

### Examples of soche as make numbers Absolute.

$$\begin{array}{r} \sqrt{\cdot 12}. \\ \sqrt{\cdot 3}. \\ \hline \sqrt{\cdot 36}. \text{that is } 6. \end{array}$$

$$\begin{array}{r} \sqrt{\cdot 48}. \\ \sqrt{\cdot 3}. \\ \hline \sqrt{\cdot 144}. \text{that is } 12. \end{array}$$

$$\begin{array}{r} \sqrt{\cdot 12} \cdot \frac{1}{2}. \\ \sqrt{\cdot 4} \cdot \frac{1}{2}. \\ \hline \sqrt{\cdot 56} \cdot \frac{1}{4} \text{ that is } 7 \cdot \frac{1}{2}. \end{array}$$

$$\begin{array}{r} \sqrt{\cdot 28} \cdot \frac{1}{4}. \\ \sqrt{\cdot 7} \cdot \frac{1}{4}. \\ \hline \sqrt{\cdot 207} \cdot \frac{1}{16} \text{ that is } 14 \cdot \frac{1}{4}. \end{array}$$

$$\begin{array}{r} \sqrt{\cdot 240}. \\ \sqrt{\cdot 15}. \\ \hline \sqrt{\cdot 3600}. \text{that is } 60. \end{array}$$

$$\begin{array}{r} \sqrt{\cdot 325}. \\ \sqrt{\cdot 13}. \\ \hline \sqrt{\cdot 4225}. \text{that is } 65. \end{array}$$

And generally when any number is multiplied by  
an other, if the proportion betwene those 2. numbers  
bee represented by a square number, as by. 4. 9. 16.  
25. &c. then dooe they make a square number by their  
multiplication.

### Examples



*of Surde numbers.*  
*Examples of Cubike rootes.*

|  |                                      |  |
|--|--------------------------------------|--|
| $\sqrt[3]{w. 2.} \quad 91.$              | $\sqrt[3]{w. 7. \frac{1}{3}.} \quad$ | $\sqrt[3]{w. 256.}$                    |
| $\sqrt[3]{w. 2.} \quad 12.$              | $\sqrt[3]{w. \frac{1}{3}.}$          | $\sqrt[3]{w. \frac{1}{3}.}$            |
| <hr/> $\sqrt[3]{w. 2. 109 \frac{1}{2}.}$ | <hr/> $\sqrt[3]{w. 5. \frac{1}{3}.}$ | <hr/> $\sqrt[3]{w. 190 \frac{1}{11}.}$ |

*Examples of soche as make*  
*Absolute numbers.*

|  |   |
|--|---|
| $\sqrt[3]{w. 54.}$   | $\sqrt[3]{w. 686.}$                             |
| $\sqrt[3]{w. 32.}$   | $\sqrt[3]{w. 4.}$                               |
| <hr/> $\sqrt[3]{w. 1728. \text{ that is. } 12.}$   | <hr/> $\sqrt[3]{w. 2744. \text{ that is. } 14}$ |
| $\sqrt[3]{w. 486.}$<br>$\sqrt[3]{w. 96.}$<br><hr/> $\sqrt[3]{w. 46656. \text{ that is. } 36.}$ |   |

*Examples of zenzizenzike rootes.*

|   |  |   |                            |                 |                |                 |  |  |
|---|--|---|----------------------------|-----------------|----------------|-----------------|--|--|
| $\sqrt{w. 15.}$   | $\sqrt{w. 204.}$                                     | $\sqrt{w. 162.}$                                      |                            |                 |                |                 |  |  |
| $\sqrt{w. 7.}$  | $\sqrt{w. 26.}$                                      | $\sqrt{w. 32.}$                                       |                            |                 |                |                 |  |  |
| <hr/> $\sqrt{w. 105.}$  | <hr/> $\sqrt{w. 5304.}$                              | <hr/> $\sqrt{w. 5184. \text{ that is. } \sqrt{.72.}}$ |                            |                 |                |                 |  |  |
| <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center; width: 50%;"><math>\sqrt{w. 7 \frac{1}{2}.}</math></td> <td style="text-align: center; width: 50%;"><math>\sqrt{w. 27.}</math></td> </tr> <tr> <td style="text-align: center;"><math>\sqrt{w. 4.}</math></td> <td style="text-align: center;"><math>\sqrt{w. 12.}</math></td> </tr> <tr> <td style="text-align: center;"><hr/><math>\sqrt{w. 5 \frac{1}{2}. \text{ that is. } .2 \frac{1}{2}.}</math></td> <td style="text-align: center;"><hr/><math>\sqrt{w. 324. \text{ that is. } \sqrt{.18.}}</math></td> </tr> </table> |  |   | $\sqrt{w. 7 \frac{1}{2}.}$ | $\sqrt{w. 27.}$ | $\sqrt{w. 4.}$ | $\sqrt{w. 12.}$ | <hr/> $\sqrt{w. 5 \frac{1}{2}. \text{ that is. } .2 \frac{1}{2}.}$ | <hr/> $\sqrt{w. 324. \text{ that is. } \sqrt{.18.}}$ |
| $\sqrt{w. 7 \frac{1}{2}.}$  | $\sqrt{w. 27.}$                                      |   |                            |                 |                |                 |  |  |
| $\sqrt{w. 4.}$  | $\sqrt{w. 12.}$                                      |   |                            |                 |                |                 |  |  |
| <hr/> $\sqrt{w. 5 \frac{1}{2}. \text{ that is. } .2 \frac{1}{2}.}$  | <hr/> $\sqrt{w. 324. \text{ that is. } \sqrt{.18.}}$ |   |                            |                 |                |                 |  |  |

*Examples of zenzizenzike rootes*  
*that make absolute numbers.*

|  |   |
|--|---|
| $\sqrt{w. 32.}$                              | $\sqrt{w. 128.}$                              |
| $\sqrt{w. 8.}$                               | $\sqrt{w. 32.}$                               |
| <hr/> $\sqrt{w. 256. \text{ that is. } 16.}$ | <hr/> $\sqrt{w. 4096. \text{ that is. } 64.}$ |
|  | $\text{Or. } \sqrt{w. 288}$                   |

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$\sqrt{\phantom{x}}$ . 288.

$\sqrt{\phantom{x}}$ . 72.

---

$\sqrt{\phantom{x}}$ . 20736. that is. 12.

But here is to bee noted, that if you would multiplie any *Surde* number, by an absolute number, or any *Surde* number of one denomination, by a *Surde* number of an other denomination: you must firste reduce that Absolute number to the like denomination. And so must you reduce the.2. *Surde* numbers to one denomination.

And because that this woork doeth serue often in this arte, and that in diuerse woorkes, I will set here the arte of reduction.

## Of reduction in Surdes.



**R**eduction in *Surdes*, is the bringing of sundrie denominations vnto one. Whiche in absolute numbers is thus doen. You shall multiplie the absolute number, according to the signe of the *Surde*, and then sette befoze it the like signe. So that if you would double.  $\sqrt{\phantom{x}}$ . 3. 8. that is to saie, if you would multiplie it by. 2. you must firste multiplie that. 2. squarely, and then multiplie those numbers together. What is to saie, you shall multiplie.  $\sqrt{\phantom{x}}$ . 3. 8. by.  $\sqrt{\phantom{x}}$ . 3. 4. and so is it doubled.

Like wises, to triple any Square *Surde*, is to multiplie it by. 9. And so to quadruple any Square *Surde*, is to multiplie it by. 16. And so forth.

But if you double any Cubike number, you shall multiplie it by. 8. that is the Cube of. 2. And so if you would triple a Cubike roote, you muste multiplie it by 27. And if you would quadruple it, you shall multiplie

## Of Surde numbers.

it by.64. And so of other like woordes.

Again, if you will double any *zenzizenzike* roote, you must multiplie it by.16. And if you will triple it, you shall multiplie it by.81. And so if you will *quadruple* it, you must multiplie it by 256. And in like maner ever moare, for the number absolute, you shall set his *zenzizenzike* number. Like as in Squares, for any number absolute, you shall set his square. And in Cubes you shall take his *Cube*.

Scholar. This is plaine inoughe: yet I praye you put an example of twoo, of eche kinde.

Master. Take these examples for square rootes.

$$\begin{array}{r} \sqrt{\phantom{00}}. \quad 38. \\ \quad \quad 2. \\ \hline \sqrt{\phantom{00}}. \quad 152. \end{array} \quad \begin{array}{r} \sqrt{\phantom{00}}. \quad 5. \quad 128. \\ \quad \quad \quad 6. \\ \hline \sqrt{\phantom{00}}. \quad 8. \quad 4608. \end{array} \quad \begin{array}{r} \sqrt{\phantom{00}}. \quad 3264. \\ \quad \quad \quad 12. \\ \hline \sqrt{\phantom{00}}. \quad 469976. \end{array}$$

### Examples in Cubike rootes.

$$\begin{array}{r} \sqrt[3]{\phantom{00}}. \quad 52. \\ \quad \quad 2. \\ \hline \sqrt[3]{\phantom{00}}. \quad 416. \end{array} \quad \begin{array}{r} \sqrt[3]{\phantom{00}}. \quad 163. \\ \quad \quad \quad 5. \\ \hline \sqrt[3]{\phantom{00}}. \quad 20375. \end{array} \quad \begin{array}{r} \sqrt[3]{\phantom{00}}. \quad 4806. \\ \quad \quad \quad 8. \\ \hline \sqrt[3]{\phantom{00}}. \quad 2460672. \end{array}$$

### Examples in *zenzizenzike* numbers.

$$\begin{array}{r} \sqrt{\phantom{00}}. \quad 69. \\ \quad \quad 2. \\ \hline \sqrt{\phantom{00}}. \quad 1104. \end{array} \quad \begin{array}{r} \sqrt{\phantom{00}}. \quad 251. \\ \quad \quad 4. \\ \hline \sqrt{\phantom{00}}. \quad 64256. \end{array} \quad \begin{array}{r} \sqrt{\phantom{00}}. \quad 1385. \\ \quad \quad \quad 5. \\ \hline \sqrt{\phantom{00}}. \quad 2250625. \end{array}$$

Scholar. This I perceiue well. But now in *Surde* numbers of diuerse denominations, what the order of reductiō is, I praye you to set forth with some examples

Master. These examples with their declaration, may sufficiently serue for a shewe, if I would multiplie.  $\sqrt[3]{\phantom{00}}.12.$  by.  $\sqrt{\phantom{00}}.5.$  I must firste multiplie the number of one signe, accordinge to the signe of the other

Min. y.

number,

## The Arte

number, and so alter them bothe. Whiche woork is like the reduction of fractions, to one common denomination. As here I made multiplie. 5. Cubikely, and 12. must be multiplied squarely, and then shall I adde bothe signes in one, for their common signe. So shall I haue for them the.  $\sqrt[3]{\cdot 5 \cdot \cdot \cdot}$ . roots of. 144. to be multiplied by the *zenzicubike* roote of 125. And so will there come of that multiplication, the *zenzicubike* roote of. 18000. As here by example doeth appeare.

$$\begin{array}{r} \sqrt[3]{\cdot 5 \cdot \cdot \cdot} \cdot 144. \\ \sqrt[3]{\cdot 5 \cdot \cdot \cdot} \cdot 125. \\ \hline \sqrt[3]{\cdot 5 \cdot \cdot \cdot} \cdot 18000. \end{array}$$

Like waies if I would multiplie.  $\sqrt[3]{\cdot 8 \cdot \cdot \cdot} \cdot 250.$  by  $\sqrt[3]{\cdot 34 \cdot \cdot \cdot}$ . I shall firste multiplie. 250. Cubikely, and it will bee. 15625000. And 34. must I multiplie *zenzicubikely*, and it will yelde. 1336336. Wherefore multiplying them together, and adding thereto the common denomination, it will bee the.  $\sqrt[3]{\cdot 8 \cdot \cdot \cdot} \cdot 20880250000000.$

This woork is aptly represented in figure, after this sorte. And then shall you multiplie crosse waies the number of the one, by the signe of the other. And so make you dooe in all other like numbers, of diuerse denominations.

$$\begin{array}{ccc} \sqrt[3]{\cdot 8 \cdot \cdot \cdot} & \times & \sqrt[3]{\cdot \cdot \cdot} \\ 250. & & 34. \end{array}$$

This reduction doeth serue for any other woork, as well as for multiplication.

## Of Diuision.



**D**iuision is as easie as multiplication. For in it there is noe regard had to the signes. But the one number diuided by the other as if they were numbers absolute. And then the firste signe added to the *quotiente*. For the more lighte and certaintie, I haue set here, examples of eche sorte.

And

*of Surde numbers.*

*And first examples of square rootes.*

$$\begin{array}{l} \sqrt{\cdot} 72. \\ \sqrt{\cdot} 8. \end{array} (\sqrt{\cdot} 9. \text{ that is. } 3. \quad \begin{array}{l} \sqrt{\cdot} 128. \\ \sqrt{\cdot} 4. \end{array} (\sqrt{\cdot} 32.$$

$$\begin{array}{l} \sqrt{\cdot} 457\frac{1}{2}. \\ \sqrt{\cdot} 21. \end{array} (\sqrt{\cdot} 21\frac{1}{2}.$$

*Examples of Cubike rootes.*

$$\begin{array}{l} \sqrt[3]{\cdot} 96. \\ \sqrt[3]{\cdot} 4. \end{array} (\sqrt[3]{\cdot} 24. \quad \begin{array}{l} \sqrt[3]{\cdot} 1664. \\ \sqrt[3]{\cdot} 32. \end{array} (\sqrt[3]{\cdot} 52.$$

$$\begin{array}{l} \sqrt[3]{\cdot} 5624. \\ \sqrt[3]{\cdot} 76. \end{array} (\sqrt[3]{\cdot} 74\frac{1}{2}.$$

*Examples of zenzizenzike rootes.*

$$\begin{array}{l} \sqrt[4]{\cdot} 54. \\ \sqrt[4]{\cdot} 6. \end{array} (\sqrt[4]{\cdot} 9. \text{ that is. } \sqrt{\cdot} 3.$$

$$\begin{array}{l} \sqrt[4]{\cdot} 286. \\ \sqrt[4]{\cdot} 42. \end{array} (\sqrt[4]{\cdot} 6\frac{1}{2}. \quad \begin{array}{l} \sqrt[4]{\cdot} 5892. \\ \sqrt[4]{\cdot} 54. \end{array} (\sqrt[4]{\cdot} 109\frac{1}{2}.$$

And this maie suffice for Division. The profe of it is by the contrary kinde. For Multiplication proueth Division: and Division trieth Multiplication.

