1/77-

IL NUOVO CIMENTO Vol. 42 B, N. 1 11 Novembre 1977

Time Symmetry and the Einstein Paradox.

O. Costa de Beauregard

Institut Henri Poincaré - 11 rue Pierre et Marie Curie, 75005 Paris

(ricevuto il 27 Gennaio 1977)

Summary. — The characteristic difference between the paleoquantal calculation (addition of partial probabilities) and the neoquantal one (addition of partial amplitudes) for the correlation of photon polarizations in cascade transitions is derived in terms of elementary trigonometry. This deliberate use of simple formulae aims at a transparent rendering of the change in paradigm required by the so-called EPR paradox (which is truly the 1927 Einstein paradox), namely that 1) the two photons do *not possess* polarizations of their own when leaving the source C, but *borrow* one *later*, when interacting with the analysers L and N; 2) the die is thus not cast at C, but later, at L and N; 3) the correlation between the measurements at L and N is tied through C, *in their common past*. The tight connection between this « Einstein nonseparability » and the nonlocality in Feynman's « theory of positrons » is demonstrated through an analysis of the e^+e^- annihilation into two photons. Thus the Einstein paradox *corresponds*, in the « new wavelike probability calculus », to the Loschmid and Zermelo sort of paradox in the old probability calculus. That is, it contrasts the *intrinsic time symmetry* existing at the elementary level to the *factlike macroscopic time asymmetry*. Our discussion deliberately by-passes the hidden-variable problem, our model in this being Einstein's by-passing of the mechanical aether when proposing special relativity. We believe that here today, like there in 1905, the problem is *tayloring the wording after the* (operationally good) *mathematics*. Moreover, that the change in paradigm, which is needed, comes through a victory of formalism over modelism.

1. – Introduction: cascade experiments.

The quantum-mechanical prediction ([1]) in atomic-cascade experiments (fig. 1) in which photon pairs propagating in opposite directions along an axis

([1]) See, for example, M. A. Horne: Ph. D. Thesis, Boston University (1970) (mimeographed).

x and passing linear polarizers L and N with relative angle α are counted is, in the ($J = 0$, $J = 1$, $J = 0$)-type cascade,

$$\langle 1, 1\rangle = \langle 0, 0\rangle = \tfrac{1}{2}\cos^2\alpha\,, \qquad \langle 1, 0\rangle = \langle 0, 1\rangle = \tfrac{1}{2}\sin^2\alpha \tag{1}$$

for the probabilities of answers (yes, yes) and (no, no), (yes, no) and (no, yes) respectively, and, in the (1, 1, 0)-type cascade,

$$\langle 1, 1\rangle = \langle 0, 0\rangle = \tfrac{1}{2}\sin^2\alpha\,, \qquad \langle 1, 0\rangle = \langle 0, 1\rangle = \tfrac{1}{2}\cos^2\alpha\,. \tag{2}$$

The experimental verifications are excellent [2].

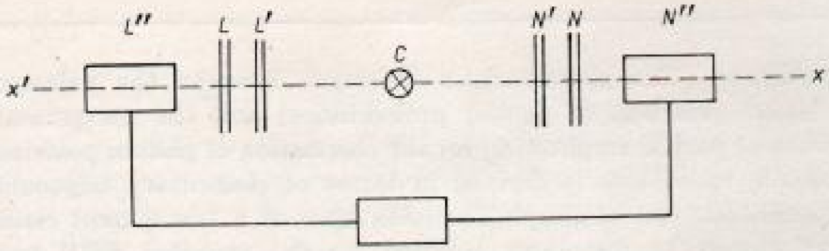

Fig. 1. – Photon polarization correlation experiments. C, atomic cascade; L', N', monochromatic filters; L and N, linear polarizers with adjustable angles A, B ($A - B \equiv \alpha$); L'', N'', coincident photodetectors.

Had these experiments been performed in the days of the old quantum mechanics, they certainly would have produced the same sort of stupefaction as the Michelson experiment did. Moreover, as will be shown, they also do require, in de Broglie's [3] words, a radical change of « our familiar notions pertaining to space and time ».

Consider for instance the case in which $\alpha = \pi/2$ with the (0, 1, 0)-type cascades. The « neoquantal » prediction is $\langle 1, 1\rangle = 0$, meaning that *all* photon pairs propagating in opposite directions along x are found with both linear polarizations, parallel to either of the two Cartesian axes y and z defined by the linear polarizers L and N. This would have been felt to be a paramount paradox [4] by any « paleoquantal » physicist, because he believed each photon pair leaving the source C to *possess* polarizations, compatible of course with the dynamics of the system, but *essentially independent of the orientations*

---

(2) S. J. Freedman and J. F. Clauser: *Phys. Rev. Lett.*, **28**, 938 (1972); J. F. Clauser: *Phys. Rev. Lett.*, **36**, 1223 (1976); E. Fry and R. C. Thompson: *Phys. Rev. Lett.*, **37**, 465 (1976).

(3) L. de Broglie: *Une tentative d'interprétation causale et non linéaire de la mécanique ondulatoire*, Chap. 12 (Paris, 1956), p. 73.

(4) Paradox: « A very surprising, but perhaps true statement » (Sense No. 1 in all dictionaries). Copernicus' heliocentrism has been a paradox.

of the polarizers L and N—and even of their presence or absence. For instance, in the case considered, parallel linear polarizations making all sort of angles with the orthogonal axes y and z should have been found, or possibly circular polarizations of equal helicities. Thus the paleoquantal prediction was (in this $\alpha = \pi/2$ case with the $(0, 1, 0)$ cascades) that a large number of (yes, yes) answers should occur. On the other hand, the subensemble of photon pairs with (parallel) linear polarizations parallel to either y or z should have been of zero measure.

The experimental fact could not be more opposite: photon pairs with the above property are *all* of the photon pairs in this experimental arrangement. This is a paradox [4] proper, the sort of which requires a change of paradigm [5].

Three major statements follow *necessarily* from the experimental [2] findings:

1) The photons in the pairs issuing from the source C *do not possess* polarizations of their own. They *borrow one later*, by interacting with the measuring devices L and N.

This, of course, is a specification of a well-known general statement in the neoquantal mechanics. The point is that *there is perhaps no more direct experimental proof of it than this one.*

2) In the chance game that is at stake, the die is cast, so to speak, *not at the beginning*, at C, but *in the end*, at L and N.

This is the very paradox EINSTEIN [6] was clever enough to point out as early as 1927 and which he [6-8], SCHRÖDINGER [9], RENNIGER [10], DE BROGLIE [3] and others rejected as unacceptable. *It is today an experimental truth* [2].

3) The correlation found to exist between the measurements at L and N is *not* tied, in space-time, along the spacelike vector LN (which is physically empty), but (fig. 2) *necessarily along the Feynman-style zigzag LCN* made of two timelike vectors (which is physically occupied). In other words, the measurements at L and N do produce the *same wave collapse* at C, *in their common past.* Or, again in other words, Einstein's prohibition to « telegraph into the past » does *not* hold at the level of the *quantal stochastic event* (the wave collapse). This statement is thus of a factlike [11] and macroscopic nature.

---

(5) *Paradox and Paradigm*, edited by R. G. COLODNY (Pittsburgh, Pa., 1973).

(6) A. EINSTEIN: in *Rapports et Discussions du V Conseil Solvay* (Paris, 1928), p. 253.

(7) A. EINSTEIN, B. PODOLSKY and N. ROSEN: *Phys. Rev.*, **47**, 777 (1935).

(8) A. EINSTEIN in *A. Einstein, Philosopher Scientist*, edited by P. A. SCHILPP (Evanston, Ill., 1949), p. 85, 683.

(9) E. SCHRÖDINGER: *Naturwiss.*, **23**, 844 (1935), see p. 845.

