

Proposal to encode Ancient Chinese Mathematical Symbols

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1. Introduction of the new block - Ancient Chinese Mathematical Symbols. Because Chinese mathematics emerged independently, when writing mathematical books, Chinese people created many special symbols. Especially in Ming and Qing Dynasty, a lot of western mathematics books were translated verbally and introduced into China by western missionaries. In order to adapt to the vertical writing of Chinese books, the Chinese native scholars modified the western mathematical symbols slightly and then a set of new symbols were made and used into books and papers. These changes are mainly reflected in the following aspects:

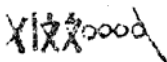
- **Mathematical Symbols.** Most of them were just rotated but some of them were reformed.
- Name of unknowns. The Latin letters and the Greek letters will be translated into the names of Chinese era, twenty-eight lunar mansions or the sixty-four hexagrams.
- Equations. The simple equations would be written vertically, and the complex equations would be written horizontally, sometimes the column could be so long to contain the equations and sometimes an equation could cross several columns.

If there are mathematical symbols which had been reformed and used in China before China drew up standards, we will put them into this new block. Considering that there are also many unexplored books, we set the new block from U+1EF00 to U+1EF3F.

1.2. Introduction of specific blocks.

(1) **Combining Diacritical Marks.** Like “point A”, “point A’” in western symbol system, when expressing some similar and different geometric points, ancient Chinese mathematician would use some left-falling strokes (撇). One to three left-falling strokes were found in *Qiuyishu Tongjie* (求一術通解).

(2) **Combining Diacritical Marks for Symbols.** The UCS contains a number of combining diacritical marks for symbols.

For example,  can be represented as “又 | 文文 ○ ○ ○ ☹”, where ☹(U+20E5) is used for indicating a negative number. There are also some marks in *Xuesuan Bitan* (學算筆談), *Siyuan Yujian* (四元玉鑑) and *Zhusuan* (珠算). In the books, $\cancel{\circ}$ is used for deleting or updating the number during the mathematical operation, and \odot is used for marking the redundant characters in the process of copying and printing, and \square is used as “!” in math, which means factorial. For example, both \square and $5!$ represent $5 \times 4 \times 3 \times 2 \times 1$.

(3) **Enclosed Ideograms.** The UCS contains a number of enclosed ideograms in BMP and SMP. In ancient Chinese math books, the ideograms enclosed by circles are also found in *Pingsanjiao Bianjiao Huqiu* (平三角邊角互求術), *Weiji Suyuan* (微積溯源), *Qiuyishu Tongjie* (求一術通解), *Duishu Xiangjie* (對數詳解), *Daishushu* (代數術) and *Zhusuan* (珠算). And the ideograms enclosed by squares are also found in *Yuanlü Kaozhen Tujie* (圓率攷真圖解), *Daishu Beizhi* (代數備旨), *Subu Yancao* (粟布演草), *Duishu Xiangjie* (對數詳解) and *Siyuan Yujian Xicao* (四元玉鑑細草). In general, most of the characters are used for marking the formula so as to quote and discuss them in later words.

(4) **Ancient Chinese Mathematical Symbols.** A brief introduction is given here.

a) $\perp\top$ and \pm means “plus” (+), \top means “minus” (-), and \pm means “plus or minus” (\pm). They were adopted in

Daiweiji Shiji (代微積拾級). For saving the space, the \perp and \top usually closed to the characters narrowly. For example “a + b” would be represented as “甲 \perp 乙”. So the usage of them obviously differ from UP TACK (\perp), DOWN TACK (\top) and PERPENDICULAR SIGN (\perp) and COUNTING RODS (\perp , \top).

- b) \downarrow and \uparrow . \downarrow is the variant of \perp , and \uparrow is the variant of \top . They were adopted in *Xingxue Beizhi* (形學備旨) and *Daishu Beizhi* (代數備旨). And the usage of \downarrow differ from NONFORKING (\downarrow).
- c) \top . \top means “find positive difference”. For example, $5 \top 3 = 3 \top 5 = 2$.
- d) { and }. They are the variants of “{” and “}”.
- e) \dots . \dots means “et cetera” (.....). The explanatory notes of *Duishu Xiangjie* (對數詳解) enumerated this symbol particularly, for it hasn’t been used before.
- f) $\dot{\perp}$ and $\dot{\top}$. $\dot{\perp}$ is deformed from the radical of 微, which means “differential” (d). $\dot{\top}$ is deformed from the radical of 積, which means “integral” (\int). And the usage of them as operators is different from them as radicals, so they should be separated.

2. Character Data.

2.1. Character position and name.

Combining Diacritical Marks Extended (U+1AC0 - U+1AFF)

	1AC0	COMBINING CHINESE PRIME
	1AC1	COMBINING CHINESE DOUBLE PRIME
	1AC2	COMBINING CHINESE TRIPLE PRIME

Combining Diacritical Marks for Symbols (U+20D0 - U+20FF)

	20F1	COMBINING LONG SOLIDUS OVERLAY
	20F2	COMBINING ENCLOSING OCTAGON
	20F3	COMBINING CHINESE FACTORIAL SYMBOL

Ancient Chinese Mathematical Symbols (U+1EF00 - U+1EF3F)

	1EF00	CHINESE PLUS SIGN
	1EF01	CHINESE MINUS SIGN
	1EF02	CHINESE PLUS-OR-MINUS SIGN
	1EF03	CHINESE VARIANT PLUS SIGN
	1EF04	CHINESE VARIANT MINUS SIGN
	1EF05	CHINESE POSITIVE DIFFERENCE SIGN
	1EF06	CHINESE LEFT CURLY BRACKET

}	1EF07	CHINESE RIGHT CURLY BRACKET
⋯	1EF08	CHINESE ELLIPSIS
↯	1EF09	CHINESE DIFFERENTIAL SIGN
∫	1EF0A	CHINESE INTEGRAL SIGN

⊕	U+5341,U+034F,U+4E00,U+034F,U+25EF
⊕	U+5341,U+034F,U+4E8C,U+034F,U+25EF
⊕	U+5341,U+034F,U+4E09,U+034F,U+25EF
⊕	U+5341,U+034F,U+56DB,U+034F,U+25EF
⊕	U+5341,U+034F,U+4E94,U+034F,U+25EF
⊕	U+5341,U+034F,U+516D,U+034F,U+25EF
⊕	U+5341,U+034F,U+4E03,U+034F,U+25EF
⊕	U+5341,U+034F,U+516B,U+034F,U+25EF
⊕	U+5341,U+034F,U+4E5D,U+034F,U+25EF
⊕	U+7532,U+034F,U+25EF
⊕	U+4E59,U+034F,U+25EF
⊕	U+4E19,U+034F,U+25EF
⊕	U+4E01,U+034F,U+25EF
⊕	U+620A,U+034F,U+25EF
⊕	U+5DF1,U+034F,U+25EF
⊕	U+5E9A,U+034F,U+25EF
⊕	U+8F9B,U+034F,U+25EF
⊕	U+5477,U+034F,U+25EF
⊕	U+20B99,U+034F,U+25EF

㒶	U+20C33,U+034F,U+25EF
㒷	U+53EE,U+034F,U+25EF
㒸	U+3595,U+034F,U+25EF
㒹	U+5B50,U+034F,U+25EF
㒺	U+4E11,U+034F,U+25EF
㒻	U+5BC5,U+034F,U+25EF
㒼	U+536F,U+034F,U+25EF
㒽	U+8FB0,U+034F,U+25EF
㒾	U+5DF3,U+034F,U+25EF
㒿	U+5348,U+034F,U+25EF
㓀	U+672A,U+034F,U+25EF
㓁	U+7533,U+034F,U+25EF
㓂	U+9149,U+034F,U+25EF
㓃	U+4EA5,U+034F,U+25EF
㓄	U+89D2,U+034F,U+25EF
㓅	U+4EA2,U+034F,U+25EF
㓆	U+6C10,U+034F,U+25EF
㓇	U+623F,U+034F,U+25EF
㓈	U+4E8C,U+034F,U+7533,U+034F,U+25EF
㓉	U+4E8C,U+034F,U+9149,U+034F,U+25EF
㓊	U+4E8C,U+034F,U+4EA5,U+034F,U+25EF
㓋	U+4E8C,U+034F,U+5929,U+034F,U+25EF
㓌	U+7532,U+034F,U+4E8C,U+034F,U+25EF

