logic, the use of van Kampen diagrams and the treatment of small cancellation theory and its applications represent very fine achievements. Much of the material has appeared only in the periodical literature until now; indeed some of the material appears here in print for the first time. The book is clearly an important contribution to the mathematical literature. But it is only fair that I add some words of warning. The authors have followed their personal interests a little too closely. As a consequence the broad scope of the subject itself is only hinted at. The book was written in two parts, the first by one author, the second by the other, and common material was simply repeated as it arose. Apparently this was intentional, allowing the reader to read each chapter as a separate entity. Nevertheless the arrangement of the material is haphazard, the exposition is very uneven, some of it is unnecessarily hard to follow, some almost impossible. There are much too many misprints, successive paragraphs are sometimes unrelated and motivation is almost totally lacking. Some of the text has not been well-worked out, the graph-theoretic-topological parts demand varying levels of topological expertise and no attempt has been made to find the general topological principles that govern much of this material as well as many of the subgroup theorems. The notion of an aspherical presentation is somehow identified with the topological notion of asphericity without sufficient justification. In spite of these very real criticisms this is still an important piece of work.

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Bounded integral operators on L^2 spaces, by P. R. Halmos and V. S. Sunder, Ergebnisse der Mathematik und ihrer Grenzgebiete no. 96, Springer-Verlag, Berlin and New York, 1978, xv + 134 pp., $16.50.

This slim volume in the Ergebnisse series (to which I shall refer as H-S) deals with bounded integral operators on L^2 spaces, that is to say, bounded linear operators $K: L^2(Y, \nu) \to L^2(X, \mu)$ of the form

$$(Kf)(x) = \int_Y k(x, y)f(y)\, d\nu(y)$$

for all $f \in L^2(Y, \nu)$, where (X, μ) and (Y, ν) are σ-finite and separable measure spaces and the integral is an "ordinary" one with respect to ν (no principal values; no L^2 limits as in the theory of Fourier transforms). Restriction to a σ-finite separable measure implies (by a well-known isomorphism theorem) that for most purposes it may just as well be assumed that ν is either Lebesgue measure in the interval [0, 1] or counting measure in **Z** (or **N**) or a finite subset of **N**. There are two recent books on integral operators of a general nature (i.e., not restricted to L^2 operators), one by the late K. Jörgens ([**2**, 1970], in German) and one by M. A. Krasnosel'skiĭ, P. P. Zabreĭko, E. I. Pustyl'nik and P. E. Sobolevskiĭ ([**3**, 1966], in Russian; English translation 1976). We may ask, therefore, if it is still possible to say something of interest about the simple L^2 case that has not been said many times before.

The H-S book shows that the answer is yes. Going one step further, we may also ask if there are any important problems, left open in H-S, which have been solved recently. The answer is yes again; the question how to recognize an integral operator is not so mysterious any more as it was before. It is interesting to observe that for the solution of the recognition problem one has to go beyond L^2 into the space of all measurable functions.

For a review of some of the most interesting points, let us be a little more precise about the definitions. Let (X, μ) and (Y, ν) be σ-finite measure spaces (not necessarily separable) and let $\mathfrak{M}(X, \mu)$ be the vector space of all complex μ-almost everywhere finite and μ-measurable functions on X (with identification of μ-almost equal functions). The vector space $\mathfrak{M}(Y, \nu)$ is defined similarly. The subset D of $\mathfrak{M}(X, \mu)$ is said to be *solid* if any function in $\mathfrak{M}(X, \mu)$ that is majorized in absolute value by a member of D belongs itself to D (i.e., if $f \in \mathfrak{M}(X, \mu)$ and $g \in D$ satisfy $|f(x)| \leqslant |g(x)|$ almost everywhere on X, then $f \in D$). Any solid linear subspace of $\mathfrak{M}(X, \mu)$ is called an *order ideal* (briefly *ideal*) in $\mathfrak{M}(X, \mu)$. All spaces $L^p(X, \mu)$, $0 < p \leqslant \infty$, are ideals.