Scholar. All this is easie enough to remember.

*Of Addition.*

Master.



ADDITION is not so easie, but hath diuerse *The firste* varieties of worke, as anon shall appere. *forme of* Whereof the firste is as easie as can bee. *Addition.* For it requireth onely the signe of addition. —+. As if I would adde.  $\sqrt{\cdot} 12.$  to  $\sqrt[3]{\cdot} 26.$

## The Arte

✓.26. I shall set it thus. ✓.26. —+— ✓.12. And so  
 ✓.20. put vnto. ✓.54. maketh. ✓.54. —+— ✓.20.  
 This forme serueth chiefly for rootes of diuerse na-  
 mes. As. ✓.38.20. —+— .ww. 30. where  
 ✓.30. is added to ✓.38.20. And so of al other.

*The seconde  
 forme.*

The seconde forme is not so easie: and yet many ti-  
 mes it is moare certaine. And this is the order of it.

You shall sette downe your .2. numbers, that you  
 would adde together, forseyng that thei be of one de-  
 nomination. Then shall you adde in plaine forme,  
 their numbers together, putting thereto the signe of  
 the roote. And kepe that as a parte of the addition.  
 Again you shall multiplie the .2. firste numbers toge-  
 ther. And their totalle you shall multiplie by .4. And  
 before that shall you sette the signe of the roote. And  
 it shall stande as the seconde parte of that addition.  
 So that those .2. partes, shall be added with the signe  
 —+—. And then is the woork eanded. Example  
 hereof. I would adde the .2. firste sommes, that is,

✓.12. to. ✓.26. wherfore I  
 set them thus. And then doe  
 I adde the bothe plainly to-  
 gether, and thei make. ✓.38  
 whiche I set by, as one part  
 of the addition. Then doe I  
 multiplie ✓.26. by. ✓.12. and  
 there riseth. ✓.312. whiche  
 I must double, or multiplie  
 by.2. And therfore seyng the

$$\begin{array}{r|l}
 \sqrt{26} \quad + \quad \sqrt{12} & \\
 \hline
 \sqrt{26}. & \\
 \sqrt{12}. & \\
 \hline
 \sqrt{312}. & \\
 \sqrt{4}. & \\
 \hline
 \sqrt{1248}. &
 \end{array}$$

woork is in square rootes, I set the square of 2. with  
 the signe of. ✓. for.2. and then multipling them to-  
 gether, I haue. ✓.1248. whiche is the seconde parte  
 of the roote. Wherfore addyng those .2. partes toge-  
 ther, with the signe. —+—. there commeth. ✓.38.  
 —+—. ✓.1248. as the totalle of that addition.

Scholar. As me thinketh, the firste forme of addi-  
 tion,

## of Surde numbers.

tion serueth better for these numbers, then this se-  
conde forme. For it is moare easie to vse, in any kinde  
of woork, and moare speedily doen: and it serueth that  
this laste number, is moare obscure then the firste.

Waite. Yet is this woork good, and very neces-  
sarie. For in these numbers, and soche other like, it  
serueth onely (as appereth) to alter the state of the nō-  
bers, whereby they maie bee commensurable, with o-  
ther, then they were before that alteration. But in  
some numbers, and that very many, it reduceth them  
to one simple forme of roote. As by the examples folo-  
wyng you shall perceiue.

An example.

$$\begin{array}{r}
 \sqrt{.28.} + \sqrt{.7.} \\
 \hline
 \sqrt{.28.} \\
 \sqrt{.7.} \\
 \hline
 \sqrt{.196.} \\
 \sqrt{.4.} \\
 \hline
 \sqrt{.784.}
 \end{array}
 \quad
 \begin{array}{r}
 28. \\
 7. \\
 \hline
 35.
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.35.} + \sqrt{.784.} \\
 \hline
 2. \sqrt{.35.} + 28. \\
 \hline
 \text{That is } \sqrt{.63.}
 \end{array}$$

The same example other  
waies wrought.

$$\begin{array}{r}
 \sqrt{.28.} + \sqrt{.7.} \\
 \hline
 \sqrt{.28.} \\
 \sqrt{.7.} \\
 \hline
 \sqrt{.196.} \text{ whose roote is } 14 \\
 14. \\
 2. \\
 \hline
 \sqrt{.35.} + 28.
 \end{array}$$

A thirde  
forme of ad-  
dition.

Where firste I haue set forth the 2. examples of one  
addition, that you maie see the agremente of the both

And firste I would adde.  $\sqrt{.28.}$  with.  $\sqrt{.7.}$  where-  
fore I doo toyne. 28. and. 7. in one somme, whiche I  
set a parte, as the firste portion of the addition. Then  
I doe multiplie. 28. by. 7. And thereof cometh. 196.  
whiche is a square nōber, and hath. 14. for his roote.  
So that I maie vse now. 2. woorkes. For other I maie  
continue my woork, as I haue doen (agreable to the  
firste example) in multipling that.  $\sqrt{.196.}$  by.  $\sqrt{.4.}$   
(whiche is but doubling) and so there cometh.  $\sqrt{.784}$   
whiche

## The Arte

whiche is a number absolut e: bicause it hath a roote,  
accoyding to his signe, whiche roote is. 28. and maie  
be set for.  $\sqrt{784}$ .

Now in the seconde woozke, bicause the first mul-  
tiplication of. 28. by. 7. doeth make a square number,  
I doe take the roote of that number for it: sayng it is  
all one thyng to saie.  $\sqrt{196}$ , and. 14. for. 14. is the  
roote of. 196. And then hauyng the roote, I muske  
double it, accoyding to the rule, or multiplye it by. 2.  
and there of commeth. 28. whiche I shall adde with  
35. And so haue 63. whose roote containeth the ad-  
dition of.  $\sqrt{28}$ . and.  $\sqrt{7}$ .

Scholar. This woozke semeth straunge: and far-  
thesse from common reason, of all other woozkes in  
this arte.

Maister. I mighte easily by demonstration make  
you, to perceiue as moche reason in this woozke, as ca-  
be in any: for it dependeth of the. 38. Theorome of the  
pathewaie. But haste of other businesse, maketh me  
to omit the demonstration at this tyme, whiche sho-  
tly you shall haue, for all the equations, and other  
woozkes likewaies.

But for this presente tyme, it shall be sufficiente to  
wozke an example in *rationall* numbers, as if they wer  
*Surde* numbers: that therby you maie perceiue the or-  
der, and the truth of the woozke.

Wherefore I take these twoo numbers.  $\sqrt{36}$ . and  
 $\sqrt{49}$ . to bee added together. Where I doe first adde  
the twoo numbers plainely together: And then make  
85. for the firste parte of the addition. Then dooe I  
multiplye. 49 by. 36. and there riseth. 1764. whiche  
is a square number. And therefore maie I vse. 2. wooz-  
kes, as you see. In the firste I multiplye that square  
number by. 2. or by.  $\sqrt{4}$ . whiche is all one: and there  
doeth amounte. 7056. a square number also, whose  
roote is. 84.

The



## Of Surde numbers.

| The firste forme.  |  | The seconde forme.  |
|--|--|---|
| $\sqrt{36} \text{---}   \text{---} \sqrt{49}$<br>$\sqrt{\phantom{00}} \phantom{00} 49$<br>$\sqrt{\phantom{00}} \phantom{00} 36$<br><hr/> $\phantom{00} 294$<br>$117$<br><hr/> $\sqrt{\phantom{00}} \phantom{00} 1764$<br>$\sqrt{\phantom{00}} \phantom{00} 4$<br><hr/> $\sqrt{\phantom{00}} \phantom{00} 7056$ |  | $\sqrt{36} \text{---}   \text{---} \sqrt{49}$<br>$\sqrt{\phantom{00}} \phantom{00} 49$<br>$\sqrt{\phantom{00}} \phantom{00} 36$<br><hr/> $\phantom{00} 294$<br>$147$<br><hr/> $\sqrt{\phantom{00}} \phantom{00} 1764$<br><b>That is. 42.</b><br>$\phantom{00} 2$<br><hr/> $\phantom{00} 84$ |
| $\sqrt{85} \text{---}   \text{---} \sqrt{7056}$<br><b>Ans.</b> $\sqrt{85} \text{---}   \text{---} 84$  |  | $\sqrt{85} \text{---}   \text{---} 84$  |
| <b>That is. <math>\sqrt{169}</math>.</b><br><b>Ans. 13.</b>  |  |   |

In the seconde woorkes I take the roote of .1764. whiche is 42 and doublyng it, I haue 84. agreable to the other woorkes. Then doe I sette those .2. numbers downe with — | —, and putte to them the signe.  $\sqrt{\phantom{00}}$ . in token that I muste take the roote of that compounde number: and not of any one parte of it.

Scholar. That haue I marked well: For 85. hath no roote, nother 84. hath any roote. But  $85 \text{---} | \text{---} 84$  that is. 169. hath. 13. for his roote.

And so I see, that the roote of. 36. whiche is. 6. And the roote of. 49. that is. 7, beeyng bothe added together will make. 13. that is the roote of. 169.

Master. Yet one other forme of easie woorkes, *Of numbers* I will shewe you, whiche is bothe pleasaunte and profitable: *commensurable, as foure be* But is not generalle, for it serueth onely for *forms.* numbers commensurable, I meane suche numbers, as by one common diuisor, maye bee brought into Square numbers. With whiche numbers, you shall woorkes thus.

## The Arte

Firste diuide theim by the common diuisor: and set for them their rootes. Then adde those. 2. rootes together, and multiplie it squarely. And that square being multiplied by the common diuisor, will byynge for the the Square of bothe the rootes. As here followeth in example.

Where I would adde  $\sqrt{384}$  vnto  $\sqrt{150}$  which numbers I doe examin, til I maie finde their commo, and leaste diuisor, whiche here is. 6. When diuidyng them by that. 6. I haue for 384. a square number. 64. And for. 150. I haue another square, that is. 25. Of

$$\begin{array}{r}
 \sqrt{384} \quad | \quad \sqrt{150} \\
 6.) \quad \begin{array}{r} 64 \\ 8 \end{array} \quad \begin{array}{r} 25 \\ 5 \end{array} \\
 \quad \quad \quad 13. \\
 \quad \quad \quad \hline 13. \\
 \quad \quad \quad \hline 169. \\
 \quad \quad \quad \quad \hline 6. \\
 \quad \quad \quad \quad \hline 1014.
 \end{array}$$

whiche bothe squares I set doune the rootes: and the common diuisor also. Then doe I adde bothe rootes together, and thereof commeth. 13. whose Square is 169. that I doe multiplie by. 6. whiche is the commo diuisor, and it will bee. 1014. whose roote doeth contain bothe the rootes befoze named. As you shall see it proued anon by Subtraction.

Scholar. In the meane season I consider, that one of these formes, maie confirme the other. And therefore if I woork this lastte example, by one of the other formes, and finde thesame totall, it must needs be that the woork is good. Whiche I proue thus.

Firste setting doune the numbers, in forme of the caselike Addition. And then addyng them together, I finde. 534. whiche I sette a side, as one parte of the number, that I doe seke for.

Then dooe I multiplie the. 2. numbers together, and thei make. 57600. whiche I dooe multiplie again by 4. And there riseth. 230400. being a square number, and hath. 480. for his roote. Wherefore I set

## of Surde numbers.

$$\begin{array}{r}
 \sqrt{384} - \text{---} - \sqrt{150} \\
 \hline
 384 \\
 150 \\
 \hline
 19200 \\
 384 \\
 \hline
 57600 \\
 4 \\
 \hline
 230400
 \end{array}$$

$$\sqrt{534} - \text{---} - \sqrt{230400} \\
 \text{Or } \sqrt{534} - \text{---} - 480.$$

That is.  $\sqrt{1014}$ .

set. 534. and. 480. together. with the signe of Addition, thus.

$534 - \text{---} - 480$ , And the roote of that number, is equale to bothe the firste rootes. But considering that bothe those numbers, which bee ioyned laste of all with  $\text{---}$ , are numbers rationally and absolutely, I make adde the in one, & so thei make

1014. agreeably to the other woork. Therefore I iudge them bothe to be good

Master. You might have wrought this woork either waies, because the firste number, that riseth of the multiplication is a square number.

Scholar. Then I perceiue, I mighte haue taken the roote of it, whiche is. 240. and doubling it, I should haue. 480. As I had in the other woork. And so all doe agree in one.

But my chief doubt now is, how to knowe those numbers that bee *commensurable*: For if I shall stande long in searchyng for that, I might sooner woork the other forme of woork, then to make that trialle of *commensurableness*.

Master. The easiест waie is, to diuide the greater number, by the lesser, as if thei were bothe numbers absolute: & the *quotiente* will declare their *Squares*. *commensurable*.

As if you doubt, whether. 384. and. 150. bee *commensurable*. diuide. 384. by. 150. and the *quotiente* will be  $2\frac{2}{3}$ , that is  $\frac{8}{3}$ . Then diuide whiche of the. 2. firste numbers you list, by his like number in the *quotiente*: And the common diuisor will amounte. So if you di-

p.n.y.      rido

## The Arte

vide the greater number. 384. by the greater number in the *quotiente*, whiche is. 64, you shall finde the new *quotiente* 6. whiche. 6. is the common number. And if you diuide. 150. by 25. the common number. 6. will be the *quotiente*.

But and if the *quotiente* be a whole number, and no fraction, and be a Square number, then is it the lesser square. Wherefore if you diuide the lesser number of the. 2. by the *quotiente*, the common number will appeare in the seconde *quotiente*. And then if you diuide the greater of the. 2. numbers, by that common number, his *quotiente* will shewe you the other Square.

And if so happen, that the *quotiente* of the first diuision be not a square number, then are those numbers *incommensurable*.

So.  $\sqrt{32}$ . and.  $\sqrt{128}$ . bee *commensurable*: and the *quotiente* of their diuision is. 4. whiche is the lesser square. And. 8. appeareth to be the common number. And the greater square is. 16.

Wherby this number it maie easily bee espied, that some numbers maie be resolued, into more squares then one. As these 2. numbers, being diuided by. 2 dooe giue. 16. and. 64. And being diuided by. 8, they being for the. 4. and. 16.

But for their addition, what Squares so euer you take, that redounds by one common diuisor, the triall will be like, and the roote one.