ㄗ	U+4E59,U+034F,U+4E8C,U+034F,U+25EF
ㄘ	U+4E19,U+034F,U+4E8C,U+034F,U+25EF
ㄙ	U+4E01,U+034F,U+4E8C,U+034F,U+25EF
ㄜ	U+620A,U+034F,U+4E8C,U+034F,U+25EF
ㄝ	U+5DF1,U+034F,U+4E8C,U+034F,U+25EF
ㄞ	U+5E9A,U+034F,U+4E8C,U+034F,U+25EF
ㄟ	U+8F9B,U+034F,U+4E8C,U+034F,U+25EF
ㄠ	U+5B50,U+034F,U+4E8C,U+034F,U+25EF
ㄡ	U+4E11,U+034F,U+4E8C,U+034F,U+25EF
ㄢ	U+5BC5,U+034F,U+4E8C,U+034F,U+25EF
ㄣ	U+536F,U+034F,U+4E8C,U+034F,U+25EF
ㄤ	U+58F9,U+034F,U+25EF
ㄥ	U+8CB3,U+034F,U+25EF
ㄦ	U+53C1,U+034F,U+25EF
ㄧ	U+5343,U+034F,U+25EF
ㄨ	U+5929,U+034F,U+25EF
ㄩ	U+5730,U+034F,U+25EF
人	U+4EBA,U+034F,U+25EF
術	U+8853,U+034F,U+25EF
又	U+53C8,U+034F,U+25EF
○	U+3007,U+034F,U+2B1C
四	U+56DB,U+034F,U+2B1C
五	U+4E94,U+034F,U+2B1C

六	U+516D,U+034F,U+2B1C
七	U+4E03,U+034F,U+2B1C
八	U+516B,U+034F,U+2B1C
九	U+4E5D,U+034F,U+2B1C
十	U+5341,U+034F,U+2B1C
士	U+5341,U+4E8C,U+034F,U+2B1C
𠂇	U+5341,U+56DB,U+034F,U+2B1C
𠂈	U+5341,U+516D,U+034F,U+2B1C
𠂉	U+5341,U+4E03,U+034F,U+2B1C
乾	U+4E7E,U+034F,U+2B1C
否	U+5426,U+034F,U+2B1C
問	U+554F,U+034F,U+2B1C
履	U+5C65,U+034F,U+2B1C
屯	U+5C6F,U+034F,U+2B1C
師	U+5E2B,U+034F,U+2B1C
步	U+6B65,U+034F,U+2B1C
比	U+6BD4,U+034F,U+2B1C
泰	U+6CF0,U+034F,U+2B1C
訟	U+8A1F,U+034F,U+2B1C
蒙	U+4749,U+034F,U+2B1C
隨	U+96A8,U+034F,U+2B1C
需	U+9700,U+034F,U+2B1C

It is important to point out that this kind of enclosed ideographic has fixed form and certain system, so they cannot be added or reduced casually.

The reason why ㊦ is different from it in the evidence figures is that, there were mistakes in books in the process of transcribing. Because the serial characters were marked in the Twelve Earthly Branches order and “巳” in the Ten Heavenly Stems cannot be in the serial for no reason, the correct character should be “己”.

2.2. Character properties.

1AC0;COMBINING CHINESE PRIME;Mn;230;NSM;;;;;N;;;;;
1AC1;COMBINING CHINESE DOUBLE PRIME;Mn;230;NSM;;;;;N;;;;;
1AC2;COMBINING CHINESE TRIPLE PRIME;Mn;230;NSM;;;;;N;;;;;
20F1;COMBINING LONG SOLIDUS OVERLAY;Mn;1;NSM;;;;;N;;;;;
20F2;COMBINING ENCLOSING OCTAGON;Me;0;NSM;;;;;N;;;;;
20F3;COMBINING CHINESE FACTORIAL SYMBOL;Mn;230;NSM;;;;;N;;;;;
1EF00;CHINESE PLUS SIGN;Sm;0;L;;;;;N;;;;;
1EF01;CHINESE MINUS SIGN;Sm;0;L;;;;;N;;;;;
1EF02;CHINESE PLUS-OR-MINUS SIGN;Sm;0;L;;;;;N;;;;;
1EF03;CHINESE VARIANT PLUS SIGN;Sm;0;L;;;;;N;;;;;
1EF04;CHINESE VARIANT MINUS SIGN;Sm;0;L;;;;;N;;;;;
1EF05;CHINESE POSITIVE DIFFERENCE SIGN;Sm;0;L;;;;;N;;;;;
1EF06;CHINESE LEFT CURLY BRACKET;Sm;0;L;;;;;N;;;;;
1EF07;CHINESE RIGHT CURLY BRACKET;Sm;0;L;;;;;N;;;;;
1EF08;CHINESE ELLIPSIS;Sm;0;L;;;;;N;;;;;
1EF09;CHINESE DIFFERENTIAL SIGN;Sm;0;L;;;;;N;;;;;
1EF0A;CHINESE INTEGRAL SIGN;Sm;0;L;;;;;N;;;;;

3. References.

劉鐸 輯. 古今算學叢書 (据微波榭本等石印). 上海: 算學書局, 光緒二十四年 (1898).

劉永錫, (美) Calvin Wilson Mateer. 形學備旨 (十卷). 重校本.

葉耀元. 形學補編 (一卷). 原稿本.

吳嘉善. 平三角術 (一卷). 白芙堂叢書本.

(英) John Fryer. 三角須知 (一卷). 原刊本.

左潛. 綴術釋明附刊誤 (二卷). 白芙堂叢書本.

曾紀鴻, 左潛, 黃宗憲. 圓率考真圖解附刊誤 (一卷). 白芙堂叢書本.

黃宗憲, 左潛. 求一術通解 (二卷). 白芙堂叢書本.

戴煦. 對數簡法附刊誤 (二卷)續對數簡法附刊誤 (一卷). 小萬卷樓叢書本.

曾紀鴻, 丁取忠. 對數詳解附刊誤 (五卷). 白芙堂叢書本.

李善蘭, (附卷) 王季同. 測圓海鏡圖表附九容公式 (一卷). 原稿本.

李冶, (校) 李銳. 益古演段 (三卷). 白芙堂叢書本.

朱世傑, (細草) 羅士琳. 四元玉鑑細草 (二十四卷). 觀我生室彙稿本.

華蘅芳, (英) John Fryer. 代數術附刊誤 (二十五卷). 江南製造局本.

[第一百十四款, 補劉彝程、沈善蒸、崔朝慶算式]

鄒立文, 生福維. 代數備旨 (六卷). 重校本.

李善蘭, (英) Alexander Wylie, (注) 湯金鑄, 李鳳苞, 華蘅芳. 代微積拾級注 (十八卷). 墨海刊本. 附三家注原稿本.

華蘅芳, (英) John Fryer. 微積溯源 (六卷). 江南製造局本.

秦九韶, (附卷) 宋景昌, (校) 鄒安邕. 數書九章十八卷附札記 (四卷). 宜稼堂叢書本校注原稿本.

(第一卷) 丁取忠, 左潛, 曾紀鴻, 吳嘉善, 李善蘭, (第二卷) 鄒伯奇, 丁取忠, 左潛, (補卷) 丁取忠. 粟布演草 (二卷) 補 (一卷). 白芙堂叢書本.

方中通. 珠算 (一卷). 隨衍室本.

徐建寅, (英) John Fryer. 運規約指 (三卷). 江南製造局本.

華蘅芳. 學算筆談 (十二卷). 原刻本.

…… (There are also 75 books collected in the series but not put to use for examples)

4. Acknowledgement.

This proposal is improved with great help and advice of Eduardo Marin and Andrew West. And the glyphs of the

symbols are modified from Symbola and Sursong.

5. Figures.

求一術通解 卷下

七算數第三

答曰積九千四百五十五歲上元太歲在庚辰

如法求得反乘率七以乘戊得 𠄎 ⑤ 申 ⑥相乘得 𠄎 ⑦ 庚 ⑧併子庚得 𠄎 ⑨以申累減辛餘 𠄎 ⑩試以金萬一千二百六十六即所求物數

今有後漢四分術木日率四千七百二十五火日率一千八百七十六土日率九千四百一十五金日率四千六百六十一水日率一千八百八十九熹平三年甲寅木日率餘五火日率餘七十五土日率餘四十金日率餘一百三十三水日率餘一十此各日率以上元盡熹平三年積算問上元以來盡熹平三年甲寅積歲幾以各日率除去所餘之數

何及上元太歲所在此題錄求一算術

草曰依術得 𠄎 ① 子 ② 𠄎 ③ 丑 ④乃以甲乙求等得 𠄎 ⑤ 約 ⑥ 乙 ⑦ 得 ⑧ 非 ⑨ 與 ⑩ 甲 ⑪ 仍有 ⑫ 等 ⑬ 知 ⑭ 不 ⑮ 約 ⑯ 乙 ⑰ 以 ⑱ 約 ⑲ 甲 ⑳ 得 ㉑ 下 ㉒ 𠄎 ㉓ 與 ㉔ 乙 ㉕ 相 ㉖ 乘 ㉗ 得 ㉘ 𠄎 ㉙ 以 ㉚ 乙 ㉛ 減 ㉜ 子 ㉝ 不 ㉞ 足 ㉟ 減 ㊱ 即 ㊲ 得 ㊳ 𠄎 ㊴ 又 ㊵ 以 ㊶ 乙 ㊷ 減 ㊸ 甲 ㊹ 之 ㊺ 約 ㊻ 數 ㊼ 不 ㊽ 足 ㊾ 減 ㊿ 即 得 𠄎 ①於丙內減去一丑餘 𠄎 ②以乙丁對列兩行求反乘率式如左

乙 左減右十 七次餘 𠄎	丁 右減左一 次餘 𠄎	甲 左減右一 次餘 𠄎	子 右減左一 次餘 𠄎
丙 左減右二 次餘 𠄎	丑 右減左一 次餘 𠄎	寅 左減右一 次餘 𠄎	卯 右減左三 次餘 𠄎

如法求得反乘率四千三百二十一以乘戊得 𠄎 ①與甲之約數相乘得 𠄎 ②併子庚得 𠄎 ③以申累減辛餘 𠄎 ④試以金日率 𠄎 累減子餘得 𠄎 與題中本位餘數合 又試以水日率

Figure 1. Example from Huang Zongxian's (黃宗憲) book *Qiyishu Tongjie* (求一術通解) vol.2 p.7 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ○, ◯ and ◌ in text.