(10) M. RENNINGER: *Physik*, **158**, 417 (1960); *Phys. Zeits.*, **136**, 251 (1963).

(11) H. MEHLBERG: in *Current Issues in the Philosophy of Science*, edited by H. FEIGL and G. MAXWELL (New York, N. Y., 1961).

This is the interpretation of the Einstein paradox I [12] have proposed quite a few times, and which now STAPP [13], BELL [14], DAVIDON [15] and others [16] are more or less advocating or pointing at.

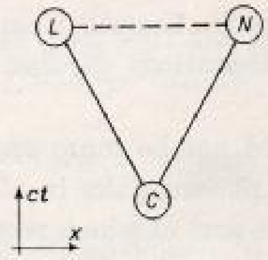

Fig. 2. – Space-time diagram of the Einstein paradox: the die is cast not at the severance point-instant C, but later, at L and N, where and when the measurements are performed. Thus the correlation between L and N is tied through C, in their common past, *via* the Feynman-style zigzag made of the timelike vectors CL and CN.

2. – Neoquantal and paleoquantal calculations for cascades.

From the two (orthogonal) pure helicity states $L_a L_b$ and $R_a R_b$ of the pairs of photons a and b, we build the two (orthogonal) P-invariant states

$$(3)\qquad \begin{cases} \dfrac{1}{2}(L_a L_b + R_a R_b) = \dfrac{1}{2}(E_a^y E_b^y + E_a^z E_b^z)\,, \\[2ex] \dfrac{1}{2}(L_a L_b - R_a R_b) = \dfrac{i}{2}[E_a^z E_b^y - E_a^y E_b^z]\,, \end{cases}$$

where the well-known formulae

$$(4)\qquad \begin{cases} \sqrt{2}\,L_a = E_a^y + iE_a^z\,, \quad \sqrt{2}\,R_a = E_a^y - iE_a^z\,, \\ \sqrt{2}\,L_b = E_b^y - iE_b^z\,, \quad \sqrt{2}\,R_b = E_b^y + iE_b^z \end{cases}$$

have been used; y and z denote arbitrary Cartesian axes orthogonal to the line of flight x of the two photons.

([12]) O. COSTA DE BEAUREGARD: *Compt. Rend.*, **236**, 1632 (1953); *Rev. Intern. Philos.*, **61-62**, 1 (1962); *Dialectica*, **19**, 280 (1965); in *Proceedings of the International Conference on Thermodynamics*, edited by P. T. LANDSBERG (London, 1970), p. 539.
([13]) H. P. STAPP: *Nuovo Cimento*, **29** B, 270 (1975).
([14]) J. S. BELL: *Epist. Lett.*, **9**, 11 (1976).
([15]) W. C. DAVIDON: *Nuovo Cimento*, **36** B, 34 (1976).
([16]) See footnote ([13]) in J. F. CLAUSER and M. A. HORNE: *Phys. Rev. D*, **10**, 526 (1974).

A and B denoting the angles with, say, the y-axis of two linear polarizers respectively inserted on the paths of the photons a and b, and setting

$$A - B = \alpha\,, \tag{5}$$

now we calculate, using the « *golden rule* » *of the neoquantal mechanics* that *partial amplitudes* rather than *partial probabilities* should be added, and their modulus squared, the overall transition probability.

Turning the polarizer A by ΔA changes the phase of the photon L_a by $-\Delta A$, and turning the polarizer B by ΔB changes the phase of the photon L_b by $+\Delta B$. Thus the transition amplitude towards the (yes, yes) answer is proportional to $\exp[i\alpha]$ for the L_aL_b pair and, similarly, to $\exp[-i\alpha]$ for the R_aR_b pair. Adding (respectively, substracting), squaring the absolute value and normalizing, we obtain the expression $\langle 1, 1\rangle$ as in (1) (respectively, as in (2)). The calculation of $\langle 0, 0\rangle$, $\langle 1, 0\rangle$ and $\langle 0, 1\rangle$ proceeds similarly and, on the whole, formulae (1) and (2) are recovered.

What is interesting is the expansion of the expressions (for, say, the (0, 1, 0) case)

$$\langle 1, 1\rangle = \langle 0, 0\rangle = \tfrac{1}{8}\,|\exp[i\alpha] + \exp[-i\alpha]|^2 = \tfrac{1}{4}\,(1 + \cos 2\alpha)\,, \tag{6a}$$

$$\langle 1, 0\rangle = \langle 0, 1\rangle = \tfrac{1}{8}\,|\exp[i\alpha] - \exp[-i\alpha]|^2 = \tfrac{1}{4}\,(1 - \cos 2\alpha)\,. \tag{6b}$$

In both of the formulae, the contribution

$$\langle 1, 1\rangle_0 = \langle 0, 0\rangle_0 = \langle 1, 0\rangle_0 = \langle 0, 1\rangle_0 = \tfrac{1}{4} \tag{7}$$

is the paleoquantal prediction, if we assume that the photon pairs do leave the source C in either the L_aL_b or the R_aR_b state. The contributions

$$\Delta\langle 1, 1\rangle = \Delta\langle 0, 0\rangle = \tfrac{1}{4}\cos 2\alpha\,, \qquad \Delta\langle 1, 0\rangle = \Delta\langle 0, 1\rangle = -\tfrac{1}{4}\cos 2\alpha \tag{8}$$

are the neoquantal corrections, containing the phase relation between the photons a and b.

Second, we use as orthogonal states the linear polarizations along y and z. The transition amplitude towards the (yes, yes) answer is $\cos A \cos B$ for the $E_a^y E_b^y$ state, $\sin A \sin B$ for the $E_a^z E_b^z$ state, $\cos A \sin B$ for the $E_a^y E_b^z$ state and $\sin A \cos B$ for the $E_a^z E_b^y$ state. Using the « golden rule », we again find formulae (1) and (2).

What is interesting is in, say, the (0, 1, 0) case the expansion of the expressions

$$\langle 1, 1\rangle = \langle 0, 0\rangle = \tfrac{1}{2}(\cos A \cos B + \sin A \sin B)^2 = \\ = \tfrac{1}{2}(\cos^2 A \cos^2 B + \sin^2 A \sin^2 B) + \tfrac{1}{4}\sin 2A \sin 2B\,, \tag{9}$$

$$\langle 1, 0\rangle = \langle 0, 1\rangle = \tfrac{1}{2}(\sin A \cos B - \cos A \sin B)^2 = \\ = \tfrac{1}{2}(\sin^2 A \cos^2 B + \cos^2 A \sin^2 B) - \tfrac{1}{4}\sin 2A \sin 2B\,. \tag{10}$$

The contributions

$$(11) \qquad \begin{cases} \langle 1, 1\rangle_0 = \langle 0, 0\rangle_0 = \frac{1}{2}(\cos^2 A \cos^2 B + \sin^2 A \sin^2 B)\,, \\ \langle 1, 0\rangle_0 = \langle 0, 1\rangle_0 = \frac{1}{2}(\sin^2 A \cos^2 B + \cos^2 A \sin^2 B) \end{cases}$$

are the paleoquantal predictions assuming that the photon pairs do leave the source as a statistical mixture with (parallel) linear polarizations parallel to either the y- or the z-axis. The contribution

$$(12) \qquad \Delta\langle 1, 1\rangle = \Delta\langle 0, 0\rangle = -\Delta\langle 1, 0\rangle = -\Delta\langle 0, 1\rangle = \tfrac{1}{4}\sin 2A \sin 2B$$

is the neoquantal correction, containing the phase relation between photons a and b [17].

In summary:

1) The Einstein paradox has its root in the replacement of the paleostochastic rule of addition of partial probabilities by the neostochastic rule of addition of partial amplitudes. This statement is either explicit or implicit in most papers dealing with the Einstein paradox, and traces the origin of the paradox [4] to a well-known specific rule of the neoquantal mechanics. DIRAC [18] and LANDÉ [19] among others are adamant on this point.

