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Bulletin 149

***Highway Curves
and
Test Track Design***

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Automotive Test Track Design

K. A. STONEX, Head, Technical Data Department
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● CONSIDERATION of automotive test track design before an audience of highway engineers is justified by the hope that some of the problems and possibly somewhat extreme treatments may be of help in the approach to highway problems.

An essential difference between the areas is that special types of construction and operation are easily possible on private road systems because of the opportunity to provide any driver training necessary and because operation can be supervised.

The special design problems of automobile test tracks may perhaps be approached best by considering the objectives of automotive proving grounds and certain characteristics of the vehicles which are tested there.

The history of the establishment and the development of the General Motors Proving Ground, which was the first and is still the largest and most highly developed, will make these objectives clear.

PROVING GROUND OBJECTIVES

The story is told that the concept of a proving ground crystallized from a series of tests conducted in 1923 when the 4-wheel braking system was being developed for adoption by Buick. The General Technical Committee, with A. P. Sloan as chairman, had witnessed a test on an experimental 4-wheel brake installation on a public road near Flint. After this demonstration the Committee suggested that certain design modifications be made, and it was agreed that the Committee would return and observe the performance of the modified system at a later date. When this time came and the Committee convened to run the test, they found that the County Road Commission had resurfaced the test strip, and it was obviously impossible to make a direct comparison with the earlier tests. That pointed out clearly the necessity for having a private road system where surfaces, gradients, and operating conditions could be controlled by the development groups, so that the test programs would be free from hazards to and interference from casual private transport and where the test conditions could be maintained at any standard desired.

Requirements of the site were convenience of access from the General Motors manufacturing plants at Detroit, Pontiac, Flint, and Lansing, and specific terrain characteristics; a tract of approximately 1,100 acres was located near Milford and purchased early in 1924.

In passing we may note that additional land has been purchased and the road system developed during the years until we now have 3,873 acres in use at the Proving Ground at Milford, 2,280 acres in the Desert Proving Ground near Mesa, Arizona, and an Engineering Test Headquarters in Manitou Springs, Colorado, at the foot of Pikes Peak. The Desert Proving Ground was established because some portion of car development work must be conducted under climatic conditions prevailing in the south and southwest and in the mountains, and the Pikes Peak facility was established for mountain and high altitude testing. The road system on the Milford Proving Ground has grown to approximately 51.1 miles of several types of surface. The Desert Proving Ground has a 5-mile circular track and a 1.2-mile straightaway. Necessary garage, laboratory, and office buildings on these Proving Grounds have more than 500,000 sq ft of floor area. We have accumulated more than 168,000,000 test miles, and we operate at a current rate of about 12,000,000 miles per year.

The primary objective in establishing the Proving Ground was to provide a place where the manufacturing divisions could carry on their development work free from interference and hazards of public highway travel and where privacy in development of new designs could be assured. General Motors divisions are autonomous, and each is responsible for design, development, manufacturing, and sale of its products. As a secondary objective, the Proving Ground Section was charged with the responsibility of conducting a comprehensive series of engineering tests on production cars of General

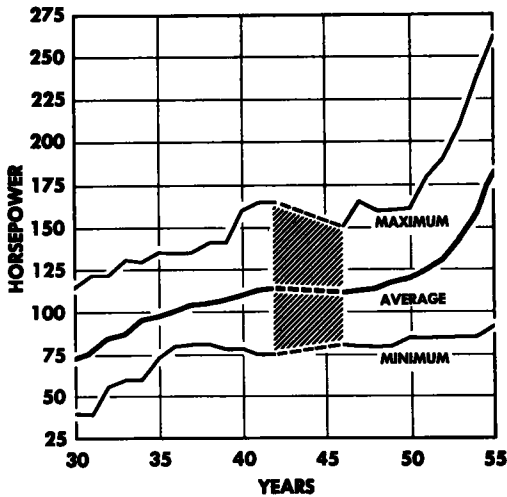


Figure 1. Rated horsepower.

In view of property valuations and the nature of the established public road systems in areas reasonably close to industrial centers, the area of such a piece of property is necessarily limited and corresponding limitations are imposed on design.

VEHICLE CHARACTERISTICS

The requirements of the road system on a proving ground are determined by the characteristics of the vehicles which are to use it.

First it must be recognized that the passenger car has developed and come in to being to the number of more than 50,000,000 units on the road today because it answers an urgent need in the mind and hearts of the American people for a means of swift, mobile, and inexpensive individual transportation. Because the passenger car provides, above all, individual transportation, it has largely replaced mass transportation facilities, which may be as fast or faster and considerably cheaper but suffer the overwhelming disadvantage of being regimented. With our families in our cars we are as free as birds, and we drive about 500 billion miles each year. The automobile is creating a significant social revolution because of the characteristics of its behavior related to flexibility and mobility.

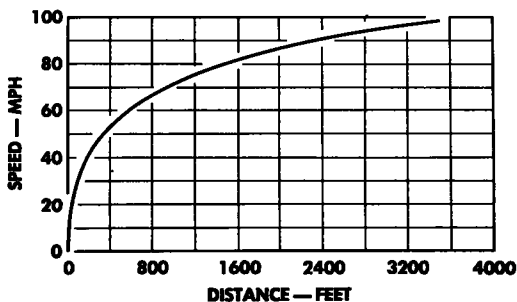


Figure 3. Speed - distance.

Motors and competitors so that the management might know at all times exactly where General Motors products stand with respect to their competition, in the eyes of the customers. In the discharge of this function, the Proving Ground staff has become established as test experts and consultants, and instrument designers, and currently about one-third of the time of the engineering departments is being spent on special tests on division development programs.

Since division development work is the primary objective, a proving ground has to be reasonably close to the manufacturing operation, and consequently site location is restricted to a relatively small part of the country.

To make maximum use of such an area it must be as compact as possible, square or rectangular in shape, and in one piece.

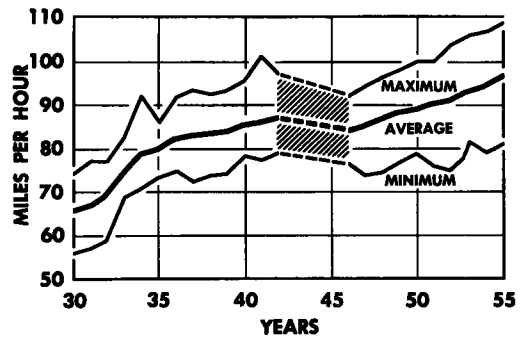


Figure 2. Maximum car speed.

These characteristics have been summarized comprehensively elsewhere (1); for our purposes it will suffice to consider trends of power and speed, and the relations of distance and speed on full throttle acceleration, to characterize the proving ground road system needs of the current vehicle and to estimate reasonable future needs.

Figure 1 shows the trend of rated horsepower from 1930 to 1955, from which we

note that there has been a steady increase, greater in the current period, but always with an upward trend.

Associated with horsepower in the popular conception is the maximum speed which is shown in Figure 2. Here we note that in 1930 the average speed of representative American cars was approximately 65 mph, and in 1955 it was approximately 97. The fastest car of the group developed about 73 mph in 1930 and about 108 mph in 1955. Both the average and the highest speed curves show an increase year by year, and it may be expected that this increase will continue, probably with a decreasing slope. Note that we had 100-mph production cars as early as 1941. We have noted several times that the values of maximum speed noted in Figure 2 are only the by-products of design compromises to achieve ever better traffic performance and fuel economy (1).

Figure 3 shows the speed-distance performance under full throttle acceleration on a level straightaway of a high-performing 1955 car. Nearly one mile is required to attain 100 mph, and evidently a straightaway of two miles or more in length is required for any constant speed observations at 100 mph.

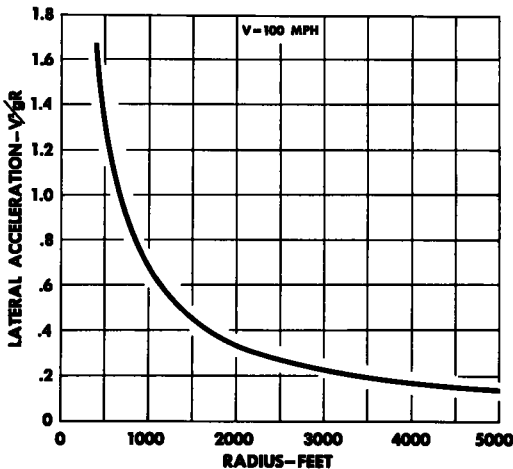


Figure 5. Lateral acceleration vs radius.

speeds but because the satisfactory development of endurance capabilities, lubrication, and cooling systems requires extremely severe testing, testing under conditions far more exacting and demanding than will ever be used by the public. A proving ground intended for realistic development work on current automobiles should provide safe speed characteristics up to something like 110 to 120 mph; from the trend of the maximum speed curve in Figure 2 it appears that the farsighted designer would allow himself a considerable margin for further increases in maximum speed.

It is my understanding that test tracks

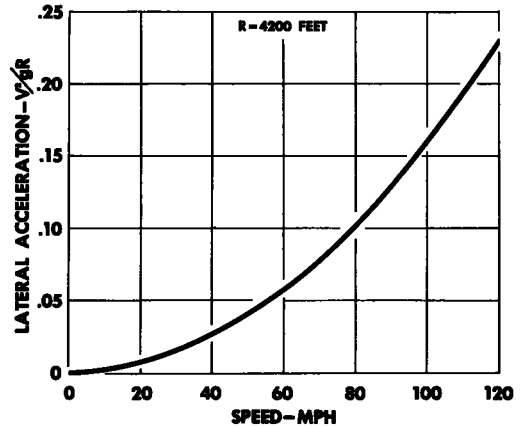


Figure 4. Desert proving ground - lateral acceleration vs speed.

AUTOMOTIVE TEST TRACK REQUIREMENTS

It seems clear, therefore, that a test track on an automobile proving ground must be so designed that it will permit development work up to the top speeds of current automobiles, not because manufacturer intends that the public will operate on those

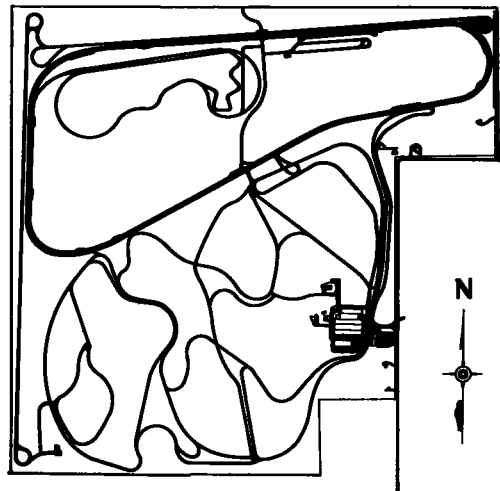


Figure 6. General Motors commercial proving ground.

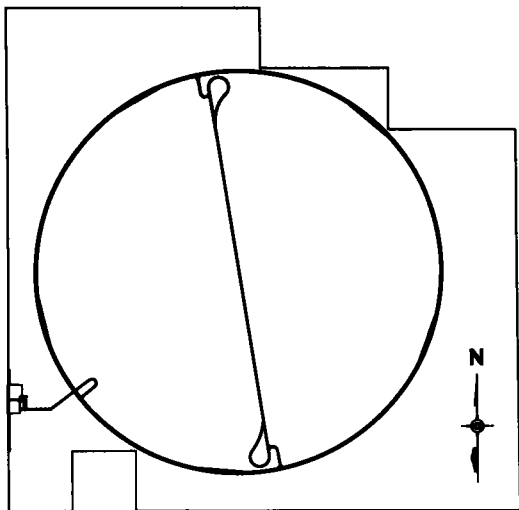


Figure 7. General Motors desert proving ground.

except that the scale on the left side will be changed as the radius varies. The relationship between radial acceleration and the radius for an arbitrary value of speed is shown in Figure 5. Here the design speed was selected as 100 mph for purposes of illustration, and this general reciprocal relation holds for all values of speed. It is evident that the radial acceleration falls rapidly as the radius is increased up to 2,000 ft or more and that there is further substantial decrease as the radius goes up to 5,000 ft. The operating and construction problems are very greatly simplified by the selection of a large radius.

Figure 6 shows the layout of the General Motors Proving Ground road system generally as it existed for 30 years; the test track is roughly triangular in shape because of the property line restrictions prevailing in 1924 and the terrain limitations. The three principal curves have radii of 1,042 ft, 676 ft, and 661 ft, and the superelevation requirements for a uniform design speed change accordingly. On other sites with different terrain characteristics and property restrictions, test tracks have been constructed which are roughly oval in shape, with straight parallel sides and symmetrical circular curves at the ends. When the General Motors Desert Proving Ground site was selected in 1951 a sufficient area was provided to permit the construction of a circular track with a radius of approximately 4,200 ft (Figure 7).

Since the Desert Proving Ground track occupies all the available space in a 2,280-acre area, it seems that a practical maximum radius of the order from 3,500 to 5,000 ft will probably confront most designers of automotive test tracks. Note the location of the engineering test straightaway in Figure 7.

The design of the cross-section of an automotive test track differs from that of the usual highway superelevation in the very important respect that equilibrium of forces parallel to the pavement surface must be provided for a wide range of speeds. This is so because the tests conducted on such a track will range over every combination of speed and throttle opening from 5 or 10 mph up to continuous full-throttle maximum speed tests, or near full-throttle tests, in development work on such things as engine bearings and engine and axle lubricants. Simultaneously there will be numerous other test schedules requiring operation of a number of cars at any and all speeds from 10 to 20 mph up to the maximum. It is imperative that side forces be balanced to eliminate steering drag and undue tire wear. In other words, over a wide speed range, the design must provide operating conditions as close as possible to those on a straight road.

In view of this predominating requirement, the ceiling on any value of cross-section slope is removed; the superelevation slope is limited only by practical problems of construction. On the Milford track, slopes as high as 77 percent are used and operating

recently completed and now under construction have neutral speed capacities of from 140 to 150 mph.

The factor which makes design of automotive test tracks distinctive in highway engineering is that the area available is always very closely restricted because of land values and uses in the only locations where proving ground operation can be conducted most effectively. This means that the most careful refinements of design are required to extend the useful life of a facility to the maximum.

The physical laws related to speed, radius, and radial accelerations are applied in Figures 4 and 5. Figure 4 shows the radial acceleration in gravity units as a function of speed for a track with a radius of 4,200 ft; this refers to the General Motors Desert Proving Ground track at Mesa, Arizona. The same general second degree relationship holds for all values of radius

procedures are established accordingly. We do not carry loads of hay around such superelevated sections at low speeds, nor do we drive on them when they are covered with ice.

The equilibrium of forces parallel to the surface pavement is expressed by the fundamental equation of superelevated curves:

$$\tan \phi = \frac{V^2}{gR}$$

where ϕ = angle of inclination of cross-section

V = speed in ft/sec

R = radius in feet

g = the acceleration due to gravity

This is derived by equating the components of centrifugal force and weight parallel to the pavement surface. In usual highway design, of course, the section is flat and ϕ is constant, but in an automotive test track or variable speed curve, ϕ varies continuously so that the higher speeds can be supported at the outer edge of the paved surface (Figure 8).

$$F \cos \phi = W \sin \phi$$

$$\frac{WV^2}{gR} \cos \phi = W \sin \phi$$

$$\frac{V^2}{gR} = \frac{\sin \phi}{\cos \phi} = \tan \phi = y'$$

SOLVE FOR V :

$$V = \sqrt{gRy'}$$

Figure 9. Equilibrium of parallel forces.

the width of the section will depend upon the degree of x in the equation of the cross-section. If the original section, for example, is an equation in x^3 , the equilibrium speed-width relationship will be linear. Other types of cross-section equations will give other types of relations as indicated in Figure 10.

Which of these relationships to select is always a problem to the designer; it could be approached more confidently if an estimate could be made of the relative proportion of high-speed traffic on the new test track.

For example, if the original equation is a quadratic and 10-mph range speed lanes are laid out, the lanes will be packed together closely at the bottom of the pavement and spread more widely toward the outer edge. If, on the other hand, a 5th degree equation is used, the low-speed lanes toward the bottom of the cross-section will be spread rather widely and the high-speed lanes toward the top super will be packed in more densely. Thus the cross-section of the quadratic equation

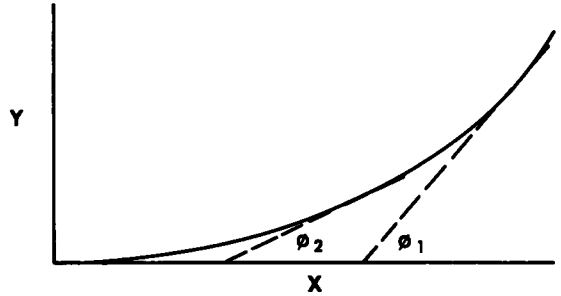


Figure 8. Variable speed super.