For brevity, I shall call any linear mapping from one vector space into another an *operator*. Let E and F be ideals in $\mathfrak{M}(Y, \nu)$ and $\mathfrak{M}(X, \mu)$ respectively. The operator K: $E \to F$ is called an *integral operator* if there exists a $(\mu \times \nu)$-measurable function $k(x, y)$ on $X \times Y$ such that, for every $f \in E$, we have

$$(Kf)(x) = \int_Y k(x, y) f(y)\, d\nu(y)$$

for almost every x (the exceptional null set depending on f, of course). The function $k(x, y)$ is called the *kernel* of K. In this case it is evident that, for every $f \in E$, the function

$$\int_Y |k(x, y) f(y)|\, d\nu(y)$$

is finite almost everywhere on X, and so $|k(x, y)|$ is the kernel of an integral operator K_a which maps E into $\mathfrak{M}(X, \mu)$. If K: $E \to F$ is an integral operator and K_a has the additional property that it maps E not only into $\mathfrak{M}(X, \mu)$ but into F as well, then I shall call K an *absolute integral operator* (from E into F). In the H-S book we meet the situation that $E = L^2(Y, \nu)$ and $F = L^2(X, \mu)$ with μ and ν not only σ-finite but also separable (and hence $L^2(X, \mu)$ and $L^2(Y, \nu)$ separable). Let us call this the separable L^2 situation. As is well known in the L^2 situation, the integral operator K is called a *Hilbert-Schmidt operator* if $|k(x, y)|^2$ is $(\mu \times \nu)$-summable and K is called a *Carleman operator* if

$$\int_Y |k(x, y)|^2\, d\nu(y)$$

is finite for almost every x.

One of the first matters to be settled in the L^2 situation is to find out whether integral operators are bounded (i.e., continuous). The answer is yes. For absolute integral operators there is a very elementary proof; for integral operators that are nonabsolute the proof is based on the closed graph theorem and on the lemma that every norm convergent sequence in $L^2(Y, \nu)$ has a dominated subsequence (i.e., dominated in absolute value by a fixed L^2 function) that converges pointwise almost everywhere on Y. There is a remark in H-S that this particular lemma does not seem to be in the conscious memory of even the experts in measure theory, so it is perhaps of some interest to observe that there exists a theorem asserting that every norm convergent sequence in a Banach lattice has a subsequence which converges in order. For $L^2(Y, \nu)$ this means that if $g_n \to g$ in norm, then there is a subsequence g_{n_j} and there is a sequence $h_j \in L^2(Y, \nu)$ such that $|g - g_{n_j}| \leq h_j \downarrow 0$ pointwise almost everywhere on Y. The boundedness proof can immediately be extended to integral operators $K: E \to F$ if the ideals E and F carry norms that make them into Banach lattices (such as, for example, L^p spaces, $1 \leq p \leq \infty$). The boundedness proof is already a first example of how much easier absolute integral operators are to handle than nonabsolute ones. On the other hand, as far as boundedness of the operator is concerned, not all order ideals in the space $\mathfrak{M}(Y, \nu)$ of measurable functions carry norms, and as I shall explain further on, it seems that in the problem how to recognize an integral operator order properties (and not norm continuity properties) play a decisive part.

Among the questions analyzed in H-S there are three major ones:

(i) which operators *can* be integral? (i.e., for which bounded operators T: $L^2(X, \mu) \to L^2(X, \mu)$ does there exist a unitary operator U such that UTU^* is an integral operator?);

(ii) which operators *must* be integral? (i.e., for which bounded operators T: $L^2(X, \mu) \to L^2(X, \mu)$ is it true that UTU^* is integral for every unitary U?);

(iii) which operators *are* integral and which operators are Carleman?

