Erratum on:
Fiscal Stimulus in a Monetary Union: Evidence from U.S. Regions
by Emi Nakamura and Jón Steinsson
American Economic Review, 104(3), 753-792, March 2014.

There were errors in lines 303, 337, and 338 of Matlab file runHetCapitalGHH.m, which produces some of the results in Table 9 of our paper. Correcting these errors slightly changes the open economy relative multiplier on output and inflation for the firm specific capital model. When prices are sticky, the output multiplier rises slightly from 1.47 to 1.74. The interpretation of these robustness results in the paper is unaffected by this small change. We have reproduced an updated version of Table 9 below with the published version also included for reference.

We would like to thank Bopjun Gwak and Paul Reimers, Ph.D. students at Goethe University in Frankfurt for finding these errors.

TABLE IX
Open Economy Relative Multiplier in Models with Variable Capital

	Output	CPI Inflation
Revised Version:		
Baseline Model (Fixed Capital)	1.42	0.17
Firm-Specific Capital Model	1.74	0.15
Regional Capital Market Model	0.98	0.09
Firm-Specific Capital Model , Flexible Prices	0.22	0.29
Published Version:		
Baseline Model (Fixed Capital)	1.42	0.17
Firm-Specific Capital Model	1.47	0.15
Regional Capital Market Model	0.98	0.09
Firm-Specific Capital Model , Flexible Prices	0.25	0.36

The table reports the open economy relative government spending multiplier for output and CPI inflation for our baseline model with GHH preferences and the two models with variable capital also with GHH preferences. Output is deflated by the regional CPI.

For Online Publication

Appendix to: Fiscal Stimulus in a Monetary Union: Evidence from U.S. Regions

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A Constant Real and Nominal Rate Monetary Policies

The paper considers specifications of monetary policy that hold the real or nominal interest rate constant in response to a government spending shock. Here, we illustrate the method used to solve for these monetary policy specifications. We do this for the case of separable preference. We use an analogous approach for the case of GHH preference.

Consider the closed economy limit of our model. A log-linear approximation of the key equilibrium conditions of this model are

$$\hat{c}_t = E_t\hat{c}_{t+1} - \sigma(\hat{r}_t^n - E_t\hat{\pi}_{t+1}), \tag{1}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\zeta\sigma^{-1}\hat{c}_t + \kappa\zeta\psi_\nu\hat{y}_t, \tag{2}$$

$$\hat{y}_t = \left(\frac{C}{Y}\right)\hat{c}_t + \hat{g}_t, \tag{3}$$

where $\zeta = 1/(1+\psi_\nu\theta)$ and $\psi_\nu = (1+\nu^{-1})/a - 1$.

Using equation (3) to eliminate $\hat{y}_t$ from equations (1) and (2) yields

$$\hat{c}_t = E_t\hat{c}_{t+1} - \sigma(\hat{r}_t^n - E_t\hat{\pi}_{t+1}), \tag{4}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\zeta_c\hat{c}_t + \kappa\zeta_g\hat{g}_t, \tag{5}$$

where $\zeta_y = \zeta(\sigma^{-1} + (C/Y)\psi_\nu)$ and $\zeta_g = \zeta\psi_\nu$. Recall that $\hat{g}_t = \rho_g\hat{g}_{t-1} + \epsilon_{g,t}$.

A.1 Fixed Real Rate Monetary Policy

An equilibrium with a fixed real interest rate must satisfy

$$\hat{c}_t = E_t \hat{c}_{t+1}, \tag{6}$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \zeta_c \hat{c}_t + \kappa \zeta_g \hat{g}_t. \tag{7}$$

We conjecture a solution of the form $\hat{c}_t^* = a_c \hat{g}_t$, $\hat{\pi}_t^* = a_\pi \hat{g}_t$. Using the method of undetermined coefficients, it is easy to verify that such an equilibrium exists with

$$a_c = 0, \quad a_\pi = \kappa \tfrac{\zeta_g}{1-\beta\rho_g}.$$

This equilibrium can be implemented with the following policy rule

$$\begin{aligned} \hat{r}_t^n &= E_t \hat{\pi}_{t+1} + \phi_\pi(\hat{\pi}_t - \hat{\pi}_t^*) \\ &= a_\pi \rho_g \hat{g}_t + \phi_\pi \hat{\pi}_t - a_\pi \phi_\pi \hat{g}_t \\ &= \phi_\pi \hat{\pi}_t - a_\pi(\phi_\pi - \rho_g)\hat{g}_t. \end{aligned} \tag{8}$$

A.2 Fixed Nominal Rate Monetary Policy

An equilibrium with a fixed nominal interest rate must satisfy

$$\hat{c}_t = E_t \hat{c}_{t+1} + \sigma E_t \hat{\pi}_{t+1}, \tag{9}$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \zeta_c \hat{c}_t + \kappa \zeta_g \hat{g}_t. \tag{10}$$

We again conjecture a solution of the form $\hat{c}_t^* = a_c \hat{g}_t$, $\hat{\pi}_t^* = a_\pi \hat{g}_t$. Using the method of undetermined coefficients, it is easy to verify that such an equilibrium exists with

$$a_c = \tfrac{\rho_g \kappa \zeta_g}{A_c}, \quad a_\pi = \kappa \tfrac{\zeta_c}{1-\beta\rho_g} a_c + \kappa \tfrac{\zeta_g}{1-\beta\rho_g}.$$

where $A_c = (1-\rho_g)(1-\beta\rho_g) - \rho_g \kappa \zeta_c$. This solution is however only valid when $A_c > 0$. For $0 < \rho_g < 1$, A_c is decreasing in ρ_g. There is a critical point at which $A_c = 0$. As ρ_g rises and A_c falls towards zero, $a_c \to \infty$. For values of ρ_g that are above this point, our solution method breaks down since a_c is infinite. Similar parameter restrictions arise in Eggertsson (2010) and Christiano, Eichenbaum, and Rebelo (2011).

In the valid range, this equilibrium can be implemented with the following policy rule

$$\begin{aligned} \hat{r}_t^n &= \phi_\pi(\hat{\pi}_t - \hat{\pi}_t^*) \\ &= \phi_\pi \hat{\pi}_t - a_\pi \phi_\pi \hat{g}_t. \end{aligned} \tag{11}$$

B Persistence of the Government Spending Shocks

We use annual data on aggregate military procurement spending to calibrate the persistence of the government spending shocks that we feed into our model. The first order autocorrelation of aggregate military procurement spending suggests a great deal of persistence. However, higher order autocorrelations suggest less persistence. To capture this behavior in a parsimonious way, we estimate a quarterly AR(1) process by simulated method of moments using the first five autocorrelations as moments.

More specifically, our procedure is the following. First, we take the log of aggregate military procurement spending and detrend it (results are insensitive to this). We then estimate regressions of the following form:

$$G_t^{agg} = \alpha_j + \beta_j G_{t-j}^{agg} + \epsilon_t,$$

where G_t^{agg} is detrended log aggregate military procurement spending. We use β_j for $j = 1, ..., 5$ as the moments in our simulated methods of moments estimation. We then simulate quarterly data from AR(1) processes with different degrees of persistence, time aggregate these data to an annual frequency, and run these same regressions on the simulated data. We then find the value of the quarterly AR(1) parameter that minimizes the sum of the squared deviations of the empirical moments from the simulated moments.

C Linear Approximation of Equation (1) for Model Regressions

To calculate the open economy relative multiplier for simulated data from our model, we must take a linear approximation of the dependent and independent variables in equation (1) so as to be able to express the variables in the regression in terms of the variables in our model. For the output regression, we approximate the specification in which we deflate regional GDP by the regional CPI (The second specification in Table 2). The linear approximation of the dependent variable is

$$\frac{\frac{P_{Ht}Y_t}{P_t} - \frac{P_{Ht-2}Y_{t-2}}{P_{t-2}}}{\frac{P_{Ht-2}Y_{t-2}}{P_{t-2}}} = \frac{Y_t}{Y_{t-2}} \frac{\Pi_{Ht}\Pi_{Ht-1}}{\Pi_t \Pi_{t-1}} - 1 = \hat{y}_t - \hat{y}_{t-2} + \hat{\pi}_{Ht} + \hat{\pi}_{Ht-1} - \hat{\pi}_t - \hat{\pi}_{t-1} + \text{h.o.t}, \quad (12)$$

where h.o.t. denotes "higher order terms." The linear approximation of the independent variable is

$$\frac{\frac{P_{Ht}G_{Ht}}{P_t^W} - \frac{P_{Ht-2}G_{Ht-2}}{P_{t-2}^W}}{\frac{P_{Ht-2}Y_{t-2}}{P_{t-2}^W}} = \frac{G_{Ht}}{Y_{t-2}}\frac{\Pi_{Ht}\Pi_{Ht-1}}{\Pi_t^W\Pi_{t-1}^W} - \frac{G_{Ht-2}}{Y_{t-2}}$$

$$= \hat{g}_t - \hat{g}_{t-2} + \left(1 - \frac{C}{Y}\right)\left(\hat{\pi}_{Ht} + \hat{\pi}_{Ht-1} - \hat{\pi}_t^W - \hat{\pi}_{t-1}^W\right) + \text{h.o.t.} \tag{13}$$

D Separable Preferences Model

The model consists of two regions that belong to a monetary and fiscal union. We refer to the regions as "home" and "foreign." Think of the home region as the region in which the government spending shock occurs – a U.S. state or small group of states – and the foreign region as the rest of the economy. The population of the entire economy is normalized to one. The population of the home region is denoted by n.

D.1 Households

The home region has a continuum of household types indexed by x. A household's type indicates the type of labor supplied by that household. Home households of type x seek to maximize their utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t(x)), \tag{14}$$

where β denotes the household's subjective discount factor, C_t denotes household consumption of a composite consumption good, $L_t(x)$ denotes household supply of differentiated labor input x. There are an equal (large) number of households of each type. The period utility function takes the form

$$u(C_t, L_t(x)) = \frac{C_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \chi\frac{L_t(x)^{1+\nu^{-1}}}{1+\nu^{-1}}. \tag{15}$$

The composite consumption good in expression (14) is an index given by

$$C_t = \left[\phi_H^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \phi_F^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{16}$$

where C_{Ht} and C_{Ft} denote the consumption of composites of home and foreign produced goods, respectively. The parameter $\eta > 0$ denotes the elasticity of substitution between home and foreign goods and ϕ_H and ϕ_F are preference parameters that determine the household's relative preference

for home and foreign goods. It is analytically convenient to normalize $\phi_H + \phi_F = 1$. If $\phi_H > n$, household preferences are biased toward home produced goods.

The subindices, C_{Ht} and C_{Ft}, are given by

$$C_{Ht} = \left[\int_0^1 c_{ht}(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad C_{Ft} = \left[\int_0^1 c_{ft}(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \tag{17}$$

where $c_{ht}(z)$ and $c_{ft}(z)$ denote consumption of variety z of home and foreign produced goods, respectively. There is a continuum of measure one of varieties in each region. The parameter $\theta > 1$ denotes the elasticity of substitution between different varieties.