2) The algebraic difference between the neostochastic and the paleostochastic transition probabilities essentially contains the off-diagonal terms. This is again, in general, a very well-known statement.

3) While the neostochastic formula is invariant with respect to changes in the representation, the paleostochastic ones are not.

4) The (noninvariant) neoquantal correction can be made zero with certain settings of the measuring apparatus, or certain choices of the representation. In these cases, the neoquantal transition probability is formally that of a classical mixture.

5) The specific difference between the new rule of addition of partial amplitudes and the old rule of addition of partial probabilities emphasizes the wavelike nature of the neostochastic theory.

---

[17] Formulae (11) and (12) are obviously not rotation invariant around the x-axis. We can rewrite (12) as $8\Delta = \cos 2\alpha - \cos 2(A+B)$, the latter term having mean value 0 by rotation around x. Thus *in the mean* $\langle\!\langle 1, 1\rangle\!\rangle_0 = \langle 1, 1\rangle - \frac{1}{8}\cos 2\alpha$. See in this respect D. BOHM and H. AHARONOV: *Phys. Rev.*, **108**, 1070 (1957).

[18] P. A. M. DIRAC: *The Principles of Quantum Mechanics*, 3rd Edition (Oxford, 1947).

[19] A. LANDÉ: *New Foundations of Quantum Mechanics* (Cambridge, 1965). A controversial, but suggestive book.

It is a *wavelike probability calculus* in which, incidentally, there must be (and there is) a tight binding between the two *macroscopic* irreversibility *facts* of wave retardation and of probability increase, as also of the two laws of *intrinsic symmetry between retarded and advanced waves*, on the one hand, and between (*blind*) *statistical prediction and retrodiction*, on the other ([20]).

This brings us back to the time and space aspect of the paradox, as emphasized first by EINSTEIN ([6]).

3. – Neoquantal and paleoquantal correlations in general.

It is surprising that the formulae presented in this section are not found (to my cognizance) in any of the competent presentations ([21]) of correlated quantal systems, although their import is implicitly stated. Our impetus for writing them down came from a *preprint* by GARUCCIO and SELLERI ([22]), but (at least in this preliminary form) their presentation was not identical with the one we are giving here.

The typical system we are discussing is described as a pure state Ψ expanded in the form of a sum of partial amplitudes

$$|\Psi\rangle = \sum_j c_j |\varphi_j\rangle |\psi_j\rangle , \tag{13}$$

where φ_j and ψ_j span two independent Hilbert spaces; the subsystems φ_j and ψ_j are thus coupled, although this coupling may very well result from an interaction that has ceased for some time—as is the case discussed in this paper. By definition,

$$w_j \equiv c_j^* c_j , \qquad \sum w_j = 1 . \tag{14}$$

A and B denoting the Hermitian operators describing two measurements to be performed upon the subsystems φ and ψ, the quantal correlated mean value (invariant under changes of the orthobases, or « co-ordinate frames ») is

$$\langle A, B\rangle = \sum\sum c_i^* c_j \langle\varphi_i|A|\varphi_j\rangle \langle\psi_i|B|\psi_j\rangle . \tag{15}$$

([20]) For an overall view and a guide in the literature of the subject, see O. COSTA DE BEAUREGARD: *Proceedings of the International Congress for Logic, Method and Philosophy of Science*, edited by Y. BAR HILLEL (Amsterdam, 1964), p. 313, or *Studium Generale*, **24**, 10 (1971).

([21]) See, for instance, F. LONDON and E. BAUER: *La théorie de l'observation en mécanique quantique* (Paris, 1939); P. A. MOLDAUER: *Phys. Rev. D*, **5**, 1028 (1972); F. J. BELINFANTE: *Measurements and Time Reversal in Objective Quantum Theory* (Oxford, 1975), p. 26.

([22]) A. GARUCCIO and F. SELLERI: *Nuovo Cimento*, **36** B, 176 (1976).

Denoting the contribution with $i = j$ as

$$\langle A, B\rangle_0 = \sum w_i \langle A_i\rangle \langle B_i\rangle \tag{16}$$

with

$$\langle A_i\rangle \equiv \langle \varphi_i|A|\varphi_i\rangle\,, \qquad \langle B_i\rangle = \langle \psi_i|B|\psi_i\rangle\,, \tag{17}$$

and the contribution with $i \neq j$ as

$$\Delta\langle A, B\rangle = \tfrac{1}{2}\sum_{i\neq j} c_i^* c_j \langle \varphi_i|A|\varphi_j\rangle \langle \psi_i|B|\psi_j\rangle + \text{c.c.} \tag{18}$$

(neither of which is co-ordinate invariant), we write

$$\langle A, B\rangle = \langle A, B\rangle_0 + \Delta\langle A, B\rangle\,, \tag{19}$$

where $\langle A, B\rangle_0$ *is the classical correlated* mean value, expressed as a *mixture*, and implying separate statistics for the subsystems φ and ψ (« local hidden variables », in the crypto-deterministic philosophy). $\Delta\langle A, B\rangle$, containing the « off-diagonal » terms (and, thus, the phase relations between the φ_i's and the ψ_i's), is the neoquantal correction which, when added to the paleoquantal correlated mean value (16), yields the neoquantal correlated mean value (15). It thus belongs typically to the *wavelike probability calculus*.

The $\Delta\langle A, B\rangle$ contribution of expression (18) is rendered *zero if one* (*a fortiori* both) of the operators A or B is diagonalized by the representation. The mean value $\langle A, B\rangle$ then assumes the form $\langle A, B\rangle_0$ it would have with mixtures. *But this is a mere* semblance, *relative* to the co-ordinate frame (and to the operator diagonalized in it).

Of course, if one of the magnitudes A or B is measured, then the phase relations are lost and the overall system becomes a mixture—although the larger system comprising also the measuring device remains in a pure state ([22]).

In summary:

1) The specific difference between the new wavelike probability calculus and the old classical one consists in the replacement of the addition rule (16) of partial probabilities by the addition rule (13) of partial probability amplitudes. The numerical difference between the corresponding correlated mean values $\langle A, B\rangle$ and $\langle A, B\rangle_0$ is the off-diagonal term $\Delta\langle A, B\rangle$ *of expression* (18).

Contrary to $\langle A, B\rangle$, neither $\langle A, B\rangle_0$ nor $\Delta\langle A, B\rangle$ are co-ordinate invariant. $\Delta\langle A, B\rangle$ is rendered zero by diagonalizing at least one of the operators A or B.

2) The Einstein paradox ([6]), belonging specifically *to the new* wavelike probability calculus, is tied with the existence of *phase relations between distant systems*, these being (as already said) *propagated both forwards and backwards in time.*

4. – The essence of the paradox.

As this point, a little fable will help us to understand matters: At midnight GMT, two travellers leave the Calcutta airport C, one for London L, the other for Nagasaki N, each carrying a closed box which contains or not the one ball which a third man, in Calcutta, has enclosed, behind a veil. Having landed at 6 GMT, each traveller opens his box, and then immediately learns what the *other* man finds.

The point is that, when made explicit, the logical inference between L and N is *not* telegraphed along the spacelike vector LN, which is physically empty, but rather along the Feynman-style zigzag LCN made of two timelike vectors, once towards the past, once towards the future (fig. 2).

There is nothing paradoxical in this, however, because what we have between L and N is pure telediction with no teleaction admixture. The die is cast at C, so to speak, and this is virtually the end of the story. From then on we have in each subsystem a « local hidden variable » with value 0 in one box, 1 in the other.