The first two questions, as formulated here, are typically L^2 questions and they get a complete answer. For the case of an atomic measure ($L^2 = l^2$) the answers are trivial; every bounded operator on l^2 is represented by a matrix and is, therefore, an integral operator. Assume now that the measure μ in X is not (purely) atomic. Then the bounded operator T (from L^2 into itself) is unitarily equivalent to an integral operator if and only if zero belongs to the right essential spectrum of T (in other words, for any bounded operator A from L^2 into itself the operator $TA - I$ is not compact; I is the identity operator). This general result was proved in the Indiana University dissertation of the junior author (1977); special cases were known earlier (J. von Neumann, 1935; J. Weidmann, 1970). The proof is based on the observation that if $\mu(X)$ is finite, then $L^2(X, \mu)$ is contained in $L^1(X, \mu)$, and so every operator in L^2 can also be regarded as an operator from L^2 into L^1. In this situation any integral operator $T: L^2 \to L^2$ (compact or not) is compact as an operator from L^2 into L^1. Using this result it is not difficult to show that any integral operator on L^2 has zero in its right essential spectrum. As a corollary one gets that T is unitarily equivalent to an integral operator if and only if T is unitarily equivalent to a Carleman operator. As for the second question, UTU^* is integral for every unitary U if and only if T is Hilbert-Schmidt (V. B. Korotkov, 1974). It was proved earlier that if UTU^* is Carleman for every unitary U, then T is Hilbert-Schmidt (G. I. Targonski, 1967).

The third question, the one asking which operators are indeed integral operators, does not receive a complete answer in H-S. The authors give a partial answer because in the last section of the book they present several necessary and sufficient conditions for an operator to be Carleman. One of these (J. Weidmann, 1970) is that the bounded operator T: $L^2 \to L^2$ is Carleman if and only if T maps every norm null sequence onto an almost everywhere null sequence (i.e., $\|g_n\| \to 0$ implies $(Tg_n)(x) \to 0$ almost everywhere). The answer for an arbitrary integral operator, although not given in H-S, is known. Except for one small but important additional condition, it is the same as for a Carleman operator. The operator T: $L^2 \to L^2$ is integral if and only if T maps every dominated norm null sequence onto an almost everywhere null sequence (i.e., $0 \leqslant |g_n| \leqslant g_0 \in L^2$ and $\|g_n\| \to 0$ implies $(Tg_n)(x) \to 0$ almost everywhere). Before going further, note that $\|g_n\| \to 0$ in L^2 implies that every subsequence of g_n contains an almost everywhere null subsequence. This last property is usually called *star convergence* of g_n to zero (notation $g_n \overset{*}{\to} 0$). In the converse direction, $g_n \in L^2$ for all n and $g_n \overset{*}{\to} 0$ does not always imply $\|g_n\| \to 0$, but $g_n \overset{*}{\to} 0$ together with $0 \leqslant |g_n| \leqslant g_0 \in L^2$ does imply $\|g_n\| \to 0$ (by the dominated convergence integration theorem). Hence, we obtain the norm free reformulation that the operator T: $L^2 \to L^2$ is integral if and only if T maps every dominated star null sequence onto an almost everywhere null sequence. For a brief discussion of this result and where it comes from I return to the more general situation that E and F are order ideals in $\mathfrak{M}(Y, \nu)$ and $\mathfrak{M}(X, \mu)$ respectively, not necessarily equipped with a norm. As mentioned in the beginning, μ and ν are σ-finite but not necessarily separable. Without restriction of the generality we may assume that the *carrier* of E is the whole set Y (i.e., there does not exist any subset Y_1 of Y of positive measure on which every $f \in E$ vanishes almost everywhere). Similarly, we assume that X is the carrier of F. As usual, the operator T: $E \to F$ is said to be *positive* if T maps nonnegative functions onto nonnegative functions and T is called *order bounded* (or *regular* in the terminology used by most Soviet authors) if $T = T_1 - T_2$ with T_1 and T_2 positive. Hence, every positive operator is order bounded. Any absolute integral operator K: $E \to F$ is order bounded since $K = K_a - (K_a - K)$ with K_a and $K_a - K$ positive operators from E into F. The set $\mathfrak{L}_b = \mathfrak{L}_b(E, F)$ of all order bounded operators (from E into F) is a *Riesz space* (*vector lattice*) under the natural ordering ($T_1 \leqslant T_2$ whenever $T_2 - T_1$ is positive) and the absolute integral operators form evidently a linear subspace $\mathfrak{L}_i = \mathfrak{L}_i(E, F)$ of $\mathfrak{L}_b(E, F)$. The linear subspace $\mathfrak{L}_i$ has additional properties; in the Riesz space terminology $\mathfrak{L}_i$ is a *band* in $\mathfrak{L}_b$. Without going into technical details, the main point here is to prove that any positive operator dominated by an integral operator is itself an integral operator (i.e., if $0 \leqslant T \leqslant K$ in $\mathfrak{L}_b$ and K is integral, then T is integral). As far as I know, there are now three different proofs for this result in the literature. One, by L. Lessner [**4**, 1978] assumes the measure ν separable. The second one, by A. V. Buhvalov ([**1**, 1974]; English translation 1978) uses lifting. The third one, finally, by A. R. Schep ([**7**, 1977]; [**8**, 1979]) is simple and makes use only of the ordinary Radon-Nikodym theorem. I briefly outline the idea behind Schep's proof. It is a technical matter to reduce the proof to the case that