者視原法數在實內有幾回也卽用其幾回之變法與暗數相呼加於實數之上逐位呼加歸除無異也

歸式

式一有銀一百二十兩二人分之問各若干曰

甲	乙	變	八
子	丑	及	
子	丑	及	
子	丑	及	
子	丑	及	

先將法二於十內減之餘八卽八爲數是爲子丑兩暗數蓋子實一作一法二也丑實二內有一回原法二也子暗數五曰五八得四十乃於子實再以變法八呼丑暗數一曰一八如上加八數已滿十曰八退二進一十而於子位五進一爲六逐位加畢視子位遞加分數爲每人各得六十兩也

朱
算

Figure 2. Example from Fang Zhongtong's (方中通) book *Zhusuan* (珠算) p.16 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing 〇 in text.

得三萬三千五百四十三為同數消左得

三藥方得九步為先開方數倍之得十八步為九步自之得八十一步為從隅加從方得九十九步先開方數減二步餘七步為後開方數自九步加二步得五十一步為曉徑合問

積加面徑減圓周餘五十六步只云虛徑多如面徑二步問三徑各幾何

答曰面徑四步 虛徑六步

通徑一十四步

曰立天元一為面徑如積求之得一百二十八為益方一十一為正隅

以平方開之得面徑

字琳案以字四步
字前字似皆羨文

二
一
三

Figure 3. Example from Zhu Shijie's (朱世傑) book *Siyuan Yujian* (四元玉鑑) vol.1.2 p.11 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing ⊙ in text.

如^甲言^乙甲^丙之和以^乙減之則等^于無也
 者其大無窮也
 如^〇言以^〇除一則其所得之數等^于
 者至此而止也所以省連書各數之繁也
 如^一可作^四
 項者式中之自成一數者也
 如^甲皆為獨項之式 如^乙

Figure 4. Example from Hua Hengfang's (華蘅芳) book *Xuesuan Bitan* (學算筆談) vol.8 p.4 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 〇 in text.

凡一綫與成角之一綫平行順其勢而成角為平行角其式同其度等

凡三角形并其三角皆足半周
 如圖甲乙丙形與乙丙平行畫戊甲己綫準前說則戊
 丙角與甲丙乙角為互角其度等以加丙甲乙角成
 戊甲乙角又己甲乙角與甲丙角為互角以加戊甲
 乙角適足半周是也

凡兩角相對則彼此互為對角
 如圖甲丙乙角與乙丙丁角相并滿一象限
 甲丙乙角為本角則乙丙丁角為餘角以乙丙丁角
 為本角則甲丙乙角為餘角
 是也

成戊(庚)己角其(庚)角與(丁)角亦為平行角也

如圖以甲丁綫與己丁綫相交而成(丁)角乃與己丁平行再畫一綫如丙乙順交於甲丁而成甲(乙)丙角其(乙)角與(丁)角為平行角或與甲丁平行再畫一綫如戊庚順交於己丁而成戊(庚)己角其(庚)角與(丁)角亦為平行角也

Figure 5. Example from Wu Jiashan's (吳嘉善) book *Pingsanjiao Bianjiao Huqiushu* (平三角邊角互求術) p.1 and p.2 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 甲, 乙, 丙, 丁, 庚 and 戊 in text.

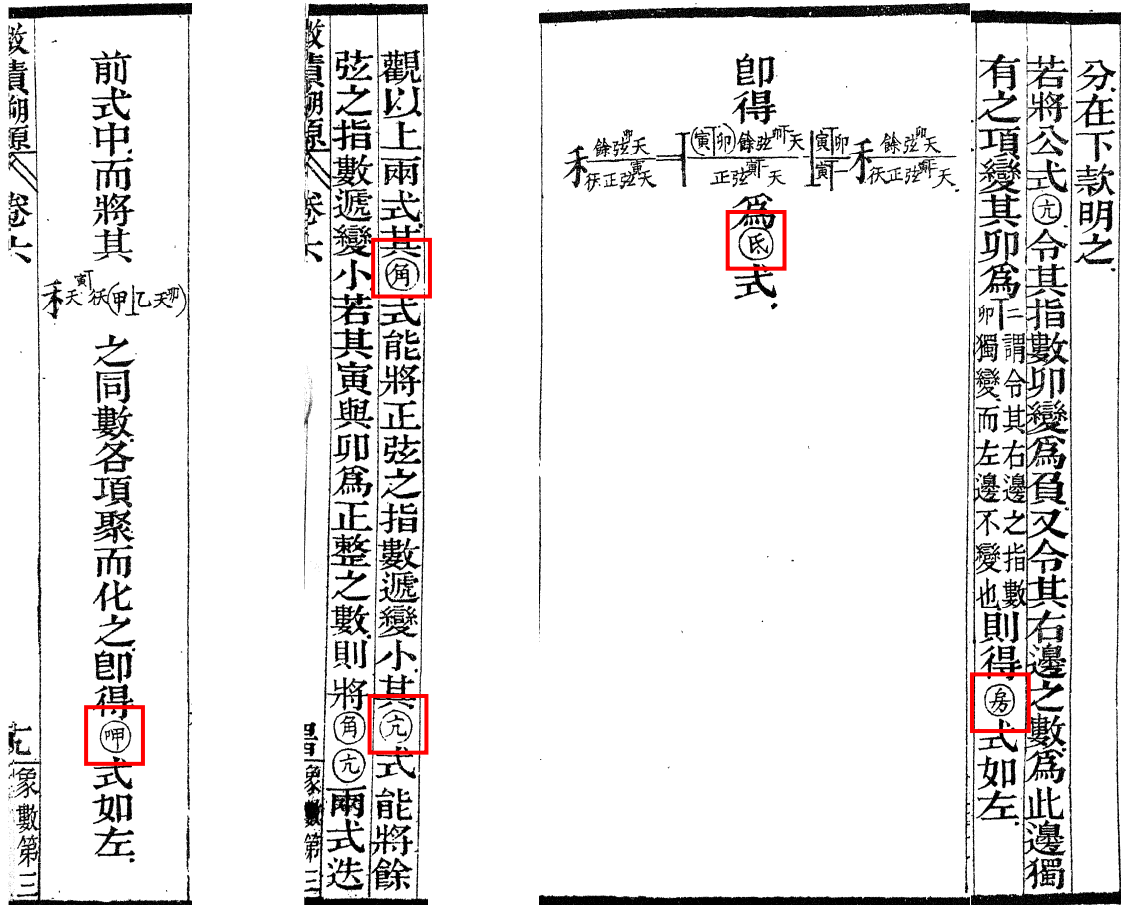


Figure 6. Example from Hua Hengfang's (華蘅芳) book *Weiji Suyuan* (微積溯源) vol.6 p.19, p.44 and p.45 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing 甲, 卯, 戌, 卯 and 卯 in text.