Goods markets are completely integrated across regions. Home and foreign households thus face the same prices for each of the differentiated goods produced in the economy. We denote these prices by $p_{ht}(z)$ for home produced goods and $p_{ft}(z)$ for foreign produced goods. All prices are denominated in a common currency called "dollars."

Households have access to complete financial markets. There are no impediments to trade in financial securities across regions. Home households of type x face a flow budget constraint given by

$$P_t C_t + E_t[M_{t,t+1} B_{t+1}(x)] \leq B_t(x) + (1-\tau_t) W_t(x) L_t(x) + \int_0^1 \Xi_{ht}(z) dz - T_t, \tag{18}$$

where P_t is a price index that gives the minimum price of a unit of the consumption good C_t, $B_{t+1}(x)$ is a random variable that denotes the state contingent payoff of the portfolio of financial securities held by households of type x at the beginning of period $t+1$, $M_{t,t+1}$ is the stochastic discount factor that prices these payoffs in period t, τ_t denotes a labor income tax levied by the government in period t, $W_t(x)$ denotes the wage rate received by home households of type x in period t, $\Xi_{ht}(z)$ is the profit of home firm z in period t and T_t denotes lump sum taxes.[1] To rule out Ponzi schemes, household debt cannot exceed the present value of future income in any state of the world.

Households face a decision in each period about how much to spend on consumption, how many hours of labor to supply, how much to consume of each differentiated good produced in the economy and what portfolio of assets to purchase. Optimal choice regarding the trade-off between current consumption and consumption in different states in the future yields the following consumption Euler equation:

$$\left(\frac{C_{t+j}}{C_t}\right)^{-\sigma^{-1}} = \frac{M_{t,t+j}}{\beta^j} \frac{P_{t+j}}{P_t} \tag{19}$$

[1] The stochastic discount factor $M_{t,t+1}$ is a random variable over states in period $t+1$. For each such state it equals the price of the Arrow-Debreu asset that pays off in that state divided by the conditional probability of that state. See Cochrane (2005) for a detailed discussion.

as well as a standard transversality condition. Equation (19) holds state-by-state for all $j > 0$. Optimal choice regarding the intratemporal trade-off between current consumption and current labor supply yields a labor supply equation:

$$\chi L_t(x)^{\nu^{-1}} C_t^{\sigma^{-1}} = (1-\tau_t)\frac{W_t(x)}{P_t}. \tag{20}$$

Households optimally choose to minimize the cost of attaining the level of consumption C_t. This implies the following demand curves for home and foreign goods and for each of the differentiated products produced in the economy:

$$C_{H,t} = \phi_H C_t \left(\tfrac{P_{Ht}}{P_t}\right)^{-\eta} \quad \text{and} \quad C_{F,t} = \phi_F C_t \left(\tfrac{P_{Ft}}{P_t}\right)^{-\eta}, \tag{21}$$

$$c_{ht}(z) = C_{Ht}\left(\tfrac{p_{ht}(z)}{P_{Ht}}\right)^{-\theta} \quad \text{and} \quad c_{ft}(z) = C_{Ft}\left(\tfrac{p_{ft}(z)}{P_{Ft}}\right)^{-\theta}, \tag{22}$$

where

$$P_{Ht} = \left[\int_0^1 p_{ht}(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \quad \text{and} \quad P_{Ft} = \left[\int_0^1 p_{ft}(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}}, \tag{23}$$

and

$$P_t = \left[\phi_H P_{Ht}^{1-\eta} + \phi_F P_{Ft}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{24}$$

The problem of the foreign household is largely analogous and we therefore refrain from describing all details for this problem. We use superscript * to denote foreign variables. The utility function of the foreign households is analogous to that of home households except that the foreign conposite consumption index is

$$C_t^* = \left[\phi_H^{*\frac{1}{\eta}} C_{Ht}^{*\frac{\eta-1}{\eta}} + \phi_F^{*\frac{1}{\eta}} C_{Ft}^{*\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{25}$$

where C_{Ht}^* is a composite index of foreign consumption of home goods and C_{Ft}^* is a composite index of foreign consumption of foreign goods. These indexes are given by analogous expressions to those in equation (17). We assume for convenience that $\phi_H^* + \phi_F^* = 1$. The foreign budget constraint is analogous to that of home households. This implies that the foreign Euler equation and labor supply equations are analogous to those of the home household. Foreign demand for home and foreign goods and for each of the differentiated products produced in the economy is given by

$$C_{H,t}^* = \phi_H^* C_t^* \left(\tfrac{P_{Ht}}{P_t^*}\right)^{-\eta} \quad \text{and} \quad C_{F,t}^* = \phi_F^* C_t^* \left(\tfrac{P_{Ft}}{P_t^*}\right)^{-\eta}, \tag{26}$$

$$c_{ht}^*(z) = C_{Ht}^*\left(\tfrac{p_{ht}(z)}{P_{Ht}}\right)^{-\theta} \quad \text{and} \quad c_{ft}^*(z) = C_{Ft}^*\left(\tfrac{p_{ft}(z)}{P_{Ft}}\right)^{-\theta}, \tag{27}$$

where

$$P_t^* = \left[\phi_H^* P_{Ht}^{1-\eta} + \phi_F^* P_{Ft}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{28}$$

It is useful to note that combining the home and foreign consumption Euler equations to eliminate the common stochastic discount factor yields

$$\left(\frac{C_t^*}{C_t}\right)^{-\sigma^{-1}} = Q_t, \tag{29}$$

where $Q_t = P_t^*/P_t$ is the real exchange rate. This is the "Backus-Smith" condition that describes optimal risk-sharing between home and foreign households (Backus and Smith, 1993). For simplicity, we assume that all households—in both regions—initially have an equal amount of financial wealth.

D.2 The Government

The economy has a federal government that conducts fiscal and monetary policy. Total government spending in the home and foreign region follow exogenous AR(1) processes. Let G_{Ht} denote government spending per capita in the home region. Total government spending in the home region is then nG_{Ht}. For simplicity, we assume that government demand for the differentiated products produced in each region takes the same CES form as private demand. In other words, we assume that

$$g_{ht}(z) = G_{Ht}\left(\frac{p_{ht}(z)}{P_{Ht}}\right)^{-\theta} \quad \text{and} \quad g_{ft}(z) = G_{Ft}\left(\frac{p_{ft}(z)}{P_{Ft}}\right)^{-\theta}. \tag{30}$$

The government levies both labor income and lump-sum taxes to pay for its purchases of goods. Our assumption of perfect financial markets implies that any risk associated with variation in lump-sum taxes and transfers across the two regions is undone through risk-sharing. Ricardian equivalence holds in our model.

We consider two specifications for tax policy. Our baseline tax policy is one in which government spending shocks are financed completely by lump sum taxes. Under this policy, all distortionary taxes remain fixed in response to the government spending shock. The second tax policy we consider is a "balanced budget" tax policy. Under this policy, labor income taxes vary in response to government spending shocks such that the government's budget remains balanced throughout:

$$nP_{Ht}G_{Ht} + (1-n)P_{Ft}G_{Ft} = \tau_t \int W_t(x)L_t(x)dx, \tag{31}$$

This policy implies that an increase in government spending is associated with an increase in distortionary taxes.

The federal government operates a common monetary policy for the two regions. This policy consists of the following augmented Taylor-rule for the economy-wide nominal interest rate:

$$\hat{r}_t^n = \rho_r \hat{r}_{t-1}^n + (1-\rho_i)(\phi_\pi \hat{\pi}_t^{ag} + \phi_y \hat{y}_t^{ag} + \phi_g \hat{g}_t^{ag}), \tag{32}$$

where hatted variables denote percentage deviations from steady state. The nominal interest rate is denoted $\hat{r}_t^n$. It responds to variation in the weighted average of consumer price inflation in the two regions $\hat{\pi}_t^{ag} = n\hat{\pi}_t + (1-n)\hat{\pi}_t^*$, where $\hat{\pi}_t$ is consumer price inflation in the home region and $\hat{\pi}_t^*$ is consumer price inflation in the foreign region. It also responds to variation in the weighted average of output in the two regions $\hat{y}_t^{ag} = n\hat{y}_t + (1-n)\hat{y}_t^*$. Finally, it may respond directly to the weighted average of the government spending shock in the two regions $\hat{g}_t^{ag} = n\hat{g}_t + (1-n)\hat{g}_t^*$. See section D.4 below, for precise definitions of these variables.

D.3 Firms

There is a continuum of firms indexed by z in the home region. Firm z specializes in the production of differentiated good z, the output of which we denote $y_{ht}(z)$. In our baseline model, labor is the only variable factor of production used by firms. Each firm is endowed with a fixed, non-depreciating stock of capital. The production function of firm z is

$$y_{ht}(z) = L_t(z)^a. \tag{33}$$

The firm's production function is increasing and concave. It is concave because there are diminishing marginal returns to labor given the fixed amount of other inputs employed at the firm. Labor is immobile across regions. We follow Woodford (2003) in assuming that each firm belongs to an industry x and that there are many firms in each industry. The goods in industry x are produced using labor of type x and all firms in industry x change prices at the same time.

Firm z acts to maximize its value,

$$E_t \sum_{j=0}^{\infty} M_{t,t+j}[p_{ht+j}(z) y_{ht+j}(z) - W_{t+j}(x) L_{t+j}(z)]. \tag{34}$$

Firm z must satisfy demand for its product. The demand for firm z's product comes from three sources: home consumers, foreign consumers and the government. It is given by

$$y_{ht}(z) = (nC_{Ht} + (1-n)C_{Ht}^* + nG_{Ht}) \left(\frac{p_{ht}(z)}{P_{Ht}}\right)^{-\theta}. \tag{35}$$

Firm z is therefore subject to the following constraint:

$$(nC_{Ht} + (1-n)C^*_{Ht} + nG_{Ht}) \left(\frac{p_{ht}(z)}{P_{Ht}}\right)^{-\theta} \leq f(L_t(z)). \tag{36}$$

Firm z takes its industry wage $W_t(x)$ as given. Optimal choice of labor demand by the firm is given by

$$W_t(x) = aL_t(z)^{a-1}S_t(z), \tag{37}$$

where $S_t(z)$ denotes the firm's nominal marginal cost (the Lagrange multiplier on equation (36) in the firm's constrained optimization problem).

Firm z can reoptimize its price with probability $1-\alpha$ as in Calvo (1983). With probability α it must keep its price unchanged. Optimal price setting by firm z in periods when it can change its price implies

$$p_{ht}(z) = \frac{\theta}{\theta-1} E_t \sum_{j=0}^{\infty} \frac{\alpha^j M_{t,t+j} y_{ht+j}(z)}{E_t \sum_{k=0}^{\infty} \alpha^k M_{t,t+k} y_{ht+j}(z)} S_{t+j}(z). \tag{38}$$

Intuitively, the firm sets its price equal to a constant markup over a weighted average of current and expected future marginal cost. Since all frims in industry x face the same wage, have the same production function and set price at the same time, they will all set the same price, produce the same amount, hire the same amount of labor and have the same marginal cost. We can therefore replace all indexes z by x.