It is precisely there that the new quantum mechanics makes the radical change, because, as pointed out by EINSTEIN [6] as early as 1927, it is claimed that the die is cast not at C, but *later*, where and when a measurement is made: at L or N—at L *and* N if *both* measurements are made.

The reason for this is the neoquantal fact that ***nonsimultaneously measurable magnitudes*** do exist—for example, linear polarizations of a photon in two nonparallel or nonorthogonal directions. Moreover, both observers at L and N can in principle decide which magnitude they will measure *after* the photon pair has left the source C (but, of course, before it reaches L and N).

For these quite compelling reasons, quantum mechanics considers that it is *at L and/or N, not at C,* that the die is cast. *In this precise sense*, what now we have between L and N is *telediction plus teleaction*—the very sort of thing that horrified EINSTEIN [6-8], SCHRÖDINGER [9], RENNINGER [10], DE BROGLIE [3] and many others, *but now is known* [2] *to be the experimental truth.*

Of course, an experiment even more crucial than those already performed would be (as emphasized first by BOHM and AHARONOV [23]) one in which the polarizers at L and N are turned after the photons of the pair have left the source C. ASPECT [24] has defined and is building such an experiment.

It is quite obvious that, if the neoquantal answer is again vindicated in this new switching experiment, this would prove *directly* the *telediction plus teleaction* existing between L and N (in the dice game we are playing). Let us bet, with the majority of theoretical physicists, that this will be the case, and proceed.

---

[23] D. BOHM and Y. AHARONOV: *Phys. Rev.*, **108**, 1070 (1957).
[24] A. ASPECT: *Phys. Lett.*, **54** A, 117 (1975); *Phys. Rev. D*, **14**, 1944 (1976).

In summary:

1) In the neoquantal dice game, « the chance occurs not when the dice are shaken in the box, but when they stop rolling on the table. However, the two issues are correlated. »

2) If two (or more) measurements are performed at L, N ... on the *same* quantal system C, they are bound to produce the *same* wave collapse at C—in their common past. This is exemplified in the observations of impacts of α-particles upon a ZnS screen by two or more observers (fig. 3).

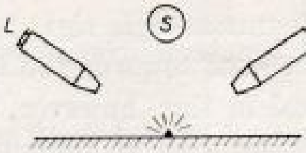

Fig. 3. – Space picture of an Einstein-style correlation between two measurements L and N performed upon the same quantal event C: impact of an α-particle (issuing from a source S) upon a ZnS screen.

3) Thus, contrary to the common feeling and to a natural assumption [4], also held inevitable by very famous physicists [3,6-9], *observers at L and N are not really independent*: *they are co-operating or competing for producing the same Ψ collapse*—in their common past. This has important philosophical implications that have been discussed elsewhere [13,25,26].

4) Thus the neoquantal stochastic event—the *transition, or wave collapse*—does not affect the future alone (as was assumed erroneously), *but, symmetrically, also the past.*

5) As will be shown in the next two sections, *this statement is written down since the beginning in the very tables of the law of the neoquantal mechanics*—but it was not received by its own followers.

6) As a corollary, physical irreversibility (in both forms of probability increase and of wave retardation) is a *factlike* (*not lawlike* [11]) *macroscopic* phenomenon implying *ensembles* (von Neumann's ensembles). Both of these formulations are *reciprocal* to each other, and are tied together by the neoquantal formalism. This I have discussed elsewhere [27].

---

[25] O. COSTA DE BEAUREGARD: *Found. Phys.*, **6**, 539 (1976); *Synthese*, **35**, 129 (1977).
[26] J. HALL, C. KIM, B. MCELROY and A. SHIMONY: *Found. Phys.* (in press).
[27] O. COSTA DE BEAUREGARD: *Cah. Phys.*, **12**, 317 (1958). See also ref. [20].

5. – Time symmetry of the wave collapse in relativistic quantum mechanics. Spinless particles.

The formulae I am presenting in this section (only outlining their demonstration) are extracted from a book now out of print [28].

By using units such that $c = 1$ and $\hbar = 1$, setting $x^4 = it$ and $\lambda, \mu, \nu, \varrho = 1, 2, 3, 4$, the Klein-Gordon equation for a free particle with unspecified spin

$$(\partial_\lambda^\lambda - m^2)\,\psi(x) = 0 \tag{20}$$

assumes in k-space the expression

$$(k_\lambda k^\lambda + m^2)\,\theta(k) = 0\,, \tag{21}$$

whence either

$$\eta(k) \equiv k_\lambda k^\lambda + m^2 = 0 \qquad or \qquad \theta(k) = 0\,. \tag{22}$$

Thus the Fourier expansion of $\psi(x)$ may be written as

$$\psi(x) = (2\pi)^{-\frac{3}{2}} \iiint_\eta \exp\,[ik_\lambda x^\lambda]\,\theta(k)\,\varepsilon(k)\,\mathrm{d}\eta\,, \tag{23}$$

where the integral is over both sheets of the mass shell $\eta(k) = 0$, because we do not exclude that the particle is endowed with an (unspecified) spin; by definition,

$$\varepsilon(k) = \begin{cases} +1 & \text{on sheet } \eta_+\,, \\ -1 & \text{on sheet } \eta_-\,, \\ 0 & \text{otherwise}\,. \end{cases} \tag{24}$$

The scalar volume element $\mathrm{d}\eta$ is defined as the length of the vector $\mathrm{d}\eta^\lambda$ through

$$\varepsilon_{\lambda\mu\nu\varrho}\,\mathrm{d}\eta^\lambda = -\,i[\mathrm{d}k_\mu\,\mathrm{d}k_\nu\,\mathrm{d}k_\varrho] \tag{25}$$

and

$$k^\lambda\,\mathrm{d}\eta = -\,m\,\mathrm{d}\eta^\lambda\,. \tag{26}$$

---

[28] O. Costa de Beauregard: *Précis de mécanique quantique relativiste* (Paris, 1967).

The reciprocal Fourier integral is, $\overleftrightarrow{\partial}_\mu$ denoting the Gordon current operator,

$$\theta(k) = -\frac{i}{2m}(2\pi)^{-\frac{3}{2}}\iiint_\sigma \exp[-ik_\lambda x^\lambda]\overleftrightarrow{\partial}_\mu \psi(x)\, \mathrm{d}\sigma^\mu . \tag{27}$$

It is over an arbitrary spacelike surface σ and is invariant if the vector k^λ is assumed to end on η; $\mathrm{d}\sigma^\lambda$ is defined as

$$\varepsilon_{\lambda\mu\nu\varrho}\, \mathrm{d}\sigma^\lambda = -i[\mathrm{d}x_\mu\, \mathrm{d}x_\nu\, \mathrm{d}x_\varrho] . \tag{28}$$

Using the well-known Dirac [18] notations, we rewrite (23) and (27) in the forms

$$\langle x\|a\rangle = \langle x\|k\rangle\langle k\|a\rangle , \tag{29}$$

$$\langle k\|a\rangle = \langle k\|x\rangle\langle x\|a\rangle \tag{30}$$

with, by definition,

$$\langle x\|k\rangle = \langle k\|x\rangle^* = \begin{cases} 2\pi^{-\frac{3}{2}} \exp[ik_\lambda x^\lambda] & \text{if } \eta(k) = 0 , \\ 0 & \text{otherwise} . \end{cases} \tag{31}$$

The double bar $\|$ in these expressions is intended to recall that they are used in connection with the second-order Gordon equation.