$$\int_{X \times Y} K(x,y)\, d(\mu \times \nu) < \infty,$$

where $K(x,y) \geqslant 0$ is the kernel of K. Then

$$\lambda(P) = \int_P K(x,y)\, d(\mu \times \nu)$$

is a finite measure on the σ-algebra of all $(\mu \times \nu)$-measurable subsets of $X \times Y$ such that λ is absolutely continuous with respect to $\mu \times \nu$. Let Γ be the semiring of all sets $A \times B$, where $A \subset X$ is μ-measurable and $B \subset Y$ is ν-measurable. For $A \times B \in \Gamma$, define

$$\lambda_1(A \times B) = \int_A (T\chi_B)(x)\, d\mu(x),$$

where χ_B is the characteristic function of B. It is easy to prove that λ_1 is a measure on Γ satisfying $\lambda_1 \leqslant \lambda$ on Γ. Now extend λ_1 by the Carathéodory procedure. Every $(\mu \times \nu)$-measurable set becomes λ_1-measurable and we get $\lambda_1 \leqslant \lambda$ on the σ-algebra of all $(\mu \times \nu)$-measurable sets, so λ_1 is absolutely continuous with respect to $\mu \times \nu$. Hence (Radon-Nikodym) there exists a $(\mu \times \nu)$-measurable $T(x,y) \geqslant 0$ such that

$$\lambda_1(P) = \int_P T(x,y)\, d(\mu \times \nu)$$

for all $(\mu \times \nu)$-measurable sets P. Applying this to $P = A \times B \in \Gamma$, it follows easily that

$$(T\chi_B)(x) = \int_Y T(x,y)\chi_B(y)\, d\nu(y)$$

for almost every x. It is then routine to show that T is an integral operator with kernel $T(x,y)$.

It is time to return to the condition that the operator $T: E \to F$ satisfies the

Condition (B): *T maps every dominated star null sequence in E onto an almost everywhere null sequence.*

An easy argument shows that any positive integral operator satisfies (B). If $T: E \to F$ is an arbitrary integral operator, then T can be regarded as an absolute integral operator from E into $\mathfrak{M}(X, \mu)$, so T_a: $E \to \mathfrak{M}(X, \mu)$ satisfies (B). It follows immediately that T itself satisfies (B). Before turning to the converse problem whether (B) implies that T is an integral operator, note that it may occur that there do not exist any nontrivial integral operators at all. If $X = Y$ is the real line with $\mu = \nu$ Lebesgue measure and if $E = F$ is the space $\mathfrak{M}(X, \mu)$ of all Lebesgue measurable functions, then the only integral operator (from E into F) is the null operator. To indicate a sufficient condition for the existence of nontrivial integral operators, let $E^{\hat{}}$ be the ideal in $\mathfrak{M}(Y, \nu)$ consisting of all functions g such that fg is ν-summable over Y for every $f \in E$. For $g \in E^{\hat{}}$ and $h \in F$ the function $h(x)g(y)$ is obviously the kernel of an absolute integral operator from E into F. If the carrier of $E^{\hat{}}$ is the whole set Y the finite linear combinations of these simple operators (integral operators of *finite rank*) are what is called *order dense* in the band $\mathfrak{L}_i$ of all absolute integral operators (i.e, $\mathfrak{L}_i$ is the smallest band containing all operators of finite rank). For $E = L^2(Y, \nu)$ and $F = L^2(X, \mu)$ the carrier of $E^\frown$ is Y because $E^\frown = E = L^2(Y, \nu)$, so the absolute integral operators in the H-S book form a band in which the finite rank operators are lying order dense. (Question for H-S readers: Is this result hidden somewhere in H-S; perhaps in Theorem 11.5?).