To provide a logical basis for the variation of the cross-section slope, it is convenient to select a cross-section of some simple mathematical form such as $y = F(x^n)$; here, of course, the value of $\tan \phi$ in the fundamental equation is the first derivative of the equation of the cross-section. This may be rewritten as

$$y' = V^2 / gR$$

The solution of this equation for V gives

$$V = \sqrt{gRy'} \quad (\text{Figure 9})$$

Since y' is a function of x , it was clear that the variation of the equilibrium speed across

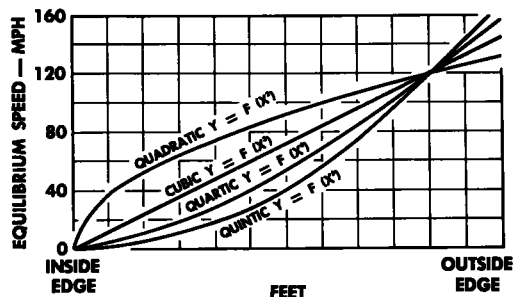


Figure 10. Equilibrium speed.

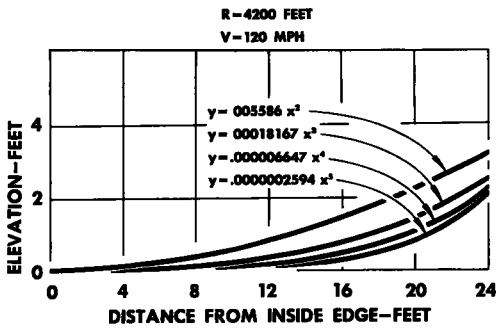


Figure 11. Cross-section elevations.

cross-section equation, and the construction problems become greater as the height at the outer edge increases. This is indicated in Figure 11, which shows the comparative cross-section elevations used in a design study of the Desert Proving Ground track, under the assumption that a design speed of 120 mph should be used. Note that the quadratic section is nearly 50 percent higher at the outer edge than that of the 5th degree equation. For curve designs using shorter radii with much higher total elevation, the differences are even greater, and the volume of earthwork required makes this consideration one of economic importance.

It may be noted, however, that the inner portions of the cross-sections computed from higher degree equations are very nearly flat, and there will usually be a serious question of whether it is really necessary to build the flat portion. The compromise in many design studies is to compute a relatively wide cross-section and use only the outer part of it, discarding several feet of the inside because of the low practical value of this portion. This compromise was reached on the Desert Proving Ground track; we computed a 4th degree cross-section 32 ft wide and used only the outer 24 ft. This gives the equilibrium speed-distance relation shown in Figure 12. At the inner edge the equilibrium speed is approximately 20 mph, while at the outer edge it is about 145 mph. The practical design speed, which is assumed to be at the middle of the outer traffic lane 3.5 ft from the outer edge, is 120 mph. This we considered would be adequate for safe operation on present vehicles and provide a considerable margin for possible increases in maximum speed during development work for the next several years. However the volume of test work exceeded expectations, and we are now reconstructing the track to increase both the surface width and the speed capacity. It will be noted, however, that any modification which discards the inner portion of cross-section results in an equilibrium speed-distance relationship which approaches the linear cubic relationship, and a cross-section elevation which approaches that of the cubic. It may be concluded, therefore, that any design using a cross-section equation of a degree higher than the third and eliminating a portion of the inner part of it will give a modified section which closely approximates the cubic, both with respect to equilibrium speed relations and cross-section elevations. The advantage of a high degree equation is that a considerable margin for higher speeds and safety is provided by a few feet of additional width at the outer edge.

There is no specific guide which can be given to the designer of the superelevated curves for automotive test tracks except that the maximum possible radius should be employed for ease of operation and simplicity and economy of construction, and that while the cross-section equation of the degree higher than the third will give no important construction economies

has more room for high-speed lanes than for low, and the cross-section in the 5th degree equation has more room for the low-speed lanes than for high. Since no one can predict with certainty what the relative load will be at any time in the future, it is perhaps safer to select an intermediate value and use a cubic cross-section. Thus, if a speed range of from 30 to 120 mph is to be covered, the speed at the centerline would be halfway between, or 75 mph.

The total volume of earthwork under the cross-section and the shoulder changes importantly with the degree of the

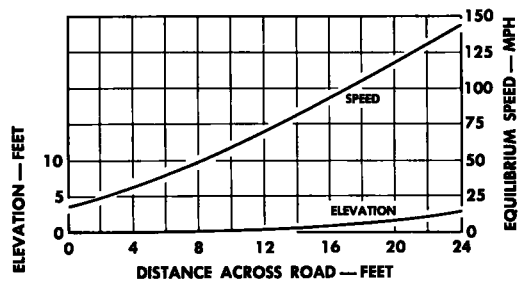


Figure 12. Circular speed loop cross-section and equilibrium speed.

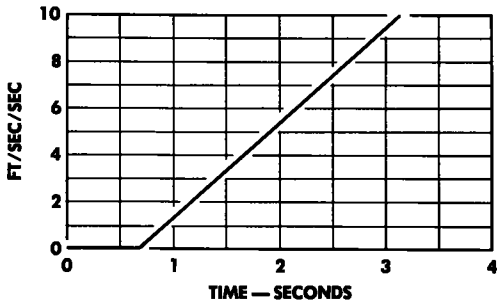


Figure 13. Radial acceleration vs time observed in sharp turn.

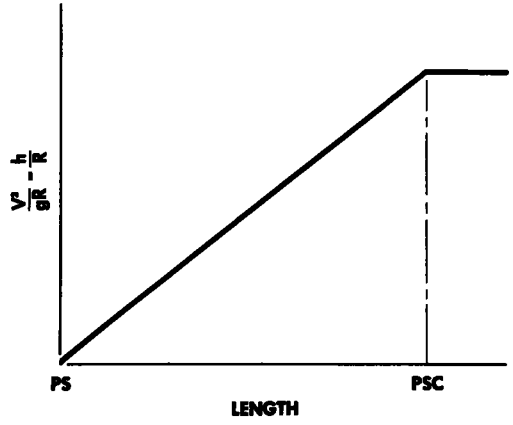


Figure 14. Cornu's spiral - typical radial acceleration vs length.

unless the whole section is used, it will give a greater margin for safety and expansion for each foot of added width. If a flat inner lane will be used extensively, significant economy is yielded by the high degree equation.

TRANSITION SPIRALS

In all layouts where tangent sections are connected to horizontal curves it is imperative that a transition spiral of some sort be incorporated. We are indebted to R. L. McNeal, retired from the Proving Ground staff, for the adaptation of the mathematical form which will give a constant rate of change of radial acceleration through this transient section. This form appears in the literature as Cornu's Spiral. It is the same transition used by Joseph Barnett in his excellent "Transition Curves for Highways." Sample computations are reproduced in the Appendix of this paper.

Figure 13 shows the radial acceleration as a function of time by an automobile during the first few seconds of a sharp turn. This makes it clear that the car will follow a transition spiral as it enters a turn and suggest the desirability of building the road along the path the car will follow. This observation is introduced because some people question the need for transition spirals.

Figure 14 shows that the rate of change of radial acceleration, or curvature, is uniform, the distinguishing characteristic of this form.

Figure 15 is a list of the spiral formu-

K - CONSTANT - $\sqrt{\frac{\pi V^2}{\alpha}}$ (FEET)

$\alpha = \frac{V^2}{R_L} - \frac{V^2}{R_t}$

L - LENGTH OF SPIRAL FROM ORIGIN TO ANY POINT - FEET

$u = \frac{L}{K}$

Z - $\frac{\pi u^2}{2}$ ANGLE BETWEEN TANGENT LINE AT ORIGIN AND TANGENT LINE AT ANY POINT ON SPIRAL - RADIAN

R - $\frac{L^2}{2Z}$ RADIUS AT ANY POINT ALONG SPIRAL - FEET

X - $K \int_0^u \cos\left(\frac{\pi u^2}{2}\right) du - K C(u)$

Y - $K \int_0^u \sin\left(\frac{\pi u^2}{2}\right) du - K S(u)$

Figure 15. Cornu's spiral formulas.

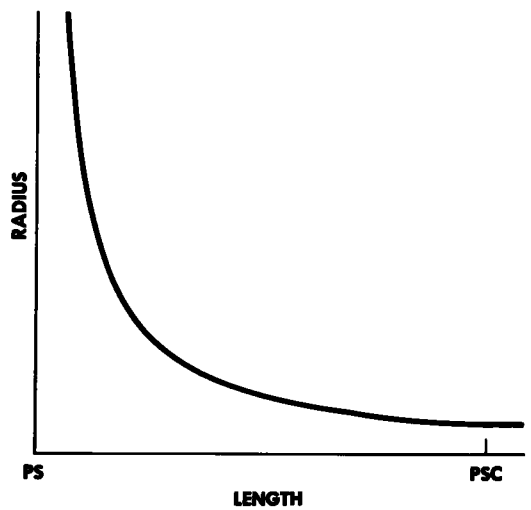


Figure 16. Cornu's spiral - typical radius vs length.

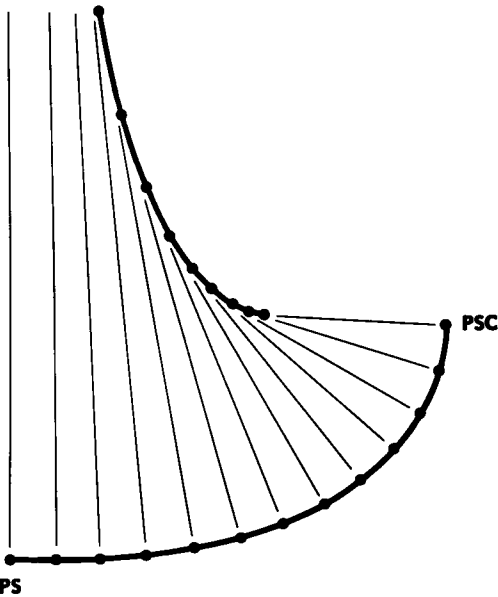


Figure 17. Typical spiral and locus of center of curvature.

rate to provide the greatest ease of passing from the condition of zero centrifugal force to the maximum centrifugal force in the full circular curve. Of course, with the proper superelevation design in the spiral, the lateral forces are always balanced so, with constant speed, this derivation results in a form which gives a constant rate of change of curvature. A significant advantage of this form is that the length of radius and the direction of the tangent may be located precisely throughout the curve, and the coordinates of all the stations can be computed precisely for any length of spiral; the radius must be known at each station to develop precise cross-section design.

There are other types of spirals in the literature, developed first for use in the layout of railroad curves and subsequently adopted in one form or another with certain variations for highway use. These do not provide for a constant rate of change in curvature; in some cases approximations are made which do not permit them to be applied on long spirals turning through considerable angle.

There is no clear criterion of the value which should be selected for the constant rate of change of radial acceleration; in McNeal's derivation the assumption was made that a rate of 3 ft/sec/sec/sec should be the maximum, but this assumption has not been verified in practice. The current highway standard, according to Barnett, is 2 ft/sec/sec/sec. For spirals into curves with a large radius it is our opinion, however, that selection of a conservative value will result in a spiral sufficiently long to permit reasonable rates of change of other elements of the transition; the designer can rest assured that the degree of permanence of his structure will depend on this value, and we are reasonably sure the value of 3 ft/sec/sec/sec is satisfactory for long radius curves on proving ground road systems. On short spirals leading into curves with radii in the neighborhood of 200 ft we have found a rate of from 6 to 8 ft/sec/sec/sec can be attained, but the operation is severe.

las; while the spiral cannot be expressed very simply in elementary functions, the computations are but little more extensive than those of other spirals. Much more information is available; for example, the radius can be computed very simply for any station, which is essential to the design of the proper cross-section. In these formulas a is the rate of change of radial acceleration and u^2 is in right angles.

Of course, for public highway alignments, the tables in "Transition Curves" eliminate the need for use of the formulas; they are required for specialized applications discussed later.

Figure 16 shows how the radius changes along the length of the spiral.

Figure 17 shows the layout of a typical spiral of this form and the locus of the center of curvature.

Figure 18 defines Z , x , and y .

In the original adaptation, which is on file in the Engineering Societies' Library (2), the assumption was made that the radial acceleration should vary at a constant

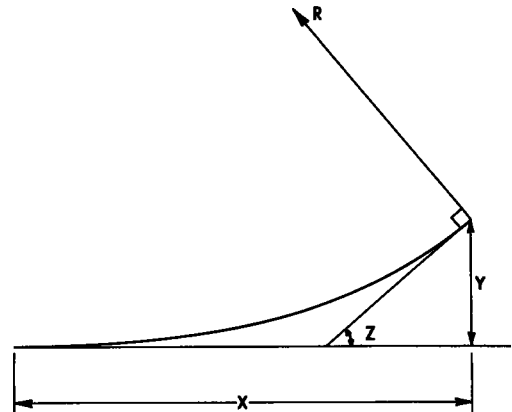


Figure 18.

It would be possible to compound the entering and leaving spirals and omit the portion of circular arc; this always requires a larger area which usually makes this design impractical, the feeling generally being that the maximum length of straight sections should be obtained.

STRAIGHTAWAY TURNAROUNDS

In automotive development work, road elements other than a test track are required for certain types of engineering tests. Level straightaways of the maximum length possible are incorporated in most proving ground layouts, because they are essential to a comprehensive test program. These roads have possibly their maximum useful application in measurements of constant speed fuel economy. On these tests, it is essential that temperatures be stabilized, and the changes in temperature during the brief interruptions of turning around at the ends of the straightaway are significant factors in widening the so-called experimental error. It is, therefore, imperative that the test car be turned around at the end of each run in the shortest possible time and at as near the test speed as possible. This gives rise to numerous interesting and difficult design problems when we consider that fuel economy observations at speeds as high as 90 or 100 mph are important on present day automobiles. As a result the designer is expected to provide a high-speed turnaround in almost no space; on a turnaround, ideal from the standpoint of the test engineer, the slope and the total height of the superelevation would go up to colossal values and the transition spirals would have excessive rates of change of curvature.

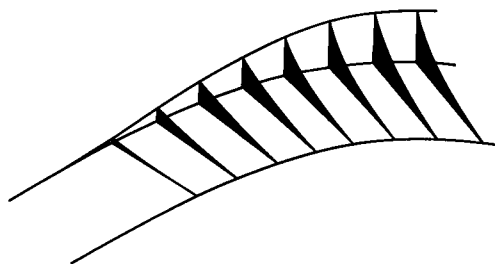


Figure 19. Typical spiral structure.

SPIRAL EASEMENT VERTICAL CURVE DESIGN

In cases such as this where the designer is cramped for room it will be found that the vertical curves leading the car up to the height of the fully superelevated structure in the high-speed lane are the critical factors in the design.

We find, for example, that the simplest type of easement from the level tangent section to the top of a superelevated curve structure would be a straight-sloped profile with simple vertical curves connecting the two horizontal levels on a structure such as shown in Figure 19. On high-speed, short-radius circular curves with short spirals, the radius of curvature in the vertical plane of both of these vertical curves must be short, and the centrifugal forces in the vertical plane are considerable. For example, as the car reaches the first concave vertical curve it will be pressed against the road surface with a force depending upon the speed of the car and the radius of the curve.

Figure 20 is used as an analogy of the case as the car climbs over the top of the second vertical curve. Figure 20 (left) illustrates an elementary problem in exterior ballistics; the projectile follows a parabolic path and climbs to a height determined by the velocity and initial angle of flight. The car at the right climbing a ramp of the same inclination at the same velocity would follow the same trajectory. If we constructed a vertical curve along the broken line, the weight of the car against the road surface would be zero, while if the ramp were continued indefinitely along the solid line, the road reaction would equal the car weight; on a vertical curve lying somewhere between, the reaction would be somewhere between zero and the car weight. The centrifugal lift means that the force in the vertical plane against the road surface is less than normal, and the gravity component down the slope which is used in the development of the fundamental equation of a superelevated curve is reduced proportionately. Therefore, the speed for balance of lateral forces parallel to the pavement surface will fall unless the superelevation slope is increased. This is a matter of considerable importance, because the critical point in operating on a high-speed turn near its capacity always occurs near the end of the transition section, and the effective operating speed of the turnaround system can be increased appreciably by careful design at this point.

We have developed a method whereby the designer can evaluate the vertical forces

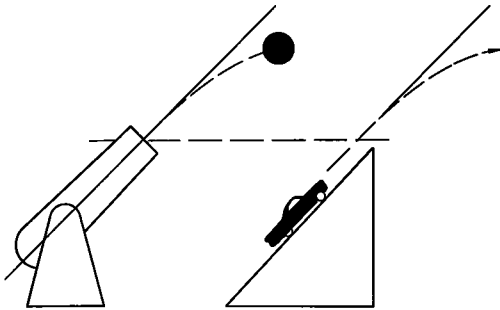


Figure 20. Vertical curve analogy.

and a criterion which permits him to select proper rates of change of the vertical forces. We have applied McNeal's transition theory to the vertical curves, which means that the vertical disturbances of the vehicle path will occur at a constant rate. This derivation permits us to evaluate these disturbances by rather simple computation, so we can select the length of the vertical transition which will keep the rate of change in vertical forces within limits we may choose. Figure 21 shows the relative value of the vertical reactions as they occur during such a transition using Cornu's Spirals.