Scholar. I praye you let me proue that varietie.

Master. Then proue it in suche numbers, where you maie finde moare varietie. As these bee.  $\sqrt{288}$ . and.  $\sqrt{1152}$ .

Scholar. If I diuide. 1152. by. 288. the *quotiente* will bee. 4. whiche I must take for the leaste square. Then by it I diuide. 288. and the *quotiente* will be. 72. as the common diuisor. By whiche if I diuide. 1152. there will rise. 16. as the seconde square. Then let I  
the

## of Surde numbers.

the nōbers in order thus.  $\sqrt{.1152} - \sqrt{.288}.$   
 And vnder. 1152. I set the  
 one Square. 16. And vnder. 288. I putte the other  
 Square. 4. And vnder eche  
 of thaim his roote. Then  
 adde I the Rootes toge-  
 ther, whiche maketh. 6.  
 whose square is. 36. And  
 that beyng multiplied by  
 72. the common number,  
 doeth yelde. 2592. whose  
 roote doeth containe bothe the other. 2. rootes by ad-  
 dition.

$$\begin{array}{r}
 \sqrt{.1152} - \sqrt{.288}. \\
 \begin{array}{r}
 16 \qquad 4. \\
 72 \mid 4 \qquad 2. \\
 \qquad 6. \\
 \qquad \underline{6.} \\
 \qquad 36. \\
 \qquad 72. \\
 \qquad \underline{72.} \\
 \qquad 252. \\
 \hline
 \sqrt{.2592}.
 \end{array}
 \end{array}$$

But now how I shall finde any other Squares in  
 those nōbers, to make any farther trial, I knowe not.

Master. Diuide alwaies one of the numbers, by  
 some square nōber, that will parte it exactly, without  
 any remainder. And marke the *quotiente*. For by it shal  
 you diuide the other nōber, and if the *quotiente* in that  
 last diuision, be a square number, then haue you your  
 purpose. Els muste you pꝛoue with an other Square  
 number.

Scholar. I vnderstande you. And therefore in these  
 numbers, I will make trialle with. 9. by whiche I di-  
 uide. 288. And finde the *quotient*. 32. Then by the same  
 32. I diuide 1152. and the *quotiente* is. 36. So haue I 9  
 and. 36. for the. 2. squares, and. 32. for the comon diui-  
 sor. Therefore I set the nōbers in order as thei ought.  
 And vnder them I place the. 2. square numbers with  
 their rootes. Then addynge the rootes together, I  
 finde. 9. whiche I multiplie square, and it yeldeth. 81.  
 that. 81. I doe multiplie by the common number. 32.  
 and there amounteth. 2592. As it did before in the o-  
 ther worke. Whereby I perceiue that these woordes  
 doe confirme one an other.

## The Arte

$$\begin{array}{r}
 \sqrt{1152} - + - \sqrt{288} \\
 32) \quad 36 \qquad 9 \\
 \qquad \quad 6 \qquad 3 \\
 \qquad \quad 9 \\
 \qquad \quad 9 \\
 \hline
 \qquad \quad 81 \\
 \qquad \quad 32 \\
 \hline
 \qquad 162 \\
 \qquad 243 \\
 \hline
 \sqrt{\quad} \quad 2592
 \end{array}$$

And therefore I will proue, how many varieties of this worke, I may finde in these numbers.

And so for my purpose, I will diuide the lesser of the .2. numbers, by as many Squares as I can, so that seemeth to be the readiest waie. And firste I proue with. 16. And so the *quotient* is. 18. by whiche.

18. I diuide. 1152. and the *quotiente* is. 64. whiche is a square n<sup>o</sup>ber. So that I haue that varietie more.

Then again I proue with. 25. But I see, that will not frame. Wherefore I assaie with. 36. And finde the *quotiente* 8. by whiche I diuide the greater square, and the *quotiente* is. 144. a square number also. And therefore I note that for an other varietie.

Thirdly, I proue with. 49. but that wil not agree. Then attempte I with. 64. And that serueth as euil. Perce that I assaie. 81. 100. and. 121. but none of them will diuide. 288. wherefore I passe vnto. 144. whiche is twise contained in 288. by that. 2. I diuide 1152. and finde the *quotiente*. 576. whiche is a Square number also. And so haue I. 3. other varieties beside the. 2. former worckes: whiche. 3. varieties, for my remembraunce I set downe, thus.

$$\sqrt{1152}.$$

# *of Surde numbers.*

$$\begin{array}{r}
 \sqrt{.1152} - | - \sqrt{.288} \\
 18) \quad 64 \qquad 16 \\
 \qquad 8 \qquad 4 \\
 \qquad \qquad 12 \\
 \qquad \qquad 12 \\
 \hline
 \qquad \qquad 144 \\
 \qquad \qquad 18 \\
 \hline
 \qquad \qquad 1152 \\
 \qquad \qquad 144 \\
 \hline
 \sqrt{.} \quad 2592
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.1152} - | - \sqrt{.288} \\
 144 \qquad 36 \\
 8) \quad 12 \qquad 6 \\
 \qquad \qquad 18 \\
 \qquad \qquad 18 \\
 \hline
 \qquad \qquad 324 \\
 \qquad \qquad 8 \\
 \hline
 \sqrt{.} \quad 2592
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.1152} - | - \sqrt{.288} \\
 2) \quad 576 \qquad 144 \\
 \qquad 24 \qquad 12 \\
 \qquad \qquad 36 \\
 \qquad \qquad 36 \\
 \hline
 \qquad \qquad 1296 \\
 \qquad \qquad 2 \\
 \hline
 \sqrt{.} \quad 2592
 \end{array}$$

Master. Then for to gratifie you, I will sette dounc. 2. other numbers with 6 varieties. Whiche maie serue to suffice for this worke, without more exāples. And because you know the order to trie the I will sette them dounc

without any explication, other declaration. As here you see.

$$\begin{array}{r}
 \sqrt{.28800} - | - \sqrt{.7200} \\
 2) \quad 14400 \qquad 3600 \\
 \qquad 120 \qquad 60 \\
 \qquad \qquad 180 \\
 \qquad \qquad 180 \\
 \hline
 \qquad \qquad 32400 \\
 \qquad \qquad 2 \\
 \hline
 \sqrt{.} \quad 64800
 \end{array}
 \quad
 \begin{array}{r}
 \sqrt{.28800} - | - \sqrt{.7200} \\
 3) \quad 3600 \qquad 900 \\
 \qquad 60 \qquad 30 \\
 \qquad \qquad 90 \\
 \qquad \qquad 90 \\
 \hline
 \qquad \qquad 8100 \\
 \qquad \qquad 81 \\
 \hline
 \sqrt{.} \quad 64800 \qquad \sqrt{.28800}
 \end{array}$$

# The Arte

|  |   |
|--|---|
| $  \begin{array}{r}  \sqrt{.28800} \text{ --- }   \text{ --- } \sqrt{.7200} \\  1600 \qquad \qquad 400 \\  18) \ 40 \qquad \qquad 20 \\  \qquad \qquad 60 \\  \qquad \qquad 60 \\  \hline  \qquad \qquad 3600 \\  \qquad \qquad 18 \\  \hline  \qquad \qquad 28800 \\  \qquad \qquad 36 \\  \hline  \sqrt{.} \ 64800  \end{array}  $ | $  \begin{array}{r}  \sqrt{.28800} \text{ --- }   \text{ --- } \sqrt{.7200} \\  900 \qquad \qquad 225 \\  32) \ 30 \qquad \qquad 15 \\  \qquad \qquad 45 \\  \qquad \qquad 45 \\  \hline  \qquad \qquad 2025 \\  \qquad \qquad 32 \\  \hline  \qquad \qquad 4050 \\  \qquad \qquad 6075 \\  \hline  \sqrt{.} \ 64800.  \end{array}  $ |
|--|---|

|  |   |
|--|---|
| $  \begin{array}{r}  \sqrt{.28800} \text{ --- }   \text{ --- } \sqrt{.7200} \\  576 \qquad \qquad 144 \\  50) \ 24 \qquad \qquad 12 \\  \qquad \qquad 36 \\  \qquad \qquad 36 \\  \hline  \qquad \qquad 1296 \\  \qquad \qquad 50 \\  \hline  \sqrt{.} \ 64800  \end{array}  $ | $  \begin{array}{r}  \sqrt{.28800} \text{ --- }   \text{ --- } \sqrt{.7200} \\  400 \qquad \qquad 100 \\  72) \ 20 \qquad \qquad 10 \\  \qquad \qquad 30 \\  \qquad \qquad 30 \\  \hline  \qquad \qquad 900 \\  \qquad \qquad 72 \\  \hline  \sqrt{.} \ 64800  \end{array}  $ |
|--|---|

Scholar. This varietie is pleasaunte.

Maister. I will satisfie your desire better at more leasure. But yet one thing more will I saie, before we can do this sort of Additio: that if you would adde any roote to it self. As.  $\sqrt{.6.}$  to.  $\sqrt{.6.}$  or.  $\sqrt{.10.}$  to.  $\sqrt{.10.}$  &c. you shall onely *quadruple* the number: and so haue you doen.

Scholar. I see good reason in that: For addition of any number to it self, is but doubling that number or multiplication by. 2. And that must be doen by that *quadruplation*, as you taught before.

Addition of  
cubike rootes

Maister. Now will I set forth the same examples of addition in *Cubike* rootes. For the worke is like vnto this laste forme in *Square* rootes, saue that the mul-  
tiplications,



## Of Surde numbers.

tiplications, whiche were Square in that woꝝke, must be Cubike in this woꝝke. And that onely in numbers *commensurable*. For numbers *incommensurable* be added with the signe. — + —. without moare woꝝke.

I call soche Cubike rootes *commensurable*, whiche be Cubike rootes divided by any common number, will make Cubike numbers in their *quotiente*. As.  $\sqrt[3]{24}$ . and.  $\sqrt[3]{81}$  whiche divided by.3. doe make.8.and.27. bothe beyng Cubike numbers. So.  $\sqrt[3]{320}$ . and.  $\sqrt[3]{135}$ . beyng divided by.5. doe make.27. and. 64. bothe Cubike numbers. Likewises.  $\sqrt[3]{2744}$ . and.  $\sqrt[3]{1000}$ . be *commensurable*, bicause thei make.343.and.125. bothe Cubike numbers: If thei be divided by.8.

Scho. I praye you make your examples with these.

Master. Where nedeth noe wordes in this woꝝke it is so like the Addition of Square rootes. And therefore marke these examples well.

$$\begin{array}{r}
 \sqrt[3]{81} \text{ — — — } \sqrt[3]{24} \\
 27 \qquad \qquad \qquad 8 \\
 3 \qquad \qquad \qquad 2 \\
 \hline
 5 \\
 5 \\
 \hline
 125 \\
 3 \\
 \hline
 \sqrt[3]{375}
 \end{array}$$

$$\begin{array}{r}
 \sqrt[3]{320} \text{ — — — } \sqrt[3]{135} \\
 64 \qquad \qquad \qquad 27 \\
 5 \qquad \qquad \qquad 3 \\
 \hline
 7 \\
 7 \\
 \hline
 343 \\
 5 \\
 \hline
 \sqrt[3]{1715}
 \end{array}$$

$$\begin{array}{r}
 \sqrt[3]{2744} \text{ — — — } \sqrt[3]{1000} \\
 343 \qquad \qquad \qquad 125 \\
 8 \qquad \qquad \qquad 5 \\
 \hline
 12 \\
 12 \\
 \hline
 1728 \\
 8 \\
 \hline
 \sqrt[3]{13824}
 \end{array}$$

Do. f. Scholar.

## The Arte

**Scholar.** Here is noe diuerſitie, from the former woꝝkes, but in ſettyng the *Cubike* roote, foꝝ the ſquare roote. And in multiplying the addition of the.2. rootes *Cubikely*.

*Another forme of addition.* **Maſter.** That is all. And therefore will I ſtande noe longer aboute it: But will proccade to an other ſorme of addition, whiche ſerueth alſo foꝝ *Cubike* rootes *commenſurable*. The rule is this. Set doune the *Cubike* rootes, with their common diuiſoꝝ, and the *Cubes* that riſe therby, and their rootes alſo. All this you did in this former woꝝke. But now peculiarly in this rule, you ſhall ſet doune.3. other numbers orderly, vnder thoſe.3. former numbers. The firſt is the ſquare of that laſt *Cubike* roote: the ſecoꝝde is the *triple* of that ſquare: and the thirde is a number reſultyng of the multiplication of that triple by the other roote.

Then take the.4. extreme numbers, that is thoſe 2 laſt numbers, and the.2. *Cubes*, and adde them together into one ſomme. And that ſomme beyng multiplied by the common diuiſoꝝ, will make a *Cubike* number, whoſe *Cubike* roote ſhall containe bothe the firſt rootes, whiche you intended to adde. Now marke theſe examples: and coſerre them well with the woꝝdes of the rule.

|                |              |                 |                |
|----------------|--------------|-----------------|----------------|
| $\sqrt{.384}$  | $\sqrt{.48}$ | $\sqrt{.15972}$ | $\sqrt{.2592}$ |
| 64             | 8            | 1331            | 216            |
| 6) 4           | 2            | 121             | 6              |
| 16             | 4            | 363             | 36             |
| 48             | 12           | 1188            | 108            |
| 48             | 96           |                 | 2178           |
|                | 216          |                 | 4913           |
|                | 6            |                 | 12             |
| $\sqrt{.1296}$ |              |                 | 9826           |
|                |              |                 | 4913           |
|                |              | $\sqrt{.58956}$ |                |
|                |              | $\sqrt{.52488}$ |                |

# of Surde numbers.

ww'. 52488. — | — ww'. 24696.

|      |         |       |
|------|---------|-------|
|      | 5832.   | 2744. |
| 9)   | 18      | 14.   |
|      | 324     | 196.  |
|      | 972     | 588.  |
|      | 10584   | 13608 |
|      | 32768.  |       |
|      | 9.      |       |
| ww'. | 294912. |       |

Scholar. In these examples I see, the woordes of your rule obserued. For vnder eche *Surde Cubike* roote, there is set a true *Cubike* number, whiche is founde by the common diuisor: then foloweth the roote of that true *Cube*: and beside it standeth the common diuisor. Then in the fourthe roome is the Square of the true *Cubike* roote. And vnder it his number tripled (as. 48 vnder. 16, and. 12. vnder. 4) whiche triple being multiplied by the roote of the other side, dooeth make the loweste number in that rowe. So. 48. multiplied by. 2. maketh. 96. whiche is set vnder the roote. 2. And. 12. multiplied by. 4. yeldeth. 48. whiche is placed vnder that. 4.