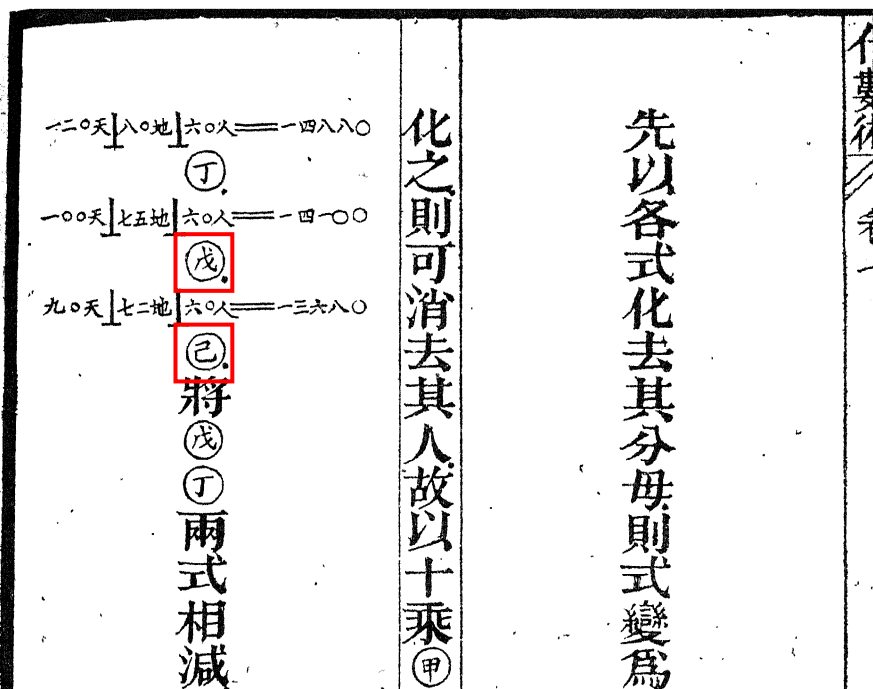


Figure 7. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.7 p.7 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing 戌 and 己 in text.

七數之賸一十七以三十五數之賸二十一問物幾何

答曰二萬一千二百六十六

草曰依術得 ㊀ ① ㊁ ② ㊂ ③ ㊃ ④ ㊄ ⑤ ㊅ ⑥ ㊆ ⑦乃以甲乙求等無等不約乘得下 ㊇ ⑧以乙減子不足減即得 ㊈ ⑨又以乙累減無等不約⑩於丙內減去一丑餘 ㊉ ⑪以乙丁對列兩行求反乘左

乙	㊀	左減右三	三	右減左二	三	左減右二
丁	㊁	次餘 ㊂	一	次餘 ㊃	七	次餘 ㊄

如法求得反乘率一十以乘戊得 ㊅ ⑥甲已相乘得下子庚得 ㊆ ⑦以申累減辛餘 ㊇ ⑧

又依術得 ㊈ ⑨ ㊉ ⑩ ㊊ ⑪ ㊋ ⑬乃以申乙求等無等不約申得下 ㊌ ⑭以乙累減子餘 ㊍ ⑮又以乙累減申餘 ㊎ ⑯

求一術通解

卷下

Figure 8. Example from Huang Zongxian's (黃宗憲) book *Qiyishu Tongjie* (求一術通解) vol.2 p.7 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ①, ②, ③ and ④ in text.

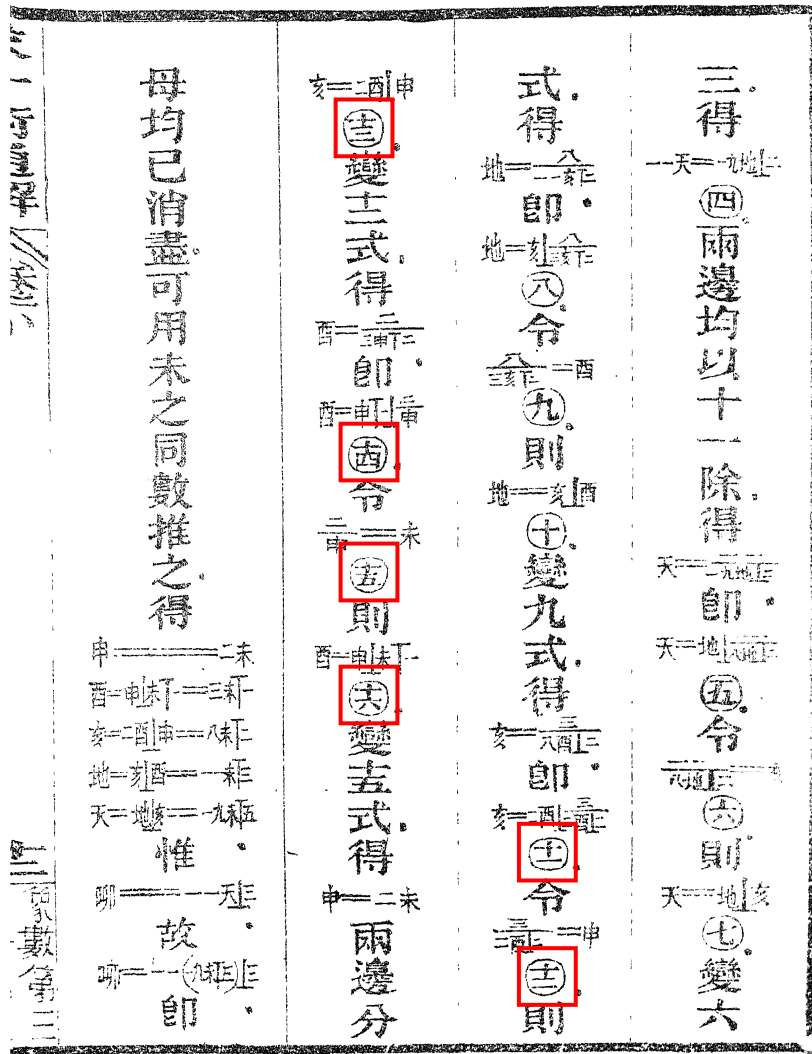


Figure 9. Example from Huang Zongxian's (黃宗憲) book *Qiyushu Tongjie* (求一術通解) vol.2 p.12 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing \oplus , \otimes , \odot , $\opl�$ and $\opl�$ in text.

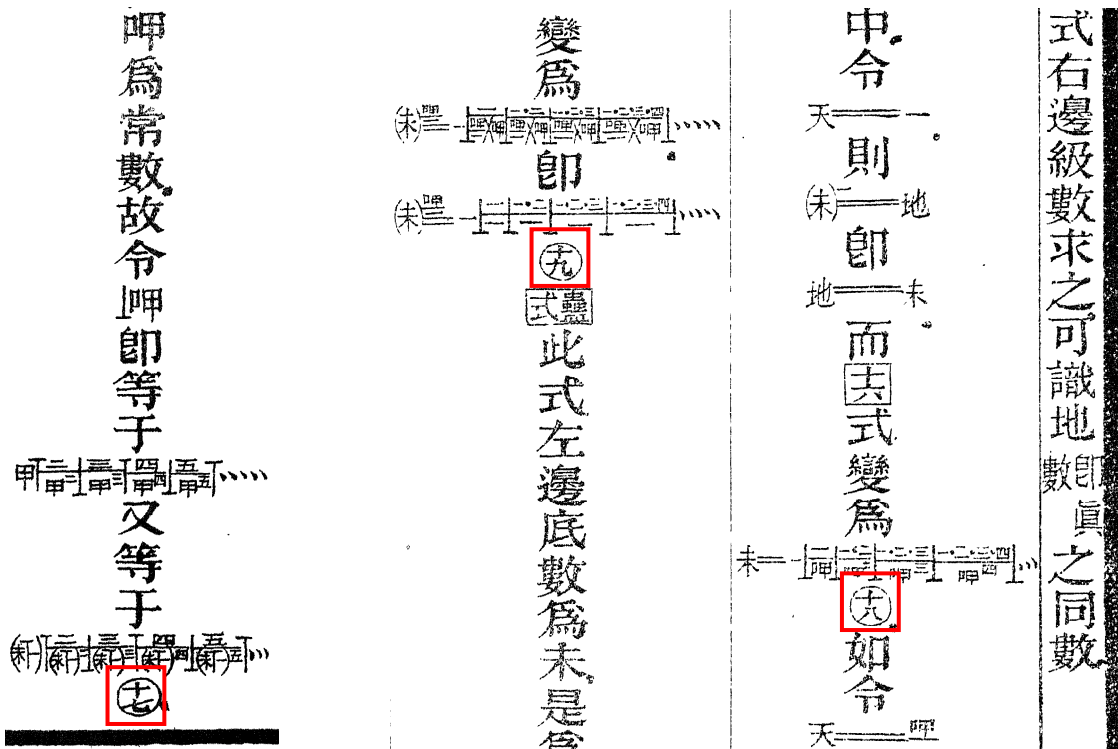


Figure 10. Example from Zeng Jihong's (曾紀鴻) book *Duishu Xiangjie* (對數詳解) vol.4 p.3 and p.4 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing \oplus , $\opl�$ and $\opl�$ in text.