Combining labor supply (20) and labor demand (37), we get that

$$\frac{S_t(x)}{P_t} = \frac{\chi}{a} \frac{L_t(x)^{\nu^{-1}-a+1} C_t^{\sigma^{-1}}}{(1-\tau_t)}.$$

Using the production function to eliminate $L_t(x)$ from this equation yields

$$\frac{S_t(x)}{P_t} = \frac{\chi}{a} \frac{y_{ht}(x)^{(\nu^{-1}-a+1)/a} C_t^{\sigma^{-1}}}{(1-\tau_t)}. \tag{39}$$

D.4 Linear Approximation

We linearize the model around a zero-growth, zero-inflation steady state. We start by deriving a log-linear approximation for the home consumption Euler equation that relates consumption growth and the return on a one-period, riskless, nominal bond. This equation takes the form $E_t[M_{t,t+1}(1+r_t^n)] = 1$. Using equation (19) to plug in for $M_{t,t+1}$ and rearranging terms yields

$$E_t \left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma^{-1}} \frac{P_t}{P_{t+1}} \right] = \frac{1}{1+r_t^n}. \tag{40}$$

The zero-growth, zero-inflation steady state of this equation is $\beta(1+\bar{r}^n)=1$. A first order Taylor series approximation of equation (40) is

$$\hat{c}_t = E_t\hat{c}_{t+1} - \sigma(\hat{r}_t^n - E_t\hat{\pi}_{t+1}), \tag{41}$$

where $\hat{c}_t = (C_t - C)/C$, $\hat{\pi}_t = \pi_t - 1$, and $\hat{r}_t^n = (1 + r_t^n - 1 - \bar{r}^n)/(1+\bar{r}^n)$.

A linear approximation of the "Backus-Smith" equation (29) is

$$\hat{c}_t - \hat{c}_t^* = \sigma\hat{q}_t, \tag{42}$$

where $\hat{q}_t = Q_t - 1$.

We next linearize equation (39) in period $t+j$ of a firm that last changed its price in period t. Let $\hat{y}_{t,t+j}(x)$ denote the percent deviation from steady state in period $t+j$ of output in industry x that last was able to change prices in period t. Let other industry level variables be defined analogously. We then get that

$$\hat{s}_{ht,t+j}(x) = \psi_v\hat{y}_{ht,t+j}(x) + \sigma^{-1}\hat{c}_{t+j} + \frac{\tau}{1-\tau}\hat{\tau}_{t+j}, \tag{43}$$

where $\hat{s}_{ht,t+j}(x)$ denotes the percentage deviation from steady state of real marginal costs in period $t+j$ of home output in industry x that last was able to change prices in period t, $\hat{\tau}_t = (\tau_t - \tau)/\tau$, and the parameter $\psi_v = (\nu^{-1}+1)/a - 1$.

The foreign analog of equation (43) is

$$\hat{s}_{ft,t+j}(x) = \hat{q}_{t+j} + \psi_v\hat{y}_{ft,t+j}(x) + \sigma^{-1}\hat{c}_{t+j}^* + \frac{\tau}{1-\tau}\hat{\tau}_{t+j}, \tag{44}$$

Recall that we have assumed $\phi_H + \phi_F = 1$ and $\phi_H^* + \phi_F^* = 1$. These assumptions imply that in steady state $P = P_H = P_F = p(z)$. We then have that in steady state $C_H = \phi_H C$, $C_F = \phi_F C$, $C_H^* = \phi_H^* C^*$, $C_F^* = \phi_F^* C^*$, $c_h(z) = C_H$, $c_f(z) = C_F$, $c_h^*(z) = C_H^*$, and $c_f^*(z) = C_F^*$. Since we assume that home and foreign households have equal initial wealth, $C = C^*$. We also assume that steady state government spending per capita is equal in the two regions, $G_H = G_F = G$. We furthermore assume that overall demand for home products as a fraction of overall demand for all products is equal to the size of the home population relative to the total population of the economy. In other words, $nC_H + (1-n)C_H^* = nC$, which implies that $\phi_H* = (n/(1-n))\phi_F$.

We take a linear approximation of all prices relative to the home price level P_t. A linear approximation of the demand for home and foreign goods by home households—equation (21)—and its foreign counterpart—equation (26)—yield

$$\hat{c}_{Ht} = \hat{c}_t - \eta\hat{p}_{Ht} \quad \text{and} \quad \hat{c}_{Ft} = \hat{c}_t - \eta\hat{p}_{Ft} \tag{45}$$

$$\hat{c}^*_{Ht} = \hat{c}^*_t - \eta(\hat{p}_{Ht} - \hat{q}_t) \quad \text{and} \quad \hat{c}^*_{Ft} = \hat{c}^*_t - \eta(\hat{p}_{Ft} - \hat{q}_t), \tag{46}$$

where $\hat{c}_{Ht} = (C_{Ht} - C_H)/C_H$, $\hat{C}_t = (C_t - C)/C$, $\hat{p}_{Ht} = P_{Ht}/P_t - 1$ and other variables are defined analogously.

It is useful to define per capita home and foreign output as

$$Y_{Ht} = \tfrac{1}{n}\int_0^1 y_{ht}(z)dz \quad \text{and} \quad Y_{Ft} = \tfrac{1}{1-n}\int_0^1 y_{ft}(z)dz, \tag{47}$$

and total output as

$$Y_t = nY_{Ht} + (1-n)Y_{Ft}.$$

In the steady state, we have $Y_H = y_h(z)/n$, $Y_F = y_f(z)/(1-n)$, and $Y = nY_H + (1-n)Y_F$.

Next we take a linear approximation of demand for home and foreign variety's—equation (35) and its foreign counterpart. The steady state of demand for home varieties is $y_h(z) = nC_H + (1-n)C^*_H + nG_H$. Using $C_H = \phi_H C$, $C^*_H = \phi^*_H C^*$, $C = C^*$, $G_H = G$, $\phi_H + \phi_F = 1$, $\phi_H* = (n/(1-n))\phi_F$, yields $Y_H = C + G$. Similar manipulations for the demand for foreign varieties yield $Y_F = C + G$. Since $Y = nY_H + (1-n)Y_F$, we thus have $Y = Y_H = Y_F$. Using these steady state relationships, we can take a linear approximation the demand in period $t+j$ for home and foreign firms in industries that last changed their prices in period t. This yields

$$\hat{y}_{ht,t+j}(x) = \phi_H\left(\frac{C}{Y}\right)\hat{c}_{Ht+j} + \frac{1-n}{n}\phi^*_H\left(\frac{C}{Y}\right)\hat{c}^*_{Ht+j} + \hat{g}_{Ht+j} - \theta\left(\hat{p}_{ht}(x) - \hat{p}_{Ht+j} - \sum_{k=1}^{j}\pi_{t+k}\right), \tag{48}$$

$$\hat{y}_{ft,t+j}(x) = \frac{n}{1-n}\phi_F\left(\frac{C}{Y}\right)\hat{c}_{Ft+j} + \phi^*_F\left(\frac{C}{Y}\right)\hat{c}^*_{Ft+j} + \hat{g}_{Ft+j} - \theta\left(\hat{p}_{ft}(x) - \hat{p}_{Ft+j} - \sum_{k=1}^{j}\pi_{t+k}\right), \tag{49}$$

where $\hat{g}_{Ht+j} = (G_{Ht+j} - G)/Y$ and $\hat{g}_{Ft+j} = (G_{Ft+j} - G)/Y$.

Given the way prices are set in the economy, home output from equation (47) can be expressed as

$$Y_{Ht} = \frac{1}{n}\sum_{j=0}^{\infty}(1-\alpha)\alpha^j y_{ht-j,t}(x).$$

A linear approximation of this equation is

$$\hat{y}_{Ht} = \sum_{j=0}^{\infty}(1-\alpha)\alpha^j \hat{y}_{ht-j,t}(x).$$

Using equation (48) to plug in for $\hat{y}_{ht-j,t}(x)$ yields

$$\begin{aligned}\hat{y}_{Ht} = \phi_H\left(\frac{C}{Y}\right)\hat{c}_{Ht} + \frac{1-n}{n}\phi^*_H\left(\frac{C}{Y}\right)\hat{c}^*_{Ht} + \hat{g}_{Ht} \\ -\theta\sum_{j=0}^{\infty}(1-\alpha)\alpha^j\left(\hat{p}_{ht-j}(x) - \hat{p}_{Ht} - \sum_{k=0}^{j-1}\pi_{t-k}\right).\end{aligned} \tag{50}$$

Using again the structure of price setting in the economy, equation (23) for the price index for home produced goods can be rewritten as

$$P_{Ht}^{1-\theta} = \sum_{j=0}^{\infty}(1-\alpha)\alpha^j p_{ht-j}(x)^{1-\theta}.$$

A linear approximation of this equation is

$$\hat{p}_{Ht} = \sum_{j=0}^{\infty}(1-\alpha)\alpha^j \left(\hat{p}_{ht+j}(x) - \sum_{k=0}^{j-1}\hat{\pi}_{t-k}\right).$$

Combining this equation and equation (50) yields

$$\hat{y}_{Ht} = \phi_H\left(\frac{C}{Y}\right)\hat{c}_{Ht} + \frac{1-n}{n}\phi_H^*\left(\frac{C}{Y}\right)\hat{c}_{Ht}^* + \hat{g}_{Ht}.$$

A similar set of manipulations for foreign output and foreign prices yields

$$\hat{y}_{Ft} = \frac{n}{1-n}\phi_F\left(\frac{C}{Y}\right)\hat{c}_{Ft} + \phi_F^*\left(\frac{C}{Y}\right)\hat{c}_{Ft}^* + \hat{g}_{Ft}.$$

Using equations (45) and (46) to eliminate $\hat{c}_{Ht}$, $\hat{c}_{Ht}^*$, $\hat{c}_{Ft}$, and $\hat{c}_{Ft}^*$ from the last two equations yields

$$\begin{aligned}\hat{y}_{Ht} = \phi_H\left(\frac{C}{Y}\right)\hat{c}_t + \frac{1-n}{n}\phi_H^*\left(\frac{C}{Y}\right)\hat{c}_t^* \\ -\eta\left(\frac{C}{Y}\right)\left(\phi_H + \frac{1-n}{n}\phi_H^*\right)\hat{p}_{Ht} + \eta\left(\frac{C}{Y}\right)\frac{1-n}{n}\phi_H^*\hat{q}_t + \hat{g}_{Ht},\end{aligned} \tag{51}$$

$$\begin{aligned}\hat{y}_{Ft} = \frac{n}{1-n}\phi_F\left(\frac{C}{Y}\right)\hat{c}_t + \phi_F^*\left(\frac{C}{Y}\right)\hat{c}_t^* \\ -\eta\left(\frac{C}{Y}\right)\left(\frac{n}{1-n}\phi_F + \phi_F^*\right)\hat{p}_{Ht} + \eta\left(\frac{C}{Y}\right)\phi_F^*\hat{q}_t + \hat{g}_{Ft}.\end{aligned} \tag{52}$$