Then those. 4. extreme numbers. 64. and. 48. & 96. doe make by addition 216. whiche somme is multiplied by. 6. that is the common diuisor, and so riseth 1296. whose *Cubike* roote comprehendeth bothe the firste rootes.

Master. The like maie you iudge of the other. 2. examples. But because you maie vnderstande the certaintie of this woork the better, I haue here sette forth. 2. examples of true *Cubike* rootes, formed like *Surde* numbers.

Do. ij.

ww'. 4096

# The Arte

$$\begin{array}{r}
 \text{w'. } 4096. \text{ --- } | \text{---. w'. } 1728. \\
 \hline
 \begin{array}{r}
 512. \\
 8) \quad 8. \\
 \quad 64. \\
 \quad 192. \\
 \hline
 864.
 \end{array}
 \qquad
 \begin{array}{r}
 216. \\
 \hline
 6. \\
 36. \\
 108. \\
 \hline
 1152.
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 2744 \\
 8 \\
 \hline
 \text{w'. } 21952
 \end{array}$$

$$\begin{array}{r}
 \text{w'. } 19683. \text{ --- } | \text{---. w'. } 3375. \\
 \hline
 \begin{array}{r}
 729 \\
 27) \quad 9 \\
 \quad 81 \\
 \quad 243 \\
 \hline
 675
 \end{array}
 \qquad
 \begin{array}{r}
 125 \\
 \hline
 5 \\
 25 \\
 75 \\
 \hline
 1215
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 2744 \\
 27 \\
 \hline
 19208 \\
 5488 \\
 \hline
 \text{w'. } 74088
 \end{array}$$

Scholar. I perceive by examination of woork in my Tables here, that 4096. is a *Cubike* number, and hath 16 for his roote. So 1728 is a *Cubike* number also, & his roote is . 12. those bothe rootes added together, doe make. 28. And that. 28. is the

*Cubike* roote to. 21952. as the firste example would.

And for the seconde example, I see likewise that 19683. hath. 27. for his *Cubike* roote. And. 3375. hath 15. for his roote. And thei bothe make. 42, which is the *Cubike* roote to. 74088. according to the woork of the seconde example.

Addition of  
zenzike  
like rootes.

Master. Saying you are conveniently instructed, in these numbers, wee will goe in hande with zenzike rootes, and their additiō: wherein is no difference of woork, but onely for the multiplicatiō, which must be agreeable to the nature of the numbers, zenzikely. And the reduction by the common diuisiō,

for,

## of Surde numbers.

For, in like forme, into *zenzizenzike* numbers, whē the firste numbers bee *commensurable*. But if they be *incommensurable*, then must the addition be wrought by the signe. —, without any other businesse.

### Examples of *zenzizenzikes* being commensurable.

|   |   |
|---|---|
| $  \begin{array}{r}  \sqrt{.648} \text{---} \sqrt{.5000} \\  81 \qquad 625 \\  8) \quad 3 \qquad 55) \quad 4 \\  \qquad 8 \qquad \qquad 10 \\  \qquad 8 \qquad \qquad 10 \\  \hline  \qquad 4096 \qquad \qquad 10000 \\  \qquad 8 \qquad \qquad 5 \\  \hline  \sqrt{.32768} \qquad \qquad \sqrt{.50000}  \end{array}  $ | $  \begin{array}{r}  \sqrt{.1280} \text{---} \sqrt{.6480} \\  256 \qquad 1296 \\  5) \quad 4 \qquad 6 \\  \qquad 10 \\  \qquad 10 \\  \hline  \qquad 10000 \\  \qquad 5 \\  \hline  \sqrt{.50000}  \end{array}  $ |
|---|---|

|  |  |
|--|--|
| $  \begin{array}{r}  \sqrt{.38416} \text{---} \sqrt{.65536} \\  2401 \qquad 4096 \\  16) \quad 7 \qquad 8 \\  \qquad 15 \\  \qquad 15 \\  \hline  \qquad 50625 \\  \qquad 16 \\  \hline  \qquad 303750 \\  \qquad 50625 \\  \hline  \sqrt{.810000}  \end{array}  $ | <p>In the firste and seconde examples the numbers are <i>Surdes</i>, but in the thirde example they are ratiounall numbers, framed like vnto <i>Surdes</i> to the intende that you mighte the better perceiue the forme of the worke. For 38416 is a <i>zenzizenzike</i> number, &amp; hath. 14. for his roote</p> |
|--|--|

So. 65536. is a *zenzizenzike* number, and hath. 16. for his roote. And these. 2. rootes do make. 30. whiche is the *zenzizenzike* roote vnto. 810000. And therefore maie it bee truly saied, that.  $\sqrt{.810000}$  doeth containe the twoo firste rootes.

Scholar. I praye you proceede to Subtraction. For all this I doe well perceiue.

# The Arte Of Subtraction. Daster.



Subtraction doeth differ from addition, in little moare then the signe ———. whiche signe serueth generally, for all numbers incommensurable. And considering there is little difficultie in Subtraction: If you remember well the arte of Addition, I wil lightly passe it ouer in the same examples, that I haue wrought in Addition, because it maie bee a ppoofe of that woork: and that woork also a confirmation of this.

Onely this shall you obserue in this rule peculiarly: that as in the seconde forme of Addition, you must adde the rootes together, before you multiplie them. So here you shall Subtracte the lesser roote, from the greater, before you doe multiplie them.

Example of Subtraction, with ———.

$\sqrt{12}$ . abated out of  $\sqrt{26}$ . maketh.  $\sqrt{26}$  ———  $\sqrt{12}$ .  
and so of other.

## Examples of the seconde forme of Subtraction

| $\sqrt{63}$ . ——— $\sqrt{28}$ . |                     | The seconde forme<br>of that woork. |
|---------------------------------|---------------------|-------------------------------------|
| 63                              | 63                  | $\sqrt{63}$ . ——— $\sqrt{28}$ .     |
| 28                              | 28                  | 63                                  |
| 504                             | 91                  | 28                                  |
| 126                             |                     | 1764                                |
| $\sqrt{1764}$                   |                     | whose roote is. 42.                 |
| $\sqrt{4}$                      |                     | 42                                  |
| $\sqrt{7056}$                   |                     | 2                                   |
| $\sqrt{91}$ ——— $\sqrt{7056}$   |                     | 84                                  |
| 2. $\sqrt{91}$ ——— 84.          | $\sqrt{91}$ ——— 84. |                                     |
| That is. $\sqrt{7}$ .           |                     |                                     |

$\sqrt{169}$

# *of Surde numbers.*

|  |  |   |
|--|--|---|
| $\sqrt{.169} \text{ --- } \sqrt{.36}$  |  | An other forme of<br>that woorkie.  |
| $\begin{array}{r} 169 \\ 36 \\ \hline 1014 \\ 507 \\ \hline \sqrt{.} 6084 \\ \sqrt{.} 4 \\ \hline 24336 \\ \sqrt{.} 205 \text{ --- } \sqrt{.} 24336. \\ 02. \sqrt{.} 205 \text{ --- } \sqrt{.} 156. \end{array}$ | $\begin{array}{r} 169 \\ 36 \\ \hline 205 \end{array}$ | $\begin{array}{r} \sqrt{.} 169. \text{ --- } \sqrt{.} 36. \\ 169 \\ 36 \\ \hline \sqrt{.} 6084 \\ \text{whose roote is. } 78. \\ 78 \\ 2 \\ \hline 156 \\ \sqrt{.} 205 \text{ --- } 156. \end{array}$ |

That is.  $\sqrt{.} 49$ .

Scholar. I see in all these examples, you take the same numbers, that you had before in Addition. And firste you set the totalle, out of whiche you abate one of the nōbers, that before were added, & the remainder bringeth forth the other. For in the firste of these. 2. examples.  $\sqrt{.} 28$ . is abated out of.  $\sqrt{.} 63$ . and there remaineth.  $\sqrt{.} 91$ . --- 84. that is.  $\sqrt{.} 7$ . for. 84. taken out of. 91. leaueth. 7. And in the seconde exāple.  $\sqrt{.} 39$  abated out of.  $\sqrt{.} 169$ . doeth leaue remainyng.  $\sqrt{.} 49$ .

Master. The thirde forme of Subtraction, is like the thirde forme of Addition: saue that we set ---. for +. And here wee muste abate the lesser roote frō the greater (as I said) before we doe multiplie that number by it self. As by this exāple, you may perceiue Where I dooe Subtracte.  $\sqrt{.} 105$ . out.  $\sqrt{.} 1014$ . and the remainder is.  $\sqrt{.} 384$ . Now marke the woorkie

|  |  |
|--|--|
| $\sqrt{.} 1014 \text{ --- } \sqrt{.} 105$  | Here you see all thinges   |
| $\begin{array}{r} 169 \\ 25. \\ \hline 6) 13 \\ 8 \\ 8 \\ \hline 64 \\ 6 \\ \hline \sqrt{.} 384 \end{array}$ | agree, with the forme of Addition, saue ---. for +. and when I begin to gather the number, that standeth in the middle, whiche I multiplie by it selfe, and I dooe not make that number, |

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number, by addyng bothe rootes together: for so. 13. and. 5. would make. 18, but I adde. 5. out of 13. and so there dooth remain. 8. with whiche I procede as I did in Addition. And then commeth forth the remainer.

$\sqrt{384}$ .

Scholar. I vnderstande it very well. And I praye you that for a prooffe, I maie varie the other examles of addition. Partly for my exercise, and partly for examination of the former additions, by the contrary kind.

Master. With good will.

Scholar. When will I set them, and worke them, as here foloweth.

But firste I will begin, with the worke of this last example, after the seconde forme of Subtraction: for a double confirmation of it.

| $\sqrt{1014} \text{ — } \sqrt{150}$     |                         | ¶ An other forme of the same worke. |  |
|---|-------------------------|-------------------------------------|--|
| 1104                                    | 1014                    | $\sqrt{1014} \text{ — } \sqrt{150}$ |  |
| 150                                     | 150                     | 1014                                |  |
| 50700                                   | 1164                    | 150                                 |  |
| 1014                                    |                         | 50700                               |  |
| $\sqrt{152100}$                         |                         | 1014                                |  |
| $\sqrt{4}$                              |                         | 152100                              |  |
| $\sqrt{608400}$                         |                         | whose roote is. 390.                |  |
| $\sqrt{1164} \text{ — } \sqrt{608400}$  |                         | 390.                                |  |
| $\text{Or } \sqrt{1164} \text{ — } 780$ |                         | 2                                   |  |
|   | That is. $\sqrt{384}$ . | $\sqrt{1164} \text{ — } 780$ .      |  |

And now here are the variations of the other examples.

$\sqrt{2592}$ .



# Of Surde numbers.

$$\sqrt{.2592} \text{ --- } \sqrt{.288.}$$

$$\begin{array}{r} 72) \quad 36 \quad 4. \\ \quad 6 \quad 2. \\ \hline \quad \quad 4 \\ \quad \quad 4 \\ \hline \quad \quad 16 \\ \quad \quad 72 \\ \hline \quad \quad \quad 32 \\ \quad \quad \quad 112 \\ \hline \sqrt{.} \quad 1152 \end{array}$$

$$\sqrt{.2592} \text{ --- } \sqrt{.288.}$$

$$\begin{array}{r} 32) \quad 81 \quad 9. \\ \quad 9 \quad 3. \\ \hline \quad \quad 6 \\ \quad \quad 6 \\ \hline \quad \quad 36 \\ \quad \quad 32 \\ \hline \quad \quad \quad 72 \\ \quad \quad \quad 108 \\ \hline \sqrt{.} \quad 1152 \end{array}$$

$$\sqrt{.2592} \text{ --- } \sqrt{.288.}$$

$$\begin{array}{r} 18) \quad 144 \quad 16. \\ \quad 12 \quad 4. \\ \hline \quad \quad 8 \\ \quad \quad 8 \\ \hline \quad \quad 64 \\ \quad \quad 18 \\ \hline \quad \quad 512 \\ \quad \quad 64 \\ \hline \sqrt{.} \quad 1152 \end{array}$$

$$\sqrt{.2592} \text{ --- } \sqrt{.288.}$$

$$\begin{array}{r} 8) \quad 324 \quad 36. \\ \quad 18 \quad 6. \\ \hline \quad \quad 12 \\ \quad \quad 12 \\ \hline \quad \quad 144 \\ \quad \quad 8 \\ \hline \sqrt{.} \quad 1152 \end{array}$$

$$\sqrt{.2592} \text{ --- } \sqrt{.288.}$$

$$\begin{array}{r} 2) \quad 1296 \quad 144. \\ \quad 36 \quad 12. \\ \hline \quad \quad 24 \\ \quad \quad 24 \\ \hline \quad \quad 576 \\ \quad \quad 2 \\ \hline \sqrt{.} \quad 1152 \end{array}$$

$$\sqrt{.2592} \text{ --- } \sqrt{.1152.}$$

$$\begin{array}{r} 2) \quad 1296 \quad 576. \\ \quad 36 \quad 24. \\ \hline \quad \quad 12 \\ \quad \quad 12 \\ \hline \quad \quad 144 \\ \quad \quad 2 \\ \hline \sqrt{.} \quad 288 \end{array}$$

Other examples varied, for proofof the like .6.  
examples in Addition.

$$\text{pp.1.} \quad \sqrt{.64800}$$

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|   |   |
|---|---|
| $  \begin{array}{r}  \sqrt{.64800} \text{ --- } \sqrt{.7200} \\  32400 \qquad 3600 \\  2) \quad 180 \qquad 60 \\  \qquad 120 \\  \qquad 120 \\  \hline  \qquad 14400 \\  \qquad 2 \\  \hline  \sqrt{.28800}  \end{array}  $ | $  \begin{array}{r}  \sqrt{.64800} \text{ --- } \sqrt{.7200} \\  8100 \qquad 900 \\  8) \quad 90 \qquad 30 \\  \qquad 60 \\  \qquad 60 \\  \hline  \qquad 3600 \\  \qquad 8 \\  \hline  \sqrt{.28800}  \end{array}  $ |
|---|---|

|  |  |
|--|--|
| $  \begin{array}{r}  \sqrt{.64800} \text{ --- } \sqrt{.7200} \\  3600 \qquad 400 \\  18) \quad 60 \qquad 20 \\  \qquad 40 \\  \qquad 40 \\  \hline  \qquad 1600 \\  \qquad 18 \\  \hline  \qquad 12800 \\  \qquad 16 \\  \hline  \sqrt{.28800}  \end{array}  $ | $  \begin{array}{r}  \sqrt{.64800} \text{ --- } \sqrt{.7200} \\  2025 \qquad 225 \\  32) \quad 45 \qquad 15 \\  \qquad 30 \\  \qquad 30 \\  \hline  \qquad 900 \\  \qquad 32 \\  \hline  \sqrt{.28800}  \end{array}  $ |
|--|--|

|  |   |
|--|---|
| $  \begin{array}{r}  \sqrt{.64800} \text{ --- } \sqrt{.7200} \\  1296 \qquad 144 \\  50) \quad 36 \qquad 12 \\  \qquad 24 \\  \qquad 24 \\  \hline  \qquad 576 \\  \qquad 50 \\  \hline  \sqrt{.28800}  \end{array}  $ | $  \begin{array}{r}  \sqrt{.64800} \text{ --- } \sqrt{.7200} \\  900 \qquad 100 \\  72) \quad 30 \qquad 10 \\  \qquad 20 \\  \qquad 20 \\  \hline  \qquad 400 \\  \qquad 72 \\  \hline  \sqrt{.28800}  \end{array}  $ |
|--|---|

*Subtraction  
of Cubike  
rootes.*

Master. Like difference is there in Subtraction of Cubike rootes commensurable. And therfore I set the examples onely, without any larger declaration.

ww.375.