The general definition of the projectors $\|x\rangle\langle x\|$ and $\|k\rangle\langle k\|$ is contained in the Parseval equality

$$-\frac{i}{2m}\iiint_\sigma \bar{\psi}_a \overleftrightarrow{\partial}_\lambda \psi_b\, \mathrm{d}\sigma^\lambda = \iiint_\eta \bar{\theta}_a \theta_b \varepsilon(k)\, \mathrm{d}\eta , \tag{32}$$

where the integral over the spacelike surface σ is invariant. Again, using Dirac's notations and also imposing the orthonormality condition, we rewrite (32) in the form

$$\langle a\|b\rangle = \langle a\|x\rangle\langle x\|b\rangle = \langle a\|k\rangle\langle k\|b\rangle = \delta_{ab} . \tag{33}$$

Substituting (27) into (23) and introducing the Jordan-Pauli propagator

$$D(x-x') \equiv \langle x\|x'\rangle = (2\pi)^{-3}\iiint_\eta \exp[ik_\lambda(x^\lambda - x'^\lambda)]\, \varepsilon(k)\, \mathrm{d}\eta \tag{34}$$

(which is odd in $x - x'$ and is zero outside the light-cone), we solve the Cauchy problem [29] in the form

$$\langle x \| a \rangle = \langle x \| x' \rangle \langle x' \| a \rangle \,, \tag{35}$$

or more explicitly

$$\psi(x) = -\frac{i}{m}\pi^{-3}\iiint_\sigma D(x-x')\,\overleftrightarrow{\partial}'_\mu \psi(x')\,\mathrm{d}\sigma'^\mu \,. \tag{36}$$

Formula (35) expresses the expansion of the wave function $\langle x \| a \rangle$ at any point-instant x upon the complete orthogonal set of Jordan-Pauli propagators $\langle x \| x' \rangle$ with apices x' or σ', the coefficients of the expansion being the values $\langle x' \| a \rangle$ of $\langle x \| a \rangle$ on σ'.

That two propagators $\langle x \| x' \rangle$ and $\langle x \| x'' \rangle$ are indeed orthogonal follows from the formula

$$\langle x' \| x'' \rangle = \langle x' \| x \rangle \langle x \| x'' \rangle \,. \tag{37}$$

According to the formulae

$$\langle x \| x' \rangle = \langle x \| k \rangle \langle k \| x' \rangle \tag{38}$$

and (similarly)

$$\langle k \| k' \rangle = \langle k \| x \rangle \langle x \| k' \rangle \,, \tag{39}$$

expressions (31) and (34) are Fourier associated. From this we deduce (by transposing a well-known argument of the nonrelativistic quantum mechanics [30]) that x^λ is the position operator of our unspecified-spin particle in the following sense.

We fix (fig. 4) an arbitrary spacelike surface σ and ask if the particle goes or not through an arbitrary, and arbitrarily small, element $\mathrm{d}\sigma^\mu$ on σ. This obviously transposes the nonrelativistic question: at some arbitrary time t, we ask if the particle is inside an arbitrary, and arbitrarily small, volume element $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$. Formulae (35) or (36) show that the eigenfunction associated with this question is the propagator $\langle x \| x' \rangle$, replacing in our case the nonrelativistic $\delta(x-x')$. Thus *the four co-ordinates* x^λ *being bound by the condition that* x *is on a fixed* σ, and considering the Fourier transforms (38) and (39), we see that, in this formalism (and within the approach of the position measurement problem we have defined), x^λ is the position operator of our (unspecified spin) particle.

---

[29] See also J. SCHWINGER: *Phys. Rev.*, **74**, 1439 (1948), p. 1451.

[30] $\langle x \| k \rangle$ is the eigenfunction of k^λ in the x representation, while $\langle k \| x \rangle = \langle x \| k \rangle^*$ is the eigenfunction of x^λ in the k representation.

As for the probability that the particle crosses a given element $d\sigma^{\mu}$ on σ, it is expressed by the left-hand side of formula (32) with $a = b$. It is the flux of the Gordon current through the element $d\sigma^{\lambda}$ (as should have been expected).

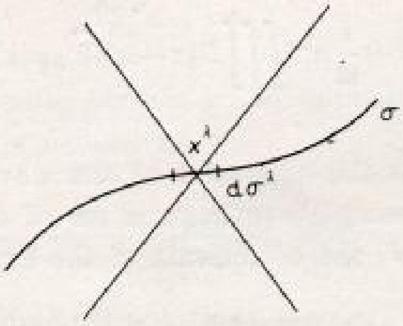

Fig. 4. – Space-time picture of a covariantly defined position measurement performed upon a Klein-Gordon particle: does the particle pass or not an arbitrary element $d\sigma^{\lambda}$ at x^{λ} on a spacelike surface σ? The corresponding eigenfunction is the Jordan-Pauli propagator with apex at x, nonzero inside *both* the future and the past light-cone. Thus the wave collapse affects symmetrically the future *and the past.*

From the various implications of the above formalism we extract, in view of our present problem, the following conclusion:

The eigenfunction associated with the relativistic position measurement (defined as crossing an element $d\sigma^{\lambda}$ at x^{λ} on a spacelike surface σ) is the Jordan-Pauli propagator with apex at x^{λ}.

This implies not only that, if found at x^{λ}, the particle has come inside the past light-cone, and will go inside the future light-cone (which is known since the early days of relativity theory), *but also that the wave collapse occurring at* x^{λ} *produces the propagator* $\langle x' \| x \rangle$ *extending into both the future and the past.*

This, of course, is the key we are proposing not for *reducing* the Einstein paradox (which is impossible, because it is a *real* paradox), *but for formalizing it.*

In summary:

The completeness of the basis for expanding the wave function at any point instant in terms of orthogonal propagators requires the presence of both retarded and advanced waves. This in turn requires the presence of both the positive and the negative frequencies in the reciprocal Fourier transforms, as shown in the very derivation of formula (36) through (23) and (27). That these two intrinsic symmetries (as opposed to large factlike macroscopic asymmetries) are tied together is made obvious by the two well-known expressions of the Jordan-Pauli propagator

$$2D(x-x') = D_{+} - D_{-} = D_{\text{ret}} - D_{\text{adv}} \cdot$$

6. – Time symmetry of the wave collapse in relativistic quantum mechanics. Spinning particles.

The formalism [28] is very similar to the preceding one. However, now the integrals over the arbitrary spacelike surface σ in space-time can be written as involving the Dirac-current rather than the Gordon-current operator, so that the normal derivative of the wave function is no longer required, but rather a linear combination of the components of the wave function.

The main new ingredient needed for our purpose is the projector projecting any solution of the Klein-Gordon equation upon a solution of the spinning-particle equation [31]. In the familiar cases of the Dirac spin-(1/2), or the Petiau-Duffin-Kemmer spin-0 or -1, particles, the expressions of this projector are (in the k representation)

$$P = \frac{i}{2k}(\gamma_\lambda k^\lambda - im)\,, \tag{40}$$

$$P = \frac{1}{2k^2}(k_\lambda k^\lambda - \beta_\mu \beta_\nu k^\mu k^\nu)\,. \tag{41}$$

Thus, by denoting the Klein-Gordon operator as G and the spinning-particle operator as S, the wave equation is written in the x representation and, by using Jauch and Rohrlich's [32] notation,

$$\begin{cases} |S\varphi\rangle = |PG\psi\rangle = i(\alpha_\lambda \partial^\lambda + m)\,\varphi = 0\,, \\ 0 = \bar{\varphi}(-\alpha_\lambda \varrho^\lambda + m)\,i = \langle\psi GP| = \langle\varphi S|\,, \end{cases} \tag{42}$$

and, in the k representation,

$$\begin{cases} |S\zeta\rangle = |PG\theta\rangle = (\alpha_\lambda k^\lambda + im)\,\zeta = 0\,, \\ 0 = \langle\zeta S| = \langle\theta GP| = \bar{\xi}(\alpha_\lambda k^\lambda + im) \end{cases} \tag{43}$$

($\alpha = \gamma$ in the Dirac case, $\alpha = \beta$ in the Petiau-Duffin-Kemmer case).