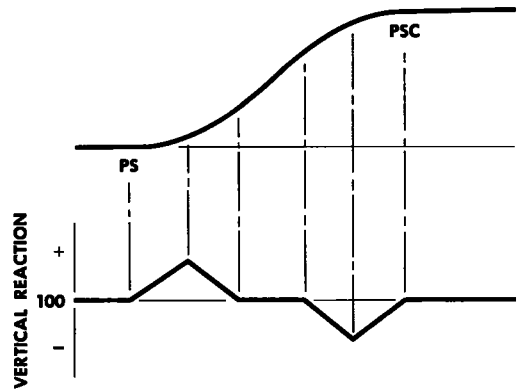


Figure 21. Easement vertical curve profile and road forces - using Cornu's spirals.

Observations made at the Proving Ground indicate that the driver is sensitive to a lateral force unbalance of about $0.1w$ producing a lateral, or radial, acceleration of $0.1g$. Figure 22 shows the effect of the vertical curve in terms of forces in the section on a car passing over the crest of a convex vertical curve leading from the transition section to the full super. The driver will be acutely aware and considerably disturbed as the component of the lift parallel to the plane of the pavement exceeds $0.1w$. The spiral in the vertical plane should therefore be selected to keep the lift component below this value or the design should account for it otherwise; this may mean that the slope of the superelevation should be increased to the point where the gravity component equals the sum of the parallel components of the lift force and the centrifugal force.

As a matter of practice in all our design studies on short-radius curves where this criterion had been employed, we have found that the length of the spiral is determined by the considerations of the vertical curve; when conservative or reasonable vertical curve rates are selected, the spiral length is such that rates in the horizontal spiral are small enough to be of no concern. It may be stated conclusively that in all high-speed, short-radius designs the considerations of the vertical curves are much more critical than those of the horizontal spiral.

In cases where drainage conditions are

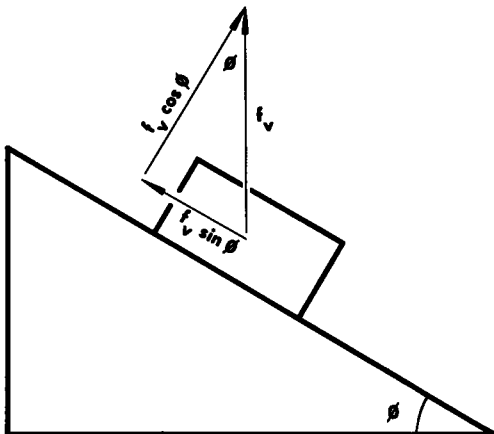


Figure 22. Effect of vertical curve forces.

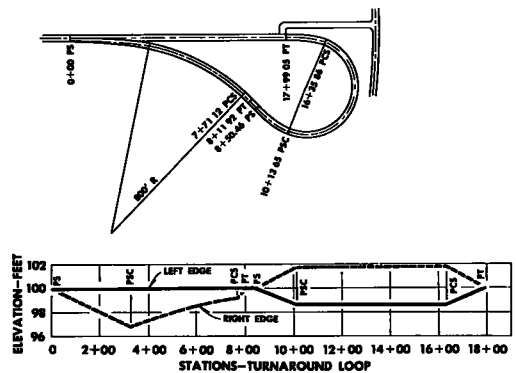


Figure 23. Desert proving ground turnaround loop - layout drawing and profiles.



Figure 24.

transition curves in the straightaway turnaround on the Desert Proving Ground were designed in this manner; the turnaround superelevation cross-section was rotated about the center of the high-speed lane so that the car traveling the high-speed lane remains on a level path. The edge profiles and layout are shown in Figure 23. The direction of travel is counterclockwise; the left edge grade is level of necessity to at least station 3 + 26 where it leaves the straightaway line.

Application to the North Turnaround

Some typical examples of proving ground design problems are found in the design of an extension and improvement of our North and South Engineering Test Straightaway. Figure 24 shows the general

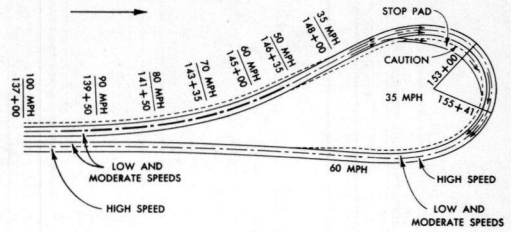
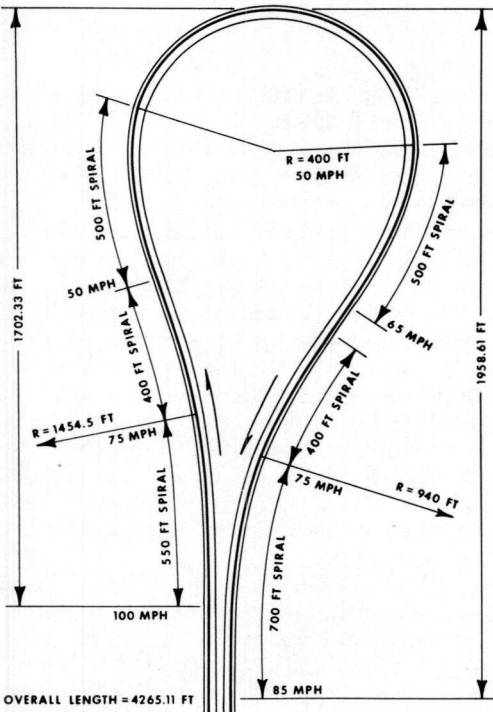


Figure 25. North turnaround.

favorable, we have been able to depress the inner edge of the superelevated structure such that the climb in the high-speed lane is negligible; this design, we have found, tends to reduce the severity of operation to a considerable degree. The

**NORTH TURNAROUND
200 FT RADIUS**

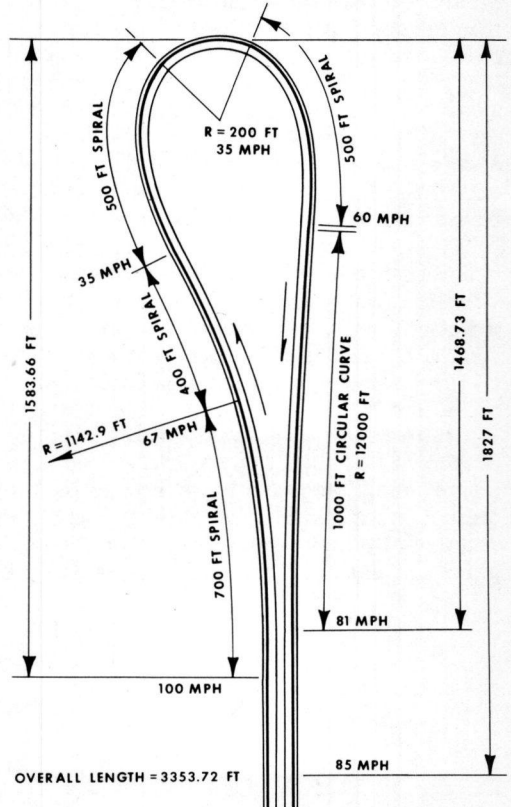


Figure 26. North turnaround - 400 ft radius. Figure 27. North turnaround - 200 ft radius.

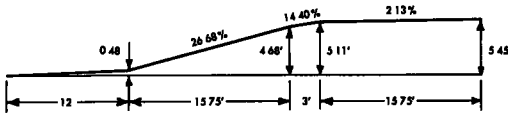


Figure 28. North turnaround - entrance spiral.

eration, the layout and traffic control pattern shown in Figure 25 were selected. Traffic will flow in a clockwise direction to separate high-speed traffic as far as possible; proximity of the property line at the south end allowed space for a median of only 20 ft.

In developing the design of the north turnaround, the principal considerations were clearance with the west property line, the choice of design speeds and radii which would minimize the length of structure, and the most economical use of the total length available for straightaway purposes. Two values of design speed and radius were studied. The first of these, using 50-mph design speed with suitable spirals, is shown in Figure 26. The unusual shape of this structure is caused by the proximity of the west property line. Here it will be noted that the total length of the curved structure is 4,265 ft, and the distance on the northbound lane from the end of the straight section to the extreme end is 1,702 ft, and the distance from the extreme end to the point where 85 mph will be obtained with a high-performing car is 1,958 ft.

Figure 27 shows corresponding values for the design speed of 35 mph using a 200-ft radius. Here again the limited clearance with the west property line made it necessary to move the extreme east portion of the curve slightly past the prolongation of the centerline of the straightaway. The total length of curved structure of this design is only 3,354 ft; the distance on the northbound lane from the end of the straight section to the extreme end is 1,584 ft; and the distance in the southbound lane from the extreme end to the 85-mph point is 1,827 ft.

Thus, with the 35-mph design speed, the total structure is shorter by more than 900 ft and both the north- and south-bound lanes are longer by more than 100 ft. It is possible that some greater economy might be gained at an even lower speed design, but practical operating considerations made it unnecessary to develop this farther. The basic design decision of clockwise operation to keep high-speed traffic separated as far as possible meant that higher speed traffic would enter the curve on the left side of the road; to avoid crossing of traffic lanes, it is kept on the left side. Obviously the left lane requires a considerably greater superelevation slope than the right. Figure 28 shows a typical superelevation cross-section on the curve to the left in the northbound lane. Because the drainage problem in this area was expensive and serious, it did not seem feasible to build the whole cross-section with a uniform slope equal to that required by high-speed traffic in the left lane, so a split section with two different planes is used. Since operation will be intermittent rather than continuous, some unbalance of lateral forces can be tolerated and it is not necessary to use a curved cross-section.

An additional 12-ft lane is added on the left side of the section to avoid serious erosion problems at the bottom of the slope. This will cover the whole length of the entering curve, and it is tapered at both ends (see Figure 30). Although it is intended primarily for erosion control,

view of the area. With a distance of approximately three miles between the north and south property lines, sustained high speed operation will be possible over a considerable part of the length. To minimize hazards it seemed essential to divide the road, and after careful consideration, the layout and traffic control pattern shown in Figure 25 were selected. Traffic will flow in a clockwise direction to separate high-speed traffic as far as possible; proximity of the property line at the south end allowed space for a median of only 20 ft.

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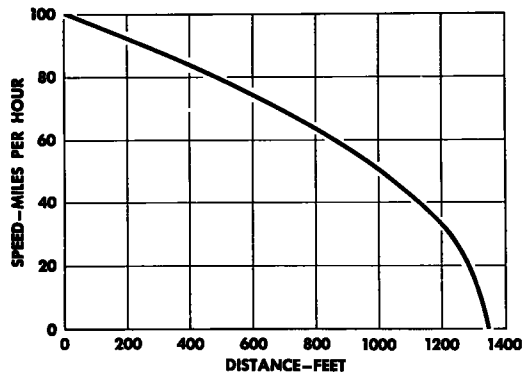


Figure 29. Speed vs distance - constant deceleration of 8 ft/sec².

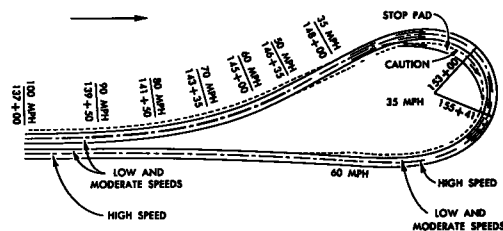


Figure 30. North turnaround.

it will be useful as a part of the operating surface, since drivers may straddle the intersection and gain some flexibility of neutral speeds.

The low and moderate speed lane on the right side has a relatively small slope, and a 3-ft wide beveled section is placed between these slopes to reduce the possibility of interference if the cars should pass from one lane to the other.

In the design of the entering spiral it was considered that the car would be operating at high speed approaching the turnaround, and a design speed at the beginning of the spiral was set arbitrarily at 100 mph with a value of $a = 4$. It is assumed that a deceleration rate of 8 ft/sec^2 can be used conservatively, and that the car would decelerate at this rate for a distance sufficient to bring it down to the design speed of 35 mph at the beginning of the curve to the right. These values have been used successfully in other recent designs. The relation of speed and distance at this rate of deceleration is shown on Figure 29.

These considerations led to a spiral with increasing curvature to the left 700 ft long followed by a 400-ft spiral unwinding to straighten the car out. On these curves a value of $a = 4$ at 100 mph was used, which means that most intense operation of the curve will occur at the beginning of the spiral, followed by decreasing intensity as speed is reduced. Superelevation slopes in the high-speed lane were computed at each station in accordance with fundamental equation and the operating speed developed in Figure 29.

We do not propose at this time to specify the speed range which will require use of the left lane; operation in this region will range from 20 to 100 mph, and what is "high speed" will depend on speed differences rather than on speed alone.

In Figure 27, the first spiral to the right leading to the short circular curve was designed on the basis of a uniform speed of 35 mph, and the rather considerable length is required primarily by the combination of vertical curve systems to reach the superelevation height without undue vertical reactions over the convex curve. It should be pointed out also that this is the only place where there is any safety margin for an operator who enters the system at too high a speed or experiences brake failure.

A section of circular arc approximately 240 ft long is used to turn the car far enough to enter a spiral out of the circular curve. The spiral out of the curve starts soon after the car starts to turn back toward the straightaway. Experience indicates that where drivers attempt to develop maximum speed after leaving a turnaround, they start to accelerate at about this position. Here we have designed the road to suit this practice. Failure to provide sufficiently for this practice is a deficiency in the design of other turnarounds in our road system.

This spiral is designed to permit full-throttle acceleration throughout its length of a current high-performing car, and it is estimated that a speed of 60 mph will be attained by the time the car reaches the end of the spiral.

Because the clearance with the west property line fence was limited and the edge of the curve system was forced across the line of the straightaway, the car is brought back to the straightaway line by means of a circular curve with a radius of 12,000 ft. The grade is brought down to the straightaway level at the beginning of this circular curve; with such a large radius it will function as a straight road.

Figure 30 shows the layout of the final design including the supplementary drainage fillets at the lower edge of the superelevated section and a stopping pad at the interior of the loop where repairs and adjustments will be made. The location of this pad was established at the point where even at high speed a car can stop safely and where the natural ground level is essentially at the grade of the inside edge of the paved surface.

Figure 31 shows the north turnaround profile.

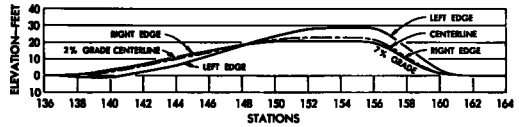


Figure 31. North turnaround profile.

Application to South Turnaround

The design of the south turnaround is distinctive because advantage was made of terrain features. A complication arose from the fact that the west property line and the access to the straightaway slowly converge north to south so that there is a distance of

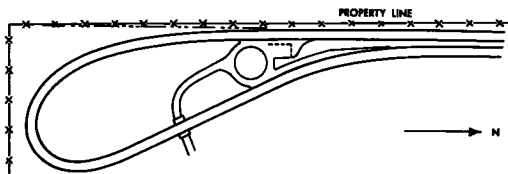


Figure 32. South turnaround.

only about 16 ft between the property line and the west edge of the new facility at the end of the former concrete pavement (Figure 32).

In order to take maximum advantage of the elevations just south of the former south turnaround, a study was made of the effect of elevation on acceleration and deceleration of cars. This relationship

is shown in Figure 33. The mathematical relation is

$$2 gh = (V_1^2 - V_0^2)$$

- where h = change in elevation - feet
- V_1 = terminal (or initial) velocity - ft/sec
- V_0 = initial (or terminal) velocity - ft/sec
- g = acceleration due to gravity

It is obvious that elevations in the order of 60 or more feet would contribute materially to changes in vehicle speed, so that it should be possible to substitute change in elevation for length of structure and conserve the 3-mile length. Figure 34 shows a layout used in design studies of the section including the turnaround loop in the old straightaway. The short turn at the right shows the old loop; the second one is a turnaround having minimum clearance with the then existing property. The chart in the lower part of this figure shows the relative elevations of the straightaway at point A, at point B which was the south property line prior to 1955, and the estimated elevation at the highest point on the private property between the Proving Ground south property line and a public highway at point C. Note that on the Proving Ground property there is a maximum change of elevation of 60 ft and on the private property an estimated change of nearly 80 ft. It was apparent that procurement of this piece of property would permit taking advantage of the considerable increase in elevation as well as lengthening the com-

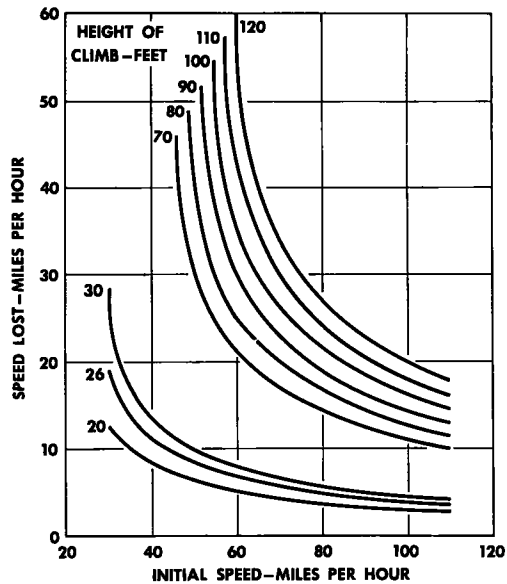


Figure 33. Decrease in speed during hill climb.