Combining equations (43), (48), and (51) and (**??**), (49), and (52) yields

$$\hat{s}_{ht,t+j}(x) = \psi_v\hat{y}_{Ht+j} - \psi_v\theta\left(\hat{p}_{ht}(x) - \hat{p}_{Ht+j} - \sum_{k=1}^{j}\hat{\pi}_{t+k}\right) + \sigma^{-1}\hat{c}_{t+j} + \frac{\tau}{1-\tau}\hat{\tau}_{t+j}, \tag{53}$$

$$\hat{s}_{ft,t+j}(x) = \hat{q}_{t+j} + \psi_v\hat{y}_{Ft+j} - \psi_v\theta\left(\hat{p}_{ft}(x) - \hat{p}_{Ft+j} - \sum_{k=1}^{j}\hat{\pi}_{t+k}\right) + \sigma^{-1}\hat{c}_{t+j}^* + \frac{\tau}{1-\tau}\hat{\tau}_{t+j}. \tag{54}$$

A linear approximation of equations (23) yields

$$\hat{\pi}_{Ht} = \tfrac{1-\alpha}{\alpha}(\hat{p}_{ht}(x) - \hat{p}_{Ht}) \quad \text{and} \quad \hat{\pi}_{Ft} = \tfrac{1-\alpha}{\alpha}(\hat{p}_{ft}(x) - \hat{p}_{Ft}) \tag{55}$$

A linear approxmation of equation (38) and its foreign counterpart yields

$$\hat{p}_{ht}(x) = (1-\alpha\beta)\sum_{j=0}^{\infty}(\alpha\beta)^j E_t \hat{s}_{ht,t+j}(x) + \alpha\beta\sum_{j=1}^{\infty}(\alpha\beta)^j E_t \hat{\pi}_{t+j}, \tag{56}$$

$$\hat{p}_{ft}(x) = (1-\alpha\beta)\sum_{j=0}^{\infty}(\alpha\beta)^j E_t \hat{s}_{ft,t+j}(x) + \alpha\beta\sum_{j=1}^{\infty}(\alpha\beta)^j E_t \hat{\pi}_{t+j}. \tag{57}$$

Using equations (53) and (54) to substitute for $\hat{s}_{ht,t+j}(x)$ and $\hat{s}_{ft,t+j}(x)$ in the last two equations yields

$$\hat{p}_{ht}(x) = (1-\alpha\beta)\zeta\sum_{j=0}^{\infty}(\alpha\beta)^j E_t\left[\psi_v\hat{y}_{Ht+j} + \theta\psi_v\hat{p}_{Ht+j} + \sigma^{-1}\hat{c}_{t+j} + \frac{\tau}{1-\tau}\hat{\tau}_{t+j}\right] + \alpha\beta\sum_{j=1}^{\infty}(\alpha\beta)^j E_t\hat{\pi}_{t+j}, \tag{58}$$

$$\hat{p}_{ft}(x) = (1-\alpha\beta)\zeta\sum_{j=0}^{\infty}(\alpha\beta)^j E_t\left[\hat{q}_{t+j} + \psi_v\hat{y}_{Ft+j} + \theta\psi_v\hat{p}_{Ft+j} + \sigma^{-1}\hat{c}^*_{t+j} + \frac{\tau}{1-\tau}\hat{\tau}_{t+j}\right] + \alpha\beta\sum_{j=1}^{\infty}(\alpha\beta)^j E_t\hat{\pi}_{t+j}, \tag{59}$$

where $\zeta = 1/(1+\psi_v\theta)$. Quasi-differencing these equations yields

$$\hat{p}_{ht}(x) - \alpha\beta E_t\hat{p}_{ht+1}(x) = (1-\alpha\beta)\zeta\left[\psi_v\hat{y}_{Ht} + \theta\psi_v\hat{p}_{Ht} + \sigma^{-1}\hat{c}_t + \frac{\tau}{1-\tau}\hat{\tau}_t\right] + \alpha\beta E_t\hat{\pi}_{t+1}, \tag{60}$$

$$\hat{p}_{ft}(x) - \alpha\beta E_t\hat{p}_{ft+1}(x) = (1-\alpha\beta)\zeta\left[\hat{q}_t + \psi_v\hat{y}_{Ft} + \theta\psi_v\hat{p}_{Ft} + \sigma^{-1}\hat{c}^*_t + \frac{\tau}{1-\tau}\hat{\tau}_t\right] + \alpha\beta E_t\hat{\pi}_{t+1}, \tag{61}$$

Using equation (55) to eliminate $\hat{p}_{ht}(x)$ and $\hat{p}_{ft}(x)$ from these equations yields

$$\pi_{Ht} - \alpha\beta E_t\pi_{Ht+1} + \frac{1-\alpha}{\alpha}(\hat{p}_{Ht} - \alpha\beta E_t\hat{p}_{Ht+1}) = \kappa\zeta\left[\psi_v\hat{y}_{Ht} + \theta\psi_v\hat{p}_{Ht} + \sigma^{-1}\hat{c}_t + \frac{\tau}{1-\tau}\hat{\tau}_t\right] + (1-\alpha)\beta E_t\hat{\pi}_{t+1}, \tag{62}$$

$$\pi_{Ft} - \alpha\beta E_t\pi_{Ft+1} + \frac{1-\alpha}{\alpha}(\hat{p}_{Ft} - \alpha\beta E_t\hat{p}_{Ft+1}) = \kappa\zeta\left[\hat{q}_t + \psi_v\hat{y}_{Ft} + \theta\psi_v\hat{p}_{Ft} + \sigma^{-1}\hat{c}^*_t + \frac{\tau}{1-\tau}\hat{\tau}_t\right] + (1-\alpha)\beta E_t\hat{\pi}_{t+1}, \tag{63}$$

where $\kappa = (1-\alpha)(1-\alpha\beta)/\alpha$. Notice that $\hat{p}_{Ht+1} - \hat{p}_{Ht} = \hat{\pi}_{Ht+1} - \hat{\pi}_{t+1}$. This implies that $\hat{p}_{Ht} - \alpha\beta\hat{p}_{Ht+1} = (1-\alpha\beta)\hat{p}_{Ht} - \alpha\beta E_t\hat{\pi}_{Ht+1} + \alpha\beta E_t\hat{\pi}_{t+1}$. Similarly,

Using these in equations (62) and (63) yields

$$\hat{\pi}_{Ht} = \beta E_t\hat{\pi}_{Ht+1} + \kappa\zeta\psi_v\hat{y}_{Ht} - \kappa\zeta\hat{p}_{Ht} + \kappa\zeta\sigma^{-1}\hat{c}_t + \kappa\zeta\frac{\tau}{1-\tau}\hat{\tau}_t, \tag{64}$$

$$\hat{\pi}_{Ft} = \beta E_t \hat{\pi}_{Ft+1} + \kappa\zeta\psi_v \hat{y}_{Ft} - \kappa\zeta\hat{p}_{Ft} + \kappa\zeta\hat{q}_t + \kappa\zeta\sigma^{-1}\hat{c}_t^* + \kappa\zeta\frac{\tau}{1-\tau}\hat{\tau}_t. \tag{65}$$

A linear approximation of equation (24) is

$$\phi_H \hat{p}_{Ht} + \phi_F \hat{p}_{Ft} = 0, \tag{66}$$

which implies

$$\hat{p}_{Ft} = -\frac{\phi_H}{\phi_F}\hat{p}_{Ht}. \tag{67}$$

First differencing equation (69) and rearranging terms yields

$$\hat{\pi}_t = \phi_H \hat{\pi}_{Ht} + \phi_F \hat{\pi}_{Ft}. \tag{68}$$

A linear approximation of equation(28) is

$$\phi_H^* \hat{p}_{Ht} + \phi_F^* \hat{p}_{Ft} = \hat{q}_t. \tag{69}$$

First differencing this equation and rearranging terms yields

$$\hat{\pi}_t^* = \phi_H^* \hat{\pi}_{Ht} + \phi_F^* \hat{\pi}_{Ft}. \tag{70}$$

E Model with GHH Preference

The second model we consider in the paper is the same as the baseline model except that households have GHH preferences. The utility function of home households is, in this case, given by

$$u(C_t, L_t(x)) = \frac{\left(C_t - \chi L_t(x)^{1+\nu^{-1}}/(1+\nu^{-1})\right)^{1-\sigma^{-1}}}{1-\sigma^{-1}}, \tag{71}$$

where ν is the Frisch elasticity of labor supply. Home household labor supply is then

$$\frac{u_\ell(C_t, L_t(x))}{u_c(C_t, L_t(x))} = (1-\tau_t)\frac{W_t(x)}{P_t},$$

where the subscripts on the function u denote partial derivatives. Notice that these partial derivatives take the form

$$\begin{aligned} u_{ct} &= \left(C_t - \chi\frac{L^{1+\nu^{-1}}}{1+\nu^{-1}}\right)^{-\sigma^{-1}}, \\ u_{\ell t} &= -\left(C_t - \chi\frac{L^{1+\nu^{-1}}}{1+\nu^{-1}}\right)^{-\sigma^{-1}} \chi L_t^{1/\nu}. \end{aligned} \tag{72}$$

This implies that home labor supply is

$$\chi L_t^{1/\nu} = (1-\tau_t)\frac{W_t(x)}{P_t}. \tag{73}$$

Labor demand and the production function of firms are the same as in the baseline model. Combining labor supply, labor demand, and the production function of the firms yields

$$\frac{S_t(x)}{P_t} = \frac{1}{(1-\tau_t)}\frac{\chi}{a} y_{ht}^{\psi_v}(x) \tag{74}$$

A linear approximation of this equation in period $t+j$ for firms that last changed their price in period t yields

$$\hat{s}_{ht,t+j}(x) = \psi_v \hat{y}_{ht,t+j}(x) + \frac{\tau}{1-\tau}\hat{\tau}_t \tag{75}$$

Using this equation and its foreign analog, we can derive Phillips curves for home and foreign PPI inflation in the same way as we did for the baseline model. These derivations yield

$$\hat{\pi}_{Ht} = \beta E_t \hat{\pi}_{Ht+1} + \kappa\zeta\psi_v \hat{y}_{Ht} - \kappa\zeta\hat{p}_{Ht} + \kappa\zeta\frac{\tau}{1-\tau}\hat{\tau}_t, \tag{76}$$

$$\hat{\pi}_{Ft} = \beta E_t \hat{\pi}_{Ft+1} + \kappa\zeta\psi_v \hat{y}_{Ft} - \kappa\zeta\hat{p}_{Ft} + \kappa\zeta\hat{q}_t + \kappa\zeta\frac{\tau}{1-\tau}\hat{\tau}_t. \tag{77}$$