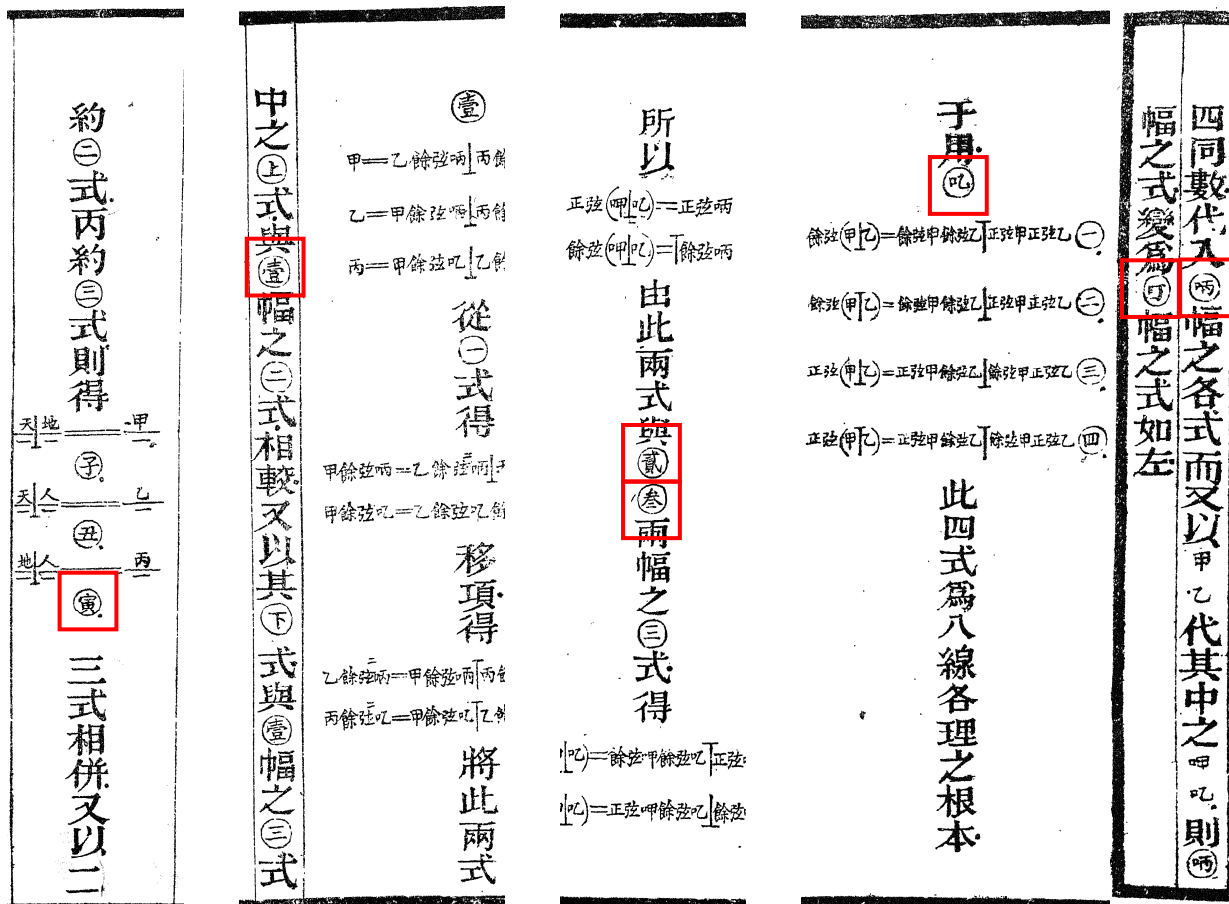


Figure 11. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.8 p.10; vol.24 p.13, p.15, p.19 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing 寅, 卯, 辰, 巳, 午 and 未 in text.

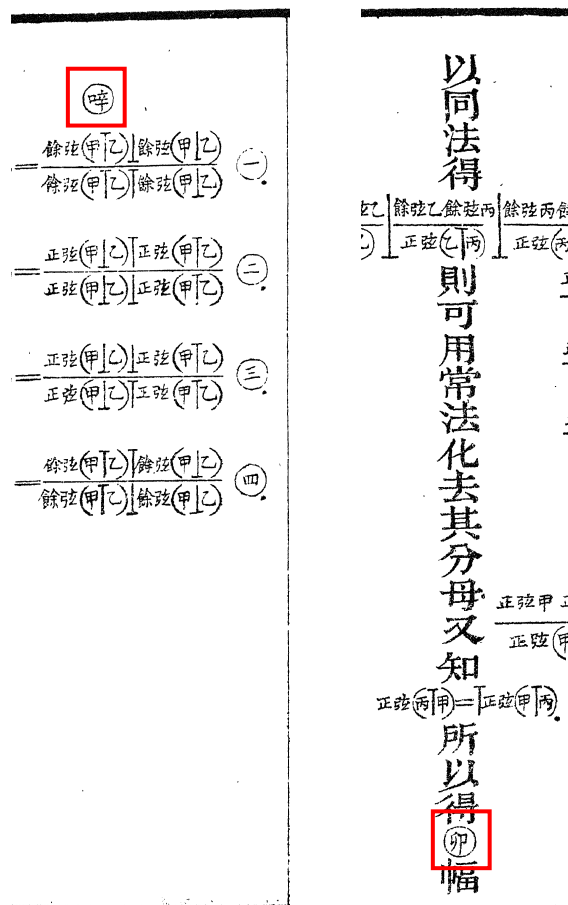


Figure 12. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.24 p.22, p.26 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing 卯 and 未 in text.

惟因^天故可由^午幅而得^亥幅^子幅之式如左

第二百五十三款 欲如前款之例求其正弦之式則可由^未幅而得^酉幅^申幅之式如左

設卯為偶數則可得^申幅之式

$$\begin{aligned} &= (T - \text{天}) \\ &= (T - \text{天}) \text{八} \\ &= (T - \text{天}) \text{四} \end{aligned}$$

使正餘弦之各方變為自小而大列之此法必辨別其弧之倍數為奇為偶

設卯為奇數則可由^午幅而得^申幅之式如左

由^戌幅之^三式得此式中若令^天代其^甲之餘弦^地代其^甲

$$\text{正弦}(\text{卯}) \text{甲} = \text{餘弦} \text{甲} \text{正弦} \text{卯}$$

之正弦則得

$$\begin{aligned} &= \text{二天正弦甲} \text{正弦} \text{卯} \\ &= \text{二天正弦} \text{二甲} \text{正弦} \text{甲} \\ &= \text{二天正弦} \text{三甲} \text{正弦} \text{二甲} \end{aligned}$$

所以從以上各式得^未幅

正弦餘弦正切餘切之各相等式因其為常用之式故列為

^辰幅如左

三五八文第廿二

Figure 13. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.24 p.27, p.29, p.31, p.32, p.33, p.34 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing ^辰, ^午, ^未, ^申, ^酉, ^戌, ^亥 and ^子 in text.

幅^三式
又^二正法甲 正法三甲 = 餘弦四甲 餘弦二甲
六正法甲 正法甲 = 三餘法二甲
八正法甲 = 餘法四甲 四餘法二甲
所以得^卯幅如左

弦之倍數明之是^亥幅如左

正初甲 ^地
正初二甲 = $\frac{1 \text{ 酉}}{2 \text{ 酉}}$
正初三甲 = $\frac{1 \text{ 酉}}{3 \text{ 酉}}$
正初四甲 = $\frac{1 \text{ 酉}}{4 \text{ 酉}}$
正初五甲 = $\frac{1 \text{ 酉}}{5 \text{ 酉}}$
如令
角 = $\frac{2 \text{ 卯}}{2}$
亢 = $\frac{2 \text{ 卯}}{3}$
辰 = $\frac{2 \text{ 卯}}{4}$
巽 = $\frac{2 \text{ 卯}}{5}$
則用角

第二百五十五款 又以同法得正弦之式為^天^天兩幅如左

Figure 14. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.24 p.35, p.37, p.39, p.40 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing ^天, ^天, ^地, ^卯 and ^卯 in text.