# of Surde numbers.

|  |   |
|--|---|
| $  \begin{array}{r}  \text{ww}.375 \text{ --- } \text{ww}.81. \\  125 \qquad 27. \\  3) \quad 5 \qquad 3 \\  \qquad \quad 2 \\  \qquad \quad 2 \\  \qquad \quad \hline  \qquad \quad 8 \\  \qquad \quad 3 \\  \qquad \quad \hline  \text{ww}. 24  \end{array}  $ | $  \begin{array}{r}  \text{ww}.1715 \text{ --- } \text{ww}.135. \\  343 \qquad 27. \\  5( \quad 7 \qquad 3. \\  \qquad \quad 4 \\  \qquad \quad 4 \\  \qquad \quad \hline  \qquad \quad 64 \\  \qquad \quad 5 \\  \qquad \quad \hline  \text{ww}. 320  \end{array}  $ |
|--|---|

$$\begin{array}{r}
 \text{ww}.13824 \text{ --- } \text{ww}.1000 \\
 1728 \qquad 125 \\
 8) \quad 12 \qquad 5 \\
 \qquad \quad 7 \\
 \qquad \quad 7 \\
 \qquad \quad \hline
 \qquad \quad 343 \\
 \qquad \quad 8 \\
 \qquad \quad \hline
 \text{ww}. 2744
 \end{array}$$

In the seconde forme of *Another*  
 addition of *Surde Cubes*, you *woorke of*  
 remember that you added *Subtraction*  
 4 numbers together. But *for Surde*  
 in Subtraction, you shall *Cubes.*  
 adde to eche roote seuerall  
 lie that, that commeth of  
 his owne multiplication,  
 with the other triple. And

then shall you Subtracte the lesser number, out of the  
 greater. And the remainder you shall multiplie by the  
 common diuisor. And so shall you haue the roote that  
 remaineth of the Subtraction. As in example,

|  |  |
|--|--|
| $  \begin{array}{r}  \text{ww}.1296 \text{ --- } \text{ww}.48. \\  216 \qquad 8 \\  6) \quad 6 \qquad 2 \\  \quad 36 \qquad 4 \\  \quad 108 \qquad 12 \\  \quad \quad 72 \qquad 216 \\  \qquad \quad 64 \\  \qquad \quad 6 \\  \qquad \quad \hline  \text{ww}. 384  \end{array}  $ | $  \begin{array}{r}  \text{ww}.58956 \text{ --- } \text{ww}.15972 \\  4913 \qquad 1331 \\  12) \quad 17 \qquad 11 \\  \quad 289 \qquad 121 \\  \quad 867 \qquad 363 \\  \quad \quad 6171 \qquad 5537 \\  \qquad \quad 216 \\  \qquad \quad 12 \\  \qquad \quad \hline  \text{ww}. 2592  \end{array}  $ |
|--|--|

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$$\begin{array}{r}
 \text{w/} 294912 \text{ --- } \text{w/} 24696 \\
 \underline{32768} \qquad \qquad \underline{2744} \\
 9) \quad 32 \qquad \qquad \qquad 14 \\
 \quad 1024 \qquad \qquad \qquad 196 \\
 \quad \underline{3072} \qquad \qquad \qquad \underline{588} \\
 \quad 18816 \qquad \qquad \underline{43008} \\
 \qquad \qquad \qquad 5832 \\
 \qquad \qquad \qquad \underline{9} \\
 \text{w/} \quad 52488
 \end{array}$$

Scholar. In all these examples I see the confirmation of the former additio. And in these laste woorkes, this I see peculiere from additio, that the Cube is added with the loweste number in

that rowe (as in the firste example. 216. is added with 72. and maketh. 288: And. 8. is added with. 216. that yeldeth. 224.) And then is the lesser abated from the greater (as. 224. from 288.) And the remainder (whiche there is. 64) set in the middle vnder bothe the reues of numbers. And then is multiplied by the common number, to make the remainder.

So in the firste example, the remainder is. w/. 384. where. w/. 48. is abated out of. w/. 1296. And in the seconde example where. w/. 15972. is subtracted out of. w/. 58956. the remainder is w/. 2592. Like wises in the thirde example. w/. 24696. is abated out of. w/ 294912 & leaueth remainyng. w/ 52488

Master. But now in addition there foloweth. 2. other examples, whiche by subtraction maie bee produced thus: as here you see.

$$\begin{array}{r}
 \text{w/} 21952 \text{ --- } \text{w/} 4096 \quad \text{w/} 74088 \text{ --- } \text{w/} 19683 \\
 \underline{2744} \qquad \qquad \underline{512} \quad \underline{2744} \qquad \qquad \underline{729} \\
 8) \quad 14 \qquad \qquad \quad 8 \quad 27) \quad 14 \qquad \qquad \quad 9 \\
 \quad 196 \qquad \qquad \quad 64 \quad \quad 196 \qquad \qquad \quad 81 \\
 \quad \underline{588} \qquad \qquad \underline{192} \quad \quad \underline{588} \qquad \qquad \underline{243} \\
 \quad 2688 \qquad \quad 4704 \quad \quad 3402 \qquad \quad 5292 \\
 \qquad \qquad \quad 216 \qquad \qquad \quad 125 \\
 \qquad \qquad \quad \underline{8} \qquad \qquad \quad \underline{27} \\
 \text{w/} \quad 1728 \qquad \qquad \text{w/} \quad 3375
 \end{array}$$

Scholar.

## of Surde numbers.

**Scholar.** I see, in these examples of Subtraction: that the firste number is the totalle, or laste number in addition. And the seconde number, whiche foloweth — is the number to be abated: and then laste and loweste of all, is the remainer, whiche was one of the firste sommes in addition.

And though there remaine. 3. other exaples of *zenzike* numbers, I see no difficultie in theini, but that I can woozke them: As here I haue set the forth.

|  |  |
|--|--|
| $  \begin{array}{r}  \sqrt{32768} \quad \sqrt{648} \\  \hline  8 \overline{) 4096} \qquad 81 \\  \underline{\phantom{0}8} \phantom{000} \\  \phantom{0}5 \phantom{00} \\  \phantom{0}5 \phantom{00} \\  \underline{\phantom{0}625} \phantom{0} \\  \phantom{0}8 \phantom{00} \\  \hline  \sqrt{5000}  \end{array}  $ | $  \begin{array}{r}  \sqrt{5000} \quad \sqrt{1280} \\  \hline  5 \overline{) 10000} \qquad 256 \\  \underline{\phantom{0}10} \phantom{000} \\  \phantom{0}6 \phantom{00} \\  \phantom{0}6 \phantom{00} \\  \underline{\phantom{0}1296} \phantom{0} \\  \phantom{0}5 \phantom{00} \\  \hline  \sqrt{6480}  \end{array}  $ |
|--|--|

$$\begin{array}{r}
 \sqrt{810000} \quad \sqrt{65536} \\
 \hline
 16 \overline{) 50625} \qquad 4096 \\
 \underline{\phantom{0}15} \phantom{000} \\
 \phantom{0}7 \phantom{00} \\
 \phantom{0}7 \phantom{00} \\
 \underline{\phantom{0}2401} \phantom{0} \\
 \phantom{0}16 \phantom{00} \\
 \hline
 \sqrt{38416}
 \end{array}$$

**Master.** Seeing you are experte inough in the 5. woozkes of these *Surdes* uncomponde, I wil teache you the like woozkes in componde *Surdes*.

**Scholar.** Is there the *Of reduction*  
 noe reduction, nother ex- *and extracti-*  
 traction of rootes, to bee *on of rootes.*

taughte in these uncomponde *Surdes*:

**Master.** As for reduction, I haue taughte you all readie in multiplication, as moche as is required in these numbers.

And for extraction of rootes, you maie sone vnderstande, that here can be none. For then were thei not *Surde* numbers. And therfoze I saied vnto you befoze,

Dp. iij. that

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that.  $\sqrt{8}$ .  $100$ . is not a *Surde* number, although it be written like a *Surde* number, because it hath a *Square* roote, according to his signe: and that is.  $10$ . Likewaies.  $\sqrt{256}$ . is no *Surde* number: for his *Square* roote is knowen to be.  $16$ .

Scholar. I might haue considered as moche, by the definition of *Surde* numbers, that their rootes can not be assigned in numbers absolute. And therefore I see that.  $\sqrt[3]{125}$ . is no *Surde* number, with his *Cubike* roote is.  $5$ . And.  $\sqrt[3]{256}$ . is a number *rationalle*, and no *Surde* number: for his *zenzizenzike* roote is.  $4$ .

Master. But.  $\sqrt[3]{64}$ . is a *Surde* number, and yet hath.  $64$ . a *Square* roote, and a *Cubike* roote also, but not a *zenzizenzike* roote, according to his signe. And therefore ought better to be written thus.  $\sqrt{8}$ .

Scholar. I praise you to procede to *Surde* numbers compounde.

### Of *Surde* numbers compounde.

Master.



*Urde* numbers compoſide, are made not onely of.  $2$ .  $3$ . or more *Surde* numbers or compoſide, but alſo of *rationalle* or *Abſtraſte* numbers ioyned with *Surde* numbers. As.  $\sqrt{10}$   
 $+\sqrt{12}$ . and.  $8$ .  $+\sqrt{6}$ . likewaies.  $\sqrt{20}$ .  $3$ . and.  $\sqrt{40}$ .

$+\sqrt{14}$ .  $3$ .

Compounde  
Surdes.

But here ſhall you marke, that I call compoſide numbers, not onely ſoche as haue the ſigne of.  $+\sqrt{\quad}$ , but alſo ſoche as haue the ſigne of.  $-\sqrt{\quad}$  for although in nature of the number  $\sqrt{10}$   $\sqrt{5}$ . be not compoſide, but abated, yet in name he is compoſide, and augmented. For.  $-\sqrt{\quad}$ . doth as well augmented the

## of Surde numbers.

the name, as — + — doeth.

Scholar. It seemeth reasonable. For when I saie,  $\sqrt{.12.} \text{ --- } \sqrt{.7.}$  the name is compounde, as well as if I had saied.  $\sqrt{.12.} \text{ --- } + \text{ --- } \sqrt{.7.}$  although the quantitie bee not so greate. For — — — doeth ener abate the quantitie of the nūber, though it do increase the name.

Master. Yet for a difference, the numbers that be compounde with — + — be called *Bimedialles*: and those *Bimedialles* that be compounde with — — —, be named *Residualles*. *Residualles*. And if the *Bimedialles* haue all their numbers and partes of one denominations, then bee they called onely by their generall name *Bimedialles*. But if their partes be of 2. denominations, then are they named *Binomialles* properly. Whobett, many vse to call *Binomialles* *Binomisalles*. all compounde numbers that haue — + —. And so wil I let the names passe.

*Euclides* definitions doe not very aptly agree to this place, as at an other tyme I will shewe you, and therefore I doe omitte them for this tyme.

But touching our principalle intent, whiche is to declare the practike woork of *Binomialles*, and *Residualles*, there is litle difficultie, if you marke well that whiche is taught before. For as *Binomialles* and *Residualles*, bee made of *Surdes*, or els of *rationalle* numbers with *Surdes*, so the woork of the compounde numbers dependeth of the woork of the simple numbers, and is all one with them. And concerning the signes — + — and. — — —. here is no moare to bee saied, then was taughte in *Coslike* numbers compounde.

Scholar. Yet of every kinde, it maie please you to set forth the some examles.

Master. I thinke that mete, without many wordes els. Not forgettyng by the waie, that *vniverſalle rootes*, are not accompted emongest these compounde *Surdes*: but are reserved to their peculiere treatice, as *rootes of compounde Surdes*.

## The Arte Of Numeration.

Numeration is moare plain, then that I neede to stande in declaring it, other waies then by examples; As here you see.

### Examples of Binomialles.

6. ————  $\sqrt{\cdot}$  8. That is 6 moze the Square roote of 8.  
 $\sqrt{\cdot}$  20 ———— .3. Is the Square roote of 20. moare .3.  
 $\sqrt[3]{\cdot}$  30 ————  $\sqrt{\cdot}$  9. Signifieth the Cubike roote of. 30.  
 moze the *zenzizenzike* roote of. 9.  
 And so of other.

### Examples of Residualles.

24. ————  $\sqrt{\cdot}$  96. That is 24. abating the roote of 96  
 $\sqrt{\cdot}$  150. ———— .9. Is the Square roote of 150. abating 9  
 $\sqrt{\cdot}$  5208 ————  $\sqrt[3]{\cdot}$  35. The *zenzizenzike* roote of. 5208.  
 saue the Square roote of. 35. And so  
 fo: the.

Scholar. So I see any Surdes maie bee compounde with other: And any nōbers *rationalle* ioined with the.

## Of Addition.

Master. Addition is as plaine. For as the partes bee, so shall the Addition bee, acco:dyng as you haue learned before.