The Parseval equality assumes the new form

$$\langle a|b\rangle = \langle a|x\rangle\langle x|b\rangle = \langle a|k\rangle\langle k|b\rangle = i\iiint_\sigma \bar{\varphi}^a \alpha^\lambda \varphi^b \,\mathrm{d}\sigma_\lambda = i\iiint_\eta \bar{\zeta}^a \alpha^\lambda \zeta^b \,\varepsilon(k)\,\mathrm{d}\eta_\lambda\,, \tag{44}$$

---

(31) See in this respect H. UMEZAWA and A. VISCONTI: *Nucl. Phys.*, **1**, 20 (1956).
(32) J. M. JAUCH and F. ROHRLICH: *The Theory of Photons and Electrons* (Cambridge, Mass., 1955).

which is much more symmetric than (32); $\varepsilon(k)$ remains defined by (24) and $d\eta_\lambda$ by (25). (44) contains the new definition of the Hermitian scalar product (where now we use single vertical bars, thus recalling that we are using a first-order wave equation).

Using formulae (31), (34) and the projector P, we get the definitions

$$\langle x|k\rangle = P\langle x\|k\rangle, \tag{45}$$

$$\langle x|x'\rangle = P\langle x\|x'\rangle, \tag{46}$$

and then write down the reciprocal Fourier transforms

$$\varphi^a(x) \equiv \langle x|a\rangle = \langle x|k\rangle\langle k|a\rangle, \tag{47}$$

$$\zeta^a(k) \equiv \langle k|a\rangle = \langle k|x\rangle\langle x|a\rangle, \tag{48}$$

and the formula solving the Cauchy problem (which is also the expansion of the wave function on the complete orthogonal set of propagators with their apices on an arbitrary spacelike surface σ)

$$\langle x|a\rangle = \langle x|x'\rangle\langle x'|a\rangle \tag{49}$$

with, as before,

$$\langle x'|x''\rangle = \langle x'|x\rangle\langle x|x''\rangle \tag{50}$$

and

$$\langle x|x'\rangle = \langle x|k\rangle\langle k|x'\rangle. \tag{51}$$

Similarly,

$$\langle k|k'\rangle = \langle k|x\rangle\langle x|k'\rangle. \tag{52}$$

Given (fig. 4) an arbitrary spacelike surface σ and, upon it, an arbitrarily small arbitrary element $d\sigma^\lambda$, we *define a position-plus-spin measurement* [33] of our particle by the question « does our particle pass or not through $d\sigma^\lambda$ ». Now the corresponding eigenfunction is the propagator $\langle x|x'\rangle$ instead of $\langle x\|x'\rangle$ of sect. **5**.

The implications remain the same as before. If the answer is *yes*, the particle has come inside the past light-cone and will go inside the future light-

[33] One need not say that this approach to the position measurement problem differs essentially from the one leading to the various definitions of the position operator of a spinning particle. No attempt is made to discuss the relation (if any) between these approaches. Also, for brevity in discourse and notation, the photon has been given a (very small) rest mass.

cone with apex x' at $d\sigma'^{\lambda}$. Moreover, the propagator $\langle x|x'\rangle$ is the *collapsed wave* corresponding to this position-plus-spin measurement. Thus the wave collapse affects symmetrically both future *and past*, and this is the key I am proposing for dealing with the Einstein paradox.

7. – Einstein nonseparability and Feynman nonlocality.

It is quite obvious that the Einstein (6,7) nonseparability between the distant measurements at L and N (fig. 2) as tied through two timelike vectors connected at C, in the past of both L and N, looks extremely akin to the Feynman (34) sort of nonlocality implied in his symmetric theory of particles and antiparticles. In order to test the content of this idea (if any), we derive in this section, using the Feynman relativistically covariant technique, the correlation between the polarizations of the photon pair issuing from the annihilation of an electron-positron pair (35). For obvious symmetry reasons, we will work in the rest frame of the overall system.

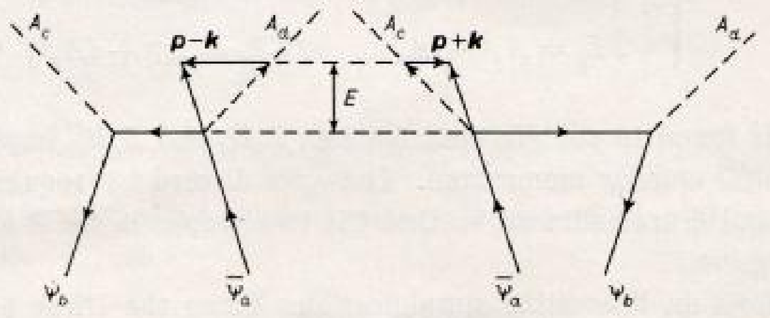

Fig. 5. – The two Feynman graphs for an e^+e^- annihilation drawn in the overall rest frame: at each vertex, there is zero energy transfer, and 3-momentum transfer $\boldsymbol{p}+\boldsymbol{k}$ and $\boldsymbol{p}-\boldsymbol{k}$, respectively.

In this frame all four particles have the same (total) energy (half of the total energy of the overall system). Moreover, they have opposite 3-momenta: $\pm\boldsymbol{p}$ for the e^-e^+ pair, denoted, respectively, as $\bar{\psi}_a$ and ψ_b, and $\pm\boldsymbol{k}$ for the photon pair, denoted as A_c and A_d. Therefore, at each vertex of the two graphs that are implied (fig. 5), there is no energy exchange, but only a 3-momentum exchange with the value $\boldsymbol{p}-\boldsymbol{k}$ in one graph and $\boldsymbol{p}+\boldsymbol{k}$ in the other. Thus with $\lambda, \mu, ... = 1, 2, 3, 4$ and $i, j, ... = 1, 2, 3$, we write down the Feynman

(34) R. P. Feynman: *Phys. Rev.*, **76**, 749 (1949), see especially p. 749.
(35) O. Costa de Beauregard: *Compt. Rend.*, **283** A, 1003 (1976); *Phys. Lett.*, **60** A, 93 (1977). Being expressed in the rest frame of the electron-positron pair, this approach is *not* directly applicable to the various experimental tests of the Einstein paradox using electron-positron annihilation.

lowest-order amplitude as

$$(53) \qquad \bar{\psi}_a \left\{ \gamma_\lambda \frac{im + (p^i + k^i)\gamma_i}{(\boldsymbol{p} + \boldsymbol{k})^2 + m^2} \gamma_\mu + \gamma_\mu \frac{im + (p^i - k^i)\gamma_i}{(\boldsymbol{p} - \boldsymbol{k})^2 + m^2} \gamma_\lambda \right\} \psi_b A_c^\lambda A_d^\mu .$$

Now we take the axis x parallel to $\pm \boldsymbol{k}$ (that is, to the photon rays) and, using gauge invariance, $A^x = 0$ and $A^4 = 0$. Then the expression $[(\boldsymbol{p} + \boldsymbol{k})^2 + m^2]^{-1} + [(\boldsymbol{p} - \boldsymbol{k})^2 + m^2]^{-1}$ factorizes, and the amplitude comes out as proportional to the sum of two terms:

$$(54) \qquad \mathscr{A}_1 = m\bar{\psi}_a \psi_b (A_c^y A_d^y + A_c^z A_d^z) + ik^x \bar{\psi}_a \gamma_{xyz} \psi_b [A_c^z A_d^y - A_c^y A_d^z] ,$$

$$(55) \qquad i\mathscr{A}_2 = \bar{\psi}_a \gamma_i \psi_b A_c^i p_j A_d^j + \bar{\psi}_a \gamma_i \psi_b A_d^i p_j A_c^j .$$

As we know (see formulae (3) and (4)), the parenthesis and the bracket in (54) are the two P-invariant amplitudes built from the left- and right-polarization states

$$(56) \qquad \begin{cases} \sqrt{2}\, L_c = A_c^y + iA_c^z , & \sqrt{2}\, R_c = A_c^y - iA_c^z , \\ \sqrt{2}\, L_d = A_d^y - iA_d^z , & \sqrt{2}\, R_d = A_d^y + iA_d^z , \end{cases}$$

where $\mathscr{A}_2$ is found to contain contributions $L_c R_d$ and $L_d R_c$, implying the presence of orbital angular momentum. These we discard by requiring from now on that $\boldsymbol{p}$ and $\boldsymbol{k}$ are collinear, so that the two graphs in fig. 4 are assumed to be plane figures.