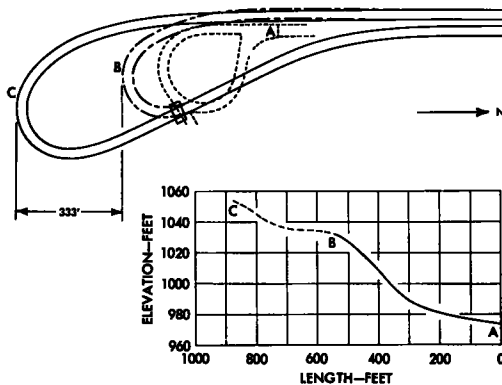


Figure 34. South turnaround.

plete structure more than 300 ft. Figure 33 suggests that the increase in elevation would give a large advantage at low speeds. This property was purchased. At the same time we decided to improve on nature and build a fill on top of the hill; design studies led to the decision that a fill of about 20 ft would be the best compromise. This put the outer edge of the structure at 100 ft above the straightaway grade, and it gives necessary clearance with the new property line and provides reasonable grades.

The next question to arise was the

choice of a design speed and radius of curvature. From Figure 33, showing the relation of height and velocity changes, it is evident that the effectiveness of any elevation is much greater at lower speeds and that the slope of the curves increases rapidly as the speed is reduced. It seemed evident that the 100-ft elevation might be much more effective in contributing speed change at a very low starting speed at the top of the hill than at more conventional speeds. The layout of the vertical curve system would, it was felt, be much simpler at a low design speed.

A study was made of the speed-distance relationships at 0, 25, and 35 mph starting speeds, using the final design elevations (Figure 35); this shows clearly that the value of design speed selected would have little relation to the terminal speed at the foot of the hill.

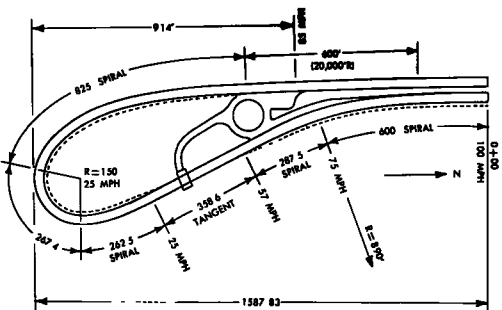


Figure 36. South turnaround.

bound lane would have to be started soon enough and thrown over rapidly enough to provide space for truck turning diameters. A second problem lay in the fact that the convergence of the straightaway and the property line made it necessary to throw the spiral leading from the turnaround arc back to the northbound lane over as rapidly as possible; the public highway along the west property line is being modernized and room is required to accommodate the difference in grade between the straightaway and the public highway. These requirements led to the consideration of compounding the horizontal spirals to increase the "throw" or deviation from the original tangent. Again it was considered that the design speed at the beginning of the first spiral would be 100 mph and moderate deceleration rates should be assumed.

If $a = 4$ at 100 mph, any reduction in speed through the spiral will reduce the value of a rapidly and operation will become much more conservative. It seemed that we could make effective use of appropriate portions of successive spirals to

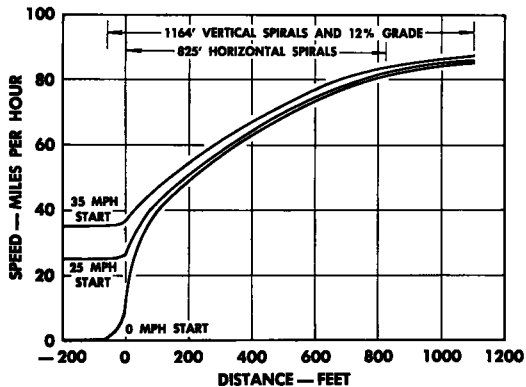


Figure 35. South turnaround exit - speed attained from various starts.

To provide some feeling of progress, and to keep the vertical curve problems simpler, a design speed of 25 mph was selected arbitrarily.

The use of the 100-ft elevation and the rather steep slopes leading up to it made it clear that we would have to provide a turnaround at a lower level for use in the wintertime and probably for use by large vehicles all year. The location and elevation selected made it possible to provide principal access to the new straightaway by means of an underpass (Figure 36). This location of the truck turnaround meant that the entering curve on the south-

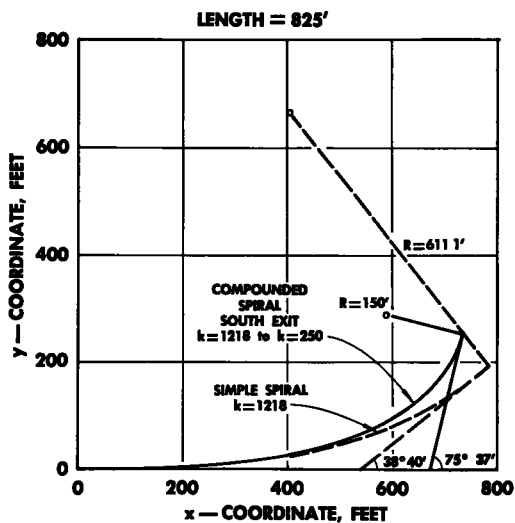


Figure 37. Comparison of simple and compounded spirals.

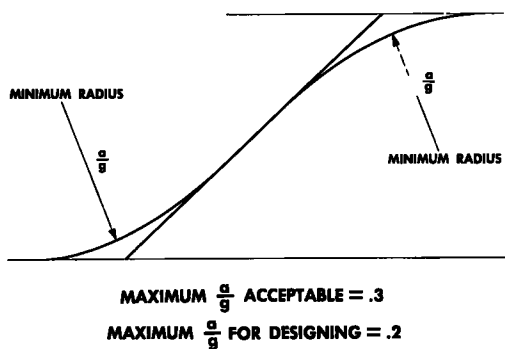


Figure 38. Maximum vertical reactions.

α remains nearly 4 on the compounded spiral, and it falls rapidly on the simple spiral. The effectiveness of this system can be appreciated by the fact that the longer uncompounded spiral would have required a total rotation of about $2\frac{1}{2}$ times to wind up at the terminal radius of the compounded spiral. The alternative to compounding would have been to start farther back and use up more straightaway.

The remainder of the horizontal design is analogous to that of the north turnaround, but modified in most details by speed differences caused by change in elevation and by property line clearance on the exit rather than the entering side (Figure 36). The deceleration to 25 mph can be accomplished easily by the help of the ascending 14 percent grade over the overpass.

The exit spiral is started as the car begins to turn toward the northbound lane, and the superelevation is based on full throttle acceleration of a current high-performing car. A split-plane cross-section with an erosion control fillet at the bottom is used on both curves. As the design developed it became apparent that clearance distance from the west property line could be obtained by incorporating 600 ft of circular curve with a radius of 20,000 ft between the end of the spiral and the straight section of the northbound lane.

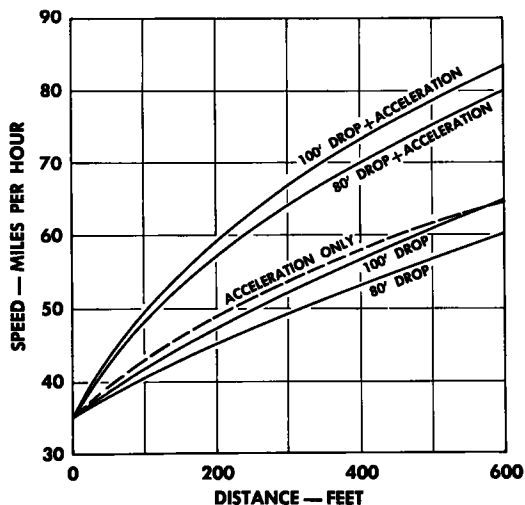


Figure 40. Effect of acceleration and drop on speed.

maintain a value of $\alpha = 4$ throughout the system and increase the curvature of the spiral much more rapidly. A technique was evolved to permit this; the results are demonstrated in Figure 37.

Note that both spirals have a value of $\alpha = 4$ at $x = 0$; the simple spiral, $K = 1,218$, turns through an angle Z of $38 \text{ deg} - 40'$ and reaches a radius of 611 ft, while in the same arc length the compounded spiral, where K varies in several steps from 1,218 to 250, develops a rotation of the tangent to where $Z = 75 \text{ deg} - 37'$ and the radius decreases to 150 ft.

With the reduction in speed, the value of

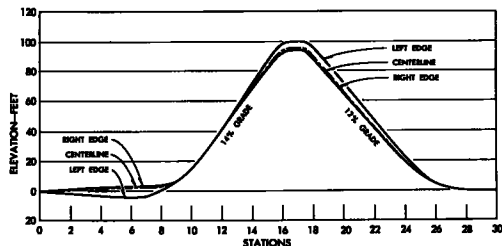


Figure 39. South turnaround profile.

as a straight section. Note that a speed of 85 mph is attained in 914 ft; this contrasts with 1,827 ft in the north turnaround.

The vertical curve system at the highest point of elevation was complicated because good design demanded that the accelerating exit spiral begin as the car begins to turn back toward the straightaway axis, and with the steep grade a short vertical curve was required to begin to make use of the drop at a low vehicle speed. This is the most critical point in design of the turnaround. It was necessary to design the convex vertical curve carefully to minimize the roller coaster effect, and no criteria had been established for guidance. A study was made of the proper values of vertical reactions by measuring tolerable and undesirable values on another part of the road system;

the consensus of opinion of the few observers was that a value of vertical acceleration of 0.3g was about as high as would be acceptable. For design purposes, in this convex vertical curve system we used values of 0.2g, recognizing that at times individual drivers might choose to exceed the design speed considerably. Cornu's spirals were used in this design. Figure 38 illustrates this concept.

Figure 39 shows the profile of the final design. It will be noted that a maximum grade of 14 percent is used on the entering side; this was required to provide sufficient clearance for the underpass access. A maximum grade of 12 percent is used on the downhill section. While the vertical scale is exaggerated, the vertical curve design problems are indicated clearly.

Figure 40 shows the expected distance-speed relationships of a current high-performing car. It shows the individual contributions to speed of the accelerating potential of the car and drops in elevation of 80 and 100 feet; after 600 ft a drop of 100 ft contributes more than the accelerating potential of a high-performing car starting at 35 mph.

As a summary, it may be said that automotive proving ground road system design uses the same principles as public highway design; the emphasis is different in that more severe operating conditions are both required and feasible - required because of area limitations and feasible because of the opportunity to train drivers and supervise operations. This results in probably greater emphasis on detail design refinement to extend service life.

REFERENCES

1. K. A. Stonex, Relation between Automobile and Highway, Proceedings, Highway Research Board, Volume 33.
2. R. L. McNeal, An Investigation of Some Problems in the Design of High Speed Tracks, GMPG No. 1. 1223 (Engineering Societies' Library).

Appendix A

Design of a Horizontal Transition Spiral

The problem was to develop a horizontal transition spiral to a circular arc having a radius of approximately 225 ft and a maximum slope in the superelevation cross-section of 30 percent; this would provide equilibrium of lateral forces parallel to the pavement at a speed of approximately 31.5 mph. The characteristics of the circular curve have already been determined by certain considerations which are not a part of this design problem.

The Cornu's Spiral formulas are:

$$K = \text{constant} = \sqrt{\frac{\pi v^3}{a}} \quad (\text{feet})$$

$$a = \frac{v^3}{RL} = \frac{v^2}{Rt}$$

L = Length of spiral from origin to any point - feet

$$u = \frac{L}{K}$$

$$Z = \frac{\pi u^2}{2} \quad \text{Angle between tangent line at origin and tangent line at any point on spiral-radians}$$

HORIZONTAL SPIRALS COMPUTED IN APPENDIX A

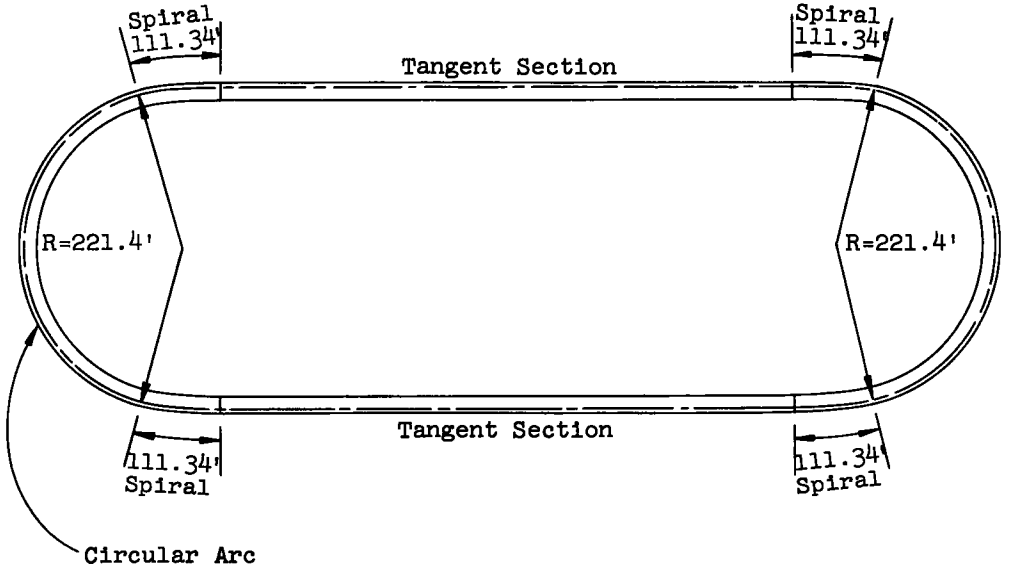
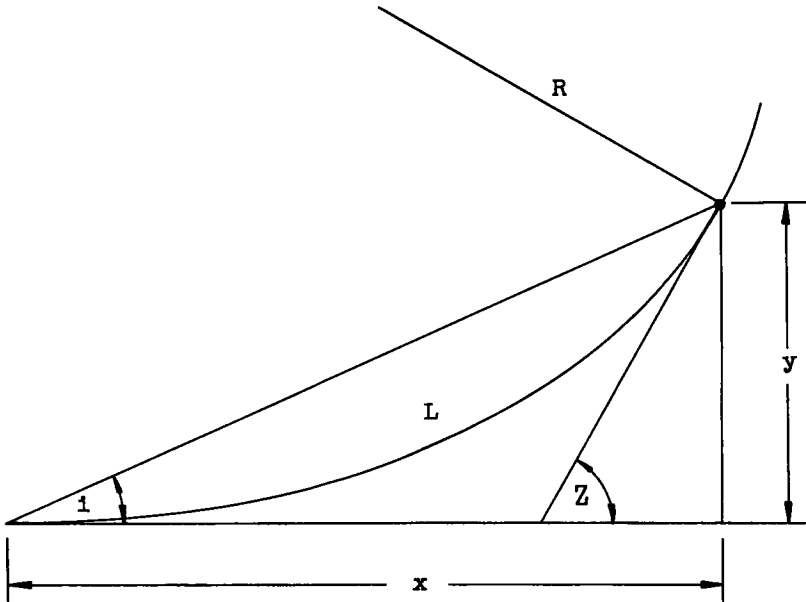


DIAGRAM OF SPIRAL CHARACTERISTICS



$$R = \frac{L}{2Z} \text{ Radius at any point along spiral - feet}$$

$$R = \frac{K}{\pi u}$$

$$x = K \int_0^u \cos\left(\frac{\pi u^2}{2}\right) du = K C(u)$$

$$y = K \int_0^u \sin\left(\frac{\pi u^2}{2}\right) du = K S(u)$$

$$C(u) = u - .24874011 u^5 + .02818550 u^9 - .00160488 u^{13} + .00005407 u^{17} -$$

$$S(u) = .52359878 u^3 - .09228059 u^7 + .00724478 u^{11} - .00031212 u^{15} + \dots 1$$

$u^2 =$ number of right angles

It was decided that a value of $\alpha = 4$ was a proper criterion. Solution for K gives:

$$K = \sqrt{\frac{\pi V^3}{\alpha}} = 278.35$$

From the formula

$$R = \frac{K}{\pi u}, \text{ for } u = .4, R = 221.41'$$

which we accept as the terminal radius of the spiral and the radius of the circular arc.