### Examples of Binomialles.

|  |  |                               |
|--|--|-------------------------------|
| $\sqrt{\cdot}$ 50 ———— 10                    | $\sqrt{\cdot}$ 15. ———— $\sqrt{\cdot}$ 15.   | $\sqrt{\cdot}$ 1264 ———— 8.   |
| $\sqrt{\cdot}$ 2. ———— .8.                   | $\sqrt{\cdot}$ 18 ———— $\sqrt{\cdot}$ 60.    | 28 ———— $\sqrt[3]{\cdot}$ 316 |
| $\sqrt{\cdot}$ 72 ———— 18                    | $\sqrt{\cdot}$ 33. ———— $\sqrt{\cdot}$ 135.  | 36 ———— $\sqrt{\cdot}$ 2844   |
| $\sqrt[3]{\cdot}$ 48. ———— $\sqrt{\cdot}$ 5. | $\sqrt[3]{\cdot}$ 32 ———— $\sqrt{\cdot}$ 10. |                               |
| $\sqrt{\cdot}$ 243. ———— $\sqrt{\cdot}$ 45.  | $\sqrt[3]{\cdot}$ 4. ———— $\sqrt{\cdot}$ 19. |                               |
| $\sqrt{\cdot}$ 1875 ———— $\sqrt{\cdot}$ 80.  | $\sqrt[3]{\cdot}$ 108 ———— $\sqrt{\cdot}$ 29 | $\sqrt{\cdot}$ 760            |

Examples



## Of Surde numbers.

### Examples of Residualles.

$$\begin{array}{r|l}
 \sqrt{.75} \text{ --- } .4 & 14 \text{ --- } \sqrt{.3} \quad | \quad 250 \text{ --- } \sqrt{.108} \\
 \sqrt{.3} \text{ --- } 1 & 16 \text{ --- } \sqrt{.27} \quad | \quad \sqrt{.44} \text{ --- } 76 \\
 \sqrt{.108} \text{ --- } 5 & 30 \text{ --- } \sqrt{.12} \quad | \quad 174 \text{ --- } \sqrt{.275}
 \end{array}$$

$$\begin{array}{r|l}
 \sqrt{.72} \text{ --- } \sqrt{.96} & \sqrt{.32} \text{ --- } \sqrt{.5} \\
 \sqrt{.9} \text{ --- } \sqrt{.6} & \sqrt{.32} \text{ --- } \sqrt{.24} \\
 \sqrt{243} \text{ --- } \sqrt{162} & \sqrt{512} \text{ --- } \sqrt{29} \text{ --- } \sqrt{480}
 \end{array}$$

### Examples of Binomialles with Residualles.

$$\begin{array}{r|l}
 \sqrt{.80} \text{ --- } 6 & 30 \text{ --- } \sqrt{.20} \quad | \quad 561 \text{ --- } \sqrt{512} \\
 \sqrt{.5} \text{ --- } 2 & 12 \text{ --- } \sqrt{.5} \quad | \quad \sqrt{288} \text{ --- } 340 \\
 \sqrt{.125} \text{ --- } 4 & 42 \text{ --- } \sqrt{.5} \quad | \quad 901 \text{ --- } \sqrt{1568}
 \end{array}$$

$$\begin{array}{r|l}
 \sqrt{.63} \text{ --- } \sqrt{160} & \sqrt{.320} \text{ --- } \sqrt{.56} \\
 \sqrt{.7} \text{ --- } \sqrt{.20} & \sqrt{.40} \text{ --- } \sqrt{.24} \\
 \sqrt{.112} \text{ --- } \sqrt{684} & \sqrt{1680} \text{ --- } \sqrt{.80} \text{ --- } \sqrt{5376}
 \end{array}$$

Scholar. I see that you make severalle Additions in all these numbers. For you adde still like numbers with their matches. So that here is nothyng diuers from the woordes of simple Surdes. Although in euery thirde example, there appeare moare difficultie, then there is in deede: When I consider the like transposition in Cosike numbers. For the woorde addeth like numbers together.

## Of Subtraction.

Master. In Subtraction there is as litle diuersitie. As these examples will sufficiently declare: whiche be set as trialles of the former Additions.

293. Examples

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### Examples of Binomialles.

$$\begin{array}{r} \sqrt{.72} \text{---} | \text{---} 18 \\ \sqrt{.2} \text{---} | \text{---} 8. \\ \hline \sqrt{.50} \text{---} | \text{---} 10 \end{array}$$

$$\begin{array}{r} 36 \text{---} | \text{---} \sqrt{2844} \\ \sqrt{.1264} \text{---} | \text{---} 8. \\ \hline 28 \text{---} | \text{---} \sqrt{.316} \end{array}$$

$$\begin{array}{r} 33 \text{---} | \text{---} \sqrt{.135} \\ 15 \text{---} | \text{---} \sqrt{.15} \\ \hline 18 \text{---} | \text{---} \sqrt{.60} \end{array}$$

$$\begin{array}{r} w\sqrt{.1875} \text{---} | \text{---} \sqrt{.80} \\ w\sqrt{.48} \text{---} | \text{---} \sqrt{.5} \\ \hline w\sqrt{.243} \text{---} | \text{---} \sqrt{.45} \end{array}$$

$$\begin{array}{r} w\sqrt{.108} \text{---} | \text{---} \sqrt{.29} \text{---} | \text{---} \sqrt{.760} \\ w\sqrt{.4} \text{---} | \text{---} \sqrt{.19} \\ \hline w\sqrt{.32} \text{---} | \text{---} \sqrt{.10} \end{array}$$

### Examples of Residualles.

$$\begin{array}{r} \sqrt{.108} \text{---} | \text{---} 5 \\ \sqrt{.3} \text{---} | \text{---} 1 \\ \hline \sqrt{.75} \text{---} | \text{---} 4 \end{array}$$

$$\begin{array}{r} 174 \text{---} | \text{---} \sqrt{.275} \\ \sqrt{.44} \text{---} | \text{---} 76 \\ \hline 250 \text{---} | \text{---} \sqrt{.108} \end{array}$$

$$\begin{array}{r} 30 \text{---} | \text{---} \sqrt{.12} \\ 14 \text{---} | \text{---} \sqrt{.3} \\ \hline 16 \text{---} | \text{---} \sqrt{.27} \end{array}$$

$$\begin{array}{r} w\sqrt{.243} \text{---} | \text{---} w\sqrt{.162} \\ w\sqrt{.9} \text{---} | \text{---} w\sqrt{.6} \\ \hline w\sqrt{.72} \text{---} | \text{---} w\sqrt{.96} \end{array}$$

$$\begin{array}{r} w\sqrt{.512} \text{---} | \text{---} \sqrt{.29} \text{---} | \text{---} \sqrt{.480} \\ w\sqrt{.32} \text{---} | \text{---} \sqrt{.5} \\ \hline w\sqrt{.32} \text{---} | \text{---} \sqrt{.24} \end{array}$$

### Examples of bothe together.

$$\begin{array}{r} \sqrt{.125} \text{---} | \text{---} 4 \\ \sqrt{.5} \text{---} | \text{---} 2 \\ \hline \sqrt{.80} \text{---} | \text{---} 6 \end{array}$$

$$\begin{array}{r} 901 \text{---} | \text{---} \sqrt{1568} \\ \sqrt{.288} \text{---} | \text{---} 340 \\ \hline 561 \text{---} | \text{---} \sqrt{.512} \\ 42. \end{array}$$

## of Surde numbers.

$$\begin{array}{rcl}
 42 - + - \sqrt{.5} & \sqrt{.112} - - - \sqrt{.648} & \\
 12 - - - \sqrt{.5} & \sqrt{.7} - + - \sqrt{.20} & \\
 \hline
 30 - + - \sqrt{.20} & \sqrt{.63} - - - \sqrt{.160} & \\
 \\
 \sqrt{.1080} - - - \sqrt{.80} - - - \sqrt{.5376} & & \\
 \sqrt{.40} - + - \sqrt{.24} & & \\
 \hline
 \sqrt{.320} - - - \sqrt{.56} & & 
 \end{array}$$

Scholar. This is as easie as Addition, saue for 3. exam-  
 ples, whiche I vnderstande not. For although I see the laste  
 example, of eche of the sortes of nom-  
 bers, to bee agreable with the like examples in Addi-  
 tion, yet I can not so well perceiue, the order of their  
 Subtraction, as I doe knowe the maner of their Ad-  
 ditiō. For by the arte of simple Surdes, I see that  $\sqrt{.10}$   
 and  $\sqrt{.19}$  doe make  $\sqrt{.29} - + - \sqrt{.760}$ . But when  
 $\sqrt{.29} - + - \sqrt{.760}$  is set as a totalle, and  $\sqrt{.19}$  to  
 be Subtracted out of it, how I shall woorke that, and  
 leaue  $\sqrt{.10}$  for the remainer, I see not.

So in the *residualles*, I knowe how  $\sqrt{.5}$  and  
 $\sqrt{.24}$  doe make  $\sqrt{.29} - + - \sqrt{.480}$ . But I knowe  
 not how  $\sqrt{.5}$  abated out of  $\sqrt{.29} - + - \sqrt{.480}$  doeth  
 make for the remainer  $\sqrt{.24}$ .

And the like doubt is in the thirde sorte of Surdes,  
 whiche are mixte numbers. For where I see in Addi-  
 tion  $- + - \sqrt{.24}$  added with  $- - - \sqrt{.56}$ . And the  
 totalle to bee  $- - - \sqrt{.80} - - - \sqrt{.5376}$ . I knowe  
 the reason of the woorke, for the signes  $- + -$  and  
 $- - -$ . by that I learned in *Cosike* numbers: And the  
 reaste is manifeste by Addition of simple Surdes. For it  
 is wrought by abatynge  $\sqrt{.24}$  out of  $\sqrt{.56}$ . But then  
 in Subtraction, how  $- + - \sqrt{.24}$  being Subtracted  
 from  $- - - \sqrt{.80} - - - \sqrt{.5376}$  shall leaue  $- \sqrt{.56}$   
 I can not iudge. And yet by the signes I gesse (as I  
 learned in *Cosike* numbers) that it is doen by Additiō,  
 because the signes doe disagree.

An. yf. Master.

## The Arte

Maſter. In that you remember the former rules, to conferre them aptly with theſe later woꝝkes, I can praiſe you well. But in that you can not vnderſtande the reaſon of that, whiche was not yet taughte you, I can not greatly blame you. Although I can not praiſe you, foꝛ that you thinke your ſelf to be cunninger then you are. Foꝛ in thoſe Additions, that you thinke your ſelf to be experte inough, I'dare ſaie, that you bee diſceiued, if you take them to bee numbers of any ſoche, as hether to hath been taughte vnto you.

Scholar. I take them foꝛ compounde *Surdes*.

Maſter. Thei are not ſo: Nother is their woꝝke agreeable, with the woꝝke of compounde *Surdes*. But thei are the rootes of compounde *Surdes*: And therfoꝛe are called *vninerſalle rootes* of *Surdes*. And accoꝛdyng to their proper nature, thei ought to bee called rootes of *Surdes*, and not *Surde* rootes. As I will tell you anon. When I will alſo diſcuſſe your doubt.

But befoꝛe I ſpeake any moare of theim, I will cande the woꝝkes of theſe compounde *Surdes*: Where of. 2. kindes yet remaine behinde.

### Of Multiplication.



Multiplicotio of compounde *Surdes*, is as eaſie as can bee. And differeth in nothyng, frō the woꝝke of ſimple *Surdes*. Onely this muſt you marke, as reaſon would, that you muſte multiplie euery parte of the one nōber, by euery parte of the other nōber: as you remember the woꝝke of compounde *Coſike* numbers.

Scholar. I praie you giue me ſome examples.

Maſter. That ſhall you haue. And that maie ſuffice foꝛ this woꝝke. Marke them well therfoꝛe.

Examles

## *of Surde numbers.*

### *Examples of Binomialles.*

$$\begin{array}{r}
 23 \text{ --- } \sqrt{.15.} \\
 6 \text{ --- } \sqrt{.8.} \\
 \hline
 138 \text{ --- } \sqrt{.120.} \\
 \text{--- } \sqrt{.540.} \text{ --- } \sqrt{.4232.} \\
 \hline
 138 \text{ --- } \sqrt{.4232} \text{ --- } \sqrt{.540} \text{ --- } \sqrt{.120.}
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.120} \text{ --- } \sqrt{.12.} \\
 \sqrt{.12} \text{ --- } \sqrt{.7.} \\
 \hline
 \sqrt{1440} \text{ --- } \sqrt{.84.} \\
 \text{--- } \sqrt{.840} \text{ --- } 12. \\
 \hline
 12 \text{ --- } \sqrt{.1440} \text{ --- } \sqrt{.840} \text{ --- } \sqrt{.84.}
 \end{array}$$

### *Examples of Residualles.*

$$\begin{array}{r}
 5. \text{ --- } \sqrt{.10.} \\
 5. \text{ --- } \sqrt{.10.} \\
 \hline
 25 \text{ --- } 10. \\
 \text{--- } \sqrt{.250} \text{ --- } 250. \\
 \hline
 35 \text{ --- } \sqrt{.1000.}
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.24} \text{ --- } \sqrt{.20.} \\
 \sqrt{.30} \text{ --- } \sqrt{.24.} \\
 \hline
 \sqrt{.720} \text{ --- } \sqrt{.480.} \\
 \text{--- } 24 \text{ --- } \sqrt{.600.} \\
 \hline
 \sqrt{.720} \text{ --- } \sqrt{.480} \text{ --- } 24 \text{ --- } \sqrt{.600.}
 \end{array}$$

### *Examples of bothe together.*

$$\begin{array}{r}
 32 \text{ --- } \sqrt{.14.} \\
 \sqrt{.124} \text{ --- } 6. \\
 \hline
 \sqrt{.126976} \text{ --- } \sqrt{.1736.} \\
 \text{--- } .192. \text{ --- } \sqrt{.504.} \\
 \hline
 \sqrt{.126976} \text{ --- } \sqrt{.1736} \text{ --- } 192 \text{ --- } \sqrt{.504} \\
 \text{--- } 29.117. \quad \sqrt{.52.}
 \end{array}$$

## The Arte

$$\begin{array}{r}
 \sqrt{\phantom{00}}. \quad 52 \overline{) 17} \\
 \underline{17} \phantom{00} \sqrt{52} \\
 \hline
 \sqrt{15028} \overline{) 289} \\
 \underline{52} \phantom{00} \sqrt{15028} \\
 \hline
 37.
 \end{array}$$

Scholar. Multiplication, as I see, is the easieſte woꝛke of all the other. So that I dooe marke the reduction, in gatherynge the totalle: whiche is easie enough to vnderstand, by that I haue learned in *Cosike* numbers. And Diuision be no harder, it maie lone be learned.