With GHH preference, the consumption Euler equation of households becomes

$$E_t\left[\beta\frac{u_c(C_{t+1}, L_{t+1}(x))}{u_c(C_t, L_t(x))}\frac{P_t}{P_{t+1}}\right] = \frac{1}{1+r_t^n}. \tag{78}$$

A linear approximation of $u_c(C_{t+1}, L_{t+1}(x))$ yields

$$u_{ct} = \frac{u_{cc}C}{u_c}\hat{c}_t + \frac{u_{c\ell}L}{u_c}\hat{\ell}_t(x). \tag{79}$$

Notice that

$$u_{cc} = -\sigma^{-1}\left(C - \chi\frac{L^{1+\nu^{-1}}}{1+\nu^{-1}}\right)^{-\sigma^{-1}-1},$$

$$u_{c\ell} = \sigma^{-1}\left(C - \chi\frac{L^{1+\nu^{-1}}}{1+\nu^{-1}}\right)^{-\sigma^{-1}-1}\chi L^{\nu^{-1}}.$$

This implies that

$$\frac{u_{cc}C}{u_c} = -\sigma^{-1}C\left(C - \chi\frac{L^{1+\nu^{-1}}}{1+\nu^{-1}}\right)^{-1} = -\sigma^{-1}\left(1 - a\mu^{-1}\left(\frac{C}{Y}\right)^{-1}(1+\nu^{-1})^{-1}\right)^{-1},$$

$$\frac{u_{c\ell}L}{u_c} = \sigma^{-1}L\left(C - \chi\frac{L^{1+\nu^{-1}}}{1+\nu^{-1}}\right)^{-1}\chi L^{\nu^{-1}} = -\frac{u_{cc}C}{u_c}\left(\frac{L}{C}\right)\chi L^{\nu^{-1}} = -\frac{u_{cc}C}{u_c}\left(\frac{C}{Y}\right)^{-1}a\mu^{-1}.$$

Let $\sigma_c^{-1} = u_{cc}C/u_c$ and $\sigma_\ell^{-1} = u_{c\ell}L/u_c$. We then have that a linear approximation of equation (78) yields

$$\hat{c}_t - \sigma_c\sigma_\ell^{-1}\hat{\ell}_t = E_t\hat{c}_{t+1} - \sigma_c\sigma_\ell^{-1}E_t\hat{\ell}_{t+1} - \sigma_c(\hat{r}_t^n - E_t\pi_{t+1}). \tag{80}$$

Let $\xi_\ell = \sigma_c\sigma_\ell^{-1} = \left(\frac{C}{Y}\right)^{-1} a\mu^{-1}$. Also, a linear approximation of the production function—equation (33)—yields $\hat{y}_{Ht} = a\hat{\ell}_t$. Let $\xi_y = \xi_\ell/a = \left(\frac{C}{Y}\right)^{-1}\mu^{-1}$. Using these conditions, we can rewrite equation (80) as

$$\hat{c}_t - \xi_y\hat{y}_{Ht} = E_t\hat{c}_{t+1} - \xi_y E_t\hat{y}_{Ht+1} - \sigma_c(\hat{r}_t^n - E_t\pi_{t+1}). \tag{81}$$

With GHH preferences, the Backus-Smith condition becomes

$$\frac{u_c(C_t^*, L_t^*(x))}{u_c(C_t, L_t(x))} = Q_t. \tag{82}$$

A linear approximation of this equation is

$$\hat{c}_t - \xi_y\hat{y}_t - \hat{c}_t^* + \xi_y\hat{y}_t^* = \sigma_c\hat{q}_t \tag{83}$$

F Model with Regional Capital Markets

This section presents an extension of the baseline model in Section 1 that incorporates investment, capital accumulation and variable capital utilization. We adopt a specification for these features that mirrors closely the specification in Christiano, Eichenbaum, and Evans (2005).

F.1 Households

Household preferences in the home region are given by equations (15)-(17) as in the baseline model. Household decisions regarding consumption, saving and labor supply are thus the same as before. However, in addition to these choices, households own the capital stock, they choose how much to invest and they choose the rate of utilization of the capital stock. Let $\bar{K}_t$ denote the physical stock capital of capital available for use in period t and I_t the amount of investment chosen by the household in period t. For simplicity, assume that I_t is a composite investment good given by an index of all the products produced in the economy analogous to equations (16)-(17) for consumption. The capital stock evolves according to

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \Phi(I_t, I_{t-1}), \tag{84}$$

where δ denotes the physical depreciation of capital and $\Phi(I_t, I_{t-1}) = \left[1 - \phi\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$, summarizes the technology for transforming current and past investment into capital. Households choose the

utilization rate u_t of the capital cost. The amount of capital services provided by the capital cost in period t is then given by $K_t = u_t \bar{K}_t$.

The budget constraint of households in the home region is given by

$$\begin{aligned} &P_t C_t + P_t I_t + P_t A(u_t)\bar{K}_t + E_t[M_{t,t+1} B_{t+1}(x)] \\ &\quad \leq B_t(x) + (1-\tau_t) W_t(x) L_t(x) + R_t^k u_t \bar{K}_t + \int_0^1 \Xi_{ht}(x) dz - T_t. \end{aligned} \tag{85}$$

The differences relative to the model presented in Section 1 are the following. First, households spend $P_t I_t$ on investment. Second, they incur a cost $P_t A(u_t)\bar{K}_t$ associated with utilizing the capital stock. Here $A(u_t)$ denotes a convex cost function. Third, they receive rental income equal to $R_t^k u_t \bar{K}_t$ for supplying $u_t \bar{K}_t$ in capital services. Here R_t^k denotes the rental rate for a unit of capital services.

In addition to equations (19)-(29) and a standard transversality condition, household optimization yields the following relevant optimality conditions. Optimal capital utilization sets the marginal cost of additional utilization equal to the rental rate on capital,

$$A'(u_t) = \frac{R_t^k}{P_t} \tag{86}$$

Optimal investment and capital accumulation imply

$$D_t \Phi_1(I_t, I_{t-1}) + \beta E_t[D_{t+1} \Phi_2(I_{t+1}, I_t)] = u_c(C_t, L_t(x)) \tag{87}$$

$$D_t = \beta(1-\delta) E_t D_{t+1} + \beta E_t[(A'(u_{t+1}) u_{t+1} - A(u_{t+1})) u_c(C_{t+1}, L_{t+1}(x))] \tag{88}$$

where D_t is the Lagrange multiplier on equation (84) and $\Phi_j(\cdot,\cdot)$ denotes the derivative of Φ with respect to its j-th argument.

F.2 Firms

The production function of firms in industry x is

$$y_t(x) = f\left(L_t(x), K_t(x)\right) \tag{89}$$

The demand for the output of firms in industry x is given by

$$y_t(x) = \left(nC_{Ht} + (1-n)C_{Ht}^* + nI_{H,t} + (1-n)I_{H,t}^* + nG_{Ht}\right) \left(\frac{p_t(x)}{P_{Ht}}\right)^{-\theta} \tag{90}$$

Optimal choice of labor and capital by firms implies

$$W_t(x) = f_\ell(L_t(x), K_t(x)) S_t(x), \tag{91}$$

$$R_t^k = f_k(L_t(x), K_t(x))S_t(x). \tag{92}$$

Optimal price setting by firms yields equation (38) as in the baseline model.

Combining labor demand, labor supply and the production function yields

$$\frac{S_t(x)}{P_t} = \frac{1}{(1-\tau_t)} \frac{v_\ell(L_t(x))}{u_c(C_t) f_\ell(f^{-1}(L_t(x), K_t(x))} \tag{93}$$

F.3 Calibration

We set the rate of depreciation of capital to $\delta = 0.025$, which implies an annual depreciation rate of 10 percent. The investment adjustment cost function is given by

$$\Phi(I_t, I_{t-1}) = \left[1 - \phi\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{94}$$

where $\phi(1) = \phi'(1) = 0$ and $\kappa_I = \phi''(1) > 0$. We set $\kappa_I = 2.5$. This is the value estimated by Christiano et al. (2005). We require that capital utilization $u_t = 1$ in steady state, assume that the cost of utilization function $A_1 = 0$ and set $\sigma_a = A''(1)/A'(1) = 0.01$. Again, this is the value estimated by Christiano et al. (2005). We assume that the production function is Cobb-Douglas with a capital share of 1/3.

F.4 Linear Approximation

A linear approximation of equation (93) and its foreign analog yields

$$\hat{s}_t(x) = \left(\frac{v_\ell L}{v_\ell} - \frac{f_{\ell\ell} L}{f_\ell}\right) \hat{\ell}_t(x) - \frac{f_{\ell k} K}{f_\ell} \hat{k}_t(x) - \frac{u_{cc} C}{u_c} \hat{c}_t + \frac{\tau}{1-\tau} \hat{\tau}_t \tag{95}$$

$$\hat{s}_t^*(x) = \left(\frac{v_{\ell\ell} L^*}{v_\ell} - \frac{f_{\ell\ell} L^*}{f_\ell}\right) \hat{\ell}_t^*(x) - \frac{f_{\ell k} K}{f_\ell} \hat{k}_t^*(x) - \frac{u_{cc} C}{u_c} \hat{c}_t^* + \frac{\tau}{1-\tau} \hat{\tau}_t + \hat{q}_t \tag{96}$$

Linearize approximations of the firm production function and capital demand equation (92) and their foreign analogs yields

$$\hat{y}_t(x) = a\hat{\ell}_t(x) + (1-a)\hat{k}_t(x) \quad \text{and} \quad \hat{y}_t^*(x) = a\hat{\ell}_t^*(x) + (1-a)\hat{k}_t^*(x) \tag{97}$$

$$\hat{r}_t^k = a\hat{\ell}_t(x) - a\hat{k}_t(x) + \hat{s}_t(x) \quad \text{and} \quad \hat{r}_t^{k*} = a\hat{\ell}_t^*(x) - a\hat{k}_t^*(x) + \hat{s}_t^*(x) \tag{98}$$

Note that there is no $\hat{q}_t$ term in the above equation, because $\hat{r}_t^k$ also is divided by P_t. Using these equation we can derive that

$$\begin{aligned} \hat{k}_t(x) &= \hat{y}_t(x) - \hat{r}_t^k + \hat{s}_t(x), \\ \hat{\ell}_t(x) &= \hat{y}_t(x) + \frac{1-a}{a}\hat{r}_t^k - \frac{1-a}{a}\hat{s}_t(x). \end{aligned}$$