Since $L = Ku$

$L = 111.34$, the total length of the spiral

Then $C(u) = .39748$

$S(u) = .03336$

$x = K C(u) = 110.64$

$y = K S(u) = 9.29$

$\tan i = y/x = .08397$

$i = 4^{\circ} 48'$

where $i =$ deflection angle to endpoint on spiral, coordinates of which are (110.64, 9.29)

Detailed computations including coordinates of each point at 10' chords, deflection angles, radius, and required slope of the cross-section follows:

$R_c = 221.41'$

$L = 111.34'$

$K = 278.35$

$V = 31.5 \text{ mph}$

$\alpha = 4.0 \text{ ft/sec}^3$

$$u = \frac{L}{K}$$

L	u	u ²	u ³	u ⁵	u ⁷
10	.03593	.0012907	.0000464	-	-
20	.07185	.0051624	.0003709	.0000019	-
30	.10778	.0116185	.0012520	.0000145	.0000002
40	.14370	.0206497	.0029674	.0000613	.0000013
50	.17963	.0322669	.0057961	.0001870	.0000060
60	.21556	.0464661	.0100162	.0004654	.0000216
70	.25148	.0632422	.0159041	.0010058	.0000636
80	.28741	.0826045	.0237414	.0019611	.0001620
90	.32333	.1045423	.0338017	.0035337	.0003694
100	.35926	.1290677	.0463689	.0059847	.0007724
111.34	.40000	.16	.064	.0102400	.0016384

$C(u) = u - .24674u^5 + \dots$

$S(u) = .5236u^3 - .092281u^7 + \dots$

L	u	.24674u ⁵	C(u)	x _h = K C(u)	.5236u ³	.092281u ⁷	S(u)	y _h = K S(u)
10	.03593	-	.03593	10.00	.00002	-	.00002	.01
20	.07185	-	.07185	20.00	.00019	-	.00019	.05
30	.10778	-	.10778	30.00	.00066	-	.00066	.18
40	.14370	.00002	.14368	39.99	.00155	-	.00155	.43
50	.17963	.00005	.17955	49.98	.00303	-	.00303	.84
60	.21556	.00011	.21545	59.97	.00524	-	.00524	1.46
70	.25148	.00025	.25123	69.93	.00833	.00001	.00832	2.32
80	.28741	.00048	.28693	79.87	.01243	.00001	.01242	3.46
90	.32333	.00087	.32246	89.76	.01770	.00003	.01767	4.92
100	.35926	.00148	.35778	99.59	.02428	.00007	.02421	6.74
111.34	.4	.00253	.39747	110.64	.03351	.00015	.03336	9.29

$V = 31.5 \text{ mph} = 46.2 \text{ fps}$

$K = 278.35$

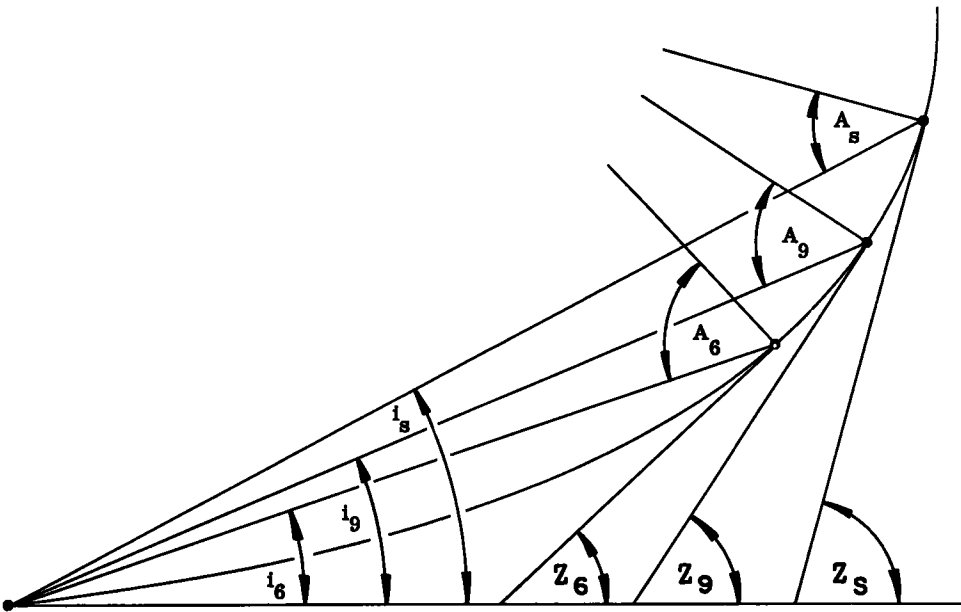
$$R = \frac{K}{\pi u} = \frac{88.601}{u}$$

$$y' = \frac{V^2}{gR} = \frac{66.37}{R}$$

L	u	x_h	y_h	$\tan i = y/x$	i	R	y'_v
10	.03593	10.00	.01	.00100	0°-03'-26.9"	2466.0	.0269
20	.07185	20.00	.05	.00250	0°-08'-35.2"	1233.0	.0538
27.83	.1	-	-	-	-	886.0	.0749
30	.10778	30.00	.18	.00600	0°-20'-37.2"	822.1	.0807
40	.14370	39.99	.43	.01075	0°-36'-57.9"	616.6	.1076
50	.17960	49.98	.84	.01681	0°-57'-47.6"	493.3	.1345
55.66	.2	-	-	-	-	443.0	.1498
60	.21556	59.97	1.46	.02435	1°-23'-41.4"	411.0	.1615
70	.25148	69.93	2.32	.03318	1°-54'-02.1"	352.3	.1884
80	.28741	79.87	3.46	.04332	2°-28'-49.7"	308.3	.2153
83.49	.3	-	-	-	-	295.3	.2248
90	.32333	89.76	4.92	.05481	3°-08'-14.5"	274.0	.2422
100	.35926	99.59	6.74	.06768	3°-52'-18.6"	246.6	.2691
111.34	.4	110.64	9.29	.08397	4°-48'-00.0"	221.5	.2996

Chord Lengths of Horizontal Spiral

L	Δ_{10}^x	Δ_{10}^y	$(\Delta x)^2$	$(\Delta y)^2$	$(\Delta x)^2 + (\Delta y)^2$	Chord = $\sqrt{(\Delta x)^2 + (\Delta y)^2}$
10	10	.01	100	.0001	100.0001	10.00
20	10	.04	100	.0016	100.0016	10.00
30	10	.13	100	.0169	100.0169	10.00
40	9.99	.25	99.80	.0625	99.8625	9.99
50	9.99	.41	99.80	.17	99.97	10.00
60	9.99	.62	99.80	.38	100.18	10.01
70	9.96	.86	99.20	.74	99.94	10.00
80	9.94	1.14	98.80	1.30	100.10	10.00
90	9.89	1.46	97.81	2.13	99.94	10.00
100	9.83	1.82	96.63	3.31	99.94	10.00
111.34	11.05	2.55	122.10	6.50	128.60	11.34



$$A = 90^\circ - (Z - i)$$

L	u^2	$Z = \frac{\pi}{2} - u^2$	Z	i	(Z - i)	A = 90° - (Z - i)
10	.0012907	.002027	0°- 6'-58.1"	0°-03'-26.9"	0°-03'-31.2"	89°-56'-28.8"
20	.0051624	.008109	0°-27'-52.6"	8'-35.2"	19'-17.4"	89°-40'-42.6"
30	.0116165	.018247	1°-02'-43.7"	20'-37.2"	42'-06.5"	89°-17'-53.5"
40	.0206497	.032437	1°-51'-30.5"	36'-57.9"	1°-14'-32.6"	88°-45'-27.4"
50	.0322669	.050685	2°-54'-14.5"	57'-47.6"	1°-56'-26.9"	88°-03'-33.1"
60	.0464661	.072989	4°-10'-54.9"	1°-23'-41.4"	2°-47'-13.5"	87°-12'-46.5"
70	.0632422	.099341	5°-41'-30.5"	1°-54'-02.1"	3°-47'-28.4"	86°-12'-31.6"
80	.0826045	.129755	7°-26'-03.9"	2°-28'-49.7"	4°-57'-14.2"	85°-02'-45.8"
90	.1045423	.164215	9°-24'-31.9"	3°-08'-14.5"	6°-16'-17.4"	83°-43'-42.6"
100	.1290677	.202740	11°-36'-58.1"	3°-52'-18.6"	7°-44'-39.5"	82°-15'-20.5"
111.34	.16	.251424	14°-24'-39.8"	4°-48'-00.0"	9°-36'-39.8"	80°-23'-20.2"

Figure A-1 shows spiral designs for various values of K through a turn of 90 deg. A spiral can be calculated for any value of K which depends on the velocity of car travel and the rate of increase of radial acceleration according to the following formula:

$$K = \sqrt{\frac{\pi V^3}{a}} \quad \text{where } V = \text{ft/sec} \\ a = \text{ft/sec}^3$$

The radius of curvature at any length along a spiral is shown in Figure A-2 for several different spiral designs denoted by K. The straight lines which radiate from the lower left corner and are denoted by u determine the angle, Z, between the line tangent to the spiral at its origin and the line tangent to the spiral at any given point.

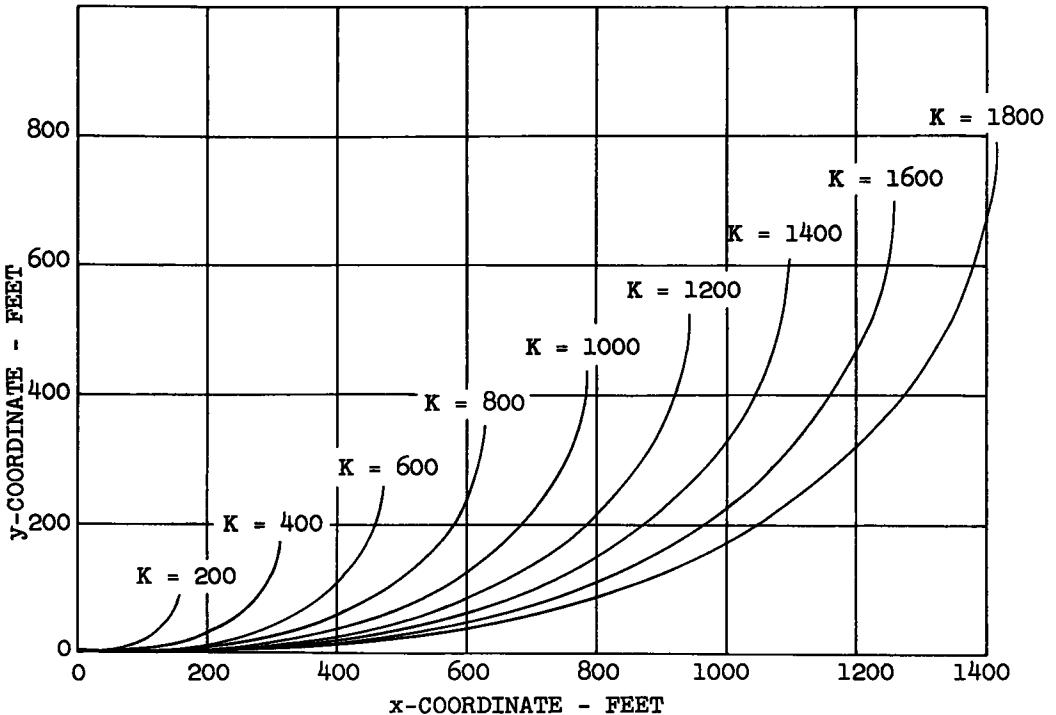


Figure A-1. Spiral designs to 90 deg.

For example, at K - 500 and R - 400', the length is L - 200' and u = 0.4.

$$z = \frac{\pi u^2}{2}$$

$$= 1.5708 (0.4)^2$$

$$= .251328 \text{ Radians}$$

A spiral can be traveled over a wide range of speeds and Figure A-3 shows the effect of changes in speed on the rate of change in lateral acceleration for given spirals. From various studies, a value $\alpha = 4.0$ has been chosen as a maximum value desired at most speeds.

Figure A-4 is the specific case of spiral constants to be used at each speed to maintain $\alpha = 4.0$, and Figure A-5 determines the length of spiral and terminal radius which may be used. The two straight-dashed lines across the family of curves show the points on each curve for cross-section slopes of 40 percent and 10 percent. Similar lines can be determined for any other slopes.

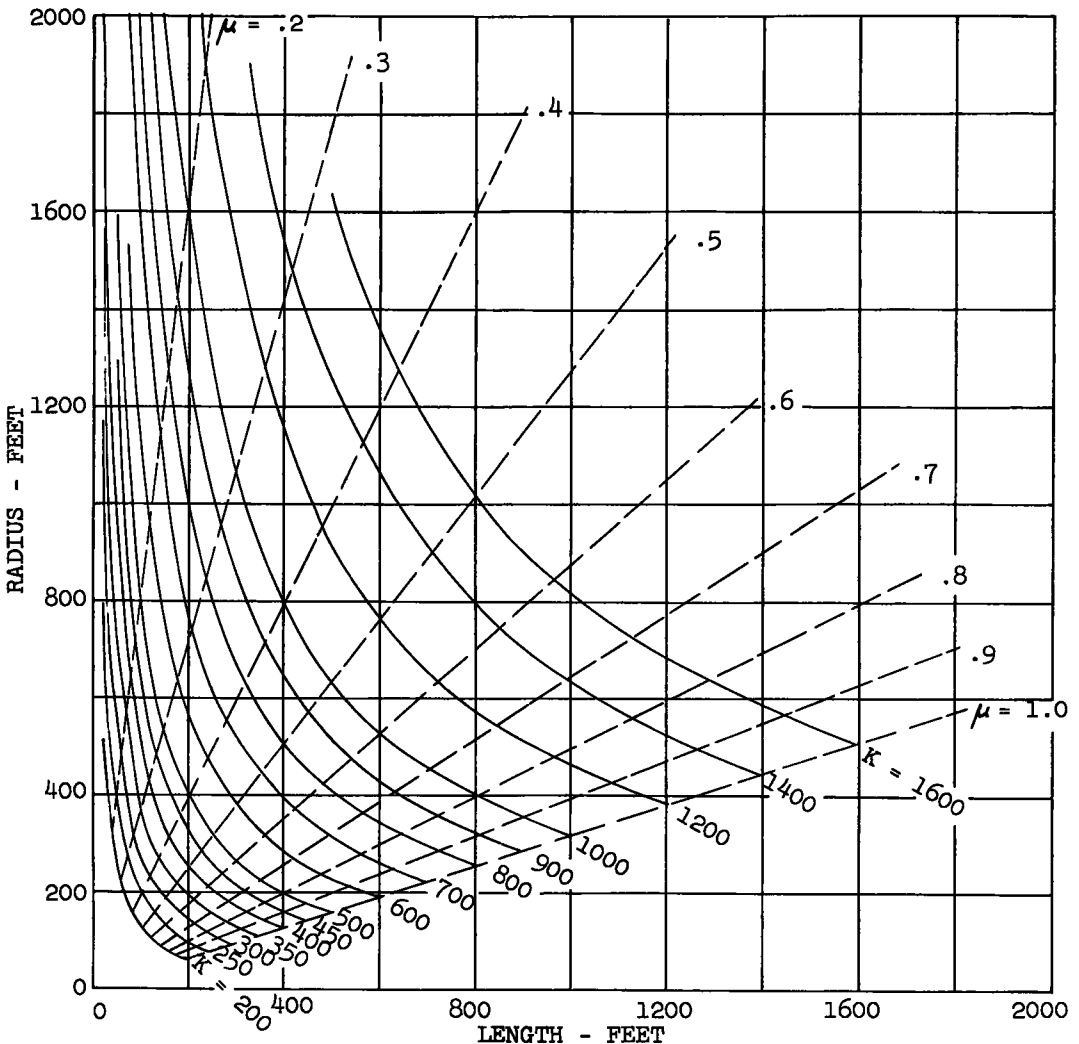


Figure A-2.