### Of Diuision.

Master.



Diuision by one simple number, is no moare difficulte: as these exam-  
ples doe declare. Where the diuisor  
is a number vncompounde.

$\sqrt{26} \overline{) 15}$  diuided by .5. doeth  
make.  $\sqrt{1 \frac{1}{5}} \overline{) 3}$ .

Againe.  $\sqrt{56} \overline{) 24}$  diui-  
ded by.  $\sqrt{6}$ . doeth yelde.  $\sqrt{9 \frac{1}{3}} \overline{) 2}$ .

And so  $\sqrt{75} \overline{) 48}$ . diuided by.  $\sqrt{3}$ . dooeth  
byng sothe.  $\sqrt{5} \overline{) 4}$ . that is. 1.

Likewises.  $\sqrt{320} \overline{) 180}$ . beynge parted by  
 $\sqrt{5}$ . doeth make the *quotiente*. 14.

Scholar. I see it so. For at the firste it is.  $\sqrt{64}$ .  
 $\overline{) 36}$ . that is. 8.  $\overline{) 6}$ . whiche maketh. 14.

Master. So maie you woꝛke all like diuisions.  
But when the diuisor is a compounde number, then  
must you vse an other meane: that is to reduce that  
compounde nōber, to a simple number: whiche thing  
you maie easly doe, by multipling any *Binomialle*, by  
his *Residuale*, or contrary waies, the *Residuale* by his  
*Binomialle*.

As

## of Surde numbers.

As  $6 \div \sqrt{10}$  multiplied by  $6 \div \sqrt{10}$  doeth make 26.

And so  $\sqrt{8} \div \sqrt{5}$  multiplied by  $\sqrt{8} \div \sqrt{5}$  doeth yelde 8. — 5. that is 3.

Scholar. I perceiue a brief waie in this multiplication: For I heade not in the firste example, to multiply 6. by  $\sqrt{10}$ . sith it would amounte to nothing. In so moche as at one multiplication, it would bee  $\div$ , and at an other. —. And so the one would abate the other, and leaue nothing for them bothe.

Master. That is well marked. And it is so generally. Therefore (as you see) the diuisor by this meanes, maie lightly be tourned into a simple number, or a plaine absolute number.

And now to make the diuidende, in the same proportion, to this new diuisor, that it was vnto the old diuisor, you shall multiply it by the same number, by whiche the diuisor was multiplied. For if any numbers bee multiplied, by one common number, their newe totalles kepe the same proportion, that was betwene the firste numbers.

Scholar. That must needs be so. For as 3. is *sesquialtera* vnto 2. so if you multiply them by 5. thei will make 15. and 10. whiche be in *sesquialtera* proportion and like waies will their proportion remain, by what so euer number thei be multiplied. Therefore it must needs be reasonable, that if the diuidende and the diuisor, be multiplied by any one number, simple or compounde, thei shall kepe the same proportion, that thei had before.

Master. For more certain vnderstandyng of this rule, take these examples. The firste is, where  $\sqrt{58} \div \sqrt{54}$  is sette to bee diuided by  $\sqrt{6} \div \sqrt{3}$ .

Here firste I multiply the diuisor by his contrarie, that is his *Binomi*:

$$\begin{array}{r} \sqrt{6} \div \sqrt{3} \\ \sqrt{6} \div \sqrt{3} \end{array}$$

$$\begin{array}{r} 6 \div 3 \\ \text{What is } 3. \end{array}$$

all

# The Arte

alle.  $\sqrt{.6}$  ———  $\cdot 3$ . And there riseth.  $\sqrt{.6}$  ———  $\cdot 3$ . that is.  $\cdot 3$   
whiche I shall kepe for the newe diuifor.

Then doe I multiplie the diuident  $\sqrt{.68}$  ———  $\sqrt{.54}$   
by the same Residuale.

$$\begin{array}{r} \sqrt{.68} \text{ --- } \sqrt{.54} \\ \sqrt{.6} \text{ --- } \sqrt{.3} \\ \hline \sqrt{.408} \text{ --- } \sqrt{.324} \\ \text{ --- } \sqrt{.204} \text{ --- } \sqrt{.162} \\ \hline \sqrt{.408} \text{ --- } \sqrt{.324} \text{ --- } \sqrt{.204} \text{ --- } \sqrt{.162} \end{array}$$

And there doth amounte, as here in woork is expresse.

$$\sqrt{.408} \text{ --- } \sqrt{.324} \text{ --- } \sqrt{.204} \text{ --- } \sqrt{.162}$$

whiche number shall be taken for the newe diuident:  
and must be diuided by.  $\cdot 3$ . that is the newe diuifor. In  
whose steede I set.  $\sqrt{.9}$ . for moare redinesse in woork.  
Therefore I set the done in order, as here foloweth.

$$\begin{array}{ccccccc} \sqrt{.408} \text{ --- } \sqrt{.324} \text{ --- } \sqrt{.204} \text{ --- } \sqrt{.162} & (\sqrt{.45\frac{1}{3}} \text{ --- } 6 \text{ --- } \sqrt{22\frac{2}{3}} \text{ --- } \sqrt{18} \\ \sqrt{.9} & \sqrt{.9} & \sqrt{.9} & \sqrt{.9} \end{array}$$

And then doe I seeke how often.  $\sqrt{.9}$ . maie bee founde  
in.  $\sqrt{.408}$ . whiche maie bee.  $45\frac{1}{3}$  of tymes. Where-  
fore I set.  $\sqrt{.45\frac{1}{3}}$  in the *quotiente*. And then doe I re-  
terate the diuifor, and sette it vnder.  $\sqrt{.324}$ . where I  
finde it.  $\cdot 36$ . tymes: and therefore set  $\cdot 6$ . for it, because  
the *quotiente* els would bee.  $\sqrt{.36}$ . whiche is iustly.  $\cdot 6$ .  
Thirdly, I remoue the diuifor vnder  $\sqrt{.204}$ . where  
it maie bee founde.  $22\frac{2}{3}$  tymes. For whiche I sette  
 $\sqrt{.22\frac{2}{3}}$  in the *quotiente*. And then set I the diuifor last  
of all vnder.  $\sqrt{.162}$ . where it is founde.  $\cdot 18$ . tymes: and  
for that cause I set  $\sqrt{.18}$ . in the *quotiente*: And so is the  
whole *quotiente*  $\sqrt{.45\frac{1}{3}} \text{ --- } 6 \text{ --- } \sqrt{22\frac{2}{3}} \text{ --- } \sqrt{18}$ .

Scholar. This diuision is straunge to credite, al-  
though it be not difficulte to woork.

Master. If you doubt of it, you maie vse the ac-  
customable trialle by the contrary kinde.

Scholar.



## of Surde numbers.

Scholar. So must it folowe, that if I dooe multi-  
 plie this *quotiente* by the firste diuisor, the firste diui-  
 dende will resulte thereof.

And for the pzoofe of that, I dooe multiplie,  
 $\sqrt{.45\frac{1}{3}}$  ———  $\sqrt{.6}$  ———  $\sqrt{.22\frac{2}{3}}$  ———  $\sqrt{.18}$  by  
 $\sqrt{.6}$  ———  $\sqrt{.3}$ . But for the moare ease, I doe tourne  
 all the mirte numbers into onely fractions. And then  
 doe I multiplie them orderly.

$$\begin{array}{r}
 \sqrt{.136\frac{8}{3}} \text{ ——— } \sqrt{.6} \text{ ——— } \sqrt{.68\frac{8}{3}} \text{ ——— } \sqrt{.18} \\
 \sqrt{.6} \text{ ——— } \sqrt{.3} \\
 \hline
 \sqrt{.816\frac{8}{3}} \text{ ——— } \sqrt{.216} \text{ ——— } \sqrt{.468\frac{8}{3}} \text{ ——— } \sqrt{.108} \\
 \sqrt{.468\frac{8}{3}} \text{ ——— } \sqrt{.108} \text{ ——— } \sqrt{.204\frac{8}{3}} \text{ ——— } \sqrt{.54} \\
 \hline
 \sqrt{.272} \text{ ——— } \sqrt{.216} \text{ ——— } \sqrt{.136} \text{ ——— } \sqrt{.108} \\
 \text{— } \sqrt{.68} \text{ ——— } \sqrt{.54} \text{ ——— } \sqrt{.136} \text{ ——— } \sqrt{.108} \\
 \hline
 \sqrt{.68} \text{ ——— } \sqrt{.54}
 \end{array}$$

First I multiplie  $\sqrt{.136\frac{8}{3}}$  by  $\sqrt{.6}$  and there cometh  
 $\sqrt{.816\frac{8}{3}}$  that is,  $\sqrt{.272}$ . Again I doe multiplie  $\sqrt{.816\frac{8}{3}}$  by  
 $\sqrt{.6}$  and it maketh  $\sqrt{.216}$ . Then I multiplie  $\sqrt{.468\frac{8}{3}}$   
 by  $\sqrt{.6}$  & it giueth  $\sqrt{.204\frac{8}{3}}$  whiche is  $\sqrt{.136}$ . Fourthly  
 $\sqrt{.18}$  multiplied by  $\sqrt{.6}$  dooeth make  $\sqrt{.108}$ . All  
 whiche I set doune with their conueniente signes.

After that I multiplie  $\sqrt{.136\frac{8}{3}}$  by  $\sqrt{.3}$  and it yeldeth  
 $\sqrt{.408\frac{8}{3}}$  that is,  $\sqrt{.136}$ . whiche I sette doune with his  
 signe ———. Then  $\sqrt{.36}$  by  $\sqrt{.3}$  maketh  $\sqrt{.108}$ . Third-  
 ly,  $\sqrt{.68\frac{8}{3}}$  by  $\sqrt{.3}$  dooeth giue  $\sqrt{.68}$ . and last of all,  $\sqrt{.18}$   
 multiplied by  $\sqrt{.3}$  bringeth forth  $\sqrt{.54}$ .

When all these be placed conueniently, I doe con-  
 sider that ———  $\sqrt{.136}$ . and ———  $\sqrt{.136}$ . maie bee  
 bothe cancelled, bicause the one doeth abate the other.  
 And like waies, ———  $\sqrt{.108}$ . and ———  $\sqrt{.108}$ . eche  
 abate other: so that thei must bothe be reiected.

When I see, that  $\sqrt{.68}$  beyng abated out of  $\sqrt{.272}$   
 there will remain  $\sqrt{.68}$ . And in like  $\sqrt{.54}$  beyng a-  
 bated out of  $\sqrt{.216}$  doeth leaue  $\sqrt{.54}$ . So that the  
 whole multiplicatio doth make iustly  $\sqrt{.68}$  ———  $\sqrt{.54}$

It. i. whiche

## The Arte

whiche is the firste diuident. And so is that diuision approued good.

*An other example.*

Master. Yet for you exercise, you shall haue some examples moare of diuision.

$\sqrt{456} \text{ --- } \sqrt{72}$ . is sette to bee diuided by  $\sqrt{18} \text{ --- } \sqrt{6}$ .

Scholar. That diuisor must I multiplie by his contrarie, whiche is the *Residuale*.  $\sqrt{18} \text{ --- } \sqrt{6}$ . And so, as you maie sone perceiue, there will rise.  $18 \text{ --- } 6$ . that is 12. whiche must be kepte for the newe diuisor.

Then shall I multiplie the former diuident, that is  $\sqrt{456} \text{ --- } \sqrt{72}$  by the same *residuale*  $\sqrt{18} \text{ --- } \sqrt{6}$

$$\begin{array}{r}
 \sqrt{456} \text{ --- } \sqrt{72} \\
 \sqrt{18} \text{ --- } \sqrt{6} \\
 \hline
 \sqrt{8208} \text{ --- } \sqrt{1296} \\
 \sqrt{432} \text{ --- } \sqrt{2736} \\
 \hline
 \sqrt{8208} \text{ --- } \sqrt{432} \text{ --- } \sqrt{2736} \text{ --- } \sqrt{1296}
 \end{array}$$

And there will rise of that multiplication, as here by example appereth  $\sqrt{8208} \text{ --- } \sqrt{432} \text{ --- } \sqrt{2736} \text{ --- } 1296$ . whiche nōber I shall diuide by. 12. that was founde for the newe diuisor. And then will the *quotiente* bee.  $\sqrt{57} \text{ --- } \sqrt{3} \text{ --- } \sqrt{19} \text{ --- } \sqrt{9}$ . As here in woorkes doeth appeare.

$$\begin{array}{r}
 \sqrt{8208} \text{ --- } \sqrt{432} \text{ --- } \sqrt{2736} \text{ --- } \sqrt{1296} (\sqrt{57} \text{ --- } \sqrt{3} \text{ --- } \sqrt{19} \text{ --- } \sqrt{9}) \\
 \sqrt{144} \quad \sqrt{144} \quad \sqrt{144} \quad \sqrt{144}
 \end{array}$$

Where I haue set.  $\sqrt{144}$ . for. 12. saying that be all one; but that.  $\sqrt{144}$ . is moare apte for this woorkes. And I haue repeated it as often tymes, as the diuisor should be remoued.

*The prooffe.*

But now to trie this woorkes, whether it bee well wroughte, I shall multiplie this *quotiente* by the firste diuident, & then ought the firste diuident to amounte.

As

## Of Surde numbers.

As here in example, you see wroughte.

$$\begin{array}{r}
 \sqrt{.57.} \text{---} | \text{---} \sqrt{.3.} \text{---} | \text{---} \sqrt{.19.} \text{---} | \text{---} \sqrt{.9.} \\
 \sqrt{.18.} \text{---} | \text{---} \sqrt{.6.} \\
 \hline
 \sqrt{.1026} \text{---} | \text{---} \sqrt{.54} \text{---} | \text{---} \sqrt{.342} \text{---} | \text{---} \sqrt{.162.} \\
 \sqrt{.342} \text{---} | \text{---} \sqrt{.18} \text{---} | \text{---} \sqrt{.114} \text{---} | \text{---} \sqrt{.54.} \\
 \hline
 \sqrt{.1026} \text{---} | \text{---} \sqrt{.18} \text{---} | \text{---} \sqrt{.114} \text{---} | \text{---} \sqrt{.162.}
 \end{array}$$

Where  $\sqrt{.54.}$  doeth cancell  $\sqrt{.54.}$  and is cancelled by it.