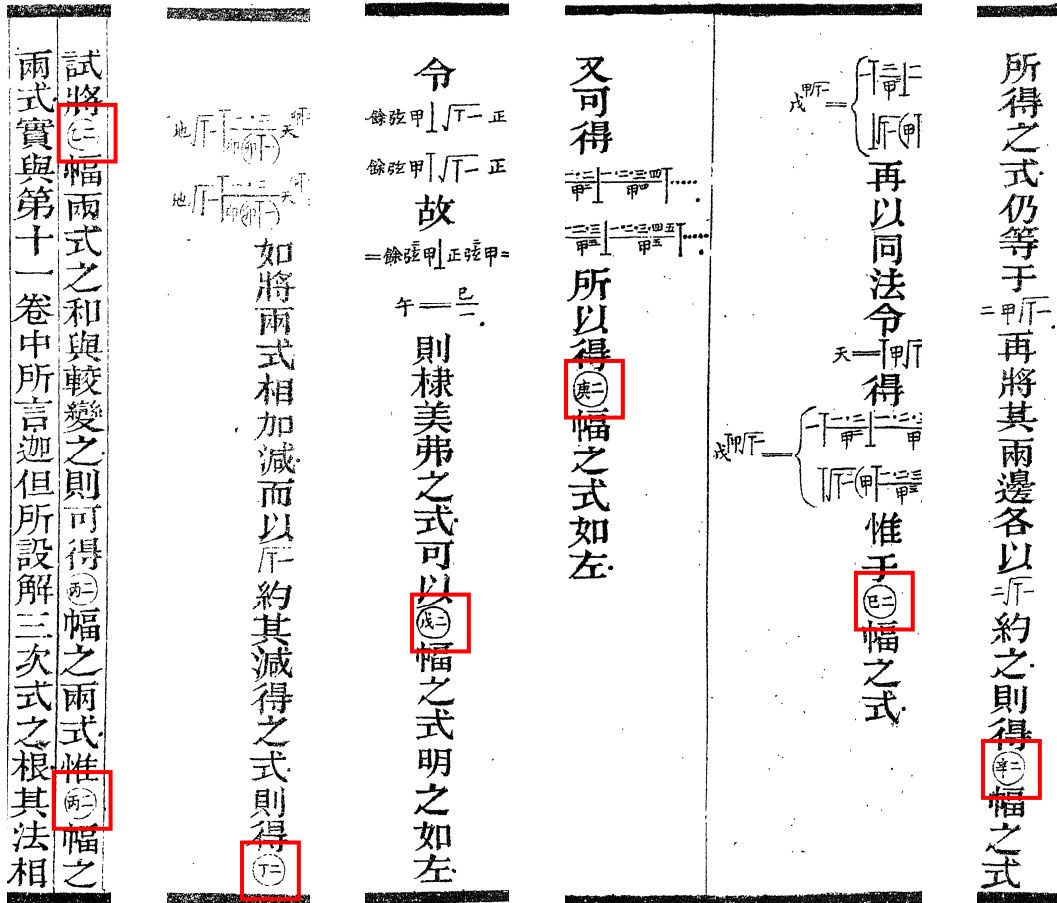


Figure 15. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.25 p.6, p.7, p.8, p.17, p.19 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing ㉓, ㉔, ㉕, ㉖, ㉗, ㉘ and ㉙ in text.

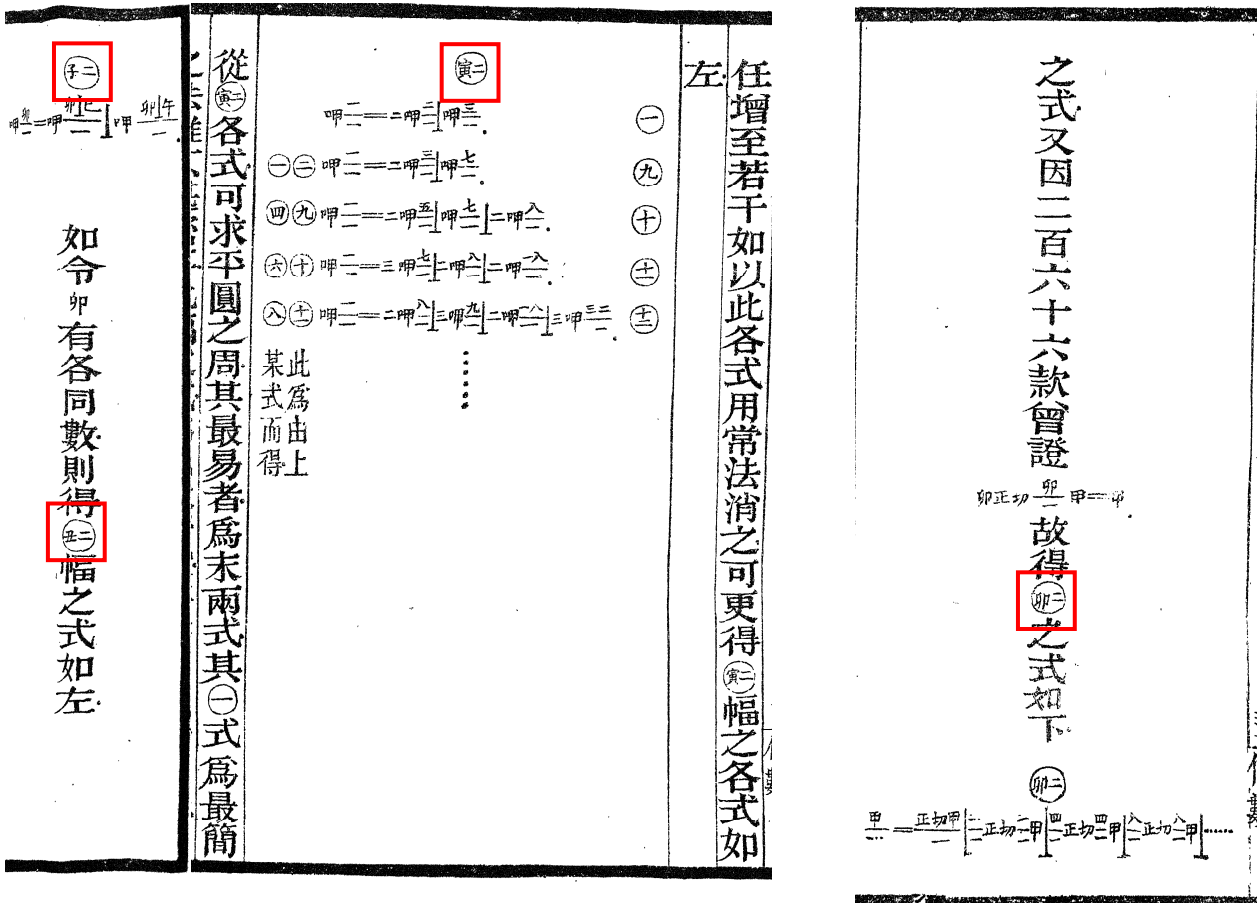


Figure 16. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.25 p.22, p.25 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing ㉚, ㉛, ㉜ and ㉝ in text.

如乙法二乘寅實五得一十則一當在卯位也甲法一乘寅實五
 得五五乃零數當在下位之下故亦在卯位上也蓋以寅為本位
 之時則卯為下位辰為下位之下也以丑為本位之時寅為下位
 卯為下位之下也

因乘定位法

式三百六十五人每人一十二兩共得四三八問四為何數日千
 數 ㊦ 通曰以法首齊實首布列甲子同位乙丑同位從丑下一位
 卯 變 八
 寅 變 三
 丑 後 四
 子 三

甲 乙 二
 甲 一

百
 四為千也但法未必單數乃可如今一十二兩
 是也若一兩二錢或一百二十兩則不同矣總
 以單數為率下則順推上則逆推可耳 ㊦ 通

曰視得數之首在實之何位上今在實之十位上又視法有幾位

Figure 17. Example from Fang Zhongtong's (方中通) book *Zhusuan* (珠算) p.5 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ㊦ , ㊦ and ㊦ in text.

數皆由二十五除得之

一除坤一數為丑一數正 三除坤二數為丑二數負 五除坤
 三數為丑三數正 七除坤四數為丑四數負 九除坤五數為
 丑五數正 十一除坤六數為丑六數負 十三除坤七數為丑
 七數正 十五除坤八數為丑八數負 十七除坤九數為丑
 五除單一為艮一數一 ㊦ 六 ㊦ 六 ㊦ 六除艮二數為艮三數
 可艮一數為艮二數 二九一六除艮二數為艮三數 二九一
 六除艮三數為艮四數 二九一六除艮四數為艮五數 二九
 一六除艮五數為艮六數 二九一六除艮六數為艮七數 二
 九一六除艮七數為艮八數除以下艮九數艮十數皆由二九一六
 一除艮一數為寅一數正 三除艮二數為寅二數負 五除艮
 三數為寅三數正 七除艮四數為寅四數負 九除艮五數為
 寅五數正 十一除艮六數為寅六數負 十三除艮七數為寅

Figure 18. Example from Zeng Jihong's (曾紀鴻) book *Yuanlü Kaozhen Tujie* (圓率攷真圖解) p.13 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ㊦ , ㊦ , ㊦ and ㊦ in text.

千隻... 卷二
 之內且欲令其答爲整數即在子丑能除絕甲也。
 十二問○有一富戶閒遊遇三乞丐遂將所帶之六十文錢分與
 之其與第二人者等於第一人之二倍與第三人者等於第一
 人之三倍問各與錢若干。
 十三問○今將甲錢分與三人與第二人等於第一人之子倍與
 第三人等於第一人之丑倍問各與錢若干。
 十四問○今有真梨者先賣去十枚後依前價又賣去一十五枚
 如是後得之錢較前多一十五文問每梨價若干。
 十五問○今有二人相去甲里同向而行一人每點鐘行子里一
 人每點鐘行丑里及其同至一處需幾點鐘。
 十六問○有一富戶臨終遺命除銀一千兩分與張王李三人與
 王姓頂與張姓三倍與李姓頂與王姓二倍問三人各宜得銀

Figure 19. Example from Zou Liwen's (鄒立文) book *Daishu Beizhi* (代數備旨) vol.3 p.13 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 圓 in text.