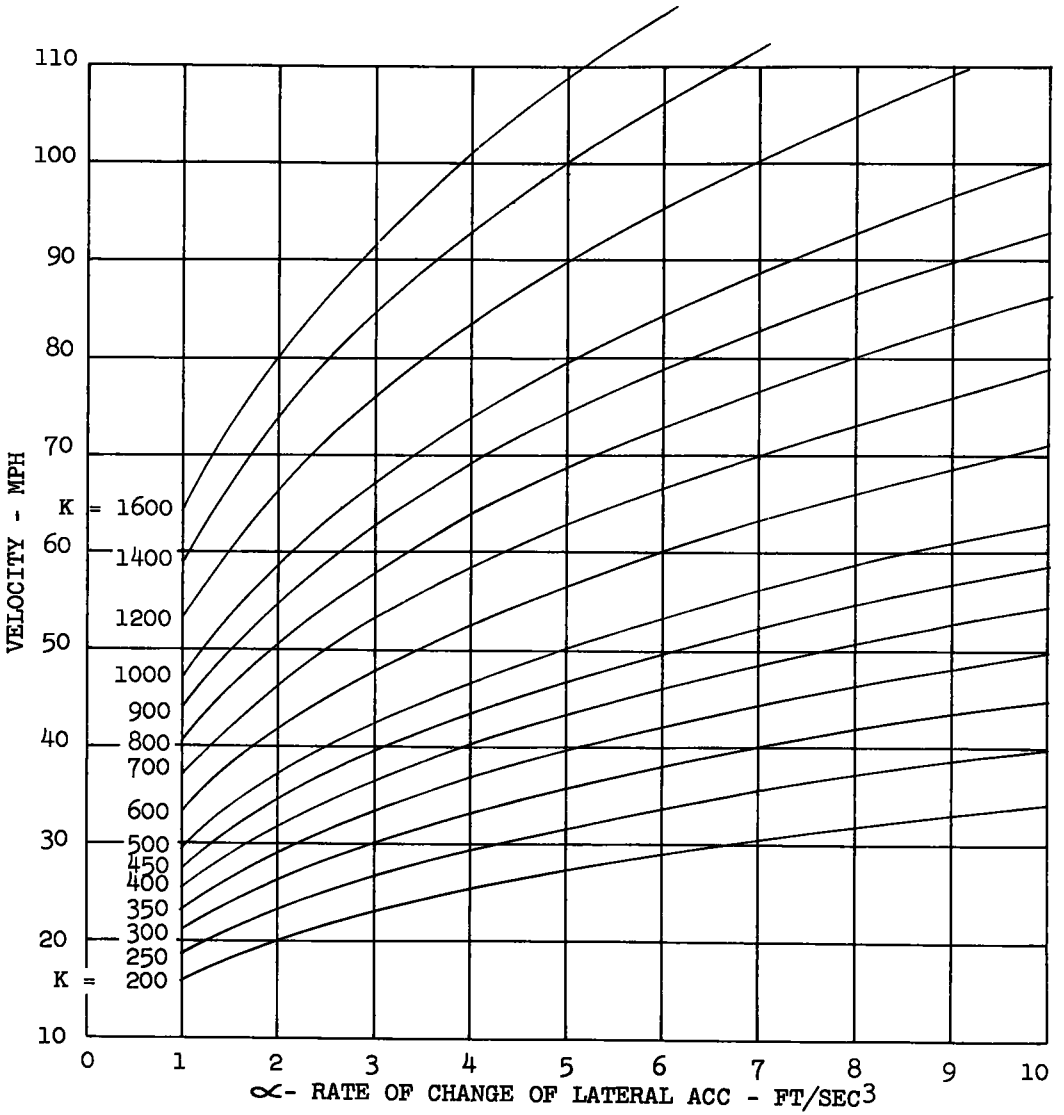


Figure A-3. At constant values of K, V is a function of α .

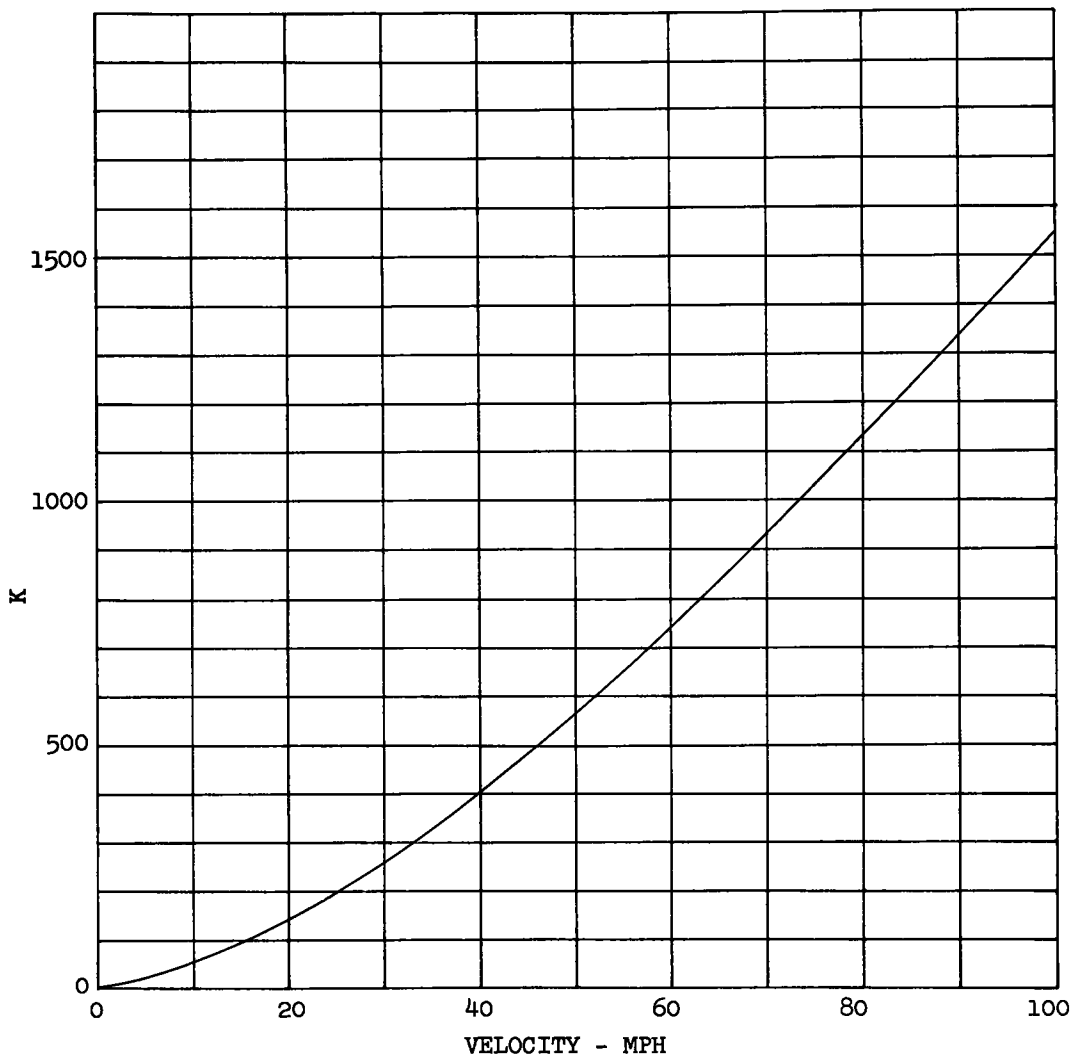


Figure A-4. Values of K at various speeds for $a = 4$.

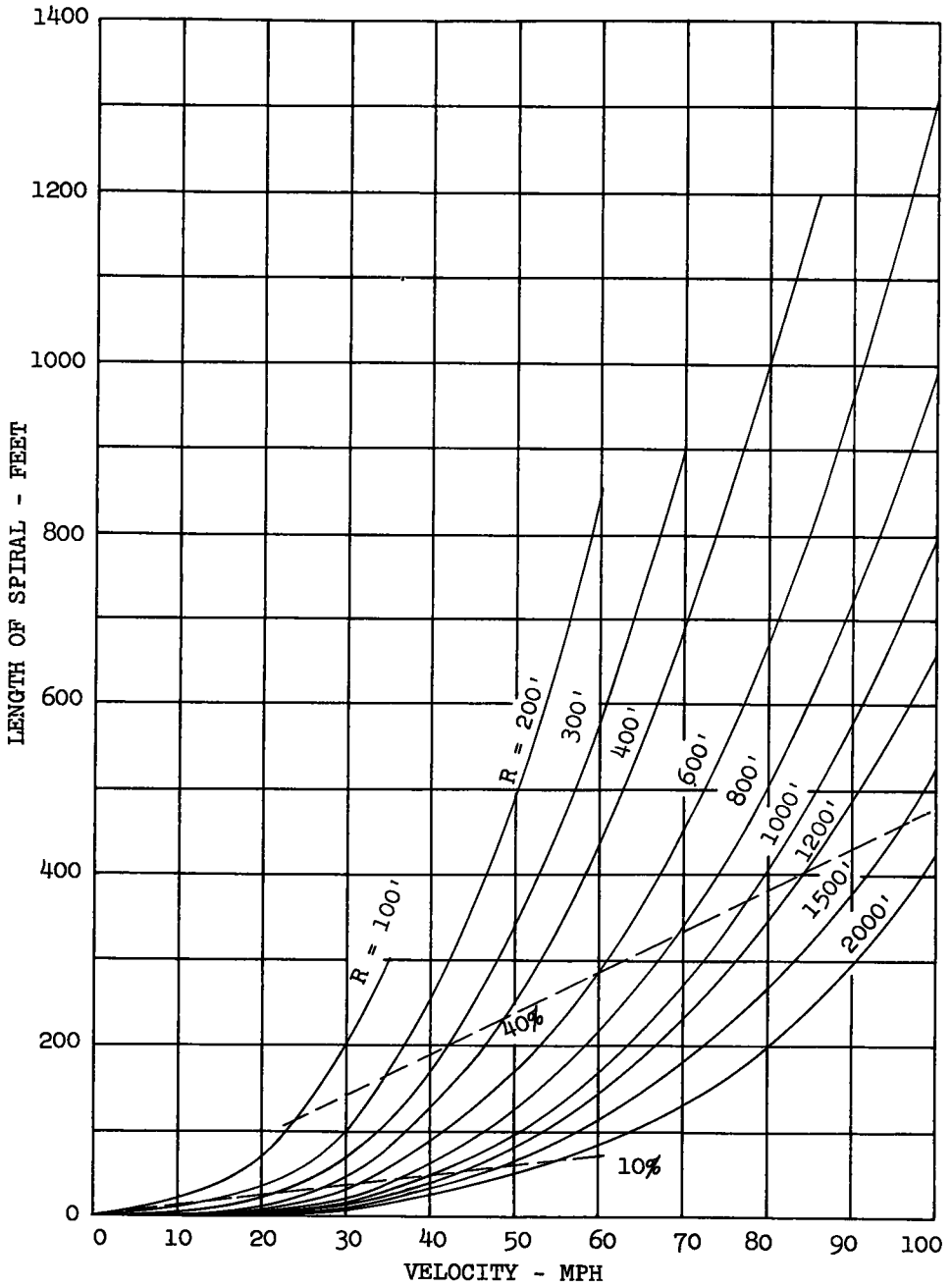


Figure A-5. Length vs speed and radius at $\alpha = 4$.

Appendix B

Design of Vertical Curves for Spiral Easement

The design problem was to develop the vertical curve detail for the horizontal spiral developed in Appendix A. Cornu's Spiral form was used to control the rate of change of vertical reactions and to provide a means of evaluating them conveniently.

The vertical system consisted of (1) a concave curve forming a transition between the horizontal tangent section and the ramp up to the top of the superelevated circular portion and (2) a convex curve between the ramp and the level at the top of the superelevated structure. Experience shows that it is convenient to divide each of these curves into two portions, and to develop Cornu's Spiral for the first portion of each curve; these spirals are then inverted to form an exit symmetrical with the entrance. In this way the radial forces on the car rise uniformly to a maximum of the midpoint of each curve and then decrease at the same uniform rate to the end, as illustrated in Figure 29; thus the vertical curves are designed in four phases, with the first and second and the third and fourth as counterparts. The first two may be identical in detail to the last two, except in direction of curvature, or they may differ; because of sensitivity of the car to decreased road reactions, where room is available, the convex curves should be longer than the concave. The convex curve may follow immediately after the concave, or there may be a length of uniform gradient between, depending upon the length of the horizontal spiral and the height of the superelevation. The vertical transition system must coincide exactly with the horizontal transition.

Following are the detailed computations:

Construct vertical spiral along center of outside lane.

$$\begin{aligned} \text{Vertical climb} &= 12' \times .05 = .6' \\ &6' \times .30 = 1.8' \\ &\quad \quad \quad \underline{\quad} \\ &2.4' \end{aligned}$$

$$L = 27.83 = 1/4 \text{ length of horizontal spiral}$$

Let $y = 1.2'$ for bottom two spirals

$$\begin{aligned} y &\cong d \cong L \sin 2Z \\ 1.2 &= 27.83 \sin 2Z \\ \sin 2Z &= .04312 \\ 2Z &= 2^\circ - 28' - 16.55'' \\ Z &= 1^\circ - 14' - 8.27'' \end{aligned}$$

$$R = \frac{L}{2Z} = \frac{27.83}{.0431316}$$

$$R = 645.23' \qquad V = 31.5 \text{ mph}$$

$$a_v = \frac{v^2}{R} = \frac{2134.44}{645.23} = 3.31 \text{ ft/sec}^2$$

$$u^2 = \frac{2Z}{\pi} = \frac{.0431316}{3.1416} = .013729$$

$$\begin{aligned} u^3 &= .001609 \\ u^5 &= .000022 \\ u^7 &= .0000003 \end{aligned}$$

$u = .11717$
 $K = L/u = 27.83 / .11717 = 237.52$

$C(u) = u - .24674u^5 + \dots$
 $= .11717 - .000005 + \dots$
 $= .11717$

$x = K C(u) = 237.52 (.11717)$
 $= 27.83$

$S(u) = .5236u^3 - .092281u^7 + \dots$
 $= .00084 - \dots$
 $= .00084$

$y = K S(u) = 237.52 (.00084)$
 $= .20$

$\sin 2Z = b / .20$
 $b = .2 (.04312)$
 $= .009$

$\cos 2Z = c / 27.83$
 $c = 27.83 (.99907)$
 $= 27.81$

$\sin 2Z = d / 27.83$
 $d = 27.83 (.04312)$
 $= 1.20$

$\cos 2Z = e / .2$
 $e = .2 (.99907)$
 $= .2$

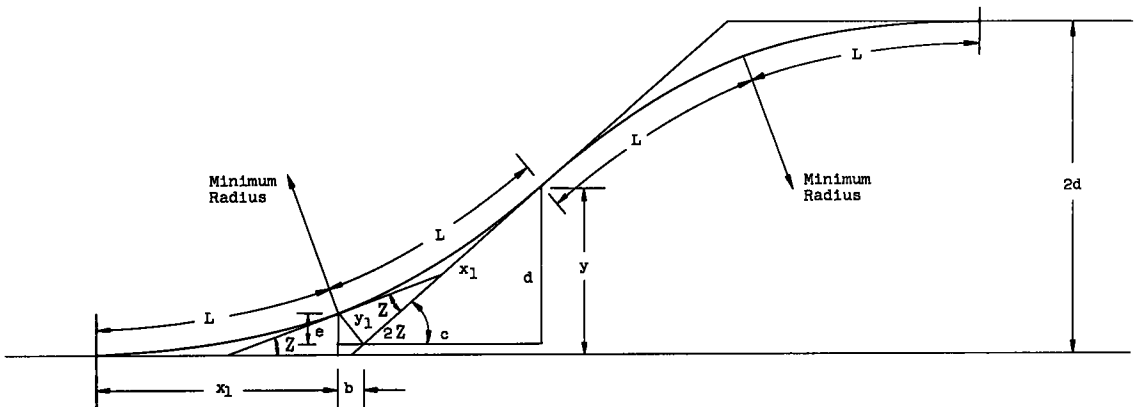
Total x for bottom spirals = 27.83 + .01 + 27.81 = 55.65
 bottom and top spirals 111.30

Total y for bottom spirals = .20 + 1.20 - .20 = 1.20
 bottom and top spirals 2.40

$L = 27.83'$ $u = L/K$
 $K = 237.52$
 $V = 31.5 \text{ mph}$
 Max. $a = 3.31 \text{ ft/sec}^2$ $a = 5.50 \text{ ft/sec}^3$

L	u	u ²	u ³	u ⁵	u ⁷
0	0	0	0	0	0
5	.02105	.0004431	.0000093	-	-
10	.04210	.0017724	.0000746	-	-
15	.06315	.0039879	.0002518	.0000010	-
20	.08420	.0070896	.0005969	.0000042	-
25	.10525	.0110776	.0011859	.0000129	-
27.83	.11717	.0137288	.0016086	.0000221	-

VERTICAL SPIRALS CALCULATED IN APPENDIX B



$$C(u) = u - .24674u^5 + \dots$$

$$S(u) = .5236u^3 - .092281u^7 + \dots$$

L	u	.24674u ⁵	C(u)	.5236u ³ = S(u)	x _v = K C(u)	y _v = K S(u)
0	0	0	0	0	0	0
5	.02105	-	.02105	.00000	5.0	.00
10	.04210	-	.04210	.00004	10.0	.010
15	.06315	-	.06315	.00013	15.0	.031
20	.08420	-	.08420	.00031	20.0	.074
25	.10525	-	.10525	.00061	25.0	.145
27.83	.11717	.00001	.11716	.00084	27.83	.200

Second of Four Phases

$$L = 27.83$$

$$K = 237.52$$

$$V = 31.5 \text{ mph}$$

$$u = L/K$$

$$\sin 2Z = \sin 2^{\circ} - 28' - 16.55'' = .04312$$

$$\cos 2Z = \cos 2^{\circ} - 28' - 16.55'' = .99907$$

Horizontal Length	Length Along Vertical Spiral	u	u ²	u ³	u ⁵
27.83 + 2.17 = 30	25.66	.10803	.0116705	.0012608	.0000147
35	20.66	.08698	.0075655	.0006580	.0000050
40	15.66	.06593	.0043468	.0002866	.0000012
45	10.66	.04488	.0020142	.0000904	.0000002
50	5.66	.02383	.0005679	.0000135	0
55.65	0	0	0	0	0

$$C(u) = u - .24674u^5 + \dots$$

$$S(u) = .5236u^3 - .092281u^7 + \dots$$

Horizontal Spiral Length	Vertical Spiral Length	C(u)	S(u)	x = K C(u)	y = K S(u)
30	25.66	.10803	.00066	25.66	.157
35	20.66	.08698	.00034	20.66	.081
40	15.66	.06593	.00015	15.66	.036
45	10.66	.04488	.00005	10.66	.012
50	5.66	.02383	.00001	5.66	.002
55.65	0	0	0	0	0

$$L \quad b = y \sin 2Z \quad c = x \cos 2Z \quad X = 55.65 - c - b \quad d = x \sin 2Z \quad e = y \cos 2Z \quad Y = 1.20 - d + e$$

30	.007	25.64	30.00	1.106	.157	.251
35	.003	20.64	35.00	.891	.081	.390
40	.002	15.65	40.00	.675	.036	.561
45	.001	10.65	45.00	.460	.012	.752
50	.000	5.65	50.00	.244	.002	.958
55.65	.000	0	55.65	0	0	1.200

Third of Four Phases

L = 27.83 u = L/K sin 2Z = .04312
 K = 237.52 cos 2Z = .99907
 V = 31.5 mph

Horizontal Length	Length Along Vertical Spiral	u	u ²	u ³	u ⁵
55.65 + 4.35 = 60	4.36	.01836	.0003371	.0000062	-
65	9.36	.03941	.0015531	.0000612	-
70	14.36	.06046	.0036554	.0002210	.0000008
75	19.36	.08151	.0066439	.0005415	.0000036
80	24.36	.10256	.0105186	.0010788	.0000113
83.47	27.83	.11717	.0137288	.0016086	.0000221

C(u) = u - .24674u⁵ + ...