Using these equations and the functional form of the utility function—equation (15)—and the production function, we can rewrite equation (95) as

$$\begin{aligned}\hat{s}_t(x) &= (\nu^{-1}+1-a)\left(\hat{y}_t(x)+\frac{1-a}{a}\hat{r}_t^k-\frac{1-a}{a}\hat{s}_t(x)\right)\\ &\qquad -(1-a)\left(\hat{y}_t(x)-\hat{r}_t^k+\hat{s}_t(x)\right)+\sigma^{-1}\hat{c}_t+\frac{\tau}{1-\tau}\hat{\tau}_t\\ &= \psi_v\hat{y}_t(x)+\sigma^{-1}\psi_c\hat{c}_t+\psi_k\hat{r}_t^k+\psi_c\frac{\tau}{1-\tau}\hat{\tau}_t, \end{aligned} \tag{99}$$

where $\psi_v \equiv \frac{a\nu^{-1}}{(1-a)\nu^{-1}+1}$, $\psi_k \equiv \frac{(1+\nu^{-1})(1-a)}{(1-a)\nu^{-1}+1}$ and $\psi_c \equiv \frac{a}{(1-a)\nu^{-1}+1}$. Correspondingly, we can rewrite equation (96) as

$$\hat{s}_t^*(x) = \psi_v\hat{y}_t^*(x)+\sigma^{-1}\psi_c\hat{c}_t^*+\psi_k\hat{r}_t^{k*}+\psi_c\hat{q}_t+\psi_c\frac{\tau}{1-\tau}\hat{\tau}_t \tag{100}$$

A Linear approximation firm demand—equation (118)—yields

$$\hat{y}_{Ht} = \phi_H\frac{C}{Y}\hat{c}_{Ht}+\phi_H\frac{I}{Y}\hat{i}_{Ht}+\phi_H^*\frac{1-n}{n}\frac{C}{Y}\hat{c}_{Ht}^*+\phi_H^*\frac{1-n}{n}\frac{I}{Y}\hat{i}_{Ht}^*+\hat{g}_{Ht}, \tag{101}$$

$$\hat{y}_{Ft} = \phi_F^*\frac{C}{Y}\hat{c}_{Ft}^*+\phi_F^*\frac{I}{Y}\hat{i}_{Ft}^*+\phi_F\frac{n}{1-n}\frac{C}{Y}\hat{c}_{Ft}+\phi_F^*\frac{n}{1-n}\frac{I}{Y}\hat{i}_{Ft}+\hat{g}_{Ft}, \tag{102}$$

where

$$\hat{y}_{t,t+j} = \hat{y}_{Ht+j}-\theta\left(\hat{p}_t-\hat{p}_{Ht+j}-\sum_{k=1}^{j}\pi_{t+k}\right) \tag{103}$$

$$\hat{y}_{t,t+j}^* = \hat{y}_{Ft+j}-\theta\left(\hat{p}_{ft}-\hat{p}_{Ft+j}-\sum_{k=1}^{j}\pi_{t+k}\right) \tag{104}$$

Here we have made use of the fact that in steady state $I_H = \phi_H I$, $I_F = \phi_F I$, $I_H^* = \phi_H^* I^*$, and $I_F^* = \phi_F^* I^*$.

Substituting these expressions into equations (99)-(100) yields

$$\hat{s}_{t,t+j} = \psi_v\hat{y}_{Ht+j}-\psi_v\theta\left(\hat{p}_t-\hat{p}_{Ht+j}-\sum_{k=1}^{j}\pi_{t+k}\right)+\sigma^{-1}\psi_c\hat{c}_{t+j}+\frac{\tau}{1-\tau}\psi_c\hat{\tau}_{t+j}+\psi_k\hat{r}_{t+j}^k$$

$$\hat{s}_{t,t+j}^* = \psi_v\hat{y}_{Ft+j}-\psi_v\theta\left(\hat{p}_t^*-\hat{p}_{Ft+j}-\sum_{k=1}^{j}\pi_{t+k}\right)+\sigma^{-1}\psi_c\hat{c}_{t+j}^*+\frac{\tau}{1-\tau}\psi_c\hat{\tau}_{t+j}+\psi_k\hat{r}_{t+j}^{k*}+\psi_c\hat{q}_{t+j}$$

Applying the same method as we did in Section 1 to derive Phillips curves yields

$$\pi_{Ht} = \beta E_t\pi_{Ht+1}+\kappa\zeta\left(\psi_v\hat{y}_{Ht}-\hat{p}_{Ht}+\sigma^{-1}\psi_c\hat{c}_t+\psi_c\frac{\tau}{1-\tau}\hat{\tau}_t+\psi_k\hat{r}_t^k\right) \tag{105}$$

$$\pi_{Ft} = \beta E_t\pi_{Ft+1}+\kappa\zeta\left(\psi_v\hat{y}_{Ft}-\hat{p}_{Ft}+\sigma^{-1}\psi_c\hat{c}_t^*+\psi_c\frac{\tau}{1-\tau}\hat{\tau}_t+\psi_k\hat{r}_t^{k*}+\psi_c\hat{q}_t\right) \tag{106}$$

We also have that

$$\hat{i}_t = \phi_H\hat{i}_{Ht}+\phi_F\hat{i}_{Ft} \quad \text{and} \quad \hat{i}_t^* = \phi_H^*\hat{i}_{Ht}^*+\phi_F^*\hat{i}_{Ft}^*$$

$$\hat{i}_{Ht} = \hat{i}_t - \eta\hat{p}_{Ht} \quad \text{and} \quad \hat{i}_{Ft} = \hat{i}_t - \eta\hat{p}_{Ft}$$

$$\hat{i}^*_{Ht} = \hat{i}^*_t - \eta(\hat{p}_{Ht} - \hat{q}_t) \quad \text{and} \quad \hat{i}^*_{Ft} = \hat{i}^*_t - \eta(\hat{p}_{Ft} - \hat{q}_t)$$

Analogously to (51) and (52) in Section 1, aggregate output is

$$\begin{aligned}\hat{y}_{Ht} &= \left(\frac{C}{Y}\right)\phi_H\hat{c}_t + \frac{1-n}{n}\left(\frac{C}{Y}\right)\phi^*_H\hat{c}^*_t + \left(\frac{I}{Y}\right)\phi_H\hat{i}_t + \frac{1-n}{n}\left(\frac{I}{Y}\right)\phi^*_H\hat{i}^*_t \\ &\quad -\eta\left(\frac{C+I}{Y}\right)\left(\phi_H + \frac{1-n}{n}\phi^*_H\right)\hat{p}_{Ht} + \eta\left(\frac{C+I}{Y}\right)\frac{1-n}{n}\phi^*_H\hat{q}_t + \hat{g}_{Ht}, \qquad (107)\end{aligned}$$

$$\begin{aligned}\hat{y}_{Ft} &= \left(\frac{C}{Y}\right)\phi^*_F\hat{c}^*_t + \frac{n}{1-n}\left(\frac{C}{Y}\right)\phi_F\hat{c}_t + \left(\frac{I}{Y}\right)\phi^*_F\hat{i}^*_t + \frac{n}{1-n}\left(\frac{I}{Y}\right)\phi_F\hat{i}_t \\ &\quad -\eta\left(\frac{C+I}{Y}\right)\left(\frac{n}{1-n}\phi_F + \phi^*_F\right)\hat{p}_{Ht} + \eta\left(\frac{C+I}{Y}\right)\phi^*_F\hat{q}_t + \hat{g}_{Ft}. \qquad (108)\end{aligned}$$

A linear approximation of $K_t = u_t\bar{K}_t$ and its foreign counterpart yields

$$\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_t. \qquad (109)$$

$$\hat{k}^*_t = \hat{u}^*_t + \hat{\bar{k}}^*_t. \qquad (110)$$

A linear approximation of $A^{'}(u_t) = R^k_t/P_t$ and its foreign counterpart yields

$$\sigma_a\hat{u}_t = \hat{r}^k_t. \qquad (111)$$

$$\sigma_a\hat{u}^*_t = \hat{r}^{k*}_t - \hat{q}_t. \qquad (112)$$

where $\sigma_a = A^{''}/A^{'}$ and we use the fact that $u = 1$ in steady state.

Assume that $\phi(1) = \phi'(1) = 0$ and $\phi''(1) = k_I > 0$. A linear approximation of equation (87) and its foreign counterpart yields

$$\hat{d}_t + k_I\left[\beta\left(E_t\hat{i}_{t+1} - \hat{i}_t\right) - \left(\hat{i}_t - \hat{i}_{t-1}\right)\right] + \sigma_c^{-1}\hat{c}_t = 0 \qquad (113)$$

$$\hat{d}^*_t + k_I\left[\beta\left(E_t\hat{i}^*_{t+1} - \hat{i}^*_t\right) - \left(\hat{i}^*_t - \hat{i}^*_{t-1}\right)\right] + \sigma_c^{-1}\hat{c}^*_t = 0 \qquad (114)$$

Assume that in steady state $A(u) = 0$, $A'(u) = \frac{R^k}{P}$. A linear approximation of equation (88) and its foreign counterpart yields

$$\hat{d}_t = \beta(1-\delta)E_t\hat{d}_{t+1} + (1-\beta(1-\delta))\left(E_t\hat{r}^k_{t+1} - \sigma_c^{-1}E_t\hat{c}_{t+1}\right) \qquad (115)$$

$$\hat{d}^*_t = \beta(1-\delta)E_t\hat{d}^*_{t+1} + (1-\beta(1-\delta))\left(E_t\hat{r}^{k*}_{t+1} + E_t\hat{\pi}_{t+1} - q_t - E_t\hat{\pi}^*_{t+1} - \sigma_c^{-1}E_t\hat{c}^*_{t+1}\right) \qquad (116)$$

G A Model with Firm-Specific Capital

This appendix presents a model of investment and capital accumulation that mirrors closely the specification in Woodford (2003, 2005). In this model, firms own their capital stock and face adjustment costs at the firm level in adjusting their capital stock. Household behavior is governed by the same equations as in our baseline GHH model presented in section E. As in our baseline model, firms belong to industries x, which make use of a specific type of labor. The production function of firms in industry x is

$$y_t(x) = f(L_t(x), K_t(x)). \tag{117}$$

The demand for the output of firms in industry x is given by

$$y_t(x) = (nC_{Ht} + (1-n)C^*_{Ht} + nI_{H,t} + (1-n)I^*_{H,t} + nG_{Ht}) \left(\frac{p_{ht}(x)}{P_{Ht}}\right)^{-\theta}. \tag{118}$$

Firms' optimal choice of labor demand implies

$$W_t(x) = f_\ell(L_t(x), K_t(x))S_t(x). \tag{119}$$

Each firm faces convex adjustment costs for investment. A firm that would like to choose a capital stock $K_{t+1}(x)$ for period $t+1$ must invest $I(K_{t+1}(x)/K_t(x))K_t(x)$ at time t. For simplicity, assume that I_t is a composite investment good given by an index of all the products produced in the economy analogous to equations (16)-(17) for consumption. We assume that $I(1) = \delta$, $I'(1) = 1$, and $I''(1) = \epsilon_\psi$. Optimal investment by firms implies

$$\begin{aligned} I'\left(\frac{K_{t+1}(x)}{K_t(x)}\right) + E_t M_{t,t+1}\frac{P_{t+1}}{P_t}\left[I\left(\frac{K_{t+2}(x)}{K_{t+1}(x)}\right) - I'\left(\frac{K_{t+2}(x)}{K_{t+1}(x)}\right)\frac{K_{t+2}(x)}{K_{t+1}(x)}\right] \\ = E_t M_{t,t+1}\frac{P_{t+1}}{P_t}\frac{R^k_{t+1}(x)}{P_{H,t+1}}\frac{P_{H,t+1}}{P_{t+1}}, \end{aligned} \tag{120}$$

where

$$R^k_t(x) = f_k(L_t(x), K_t(x))S_t(x). \tag{121}$$

Firm price setting is given by equation (38). This model has two new parameters relative to our baseline model. We follow Woodford (2003, 2005) in setting $\delta = 0.012$ and $\epsilon_\psi = 3$. Our results are virtually identical if we instead set $\delta = 0.025$ and $\epsilon_\psi = 2.5$. We assume that the production function is Cobb-Douglas with a capital share of 1/3.