負實因方大實小不能進步又單數無可商乃退步之
 如下式
 實減餘 方加得加得 廉加得加得加得 隅
 初商一正以商二乘隅得 四八八正以加廉得 六五五八正
 以商二乘之得 一六一六正以加方得 四八八正以商
 二乘之得 九七六三二正以減負實餘 三三三六八負爲次商
 實又以商二乘隅得 四八八正以加廉得 七四六正以商
 二乘之得 一四九二正以加方得 六三九九八正退一位得
 四二九九八正爲次商方 又以商二乘隅得 四八八正以

Figure 20. Example from Zeng Jihong's (曾紀鴻) book *Subu Yancao* (粟布演草) vol.1 p.3 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 四, 正 and 退 in text.

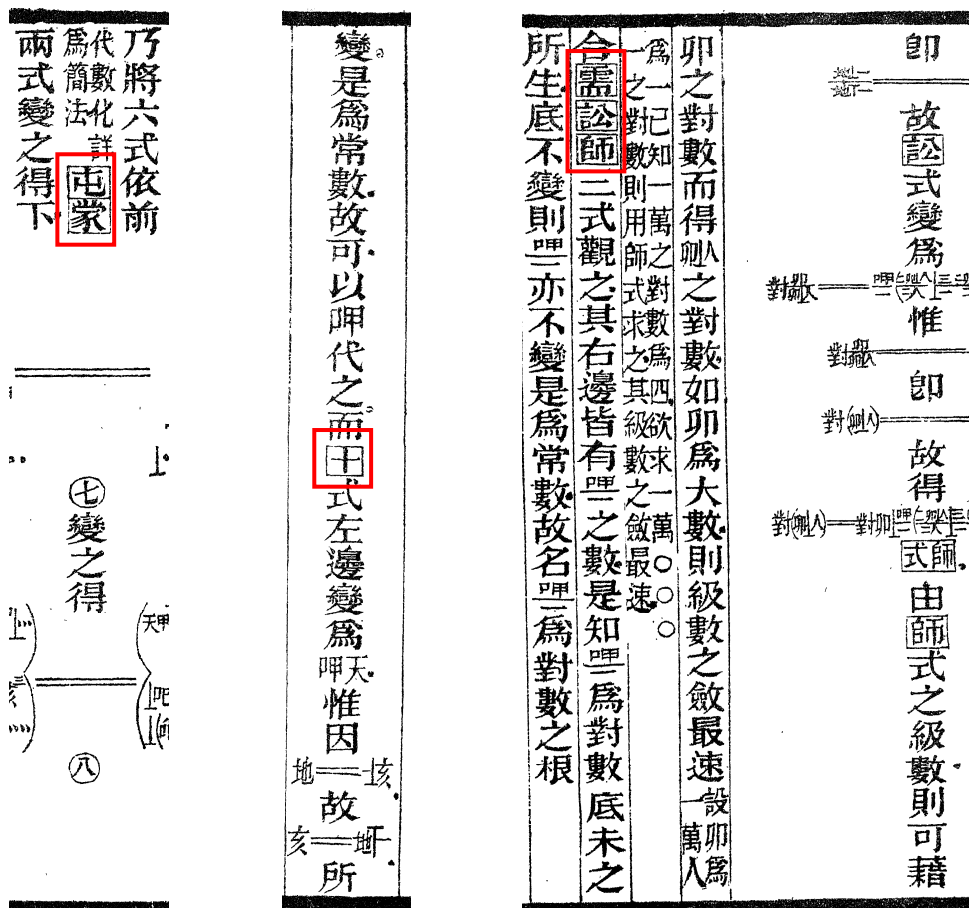


Figure 21. Example from Zeng Jihong's (曾紀鴻) book *Duishu Xiangjie* (對數詳解) vol.2 p.7, p.8 and p.10 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 𠄎, 𠄏, 𠄐, 𠄑 and 𠄒 in text.

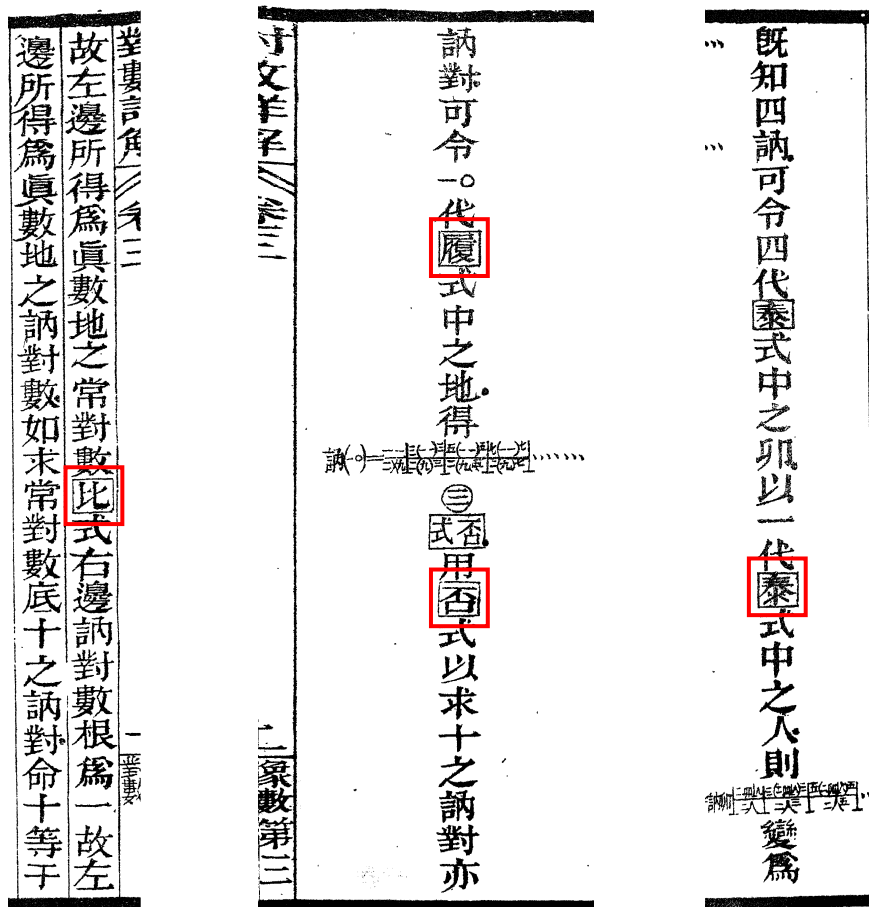


Figure 22. Example from Zeng Jihong's (曾紀鴻) book *Duishu Xiangjie* (對數詳解) vol.3 p.1, p.2 and p.3 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 𠄓, 𠄔, 𠄕 and 𠄖 in text.

如將九條式隨式中之呬以爻代之得式變之得式設令式則式

數指若干用式式右邊級數求之可識地式數式之同數

如于式之式中令式則式而式變為式如令式則式變

式式與小畜式之訥對數式既已明呬之同數為底之訥對又知天為對數未式之

又式又式所以式母子相同式用卯者可汰之得下式前三式借卯指數入算者

將式式依式式二項例詳之為級數得式

地式上式即式惟因式

Figure 23. Example from Zeng Jihong's (曾紀鴻) book *Duishu Xiangjie* (對數詳解) vol.4 p.2, p.3, p.4 and p.6 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 固, 固, 因, 固, 固 and 固 in text.

如前題

亦可令式故式為句股之通

邊三角形之每邊若干 答式式

圓徑求所容等邊三角形之式為式

除作式

取一點至三邊作三垂線三垂線之

李鳳苞曰凡三垂線與三邊總相

正方對角線與一邊之較丁求邊

華蘅芳曰

從句股形二銳角至平分句股二

天式天式天式天式天式天式天式天式天式天式

Figure 24. Example from Alexander Wylie and Li Shanlan's (李善蘭) book *Daiweiji Shiji Zhu* (代微積拾級注) vol.1 p.10 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing \perp , \perp , and \perp in text.