S(u) = .5236u³ - .092281u⁷ + ...

Horizontal Spiral Length	Vertical Spiral Length	C(u)	S(u)	x = K C(u)	y = K S(u)
60	4.36	.01836	0	4.36	0
65	9.36	.03941	.00003	9.36	.007
70	14.36	.06046	.00012	14.36	.029
75	19.36	.08151	.00028	19.36	.067
80	24.36	.10256	.00056	24.36	.133
83.47	27.83	.11717	.00084	27.83	.200

L b = y sin 2Z c = x cos 2Z X = 55.65 + b + c d = x sin 2Z e = y cos 2Z Y = 1.20 + d - e

60	.000	4.36	60.01	.188	0	1.388
65	.000	9.35	65.00	.404	.007	1.597
70	.001	14.35	70.00	.619	.029	1.790
75	.003	19.34	74.99	.835	.067	1.968
80	.006	24.34	80.00	1.050	.133	2.117
83.47	.009	27.80	83.46	1.200	.200	2.200

Fourth of Four Phases

L = 27.83 u = L/K C(u) = u - .24674u⁵ + ...
 K = 237.52 S(u) = .5236u³ - .092281u⁷ + ...
 V = 31.5 mph

Horizontal Length	Length Along Vertical Spiral	u = C(u)	u ²	u ³	S(u)	x = K C(u)	y = K S(u)
83.47 + 6.53 = 90	21.30	.08968	.0080425	.0007213	.00038	21.30	.090
95	16.30	.06863	.0047101	.0003233	.00017	16.30	.040
100	11.30	.04757	.0022629	.0001076	.00006	11.30	.014
105	6.30	.02652	.0007033	.0000187	.00001	6.30	.002
111.30	0	0	0	0	0	0	0

Horizontal Length	$X = 111.30 - x$	$Y = 2.40 - y$
90	90.00	2.310
95	95.00	2.360
100	100.00	2.386
105	105.00	2.398
111.30	111.30	2.40

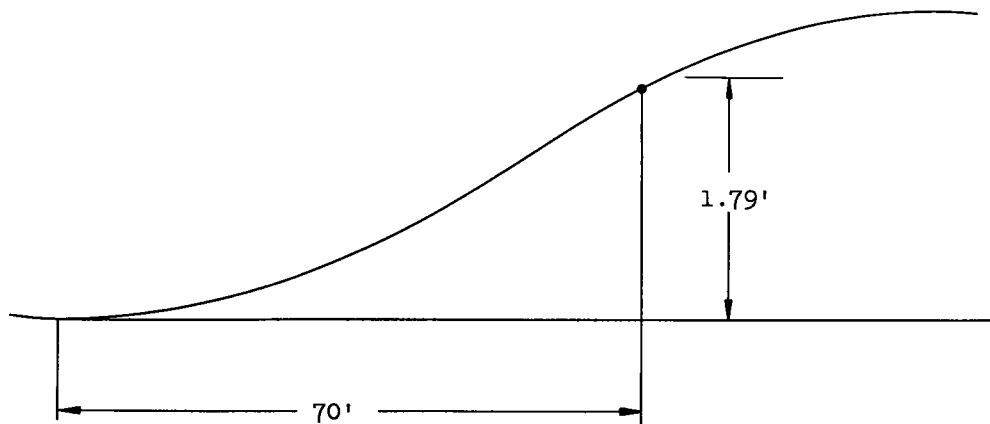
Appendix C

Design of Superelevated Cross-Section in a Spiral Easement

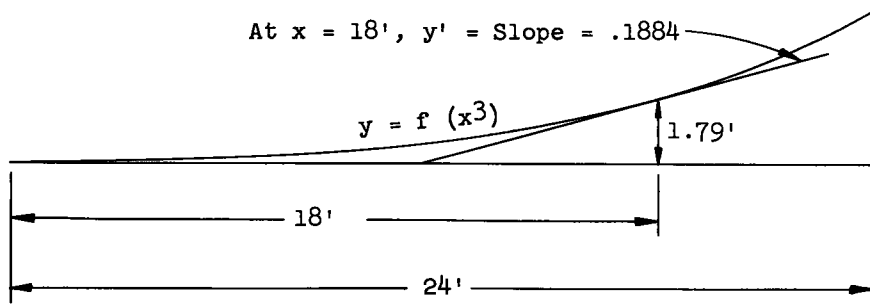
The design problem was to develop the cross-section detail for a station in the spiral easement developed in Appendix A and B. For illustration the point at $L = 70'$ was selected; a cross-section of the form $y = f(x^3)$ was selected arbitrarily.

The conditions were that a cross-section was required having a slope in the middle of the outer traffic lane which would balance the component of centrifugal force in the

VERTICAL CURVE FROM APPENDIX B



CROSS SECTION AT $L = 70'$, COMPUTED IN APPENDIX C



pavement plane as established in Appendix A and an elevation as established in Appendix B. The pavement width was selected arbitrarily as 24 ft; this put the center of the outer lane at 18 ft from the inner edge of the pavement.

It was found that the origin of the cross-section equation meeting the conditions

$$\begin{aligned} @ x_0 = 18, \text{ of: } y_0 &= 1.79 \text{ ft} \\ y' &= .1884 \end{aligned}$$

was 12.66 ft inside the inner edge of the pavement; that is, the equation was:

$$\begin{aligned} y &= m (x_0 + a)^3 - ma^3 \\ a &= 12.66 \\ m &= .000066806 \end{aligned}$$

In the design of cross-sections at successive stations along a spiral easement, the path providing equilibrium at design speed is always at a uniform distance from the pavement edge, but paths providing equilibrium at speeds below the design speed will not be at uniform distances from the edge; this occurs because section elevations are established by requirements of the vertical curve system. This means that a car following a traffic lane in the central or lower portion of the pavement will be subjected to a varying degree of lateral force unbalance; this condition is unavoidable, and it is unimportant at speeds below the design speed.

Computations of this design detail follow:

$$\begin{aligned} y &= m (x_0 + a)^3 - ma^3 \\ y' &= 3m (x_0 + a)^2 \end{aligned}$$

$$@ L = 70 \quad y = 1.79 \quad @ x_0 = 18 \quad \text{From Vertical Spiral}$$

$$y' = .1884 \quad @ x_0 = 18 \quad \text{From Horizontal Spiral}$$

$$\begin{aligned} 1.79 &= m (18+a)^3 - ma^3 \\ .1884 &= 3m (18+a)^2 \end{aligned}$$

$$\frac{1.79}{.1884} = \frac{m (18+a)^3 - ma^3}{3m (18+a)^2}$$

$$9.5011 = \frac{5832 + 972a + 54a^2 + a^3 - a^3}{3 (18+a)^2}$$

$$9.5011 = \frac{18 (324 + 54a + 3a^2)}{3 (324 + 36a + a^2)}$$

$$9.5011 = \frac{18 (108 + 18a + a^2)}{324 + 36a + a^2}$$

$$.5278 (324 + 36a + a^2) = 108 + 18a + a^2$$

$$171.0072 + 19.0008a + .5278a^2 = 108 + 18a + a^2$$

$$.4722a^2 - 1.0008a - 63.0072 = 0$$

$$a = + 1.0008 \pm \sqrt{\frac{1.00160064 + 119.0079936}{.9444}}$$

$$= + 1.0008 \pm \sqrt{\frac{120.0096}{.9444}}$$

$$= + 1.0008 \pm 10.9549$$

$$= \frac{11.9557}{.9444} \text{ or } \frac{-9.9541}{.9444}$$

$$= 12.6596 \text{ or } -10.5401$$

$$a = 12.66$$

$$.1884 = 3m (18 + 12.66)^2$$

$$m = \frac{.0628}{(30.66)^2}$$

$$= \frac{.0628}{940.0356}$$

$$= .000066806$$

$$y_0 = .000066806 (x_0 + 12.66)^3 - .000066806 (12.66)^3$$

$$= .000066806 (x_0 + 12.66)^3 - .136$$

$$y' = .000200418 (x_0 + 12.66)^2$$

$$@ x_0 = 18$$

$$y_0 = 1.925 - .136$$

$$= 1.789$$

$$y' = .1884$$

x_0	y	y'
0	0	.032
2	.074	.043
4	.173	.056
6	.298	.070
8	.453	.086
10	.641	.103
12	.866	.122
14	1.130	.142
16	1.437	.164
18	1.789	.188
20	2.191	.214
22	2.646	.241
24	3.155	.269

Human Sensitivity to Motion as a Design Criterion for Highway Curves

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Human perception thresholds to motions and the various characteristics of motions in six degrees of freedom are presented. Experiments made to establish these thresholds for disturbances of the duration or frequency that might arise from highway geometry — durations in the range of one to several hundred seconds — are described.

Analysis of threshold data shows that on highway curves the detection of direction changes, and comfort levels as well, are determined by changes in acceleration, both angular and translational; and that on roads meeting AASHO design standards, critical rates of change are reached before the accelerations or forces themselves reach perceptible or disturbing levels.

Applications of motion sensitivity criteria in the design of vertical and horizontal highway curves and transitions are developed, with examples from Ford Motor Company's Arizona and Michigan proving grounds. The resulting geometric features of the high-speed road systems differ in several important respects from conventional highway practice and previous automotive test track designs.

● SHORTLY after the first automobiles were built, suspensions were employed, not so much for comfort, in the beginning, as for survival — to keep the passengers from being thrown out or maimed. As cars and road surfaces improved, users became more critical of ride, and standards of judgment changed. The comfortable was sought instead of the tolerable, then luxury in preference to comfort.

Today, ride engineers are a special breed pursuing the diminishing return, using instruments and apparatus to measure motions so slight and differences so subtle that their own physique can no longer register them with sufficient accuracy or objectivity. Quantitative riding comfort and vibration limits have been formulated which are useful design tools.

Highway engineering has evolved along much the same lines. At first it was enough to clear the trees and scrape a path. Then surfaces were smoothed, summits shaved, valleys filled, and alignment eased to speed the passage. Today highway technology has progressed to where standards are defined for gradients, curvature, and other features. With the increased understanding of the dynamic aspects of highway geometry, it is no longer a question of whether transitions, superelevations, vertical curves and easements should be used, but rather what the best forms are and what design criteria can be used.

In designing high-speed test road systems for an automotive proving ground, these questions become especially critical. It is important in test work to be able to operate under stabilized conditions at all possible vehicle speeds. What really is wanted is a straight road, indefinitely long; yet this must be built in a limited space, with curves sharper than any highway designer would consider feasible for speeds up to twice what might ordinarily be considered reasonable and proper. These curves should be designed in such a way that neither the car nor its occupants can tell that the vehicle is in a turn and not on a straight level road. There should be no steering drag, no undue tire wear, no perceptible motions, forces, or changes in force above those which would be experienced on a straight level road.

It is probably natural that automobile engineers faced with a highway design problem of this sort would consider highway geometry problems as an extension of the ride problem. Dips and turns are just big bumps in the road. Traveling across mountains and valleys is just a vibration, although a very large-amplitude low-frequency one. If the suspensions can not smooth such bumps, maybe the bumps themselves, since they are intentional ones, can be shaped to give the ride motions preferred. Possibly comfort

criteria established for smaller highway faults, irregularities and roughness can be extrapolated to govern the character of the slower motions and disturbances designed into the highway.

There was confidence that a surface shape could be developed to produce any desired redirection of a vehicle within any given limits to the velocities, accelerations, or rates of change involved. Analysis of existing high-speed road designs showed several approaches. Most were aimed at neutralizing lateral forces; some were concerned with vertical accelerations and rates, and others considered roll motions as well.

There were some published data showing how particular motion characteristics could be minimized to make best use of a given space. But a superelevated spiral curve involves a complex motion in six degrees of freedom. It is not clear which are the critical motions, or what the relationship between them should be to achieve a balanced design within a given space; not could it be determined how much of this given space to ask for.

A rational determination of curve dimensions and straight-to-curve transition geometry, therefore, seems to start with an evaluation of human sensitivity to all the perceptible characteristics of motions for the six possible degrees of freedom and with the establishment of suitable working limits to the magnitude of these characteristics.

Perception Threshold Data

Two different parameters for measuring body sensitivity are commonly used. One employs a scale with different degrees of comfort from luxury through varying stages of disturbance to pain. The other ranges from perception through various degrees of injury to death. It should be noted that the first, or psychological scale, calls for an opinion or conclusion on the part of the witness; the second, or physiological evaluation, which is preferred, does not.

Engineers prefer to think of human sensory mechanisms as a bunch of instruments — accelerometers, both angular and linear, in the inner ear; some force, pressure, or load cells in the skin; and some stress and strain gages in the muscles. Since people come in only two models, it was felt that better consistency would be found in determining the limiting sensitivity of their instruments (that is, the point at which awareness of motion first occurred) than in establishing some vague value of exposure which, in the group opinion, was judged comfortable. It also was decided to ignore visual stimuli, since it did not appear feasible on a highway to eliminate the impression of motion gained by watching a changing landscape.

The first search for perception threshold data was disconcerting. The human instruments did not have a very flat frequency response. There are apparently many distinct types of neural receptors, each tuned to its own range and variety of stimulation. For example, there are seven specific kinds of nerve endings in the skin alone. These receptors also can adapt to certain levels of stimuli under continuous non-changing excitation, and under these conditions cease to give their characteristic sensation.

In this respect, they are much like some test instruments, which will measure strain or acceleration under dynamic conditions, but will not respond at all to steady or low-frequency signals. A force change on the body equivalent to forty G's, which crash people say will produce "irreversible injury" (i. e., death) if sustained for a few milliseconds, can be endured with no especial harm if developed gradually over a long time, for instance 20 years, as any 5-lb baby who is now a 200-lb man can testify. It was also apparent that the skin and kinesthetic or muscle sensations would be very much influenced by the manner of support of the body. Seated, the unit pressures and muscular tensions are less, and forces and accelerations can

Degree of Freedom	Motion Characteristic			
	Position x	Velocity $\frac{dx}{dt}$	Acceleration $\frac{d^2x}{dt^2}$	Rate of Change of Acceleration $\frac{d^3x}{dt^3}$
Translational				
1) Longitudinal	Not Sensed	Not Sensed	$\pm 1.0 \text{ ft/sec}^2$	$\pm 0.5 \text{ ft/sec}^3$
2) Lateral	Not Sensed	Not Sensed	$\pm 0.6 \text{ ft/sec}^2$	$\pm 0.3 \text{ ft/sec}^3$
3) Vertical	Not Sensed	Not Sensed	$\pm 4.0 \text{ ft/sec}^2$	$\pm 0.8 \text{ ft/sec}^3$
Rotational				
4) Yaw	Not Sensed	$\pm 5 \text{ deg/sec}$	$\pm 2 \text{ deg/sec}^2$	$\pm 1 \text{ deg/sec}^3$
5) Roll	$\pm (1.1 \text{ deg})^a$	$\pm 8 \text{ deg/sec}$	$\pm 4 \text{ deg/sec}^2$	$\pm 2 \text{ deg/sec}^3$
6) Pitch	$\pm (1.9 \text{ deg})^a$	$\pm 12 \text{ deg/sec}$	$\pm 6 \text{ deg/sec}^2$	$\pm 3 \text{ deg/sec}^3$

^a Roll and Pitch position sensitivity due to lateral and longitudinal components of gravity force. Not sensed when balanced by lateral or longitudinal accelerations.

Conditions of Measurement:

- 1) Subjects seated in standard automobile seats
- 2) Subjects blindfolded, kinesthetic and labyrinthine cues only

Figure 1. Human perception thresholds for possible modes of motion.

be withstood which could not be taken standing up.