Assume now, to avoid complications, that Y is the carrier not only of E but of $E^\frown$ as well. This guarantees, therefore, the existence of plenty of integral operators. In this situation *Buhvalov's theorem* holds, stating that condition (B) is not only necessary but also sufficient for $T: E \to F$ to be an integral operator. This was proved by Buhvalov and, independently, by Schep in the papers mentioned above. Their proofs are to some extent analogous to a proof by H. Nakano ([**6**, Theorem 5.2], 1953) about bilinear forms (this can be understood only by those who are more or less familiar with Nakano's terminology). In the case that $T: E \to F$ satisfies (B) and is also order bounded, T can be written as $T = T_1 - T_2$ with T_1, T_2 positive and satisfying (B). Hence, in this case it may be assumed that T is positive. Since the absolute integral operators form a band in the space of order bounded operators, T can now be written uniquely as $T = T' + T''$ with T' a positive integral operator and T'' positive, satisfying (B) and *disjoint* to the band of integral operators (i.e., the only positive integral operator majorized by T'' is the null operator). The proof is reduced, therefore, to showing that any positive T which satisfies (B) and is disjoint to all integral operators is the null operator. This is not easy, but the remarkable analogy with Nakano's result (referred to above) shows part of the way. If $T: E \to F$ satisfies (B) but T is not order bounded as an operator from E into F, then it follows from condition (B) that T is order bounded as an operator from E into $\mathfrak{M}(X, \mu)$; the proof is not trivial. Anyhow, in view of the result in the order bounded case, it follows that T is an integral operator also in this more general case.

Altogether this is a rather formidable structure which, except for some elementary facts from the theory of Riesz spaces, can be expressed in purely measure theoretic classical terms. If one should desire so, even the Riesz space terminology can be avoided (at the cost of more words). It will be evident that it took some time to develop all this. Let me still mention the pioneering work of the late G. Ya. Lozanovskiĭ [**5**, 1966]. In relation to the H-S book several questions arise. Is it possible, for example, to simplify matters if the theory is restricted to L^2? I believe not. And to which extent is it possible to drop the separability assumption in H-S?

I return for a moment to the situation that E is a ideal in $\mathfrak{M}(Y, \nu)$. If E is equipped with a norm ρ such that (E, ρ) is a Banach lattice, then the corresponding ideal $E^\frown$ is a closed linear subspace of the Banach adjoint space E^* such that the norm ρ^* in E^*, restricted to $E^\frown$, makes $(E^\frown, \rho^*)$ into a Banach lattice (if, for example, $E = L^p$ for some p satisfying $1 \leqslant p \leqslant \infty$, then $E^\frown = L^q$ for $p^{-1} + q^{-1} = 1$). The measurable function $k(x, y)$ on $X \times Y$ is now called a *generalized Carleman kernel* if $\rho^*\{k(x, \cdot)\}$ exists as a finite number for almost every x and $\rho^*\{k(x, \cdot)\} \in \mathfrak{M}(x, \mu)$, i.e., $\rho^*\{k(x, \cdot)\}$ is a measurable function of x. It can be proved that the following conditions for the operator $K: E \to \mathfrak{M}(X, \mu)$ are equivalent:

(i) K is an integral operator with generalized Carleman kernel.

(ii) There exists a finite nonnegative measurable function Ω on X such that $|(Kg)(x)| \leqslant \Omega(x) \cdot \rho(g)$ almost everywhere for every $g \in E$ (compare this with Theorem 17.2 in H-S).

(iii) K maps every norm null sequence in E onto an almost everywhere null sequence (compare this with Theorem 17.7 in H-S).

In the particular case that $E = L^2(Y, \nu)$, there is one more equivalent condition, as follows.