丁_下四乙_上六甲丙與五乙_下二甲丙_下一二丁與四乙_下丁
 問相加若干
 七問○今有七甲乙_下子_上亥與_上四甲乙_下子_上亥與一
 二甲乙_下一四子_上人與_上六子_上二亥問四式之和若干
 八問○今有六天_下五乙_下八_上甲及_上五甲_下四天下_下四乙_下
 三問二式之和若干
 九問○今有甲_下二乙_下三丙_下一〇及_上二乙_下四甲_下五丙_下
 一〇問相加若干
 十問○今有(一五甲_下八乙_下三二丙_下一)二乙_下丙_下加(一
 九乙_下四甲_下一甲_下二乙_下丙_下)加(甲_下二九甲_下丙_下
 一二乙_下丙_下五乙_下丙_下)加(九甲_下丙_下一四乙_下丙_下乙_下丙_下)其總
 若干
 十一問○今有(五甲_下乙_下八甲_下乙_下天也)加(四甲_下乙_下天
 也)加(四甲_下乙_下天也)加(四甲_下乙_下天也)加(四甲_下乙_下天也)
 十四○相交之兩直線不能各與他直線平行
 號
 形學中因欲省文並欲令學者易明其理即藉用數學之諸號以
 及代數變式之法
 等號二○此號寫於兩幾何之間即顯此幾何等於彼幾何譬如
 甲二丙意即甲等於丙也
 不等號八或 \vee ○此號寫於兩幾何之間即顯此幾何與彼幾何
 不等譬如甲 \wedge 乙意即甲小於乙又甲 \vee 乙意即甲大於乙也
 總之不等號其口必向幾何之大者
 正號十今改用上○此號寫於兩幾何之間即顯此幾何加彼幾
 何譬如丙_下丁_下意即丙加丁也
 負號一今改用上○此號寫於兩幾何之間即顯此幾何減彼幾
 何譬如甲_下丁_下意即甲減丁也
 多算備旨 卷一
 十一 象數第三

Figure 25. Example from Zou Liwen's (鄒立文) book *Xingxue Beizhi* (形學備旨) vol.1 p.11 and *Daishu Beizhi* (代數備旨) vol.1 p.19 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing \downarrow and \uparrow in text.

如 \mathbb{I} 言以乙與甲相加也其甲字之左不作 \mathbb{I} 號亦與有 \mathbb{I} 號
 者無異
 丁者負也減也
 如 \mathbb{I} 言于甲數內減去乙數也
 丁者相減也凡言相減者必以小數減其大數
 如 \mathbb{I} 則當以二減三得一如 \mathbb{I} 亦當以二減三得一
 丁者左右兩數相乘也
 如 \mathbb{I} 則爲六
 若以兩元相乘者可將兩元並書之而中間不必作 \mathbb{I} 如甲乙
 相乘則作 \mathbb{I} 其意與 \mathbb{I} 無異
 者以右約左也
 如 \mathbb{I} 言以甲爲法而約其乙也
 一者以上約下也猶言分之也
 學算筆談 卷八 第一

Figure 26. Example from Hua Hengfang's (華蘅芳) book *Xuesuan Bitan* (學算筆談) vol.8 p.1 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing \mathbb{I} in text.

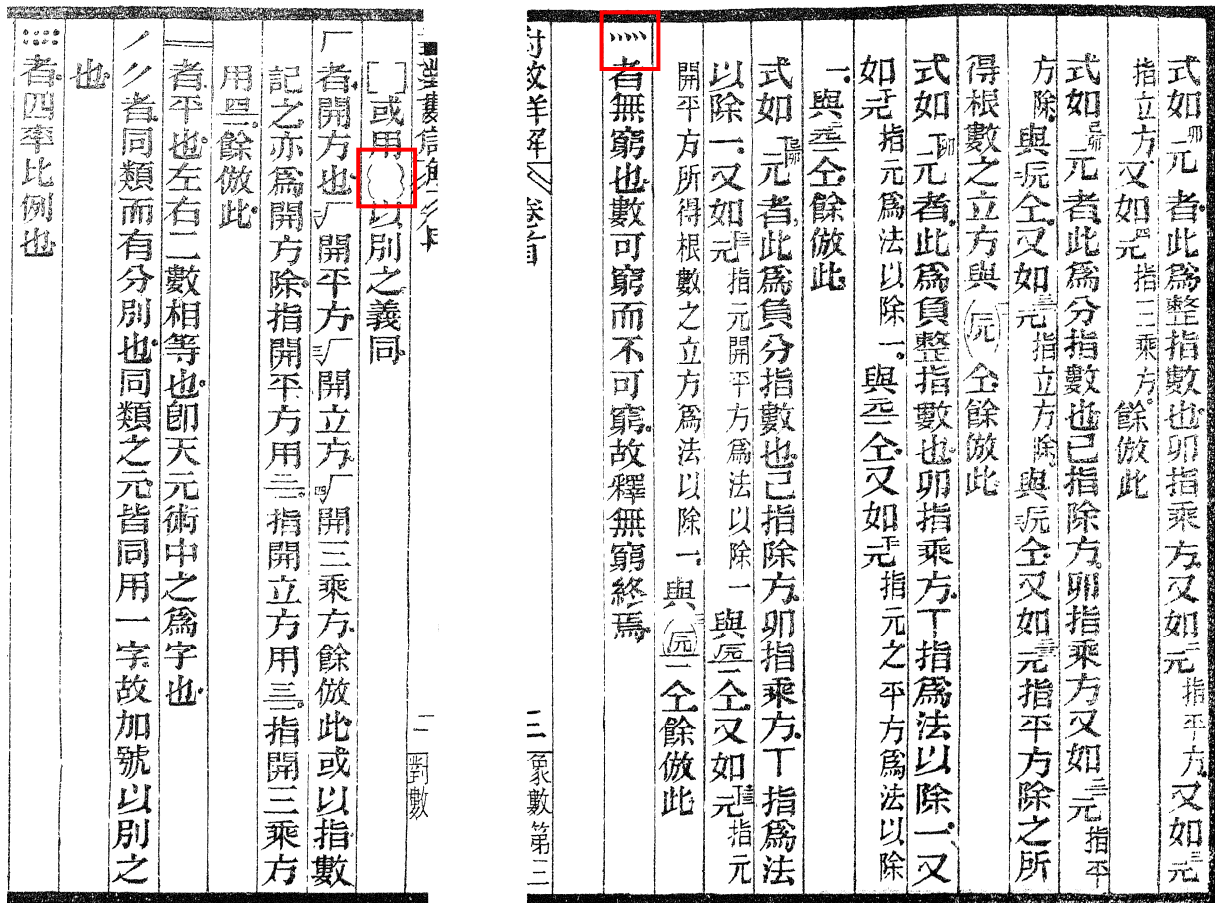


Figure 27. Example from Zeng Jihong's (曾紀鴻) book *Duishu Xiangjie* (對數詳解) vol.1 p.2 and p.3 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing {, } and ∞ in text.

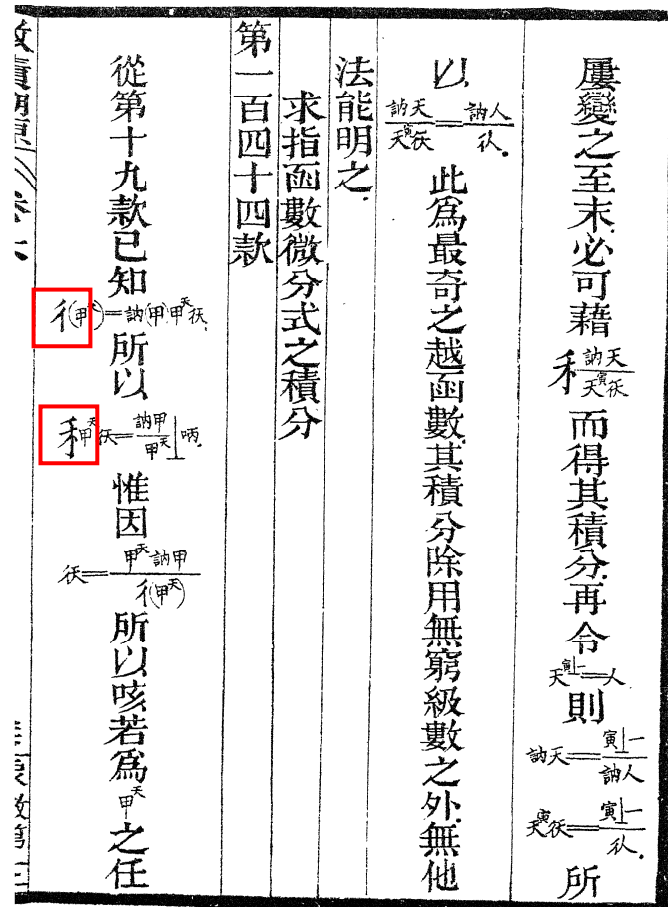


Figure 28. Example from Hua Hengfang's (華蘅芳) book *Weiji Suyuan* (微積溯源) vol.6 p.36 (Shanghai: Arithmetic Publishing House (算学书局), 1898), showing † and ‡ in text.