This allows an interesting simplification. Using the Dirac equation

$$(57) \qquad (\gamma_x p^x - im)\psi_b = 0 ,$$

the definition

$$(58) \qquad \beta \equiv p_x / k_x$$

and the expressions

$$(59) \qquad (\,) = L_c L_d + R_c R_d , \quad [\,] = L_c L_d - R_c R_d$$

for the parenthesis and the bracket in (54), we rewrite $\mathscr{A}_1$ as

$$(60) \qquad \mathscr{A}_1 = \beta \bar{\psi}_a \psi_b (L_c L_d + R_c R_d) + i\bar{\psi}_a \gamma_{zy} \psi_b [L_c L_d - R_c R_d] .$$

At this point, the result we were aiming at is established. If either $\bar{\psi}_a \gamma_{yz} \psi_b$ or $\bar{\psi}_a \psi_b$ is zero, the photon pair amplitude is the P-invariant state appearing, respectively, in the $(0, 1, 0)$ and the $(1, 1, 0)$ atomic cascades (discussed in sect. **1** and **2**), so that the tight binding between the Einstein nonseparability and the Feynman nonlocality concepts is obvious.

What now we add does not belong strictly to our subject.

If $\bar{\psi}_a\psi_b$ and $\bar{\psi}_a\gamma_{yz}\psi_b$ are both nonzero, the amplitude $\mathscr{A}_1$ is not P-invariant ([36]).

The two pure-helicity photon states being

$$\left\{\begin{array}{lll} L_cL_d & \text{with} & R_cR_d = 0\,, \\ R_cR_d & \text{with} & L_cL_d = 0\,, \end{array}\right. \tag{61}$$

the corresponding e^+e^- states are

$$\beta\bar{\psi}_a\psi_b \pm i\bar{\psi}_a\gamma_{yz}\psi_b \qquad \text{with} \qquad \beta\bar{\psi}_a\psi_b \mp i\bar{\psi}_a\gamma_{yz}\psi_b = 0\,. \tag{62}$$

To study their implications, we use the standard representation where γ_4, $-i\gamma_{yz}$ (spin operator along x) and $-i\gamma_{4yz}$ (magnetic-moment operator along x) are diagonal with traces $+1+1-1-1$, $+1-1+1-1$, $+1-1-1+1$, respectively, and use the relations

$$\left\{\begin{array}{ll} a_3 = \beta a_1 = \beta l_a\,, & a_4 = -\beta a_2 = -\beta r_a\,, \\ b_1 = -\beta b_3 = -\beta l_b\,, & b_2 = \beta b_4 = \beta r_b \end{array}\right. \tag{63}$$

between the « large » and « small » real amplitudes of ψ_a and ψ_b. We then rewrite formula (62) as

$$(1 \pm \beta)\, l_a l_b = (1 \mp \beta)\, r_a r_b \tag{64}$$

(the upper and lower signs being associated).

This is the helicity condition on the e^+e^- pair that is associated with the pure-helicity condition on the two photon state. In the extreme relativistic limit, $\beta = \pm 1$, this condition is either $r_a r_b = 0$ or $l_a l_b = 0$. The case in which the e^+e^- pair annihilates at rest, $\beta = 0$, is P invariant.

Incidentally, very similar formulae and conclusions can be derived by using the Petiau-Duffin-Kemmer algebra, and they would be significant for charged particles of spin 1.

In summary:

There is a very tight connection between the Einstein ([6,7]) nonseparability and the Feynman ([34]) nonlocality concepts, showing again that the « paradox » under discussion belongs to the neoquantal wavelike probability calculus, *and* that it implies intrinsic time symmetry at the elementary level.

([36]) It is, of course, PC invariant, as most easily seen by exchanging the convention in which the e^- has positive and the e^+ negative energy against the opposite one.

8. – Conclusion.

All formulae in this paper are simple, and either well known, or easily derivable from known formulae.

When dealing with a true, or real paradox (4), that is, when aiming at formulating a new paradigm (5), it is *extremely* appropriate to use formulae the simplest as possible, and the closest as possible to the (paradoxical) experimental facts. This is what EINSTEIN did when proposing the special relativity theory, and what DE BROGLIE did when proposing wave mechanics.

No discussion is found in this paper on the hidden-variable problem, as discussed in depth by BELL, CLAUSER, HORNE, SHIMONY, HOLT, D'ESPAGNAT (37) and others. The reason for this is that, important as these works have been historically and in helping elucidating the problem, the whole so-called hidden-variable approach suffers, as it seems to me, from the same drawback as the deceased theories of the luminiferous aether: too much faith in mechanistic realism, unnecessary complication, and insufficient faith in the operational formalism.

We deem that (contrary to a widespread belief) the most important changes in paradigm result from *a victory of formalism over modelism rather than the contrary*. Thus, they consist in *understanding the true meaning of the operational formulae as they stand.* They « unveil the Sense of the Scriptures » by strictly *tayloring the wording after the mathematics.*

The paradigm we are proposing—which, in de Broglie's (3) words, obviously upsets « our familiar notions concerning space and time »—is *complete time symmetry in the quantal stochastic event*, the *transition*, or *wave collapse.*

Of course, this raises the question of how to reconcile this *intrinsic time symmetry at the elementary level* with the *factlike macroscopic time asymmetry.*

It is well known that an analogous problem existed in classical statistical mechanics, where it gave rise to the famous Loschmid and Zermelo paradoxes. It even existed in the classical probability calculus itself, where predictive and retrodictive problems (« problems in the probability of causes ») were treated by quite different methods, contrasting the *intrinsic time symmetry of the transition probabilities* existing in most cases (20). Thus, what we have here is a transposition of an old problem inside the field of the new wavelike probability calculus.

Of course, quite a few new elements are brought in together with this transposition, of which we mention only two. The first one is, of course, that the « factlike (11) physical irreversibility », certainly absent at the elementary

---

(37) We quote for instance B. D'ESPAGNAT: *Phys. Rev. D*, **11**, 1424 (1975) as one of the later papers, and one containing many references to the literature.

The classical answer was when the photons issuing from L impinge upon N. This is, however, a *macroscopical prejudice*, impressed upon our minds by our familiarity with the *factlike* (11) preponderance of retarded over advanced waves.

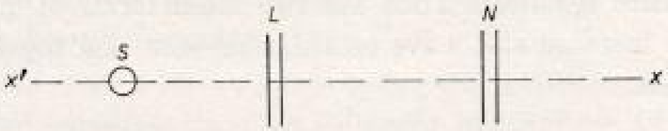

Fig. 6. – Quantal transition of photons issuing from a source S and passing successively two linear polarizers with angles A and B ($\alpha \equiv A - B$).

From the neoquantal mathematical formalism (not to speak of the very successful experimental proofs (2) of the Einstein paradox) *a very different concept* follows. The transition occurs «somewhere in between» L and N, and consists in $\cos^2 \alpha$ of the photons jumping from the retarded wave having passed L into the advanced wave that will pass N.