Combining labor demand and labor supply yields

$$aL_t(x)^{a-1}K_t(x)^{1-a}(1-\tau_t)\frac{S_t(x)}{P_{Ht}}\frac{P_{Ht}}{P_t} = \chi L_t(x)^{1/\nu} \tag{122}$$

G.1 Linear Approximation

Notice that

$$K_{t+1}(x) = (1-\delta)K_t(x) + I_t(x) \tag{123}$$

In steady state, we have $I/K = \delta$. A linear approximation of equation (123) then yields

$$\hat{k}_{t+1}(x) = (1-\delta)\hat{k}_t(x) + \delta\hat{I}_t(x). \tag{124}$$

Similarly, for the aggregate home capital stock we have

$$\hat{k}_{Ht+1} = (1-\delta)\hat{k}_{Ht} + \delta\hat{I}_{Ht} \tag{125}$$

A linear approximation of equation (122) yields

$$\hat{s}_t(x) = (\nu^{-1} + 1 - a)\hat{\ell}_t(x) - (1-a)\hat{k}_t(x) - \hat{p}_{Ht} + \frac{\tau}{1-\tau}\hat{\tau}_t \tag{126}$$

A linear approximation of equation (121) yields

$$\hat{r}^k_t(x) = \hat{s}_t(x) + a\hat{\ell}_t(x) - a\hat{k}_t(x) \tag{127}$$

Here, in contrast to earlier models, we are deflating nominal variables by P_{Ht}.

From equation (**??**) we have that in steady state $R^k/P = \delta + 1/\beta - 1$. A linear approximation of equation (**??**) then yields

$$\begin{aligned}\hat{u}_{ct} + \epsilon_\psi\left(\hat{k}_{t+1}(x) - \hat{k}_t(x)\right) + E_t\hat{u}_{ct+1} + \beta\epsilon_\psi\left(E_t\hat{k}_{t+2}(x) - \hat{k}_{t+1}(x)\right)\\ +(1-\beta(1-\delta))\left(E_t\hat{r}^k_{t+1}(x) + E_t\hat{p}_{Ht+1}\right)\end{aligned} \tag{128}$$

A linear approximation of the production function yields

$$\hat{y}_t(x) = a\hat{\ell}_t(x) + (1-a)\hat{k}_t(x) \tag{129}$$

Combining equations (126) and (129) yields

$$\hat{s}_t(x) = \frac{\nu^{-1}+1-a}{a}\hat{y}_t(x) - \frac{(1-a)(\nu^{-1}+1)}{a}\hat{k}_t(x) - \hat{p}_{Ht} + \frac{\tau}{1-\tau}\hat{\tau}_t \tag{130}$$

Adopting the notation used in Woodford (2005), this equation becomes

$$\hat{s}_t(x) = \bar{\omega}\hat{y}_t(x) - (\bar{\omega} - \bar{\nu})\hat{k}_t(x) - \hat{p}_{Ht} + \frac{\tau}{1-\tau}\hat{\tau}_t \tag{131}$$

where $\bar{\omega} = (\nu^{-1} + 1 - a)/a$, $\bar{\nu} = \nu^{-1}$. Taking an average across all home firms we get

$$\hat{s}_{Ht} = \bar{\omega}\hat{y}_{Ht} - (\bar{\omega} - \bar{\nu})\hat{k}_{Ht} - \hat{p}_{Ht} + \frac{\tau}{1-\tau}\hat{\tau}_t \tag{132}$$

and

$$\hat{s}_t(x) = \hat{s}_{Ht} + \bar{\omega}(\hat{y}_t(x) - \hat{y}_{Ht}) - (\bar{\omega} - \bar{\nu})(\hat{k}_t(x) - \hat{k}_{Ht}) \tag{133}$$

Now using the demand curve for home firms we get that

$$\hat{s}_t(x) = \hat{s}_{Ht} - \bar{\omega}\theta\hat{p}_t(x) - (\bar{\omega} - \bar{\nu})\tilde{k}_t(x) \tag{134}$$

where $\tilde{k}_t(x) = \hat{k}_t(x) - \hat{k}_{Ht}$.

A linear approximation of the firms' price setting equation yields

$$\hat{E}_t^x \sum_{j=0}^{\infty} (\alpha\beta)^j \left(\hat{p}_t(x) - \sum_{k=1}^{j} \pi_{Ht+k} - \hat{s}_{t+j}(x) \right) = 0 \tag{135}$$

where following Woodford (2005) we use $\hat{E}_t^x$ to denote an expectation conditional on the state of the world at date t, but integrating only over those future states in which firms in industry x have not reset their prices since period t. For aggregate variables $\hat{E}_t^x x_T = E_t x_T$. For firm specific variables, this is not the case.

Substituting for marginal costs in the above equation we get

$$\hat{E}_t^x \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\hat{p}_t(x) - \sum_{k=1}^{j} \pi_{Ht+k} - \hat{s}_{Ht+j}(x) + \bar{\omega}\theta \left(\hat{p}_t(x) - \sum_{k=1}^{j} \pi_{t+k} \right) + (\bar{\omega} - \bar{\nu})\tilde{k}_{t+j}(x)) \right] = 0 \tag{136}$$

Thus,

$$(1 + \bar{\omega}\theta)\hat{p}_t(x) = (1 - \alpha\beta)\hat{E}_t^x \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\hat{s}_{Ht+j} + (1 + \bar{\omega}\theta) \sum_{k=1}^{j} \pi_{Ht+k} - (\bar{\omega} - \bar{\nu})\tilde{k}_{t+j}(x) \right] \tag{137}$$

Notice that unlike in the heterogeneous labor model $\hat{p}_t(x)$ is not independent of x. This is due to the presence of the $\tilde{k}_{t+j}(x)$ term on the right hand side. Notice also that $\hat{E}_t^x \tilde{k}_{t+j}(x)$ depends on $\hat{p}_t(x)$, we need to determine this dependence to be able to solve for $\hat{p}_t(x)$.

Combining (127) and (131), we have

$$\hat{r}_t^k(x) = \bar{\omega}\hat{y}_t(x) - (\bar{\omega} - \bar{\nu})\hat{k}_t(x) - \hat{p}_{Ht} + a\hat{\ell}_t(x) - a\hat{k}_t(x) + \frac{\tau}{1-\tau}\hat{\tau}_t \tag{138}$$

Using the product function to eliminate $\hat{\ell}_t(x)$ yields

$$\hat{r}_t^k(x) = \rho_y\hat{y}_t(x) - \rho_k\hat{k}_t(x) - \hat{p}_{Ht} + \frac{\tau}{1-\tau}\hat{\tau}_t \tag{139}$$

where $\rho_y = \bar{\omega} + 1$ and $\rho_k = \rho_y - \bar{\nu}$ as in Woodford (2005).

Thus, aggregating equation (128) yields

$$\hat{u}_{ct} + \epsilon_\psi(\hat{k}_{Ht+1} - \hat{k}_{Ht}) = E_t\hat{u}_{ct+1} + \beta\epsilon_\psi(E_t\hat{k}_{Ht+2} - \hat{k}_{Ht+1}) \\ +(1-\beta(1-\delta))(\rho_y E_t\hat{y}_{Ht+1} - \rho_k\hat{k}_{Ht+1} + \frac{\tau}{1-\tau}\hat{\tau}_t) \quad (140)$$

Here our expression differs from Woodford (2005) in the coefficient on $E_t\hat{u}_{ct+1}$. This difference arises because we are using GHH preferences.

Combining this expression with the one on (128) yields

$$\begin{aligned} \epsilon_\psi\left(\tilde{k}_{t+1}(x) - \tilde{k}_t(x)\right) &= \beta\epsilon_\psi\left(E_t\tilde{k}_{t+2}(x) - \tilde{k}_{t+1}(x)\right) \\ &\quad + (1-\beta(1-\delta))\left(\rho_y E_t\left(\hat{y}_{t+1}(x) - \hat{y}_{Ht+1}\right) - \rho_k\tilde{k}_{t+1}(x)\right) \end{aligned} \quad (141)$$

Rearranging and using the firm's demand curve

$$(1-\beta(1-\delta))\,\rho_y\theta\epsilon_\psi^{-1}E_t\hat{p}_{t+1}(x) = \beta E_t\tilde{k}_{t+2}(x) \\ -\left(1+\beta+(1-\beta(1-\delta))\rho_k\epsilon_\psi^{-1}\right)\tilde{k}_{t+1}(x) + \tilde{k}_t(x) \quad (142)$$

or

$$\Theta E_t\hat{p}_{t+1}(x) = E_t\left[Q(L)\tilde{k}_{t+2}(x)\right] \quad (143)$$

where

$$\Theta = (1-\beta(1-\delta))\,\rho_y\theta\epsilon_\psi^{-1}$$

$$Q(L) = \beta - \left[1+\beta+(1-\beta(1-\delta))\rho_k\epsilon_\psi^{-1}\right]L + L^2$$

Notice that $Q(0) = \beta > 0$, $Q(\beta) < 0$, $Q(1) < 0$ and $Q(n) > 0$ for large n. So,

$$Q(L) = \beta(1-\mu_1 L)(1-\mu_2 L) \quad (144)$$

with μ_1, μ_2 real and $0 < \mu_1 < 1 < \beta^{-1} < \mu_2$.

Using the argument on page 13 of Woodford (2005) we have

$$\hat{p}_t(x) = \hat{p}_{Ht} - \psi\tilde{k}_t(x) \quad (145)$$

i.e. the choice of a price to set by firms in industry x is a function of aggregate variables and industry x's capital stock. Here, ψ is a coefficient to be determined below. Also, note that $\tilde{k}_t(x) = 0$ on average for firms that reset their prices at time t because of the Calvo assumption. Thus $\hat{p}_t$ is the average relative price set by firms at time t.