(iv) If $\{e_\alpha\}$ is any orthonormal set in $L^2(Y, \nu)$, then

$$\sup\left(\sum_{i=1}^{n} |Ke_{\alpha_i}(x)|^2 : n \in \mathbf{N}, (\alpha_1, \ldots, \alpha_n) \in \{\alpha\}\right) \in \mathfrak{M}(X, \mu).$$

If the orthonormal set $\{e_\alpha\}$ in (iv) is an orthonormal basis, then the supremum in (iv) is (for almost every x) exactly the integral over Y of $|k(x, y)|^2$, so different orthonormal bases give the same supremum (compare this with Theorem 17.5 in H-S). These results about generalized Carleman operators are due to A. R. Schep and will be published in a forthcoming paper.

Finally, let us take a look at adjoint operators. If $K: L^2(Y, \nu) \to L^2(X, \mu)$ is an absolute integral operator with kernel $k(x, y)$, then the Banach adjoint operator K^* is also an integral operator, possessing $k(y, x)$ as kernel. If K is a nonabsolute integral operator, the situation is not so simple. In Theorem 7.5 of H-S it is proved that K^* is an integral operator if and only if $k(y, x)$ is the kernel of some integral operator (from $L^2(X, \mu)$ into $L^2(Y, \nu)$), and, in that case, $k(y, x)$ is the kernel of K^*. In the more general situation that E is an ideal in $\mathfrak{M}(Y, \nu)$ such that E and $E^\frown$ have Y as carrier and F is an ideal in $\mathfrak{M}(X, \mu)$ such that F and $F^\frown$ have X as carrier, it is not difficult to prove that if $K: E \to F$ is an absolute integral operator with kernel $k(x, y)$, then there exists an absolute integral operator $K^\frown: F^\frown \to E^\frown$ (the restriction of the *order adjoint* of K to $F^\frown$) possessing $k(y, x)$ as kernel. Unless there are norms available, it seems not immediately clear how to extend this to nonabsolute integral operators.

The H-S book is written in a lively style, as was to be expected. It is a mine of information for any analyst interested in operators, in particular operators on L^2 spaces. The theory is illustrated by examples as well as by counterexamples; open problems are mentioned and sometimes analyzed. A list of bibliographical notes gives information about the history of the subject. The preface ends with the remark that the book contains only a part of a large subject, with only one of several approaches, and with explicit mention of only a few of the many challenging problems that are still open. I agree, but I hope that nevertheless it will be clear from my comments in this review that I believe the present contribution to operator theory by Halmos and Sunder is a valuable one.

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The metric theory of Banach manifolds, by Ethan Akin, Lecture Notes in Math., vol. 662, Springer-Verlag, Berlin and New York, 1978, xix + 306 pp., $13.50.

Few books have been written on the subject of an abstract model for the structure of manifolds of maps; the present book is one of this select group. Most of the books and papers in global analysis have been concerned with the point-set or differential topology of abstract Banach manifolds, with applications to various equations from physics, or else with the introduction and investigation of new examples of manifolds of maps which may play a role in future investigations. While this mass of literature is for the most part not concerned with the structure of manifolds of maps itself, it has nonetheless helped to shed some light on the nature of this still poorly-understood structure (and, equally important, on what this structure is not). Before discussing *The metric theory of Banach manifolds*, let me review what the literature of global analysis has told us thus far about the nature of manifolds of maps.

Let us assume for the moment that M_1 and M_2 are smooth finite-dimensional manifolds, and that M_1 is compact (possibly with boundary). Then all the standard examples of manifolds of maps from M_1 to M_2 contain the C^∞ maps, are infinite-dimensional manifolds, and are sandwiched as topological spaces between $C^\infty(M_1, M_2)$ and $C^0(M_1, M_2)$. By a result due to Palais [**14**, Theorem 16], it follows that all of these manifolds are of the same homotopy type as $C^0(M_1, M_2)$. But it is also well known to infinite-dimensional topologists [**9**] that any two homotopically equivalent topological manifolds, each of which is modeled on a separable infinite-dimensional Fréchet space (not necessarily the same space), are homeomorphic. While there exist important examples of manifolds of maps which are not separable, it still follows that most of the interesting spaces of maps from M_1 to M_2 are homeomorphic to $C^0(M_1, M_2)$. Thus nothing is gained topologically by investigating any Fréchet manifold of maps from M_1 to M_2 other than $C^0(M_1, M_2)$. Any gain is going to come from the analytical structure on the function space. Putting