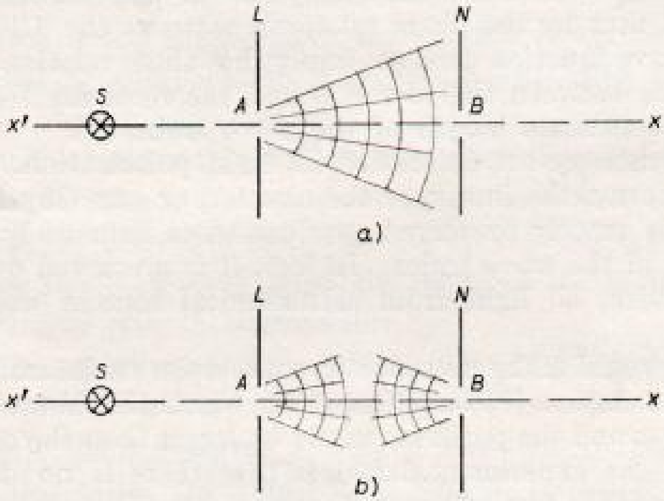

Fig. 7. – Quantal transition of photons issuing from a source S and passing successively two holes A and B in screens L and N. *a*) Classical, macroscopic concept (retarded waves); *b*) neoquantal concept of the Ψ collapse: symmetry between retarded and advanced waves.

Consider also (fig. 7) those photons emanating from a source S and passing successively two small holes A and B inside screens L and N. *Mutatis mutandis* the discourse is the same as before and, as very explicit pictures can be drawn in this case (fig. 7*a*) and *b*)), no more comment will be made.

Notes added in proofs.

1) *Pflegor and Mandel's* (40) *retrodictive correlation experiment between occupation numbers of photon waves exemplifies the time-inverted Einstein paradox: nonseparability of sources that will interfere.*

(40) R. L. Pflegor and L. Mandel: *Phys. Rev.*, **159**, 1084 (1967); *Journ. Opt. Soc. Amer.*, **58**, 946 (1968).

If we denote by x' and x'' two point-instants inside the interference region, by x_1 and x_2 two point-instants inside the sources, by n_1 and n_2 the corresponding occupation number operators, the (18) type contribution is written as

$$\Delta = \tfrac{1}{2}\langle x'|n_1|x''\rangle \otimes \langle x'|n_2|x''\rangle + \text{c.c.} \tag{65}$$

with $(r = 1, 2)$

$$\langle x'|n_r|x''\rangle = \langle x'|x_r\rangle\langle x_r|n_r|x_r\rangle\langle x_r|x''\rangle\,. \tag{66}$$

2) *The Einstein correlation (either predictive* (2) *or retrodictive* (40)) *is inherent in the (relativistically covariant) S-matrix formalism.*

The Feynman transition amplitude $\langle\psi_1|\psi_2\rangle$ between an initial $\psi(\sigma_1)$ and a final $\psi(\sigma_2)$ state may be expanded in the form

$$\langle\psi_1|\psi_2\rangle = \langle\psi_1|\Theta\rangle\langle\Theta|\psi_2\rangle\,, \tag{67}$$

where the complete set of orthogonal projectors $|\Theta_i\rangle\langle\Theta_i|$ is the one adapted to the problem (polarization states (2) or occupation number states (40), for instance).

Formula (67) is the expansion of either $|\Psi_2\rangle$ (in predictive problems (2)) or of $|\Psi_1\rangle$ (in retrodictive problems (40)) in the form

$$|\Psi\rangle = \sum_j c_j \prod_\lambda |\varphi_{j\lambda}\rangle\,, \tag{68}$$

which is the n factor generalization of (13). For example, in quantum electrodynamics, the $|\varphi_\lambda\rangle$'s are the photon $|A\rangle$, the electron $|\bar\psi\rangle$ and the positron $|\psi\rangle$ states.

The « paradox » is, of course, that the correlation exists in the absence of « present » interaction, if there is either a past (2) or a future interaction (40).

3) By using formula (26) in the form $k_n\,\mathrm{d}\eta^\lambda = m\,\mathrm{d}\eta$, the Parseval equality (32) is cast in the more symmetric form

$$-\frac{i}{2}\iiint_\sigma \bar\varphi_a \overleftrightarrow{\partial}_\lambda \varphi_b\,\mathrm{d}\sigma^\lambda = \iiint_\eta \bar\theta_a k_\lambda \theta_b\,\varepsilon(k)\,\mathrm{d}\eta^\lambda\,. \tag{69}$$

● RIASSUNTO (*)

Si deriva la caratteristica differenza tra il calcolo paleoquantico (somme di probabilità parziali) e quello neoquantico (somme di ampiezze parziali) per la correlazione di polarizzazioni di fotoni in transizioni a cascata sulle basi della trigonometria elementare. Questo uso deliberato di formule semplici mira a rendere chiaro il cambiamento di paradigma richiesto dal cosiddetto paradosso di EPR (che è realmente il paradosso di Einstein del 1927), cioè che 1) i due fotoni non posseggono polarizzazioni in proprio quando lasciano la sorgente C, ma ne prendono in prestito una più tardi, quando inter-

(*) *Traduzione a cura della Redazione.*

agiscono con gli analizzatori L e N; 2) il dado non è tratto in C, ma più tardi, in L e N; 3) la correlazione tra misurazioni in L e N è legata tramite C nel loro comune passato. La salda correlazione tra questa « non separabilità di Einstein » e la non località nella « teoria dei positoni » di Feynmann è dimostrata attraverso un'analisi dell'annichilazione di e^+e^- in due fotoni. Così il paradosso di Einstein corrisponde, nel « nuovo calcolo della probabilità ondulatoria », al tipo di paradosso di Loschmid e Zermelo nel vecchio calcolo delle probabilità. Cioè esso contrappone l'intrinseca simmetria temporale che esiste al livello elementare alla macroscopica asimmetria temporale fattuale. La nostra discussione trascura deliberatamente il problema delle variabili nascoste, prendendo come modello in ciò la trascuratezza di Einstein riguardo l'etere meccanico, nel proporre la sua relatività ristretta. Crediamo che qui oggi, come là nel 1905, il problema sia costruire il discorso secondo la matematica (valida dal punto di vista operativo). E in più il fatto che il cambiamento di paradigma, che è necessario, avvenga attraverso una vittoria del formalismo sul modellismo.

Симметрия времени и парадокс Эйнштейна.

Резюме (*). — С помощью элементарной тригонометрии определяется характерное различие между палеоквантовым вычислением (суммирование парциальных вероятностей) и неоквантовым вычислением (суммирование парциальных амплитуд) для корреляции поляризаций фотонов в каскадных переходах. Указанное обдуманное использование простых формул имеет цель — сформулировать в явном виде изменение в парадигме, требуемое так называемым EPR парадоксом (который является парадоксом Эйнштейна), а именно: 1) два фотона не обладают поляризациями, когда испускаются источником C, но позже преобретают поляризацию, когда взаимодействуют с анализаторами L и N; 2) цвет не дает информацию об источнике C, а об анализаторах L и N; 3) корреляция между измерениями в L и N связана через C, *их общим прошлым*. Анализ e^+e^- аннигиляции в два фотона демонстрирует тесную связь между этой « Эйнштейновской неразличимостью » и нелокальностью в Фейнмановской « теории позитронов ». Таким образом, парадокс Эйнштейна при « новых волноподобных вероятностных вычислениях » соответствует парадоксу Лошмида и Цермело в старых вероятностных вычислениях. Указанное обстоятельство сопоставляет *внутреннюю симметрию времени*, которая существует на элементарном уровне, с *действительной макроскопической асимметрией времени*. Наша дискуссия умышленно не затрагивает проблему скрытых переменных. Мы считаем, что сегодня, как и в 1905, проблема состоит в приспособлении формулировок. Таким образом, это изменение в парадигме, которое является необходимым, происходит через победу формализма над моделизмом.

(*) *Переведено редакцией.*