So  $\sqrt{.342.}$  and  $\sqrt{.342.}$  exclude one another, and therefore must bee bothe relected. And then remaineth onely,

$\sqrt{.1026} \text{---} | \text{---} \sqrt{.18.} \text{---} | \text{---} \sqrt{.114} \text{---} | \text{---} \sqrt{.162.}$   
 Whiche numbers I dooe well examine: and finde that  $\sqrt{.114.}$  beyng abated out of  $\sqrt{.1026.}$  there will remaine  $\sqrt{.456.}$  Again if  $\sqrt{.18.}$  be subtracted out of  $\sqrt{.162.}$  there will reste  $\sqrt{.72.}$  And so is that whole multiplicatio onely  $\sqrt{.456} \text{---} \sqrt{.72}$  agreable to the firste diuidende. Whereby it is manifeste, that the former diuision was good.

Master. How can you worke this example?

Where  $.24.$  is set to be diuided by  $.3.$   $\text{---} | \text{---} \sqrt{.8.}$

*The thirde example.*

Scholar. I must still obserue the generalls rule. And multiplie bothe those numbers, by the contrarie of the diuisor, that is, by the *residuale*.  $.3 \text{---} | \text{---} \sqrt{.8.}$  And

|  |   |
|--|---|
| $  \begin{array}{r}  24. \\  3 \text{---}   \text{---} \sqrt{.8} \\  \hline  72 \text{---}   \text{---} \sqrt{.4608}  \end{array}  $ | <p>of the firste multiplication of it, with the diuidende. <math>24.</math> there resteth <math>.72 \text{---}   \text{---} \sqrt{.4608.}</math> Of the seconde multiplication <math>.3. \text{---}   \text{---} \sqrt{.8}</math> tion, where the <i>Binomialle</i> is multiplied by the <i>Residuale</i>, that is his contrary, the totalle will be <math>.9 \text{---}   \text{---} 8.</math> that is but <math>.1.</math> That is. <math>.1.</math> And therefore sayng <math>.1.</math> doeth nother multiplie nor diuide, the former number.</p> |
|--|---|

That is.  $.72 \text{---} | \text{---} \sqrt{.4608.}$  is the *quotiente*, when  $.24.$  is diuided by  $.3. \text{---} | \text{---} \sqrt{.8.}$

# The Arte

*The prooffe.*

For p<sup>ro</sup>ofe whereof, I multiplie 72 —  $\sqrt{\cdot 4608}$   
that is the qu<sup>o</sup>ti<sup>e</sup>nte, by. 3. —  $\sqrt{\cdot 8}$ . And there riseth  
216 —  $\sqrt{\cdot 41472}$  —  $\sqrt{\cdot 41472}$  —  $\sqrt{\cdot 36864}$ .  
whereof. 2. numbers differing but by — & —  
mu<sup>l</sup>t<sup>i</sup>c<sup>i</sup>e bothe bee reiecte<sup>d</sup>, as numbers superfluous.

|  |  |
|--|--|
| $\begin{array}{r} 72 \text{ — } \sqrt{\cdot 4608}. \\ 3 \text{ — } \sqrt{\cdot 8}. \\ \hline 216 \text{ — } \sqrt{\cdot 41472} \\ \sqrt{\cdot 41472} \text{ — } \sqrt{\cdot 36864} \\ \hline 216 \text{ — } .192. \end{array}$ | <p>Then. 36864. is a square<br/>number, and hath. 192<br/>fo<sup>r</sup> his roote. Wherfoze<br/>the whole number is,<br/>216 — 192 that is (as<br/>it is man<sup>i</sup>feste inough)<br/>24. And so is the whole<br/>woozke p<sup>ro</sup>oued good.</p> |
| <p>That is. 24.</p>  |  |

*The fourth  
example.*

Ma<sup>st</sup>er. You shall haue one ex<sup>a</sup>mple moare, and  
then will I make an ende of diu<sup>i</sup>sion.

When  $\sqrt{\cdot 6570}$ . —  $\sqrt{\cdot 254}$ . is p<sup>ro</sup>ponde<sup>d</sup> to  
bee diu<sup>i</sup>ded by  $\sqrt{\cdot 54}$  —  $\sqrt{\cdot 6}$ . I would knowe the  
qu<sup>o</sup>ti<sup>e</sup>nte.

Scholar. I see the netwe diu<sup>i</sup>so<sup>r</sup> will be. 54 — 6.  
that is. 48.

And then fo<sup>r</sup> to finde a diu<sup>i</sup>de<sup>n</sup>de conueniente, I  
shall multiplie the fir<sup>st</sup>e  
diu<sup>i</sup>de<sup>n</sup>de, by the contra<sup>r</sup>  
ie of the fir<sup>st</sup>e diu<sup>i</sup>so<sup>r</sup>,  
that is by  $\sqrt{\cdot 54}$  —  $\sqrt{\cdot 6}$   
And there will rise, as  
you see.  $\sqrt{\cdot 354780}$ .

—  $\sqrt{\cdot 13716}$  —  $\sqrt{\cdot 39420}$  —  $\sqrt{\cdot 1524}$ .  
That diu<sup>i</sup>de<sup>n</sup>de mu<sup>st</sup> be diu<sup>i</sup>ded by. 48. o<sup>r</sup> moare ap<sup>p</sup>  
ty by.  $\sqrt{\cdot 2304}$ . And the qu<sup>o</sup>ti<sup>e</sup>nte will bee.

$\sqrt{\cdot 153\frac{80}{192}}$  —  $\sqrt{\cdot 5\frac{181}{192}}$  —  $\sqrt{\cdot 17\frac{63}{176}}$  —  $\sqrt{\cdot 1\frac{17}{192}}$ .

As here appeareth in woozke.

|                       |   |                      |   |                     |   |                       |   |
|-----------------------|---|----------------------|---|---------------------|---|-----------------------|---|
| $\sqrt{\cdot 354780}$ | — | $\sqrt{\cdot 13716}$ | — | $\sqrt{39420}$      | — | $\sqrt{1524}$         | ( $\sqrt{153\frac{80}{192}}$ — $\sqrt{5\frac{181}{192}}$ — $\sqrt{17\frac{63}{176}}$ — $\sqrt{1\frac{17}{192}}$ ) |
| $\sqrt{\cdot 2304}$ . |   | $\sqrt{\cdot 2304}$  |   | $\sqrt{\cdot 2304}$ |   | $\sqrt{\cdot 2304}$ . |   |

*The prooffe.*

And that this woozke is good, I will p<sup>ro</sup>ue it by  
multiplication.

## of Surde numbers.

multiplication. As the example folowynge dooeth declare. ¶ Here by the firste multiplication there cometh. 8. numbers, that is. 4. with. —+—. and. 4. with ———.

$$\begin{array}{ccccccc}
 \sqrt{\cdot \frac{29565}{192}} & -+ & \sqrt{\cdot \frac{1141}{192}} & -+ & \sqrt{\cdot \frac{3285}{192}} & -+ & \sqrt{\cdot \frac{127}{192}} \\
 \sqrt{\cdot 54} & -+ & \sqrt{\cdot 6} & & & & \\
 \hline
 \sqrt{\cdot \frac{1596510}{192}} & -+ & \sqrt{\cdot \frac{61722}{192}} & -+ & \sqrt{\cdot \frac{177390}{192}} & -+ & \sqrt{\cdot \frac{6858}{192}} \\
 -\sqrt{\cdot \frac{177390}{192}} & -+ & -\sqrt{\cdot \frac{6858}{192}} & -+ & -\sqrt{\cdot \frac{19710}{192}} & -+ & -\sqrt{\cdot \frac{761}{192}} \\
 \hline
 \sqrt{\cdot \frac{1596510}{192}} & -+ & \sqrt{\cdot \frac{1712}{192}} & -+ & \sqrt{\cdot \frac{19710}{192}} & -+ & \sqrt{\cdot \frac{761}{192}} \\
 \hline
 \sqrt{\cdot 6570} & -+ & \sqrt{\cdot 254} & & & & 
 \end{array}$$

And bicause the firste nōber with ———, is equalle to the thirde with —+—, therfoze thei bothe must be reiected. Again in as moche as the seconde nōber with ——— is equalle to the fourthe number with —+—, thei bothe shall bee cancelled. And then remaineth. 2. numbers with —+—, and other. 2. with ———.

So if you abate the thirde ——— out of the firste —+—, the *quotiente* will be.  $\sqrt{\cdot 6570}$ .

Likewaies if you abate the fourthe ——— out of the seconde —+—, the *quotiente* will yelde.  $\sqrt{\cdot 254}$ .

And thei bothe will make the firste diuidende.  $\sqrt{\cdot 6570}$ . ¶ Herby the former diuision is approued good. Pasther. This shall suffice for diuision.

## Of extraction of rootes.

**T**he nexte woork is extraction of rootes: whiche you maye very easilie woork, by puttynge the signe of the roote, that you desire, befoze the whole number. As if you would haue the square roote of  $\sqrt{\cdot 10}$  —+—  $\sqrt{\cdot 5}$ . this is it  $\sqrt{\cdot 10}$  —+—  $\sqrt{\cdot 5}$ . The Cubike roote of the same nobre is.  $\sqrt[3]{\cdot 10}$  —+—  $\sqrt[3]{\cdot 5}$ . And the zenizenze roote of it is  $\sqrt[4]{\cdot 10}$  —+—  $\sqrt[4]{\cdot 5}$ . But if you will haue the square roote of.  $\sqrt{\cdot 10}$  —+—  $\sqrt{\cdot 5}$

Ar. 13.      it

## The Arte

it is.  $\sqrt{10} - + - \sqrt{5}$ . And his Cubike roote is.  $\sqrt[3]{10} - + - \sqrt{5}$ . Likewise his *zenzizenzike* roote is  $\sqrt[4]{10} - + - \sqrt{5}$ .

So of.  $\sqrt[3]{18}$  ——— 2. the Square roote is  $\sqrt{18}$  ——— 2. The Cubike roote is.  $\sqrt[3]{18}$  ——— 2. And the *zenzizenzike* roote is.  $\sqrt[4]{18}$  ——— 2.

Scholar. Hereby I perceiue that the later parte of the cōposition, is not varied at all, but onely the firste parte taketh vnto it the signe of the roote. And that signe is referred to the whole compounde number.

*Vniuersalle  
rootes.*

Master. These rootes therefore bee called *vniuersalle rootes*, because they are the rootes, not of the seuerall partes of the compounde nōber, but of the whole compounde number. And that is the difference, betwene the common *Surde* numbers, and *vniuersalle rootes*. For if  $\sqrt{24} - + - \sqrt{144}$ , be sette for a common *Surde* number, then doeth it betoken, that I must take 2. rootes, that is.  $\sqrt{24}$ . and  $\sqrt{144}$ , and ioine them together. But if it stande for an *vniuersalle roote*, it representeth the roote of this whole number.  $24 - + - \sqrt{144}$ . which is 6. for the whole Square is. 36.

Scholar. I perceiue it well. For.  $\sqrt{144}$ . being 12, that. 12. with. 24. dooeth make. 36. And therefore must the *vniuersalle roote* of.  $24 - + - \sqrt{144}$ . bee. 6. And so  $\sqrt{24} - + - \sqrt{144}$ . is iust. 6.

But if.  $\sqrt{24} - + - \sqrt{144}$ . doe stande for a common *Surde* number compounde: then is it made of. 2. rootes, that is  $\sqrt{24}$ . which is almoste. 5. and  $\sqrt{144}$  being. 12. And so the whole compounde roote, in that sorte is almoste. 17. And is nigh. 3. tymes so moche as the same number, being an *vniuersalle roote*.

Master. Because you maie perceiue it the better, I will put an example in *Square* numbers, made like *Surdes*. As this.  $\sqrt{81} - + - \sqrt{36}$  if it be an *vniuersalle roote*, then it is equall to 10. For I must take first the roote of the laste number, which is. 19. And adde it  
with

## of Surde numbers.

with. 81. wherby there amounteth. 100. whose roote is. 10. But if it stand after the common sorte of *Surde numbers*, it betokeneth the roote of. 81. and the roote of. 361. (that is. 9. and. 19) to bee added together. And so they make. 28. whiche is farre about. 10.

But farther now, if it stande for a common *Surde number*: And I would haue the *Square roote* of it, then is that.  $\sqrt{\sqrt{81} + \sqrt{361}}$ . And betokeneth the *Square roote* of the *Square roote* of. 81. and the *Square roote* of. 361. added together, that is the *Square roote* of. 28. But moſte generally and moſte aptly, it betokeneth the roote of the *vnuerſalle roote* of. 81. &  $\sqrt{361}$ .

Scholar. Now I perceiue that in Addition, and Subtraction of *Surdes*, the last numbers that did result of that woorkes, were *vnuerſalle rootes*.

Maſter. You ſaie truth. But haſke what meaneth that haſtie knocking at the doore?

Scholar. It is a meſſenger.

Maſter. What is the meſſage: tel me in mine care

What ſir is that the mater: When ſir there noe reioyce, but that I muſt neglect all ſtudies, and teaching, ſo, to withſtande thoſe danngers. My fortune is not ſo good, to haue quiete tyme to teache.

Scholar. But my fortune and my fellowes, is moche worſe, that your vniquietnes, ſo hindereth our knowledge. I praie God amende it.

Maſter. I am inforced to make an excuſe of this mater: But yet will I promiſe you, that whiche you ſhall chalenge of me, when you ſee me at better leiſure: That I will teache you the whole arte of *vnuerſalle rootes*. And the extraction of rootes in all *Square Surdes*: With the demonſtration of them, and all the former woorkes.

If I mighte haue been quietly permitted, to reſte but a litle while longer, I had determined not to haue ceaſed, till I had ended all theſe thinges at large. But

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now farewell. And applie your studie diligently in this that you haue learned. And if I male gette any quietnesse reasonable, I will not forget to perfoyme my promise with an augmentation.

Scholar. My harte is so oppressed with pēisenes, by this sodaine vnquietnesse, that I can not expresse my grief. But I will pzeale, with all them that loue honeste knowlodge, that God of his mercie, will sone ende your troubles, and graunte you soche reste, as your trauell doeth merite.

And al that loue learning: saie thereto. Amen.

Paster. Amen,  
and Amen.



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