A linear approximation of the home price index yields

$$\hat{p}_{Ht} = \frac{\alpha}{1-\alpha}\pi_{Ht} \tag{146}$$

Let's now introduce the notation $\tilde{p}_t(x)$ for a generic relative price. This contrasts $\hat{p}_t(x)$, which we have been using to denote the optimal price set at time t. Notice that

$$E_t\tilde{p}_{t+1}(x) = \alpha\left[\tilde{p}_t(x) - E_t\pi_{Ht+1}\right] + (1-\alpha)E_t\hat{p}_{t+1}(x) \tag{147}$$

and using the last three equations we get

$$\begin{aligned} E_t\tilde{p}_{t+1}(x) &= \alpha(\tilde{p}_t(x) - E_t\pi_{Ht+1}) + (1-\alpha)E_t[\hat{p}_{Ht+1}(x) - \psi\tilde{k}_{t+1}(x)] \\ &= \alpha\tilde{p}_t(x) - (1-\alpha)\psi E_t\tilde{k}_{t+1}(x) \end{aligned} \tag{148}$$

Again, Woodford (2005) argues that

$$\tilde{k}_{t+1}(x) = \lambda\tilde{k}_t(x) - \gamma\tilde{p}_t(x) \tag{149}$$

where λ and γ are to be determined. The algebra on pages 14-15 in Woodford (2005) applies to our model. We plug the equation (148) into the last equation and get

$$E_t\tilde{k}_{t+2}(x) = [\lambda + (1-\alpha)\gamma\psi]\tilde{k}_{t+1}(x) - \alpha\gamma\tilde{p}_t(x) \tag{150}$$

Using this to substitute for $E_t\tilde{k}_{t+2}(x)$ in equation (142), and again using (148) to substitute for $E_t\tilde{p}_{t+1}(x)$, we obtain a linear relation of $\tilde{k}_t(x)$ and $\tilde{p}_t(x)$. For convenience, denote $A \equiv 1+\beta+(1-\beta(1-\delta))\rho_k\epsilon_\psi^{-1}$, thus $Q(L) = \beta - AL + L^2$. Equation (142) becomes,

$$\begin{aligned} \Theta\alpha\tilde{p}_t(x) - \Theta(1-\alpha)\psi E_t\tilde{k}_{t+1}(x) = \lambda^{-1}E_t\tilde{k}_{t+1}(x) + \lambda^{-1}\gamma\tilde{p}_t(x) \\ -A\tilde{k}_{t+1}(x) + \beta[\lambda + (1-\alpha)\gamma\psi]\tilde{k}_{t+1}(x) - \alpha\beta\gamma\tilde{p}_t(x) \end{aligned}$$

For the conjectured solution (149) to satisfy this equation, we need the coefficient in front of $\tilde{p}_t(x)$ to satisfy

$$(1-\alpha\beta\lambda)\gamma = \Theta\alpha\lambda. \tag{151}$$

Using this equation the coefficient in front of $E_t\tilde{k}_{t+1}(x)$ becomes

$$\begin{aligned} \Theta(1-\alpha)\psi\lambda + 1 - A\lambda + \beta\lambda^2 + (1-\alpha)\beta\lambda\psi\gamma &= 0 \\ => \Theta(1-\alpha)\psi\lambda + (1-\alpha\beta\lambda)(1 - A\lambda + \beta\lambda^2) &= 0 \\ => (\beta^{-1} - \alpha\lambda)Q(\beta\lambda) + (1-\alpha)\Theta\psi\lambda &= 0 \end{aligned} \tag{152}$$

Now, returning to optimal price setting, we focus on the term

$$\hat{E}_t^x \sum_{j=0}^{\infty} (\alpha\beta)^j \tilde{k}_{t+j}(x) \tag{153}$$

Since

$$\tilde{k}_{t+1}(x) = \lambda \tilde{k}_t(x) - \gamma \tilde{p}_t(x) \tag{154}$$

we have

$$\hat{E}_t^x \tilde{k}_{t+j+1}(x) = \lambda \hat{E}_t^x \tilde{k}_{t+j}(x) - \gamma \left(\tilde{p}_t(x) - E_t \sum_{k=1}^{j} \pi_{Ht+k} \right) \tag{155}$$

for all $j \leq 0$, and using $\tilde{p}_t(x) - E_t \sum_{k=1}^{j} \pi_{Ht+k} = \hat{E}_t^x \tilde{p}_{t+j}(x)$. Notice that

$$\begin{aligned}
\tilde{k}_{t+1}(x) &= \lambda \tilde{k}_t(x) - \gamma \tilde{p}_t(x) \\
\tilde{k}_{t+2}(x) &= \lambda^2 \tilde{k}_t(x) - \gamma\lambda \tilde{p}_t(x) - \gamma \tilde{p}_{t+1}(x) \\
\tilde{k}_{t+3}(x) &= \lambda^3 \tilde{k}_t(x) - \gamma\lambda^2 \tilde{p}_t(x) - \gamma\lambda \tilde{p}_{t+1}(x) - \gamma \tilde{p}_{t+2}(x)
\end{aligned}$$

so

$$\begin{aligned}
\hat{E}_t^x \sum_{j=0}^{\infty} (\alpha\beta)^j \tilde{k}_{t+j}(x) &= \frac{\tilde{k}_t(x)}{1-\alpha\beta\lambda} - \frac{\gamma\alpha\beta}{1-\alpha\beta\lambda} \hat{E}_t^x \sum_{j=0}^{\infty} (\alpha\beta)^j \tilde{p}_{t+j}(x) \\
\hat{E}_t^x \sum_{j=0}^{\infty} (\alpha\beta)^j \tilde{p}_{t+j}(x) &= \sum_{j=0}^{\infty} (\alpha\beta)^j \left(\tilde{p}_t(x) - E_t \sum_{k=1}^{j} \pi_{Ht+k} \right)
\end{aligned}$$

In addition using the fact that, $\hat{E}_t^x \sum_{j=0}^{\infty} (\alpha\beta)^j \sum_{k=1}^{j} \pi_{Ht+k} = \frac{1}{1-\alpha\beta} E_t \sum_{j=1}^{\infty} (\alpha\beta)^j \pi_{Ht+j}$, we have

$$\hat{E}_t^x \sum_{j=0}^{\infty} (\alpha\beta)^j k_{t+j}(x) = \frac{\tilde{k}_t(x)}{1-\alpha\beta\lambda} - \frac{\gamma\alpha\beta}{(1-\alpha\beta)(1-\alpha\beta\lambda)} \tilde{p}_t(x) + \frac{\gamma\alpha\beta}{(1-\alpha\beta)(1-\alpha\beta\lambda)} E_t \sum_{j=1}^{\infty} (\alpha\beta)^j \pi_{Ht+j} \tag{156}$$

Noting that for firms reoptimizing their price at time t, $\tilde{p}_t(x) = \hat{p}_t(x)$. Therefore, combining equation (137) and the last equation yields

$$\begin{aligned}
(1+\bar{\omega}\theta)\hat{p}_t(x) = (1-\alpha\beta) E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \hat{s}_{Ht+j} + (1-\alpha\beta)(1+\bar{\omega}\theta) E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \sum_{k=1}^{j} \pi_{Ht+k} \\
- \frac{(1-\alpha\beta)(\bar{\omega}-\bar{\nu})}{1-\alpha\beta\lambda} \tilde{k}_t(x) + \frac{\gamma\alpha\beta(\bar{\omega}-\bar{\nu})}{1-\alpha\beta\lambda} \hat{p}_t(x) - \frac{\gamma\alpha\beta(\bar{\omega}-\bar{\nu})}{1-\alpha\beta\lambda} E_t \sum_{j=1}^{\infty} (\alpha\beta)^j \pi_{t+j}
\end{aligned}$$

Thus,

$$\phi \hat{p}_t(x) = (1-\alpha\beta) E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \hat{s}_{Ht+j} + \phi \sum_{j=1}^{\infty} (\alpha\beta)^j E_t \pi_{Ht+j} - (\bar{\omega}-\bar{\nu}) \frac{1-\alpha\beta}{1-\alpha\beta\lambda} \tilde{k}_t(x) \tag{157}$$

where $\phi = 1 + \bar{\omega}\theta - (\bar{\omega} - \bar{\nu})\frac{\gamma\alpha\beta}{1-\alpha\beta\lambda}$.

For this last equation to be consistent with our conjecture (145), we must have

$$\phi\hat{p}_{Ht} = (1-\alpha\beta)\sum_{j=0}^{\infty}(\alpha\beta)^j E_t\hat{s}_{Ht+j} + \phi\sum_{j=1}^{\infty}(\alpha\beta)^j E_t\pi_{Ht+j} \tag{158}$$

and

$$\phi\psi = (\bar{\omega} - \bar{\nu})\frac{1-\alpha\beta}{1-\alpha\beta\lambda} \tag{159}$$

This last equation along with (151) and (152) comprise a system of three equation in the three unknown coefficients λ, γ, and ψ. Woodford (2005, pages 17-18) describes an algorithm for solving these three equations. The following explains how to reduce this system of equations to a single equation for λ. For $\lambda \neq 0$, (151) can be solved for ψ.

$$\psi(\lambda) = -\frac{(\beta^{-1} - \alpha\lambda)Q(\beta\lambda)}{(1-\alpha)\Theta\lambda} \tag{160}$$

Similarly, (152) defines a function

$$\gamma(\lambda) = \frac{\Theta\alpha\lambda}{1-\alpha\beta\lambda} \tag{161}$$

Substituting these functions for ψ and γ in (159), we get the equation to solve for λ:

$$V(\lambda) = \left[(1+\bar{\omega}\theta)(1-\alpha\beta\lambda)^2 - \alpha^2\beta(\bar{\omega}-\bar{\nu})\Theta\lambda\right]Q(\beta\lambda) + \beta(1-\alpha)(1-\alpha\beta)(\bar{\omega}-\bar{\nu})\Theta\lambda = 0 \tag{162}$$

Quasi-differencing the expression for $\hat{p}_{Ht}$, equation (158), yields

$$\hat{p}_{Ht} - \alpha\beta E_t\hat{p}_{Ht+1} = (1-\alpha\beta)\phi^{-1}\hat{s}_{Ht} + \alpha\beta E_t\pi_{Ht+1} \tag{163}$$

Using equation (146) to plug in for $\hat{p}_t$ yields

$$\frac{\alpha}{1-\alpha}\pi_{Ht} - \frac{\alpha^2\beta}{1-\alpha}E_t\pi_{Ht+1} = (1-\alpha\beta)\phi^{-1}\hat{s}_{Ht} + \alpha\beta E_t\pi_{Ht+1} \tag{164}$$

$$\pi_{Ht} = k\phi^{-1}\hat{s}_{Ht} + \beta E_t\pi_{Ht+1} \tag{165}$$

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