The thresholds (Figure 1) began with a correlation of values obtained by other investigators, although these appeared inadequate in many respects. Ride motion analyses had been confined generally to frequencies above one cycle per second, and extrapolations to slower motions could be misleading, as there are infinitely more motions differing by a given time or length interval below one cycle per second than there are above this frequency.

Therefore, the data would have to be checked for motions of the time and amplitude range encountered in highway geometry disturbances. The subjects should be seated in an automobile seat, preferably in an automobile. Particular attention would have to be given to rates of change of acceleration or forces, as the first perception of these slow motions would probably come from detection of a change, even though the level of the force stimulation itself might be unrecognized.

The procedure for checking the tentative table of thresholds was quite simple. Briefly, it amounted to simply driving over as many different examples of highway curves a and transitions as were available, gradually increasing the speeds until a blindfolded passenger perceived that he was changing direction. From the known geometry of these road features, it was possible to calculate the velocities, accelerations, and rates of change of acceleration (hereafter referred to as "jerk," for want of a better name) which were present at the critical speed, for each of the six modes of motion. The table was revised whenever an imperceptible maneuver showed a particular characteristic with a higher value than that already tabulated. It was reasoned that if no characteristic on a new design exceeded the values reached without perception on other existing road features, the new design likewise should be imperceptible.

Among the road facilities used for this purpose were some superelevated transition curves at the ends of the straightaway at Dearborn; some flat curves of varying radii; a 350-ft radius parabolic section curve, or "bowl"; some crowned roads, or side slopes; all with precisely known geometry. The opportunity also has been utilized of making limited observations on some competitors' tracks, and on the larger radius and longer transitions of Ford's recently completed Arizona Proving Ground track.

In most cases it was possible to obtain further road checks of threshold limits for individual motion characteristics, uncomplicated by cues from motions in other degrees of freedom, either by eliminating these other motions, balancing them, or keeping them below the appropriate threshold. Thus, longitudinal acceleration was checked on a level straightaway. With the perceptible acceleration determined, various rates of change were tried by uniformly increasing and decreasing acceleration without allowing the maximum to approach the value previously established as a force limit. Likewise, lateral acceleration limits were found by driving slowly in a large-radius flat turn until the subjects felt the pull of centrifugal force. The vertical threshold was confirmed by very slowly gathering speed in a large-radius banked bowl, allowing the car to find its equilibrium path so that lateral forces would be balanced, until the passenger noticed that he felt heavier.

At slow speeds and large radii, centrifugal forces are relatively large and angular motions relatively negligible. By shifting to the skid pad, and operating very slowly in small-radius turns, yaw motions were investigated with centrifugal forces reduced to a negligible value. In addition to road tests in automobiles, some checks on angular motions were made in the laboratory with an automobile seat mounted on a teeter-totter for roll and pitch, and on a rotary table for turning movements.

Some caution is in order in accepting the values given in Figure 1. To begin with, the number of subjects used in the check tests unfortunately was small. A maximum of twelve people was used in estimating vertical, roll, and yaw thresholds which were of most concern in the special design problem. Even with so few subjects, the scatter was wide, with thresholds reported by different individuals varying by factors as large as two to one. Even on the smoothest roadways, extraneous motions and vibrations of the car produced a masking effect, somewhat like trying to feel the rise and fall of the tide in a rowboat on a choppy sea. A complex motion, with all components below the threshold values, might also produce a resultant larger than any of the limits shown, and so might be sensed. However, the manner of support in a car seat makes the

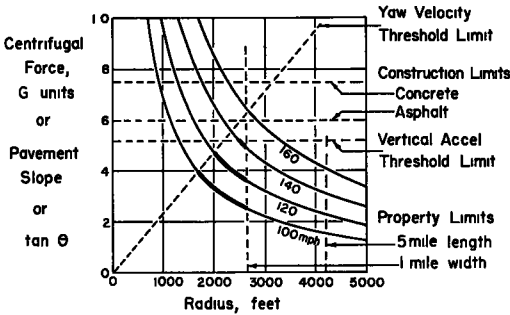


Figure 2. Limitations on turn radius and bank for various design speeds.

isolating particular characteristics for measurement, will be interested in further developing a table of perception thresholds.

APPLICATION TO TRACK DESIGN

The manner of applying motion threshold data to highway design can be illustrated by the design of Ford Motor Company's high-speed test track at the new proving ground in Romeo, Michigan. It was known, from the type of tests to be performed, that a track five miles long was wanted. It also had been determined that it should be suitable for speeds up to 140 mph without side force on the tires, and that it should have an oval form with two straight legs as long as possible. The first step was to determine the approximate shape, and the radius of the turns.

Figure 2 shows the forces acting on a car at equilibrium speed in a banked turn. The equilibrium, or neutral speed for a banked turn, of course, is that speed where centrifugal force and the vehicle weight combine to produce a resultant force which is perpendicular to the pavement. Wheel reactions on both sides of the car are equal, and likewise normal to the road surface.

The passenger in such a situation perceives the resultant force as his apparent weight, and has no tendency to slide laterally on the seat. There are no transverse forces between the tire and the pavement. The difference between his true weight and his apparent weight appears to the car occupant as a vertical acceleration. If this is to be imperceptible, it must be kept below the threshold value for vertical acceleration of 4 ft per sec per sec; that is, the apparent weight must not be more than about 12½ per cent larger than the acceleration of gravity. This limit is reached with an angle of about 28 degrees, or a pavement slope equal to the tangent of this angle, or 0.52.

Some of the restrictions on the choice of turn radius are shown in Figure 3, which is a plot of the pavement slope required for equilibrium as a function of curve radius for four different speeds. The slope limit of 0.52 required to keep vertical acceleration below perception is the governing limit on slope. The construction limits, or maximum practical slopes which can be formed with asphalt or concrete, are not approached. A yaw velocity threshold of 5 degrees per second also limits the choice of radius. This angular velocity is reached at 100 mph in a 1,700-ft radius, or at 160 mph in a 2,700-ft radius. Everything to the left of a diagonal line defining the yaw velocity limit involves a turning velocity which will be perceptible. The upper limit on radius is one of 4,200 ft, which is the largest that can be gotten into a 5-mile circumference. A lower limit was fixed in this case at 2,600 ft in order to keep within certain geographical limits. For a 100-mph track, the radius could be between 1,700 and 2,600 ft. For the 140-mph design speed, there is the choice of any radius

sensitivity to vertical motion and pitch substantially less than the sensitivity to lateral or roll motions. There is enough difference between thresholds in the different modes that any resultant will not be significantly larger than the limits shown.

Despite some shortcomings, it is felt that these data afford safe design criteria for highway curves, and that further checks will raise rather than lower these limits. It is hoped that others with more subjects, more time, more facilities, and more ingenuity in devising ways of

isolating particular characteristics for measurement, will be interested in further developing a table of perception thresholds.

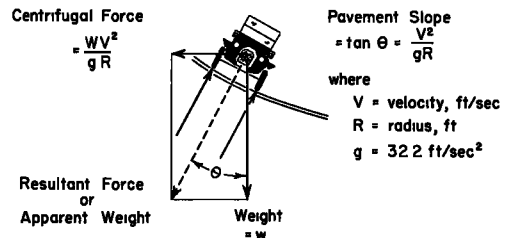


Figure 3. Forces on vehicle in superelevated curve at equilibrium or neutral speed.

between 2,500 and 2,600 ft. A 160-mph track with imperceptible curves can not be built in the space available. A radius of 2,500 ft was chosen for the 140-mph tracks at Romeo and Arizona.

TRANSITION SPIRAL DESIGN

The next step was to determine the shape of the transition spiral to connect the straight and circular portions of the 5-mile oval. Several test roads have been built recently, including a large high-speed automotive test track similar to the one planned, which have used a mathematical form known as Cornu's spiral as a basis for laying out the transition curves.

The pertinent property of the Cornu spiral is that the radius varies inversely as the distance along the spiral. The significance of this property in a highway transition is that the lateral acceleration, or centrifugal force, will increase at a constant uniform rate for a vehicle traveling along the spiral path at constant speed. The lateral "jerk" will be at a minimum.

This is a logical form for railroad and highway curves, and is the basis for several transition formulas in common use. Highway traffic and railroad trains will seldom travel at the exact neutral speed of a curve, and such design will minimize the lateral movements to which riders are especially sensitive. A train cannot choose different track superelevations appropriate to its speed. Highways, as long as they must carry two-way traffic and make both left and right turns without low- and high-speed vehicles changing lanes, are restricted to straight sections with constant superelevation slope.

On automotive test tracks, however, curved cross-sections are usually used, so that there are an infinite variety of superelevations for any speed up to the design maximum. In the transitions, the cross-sections are matched to the instantaneous radius at each station. Regardless of the horizontal shape the transition assumes, the vehicle will find an equilibrium path where lateral forces are perfectly balanced. In view of this, it would appear that variation of lateral force and even the amount of lateral force is irrelevant, since this is the one thing expected to be completely neutralized by the test track superelevation. It will be imperceptible to the vehicle and its occupants.

What cannot be balanced by superelevations, and what may, therefore, become perceptible, are the roll acceleration and roll jerk about a longitudinal axis of the vehicle as it passes from the level attitude on the straightaway to the banked attitude on the curve. These were the critical factors which limited the speed for imperceptible operation on every other track studied — except one, which avoided the problem because it was built as a circle without transitions. In the transition design for the new test tracks, therefore, the Cornu spiral was rejected and equations to control the roll motions were developed instead.

The method of developing the transition equation is shown in Figure 4. Curves representing the roll characteristics are drawn such that the acceleration increases at a uniform, minimum rate for one-fourth of the distance, decreases at the same uniform rate to a negative peak three-fourths of the way through, and finally increases again at the same rate to zero value. Velocity is found by integrating the roll acceleration curve, and finally, roll angles for various stations along the transition are obtained by integrating the velocity curve. The final bank angle of the car in the circular curve is known, from the design speed and curve radius previously selected. Peak values (threshold limits) are also established for the jerk, acceleration and velocity characteristics. With this information, it is possible to evaluate the integration constants, solve for a length for the transition which will keep the characteristics within the desired limits, and develop equations giving the best roll angle for the vehicle at any distance along the curve. Derivation and solution of these equations for the Ford track transition at Romeo are given in the appendix.

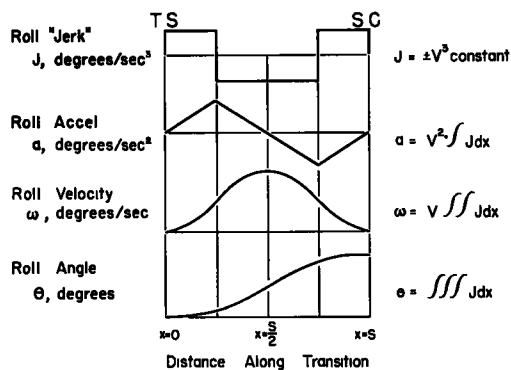


Figure 4. Roll characteristics of transition designed for minimum roll jerk.

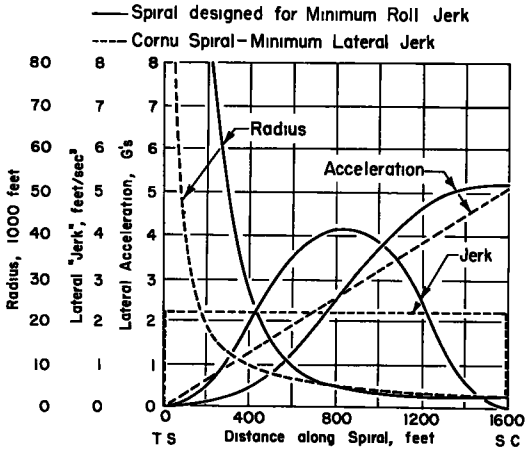


Figure 5. Lateral characteristics of transition curves.

determined by minimizing roll motions, at various distances along the curve, together with the lateral acceleration and jerk values which would occur on a flat curve of this shape. For comparison, a Cornu spiral is also shown. It should be noted that although the Cornu spiral shows a minimum jerk value, lateral acceleration and jerk are completely balanced by superelevation in both cases, and will not be perceptible. Roll characteristics of the Cornu spiral are less desirable, for the roll angle is nearly a linear function of distance along the spiral, with the result that roll velocity is nearly constant, but roll acceleration and jerk are theoretically infinite at the beginning and end of the curve.

In practice, roll velocity is developed in the space of one station, so that accelerations are finite, but very high, and can be controlled only by keeping the velocity small, and making the transition excessively long.

CROSS-SECTION SELECTION

The final design step in developing the transition is to calculate cross-sections for each station. An equation is first written for the cross-section in the curve, and the same equation used for all stations in the transition, but with the constants changed to give the proper slope at design path as determined from the required roll angle. In this manner, the slope for each lower speed lane or path will be accurately proportioned

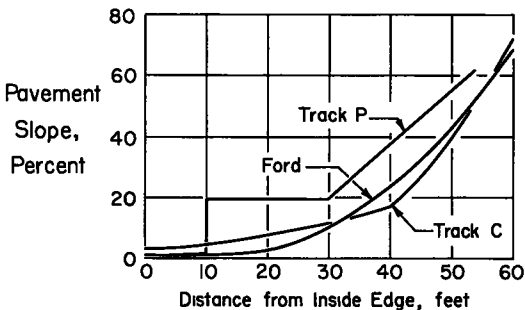


Figure 7. Pavement slope-width relationship of three high-speed track curves.

to the angles of the design path, and the motion characteristics for lower speed vehicles will show the same form, but with diminished values. These calculations again are simple but tedious, and are given in sample form in the appendix.

The choice of cross-section depends largely on anticipated usage and the speed distribution desired across the width of the track. Cross-sections of the curves of three automotive tracks are shown in Figure 6. The actual section reveals little except that Track P required considerably more earthwork than the other two.

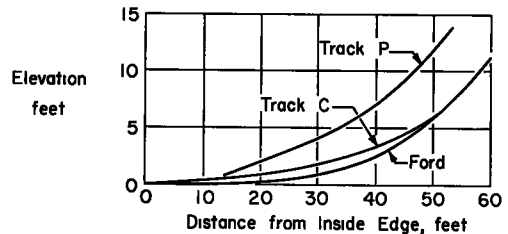


Figure 6. Cross-section comparison of three high-speed track curves.

A plot showing the slopes of these cross-sections (Figure 7) is more revealing. Track P has a greater slope over most of its width. Going one step further, and plotting the equilibrium speeds corresponding to these slopes, gives the most informative comparison (Figure 8). If the inner lane, which is nearly flat in all cases, is overlooked, it is noted that Track P was evidently designed for a preponderance of high-speed traffic, although its maximum design speed happens to be lowest because of the shorter radius. The center lanes are a straight slope, with a constant neutral speed, and the outer lanes are a quadratic form. Track C shows two straight lines, indicating that two cubic equations were used, each giving a uniform velocity distribution across part of the track. Two-thirds of the width have been devoted to low-speed operation, and a relatively narrow portion at the outer edge to very high-speed use. The Ford track uses a single cubic cross-section with a uniform velocity distribution, except for the inner lane, which must have a minimum 1 percent slope for drainage. This section was chosen because about equal volumes of extremely high-speed and low-speed (truck) traffic were anticipated on this track. With narrower pavements, higher power equations for the cross-section are commonly used. Thus, the Arizona track has a quartic, or fourth power cross-section. The speed distribution of the quartic form approximates that of Track C, in that while most of the track width is available for normal speed driving, a considerable margin for higher speeds is obtained within a limited width near the outside edge.

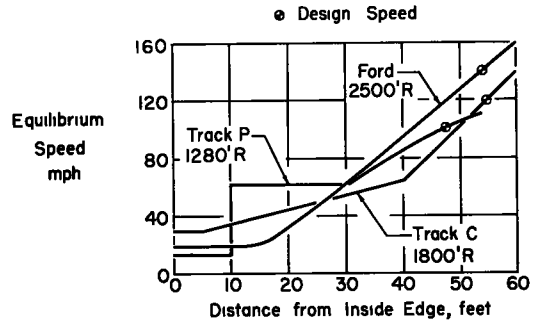


Figure 8. Equilibrium speed-width relationship of three high-speed track curves.

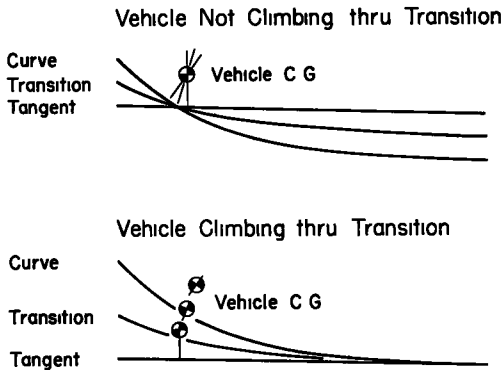


Figure 9. Possible vertical motion in transition.

Once a cross-section is decided upon, one further problem arises (see Figure 9). Should the vehicle center of gravity be kept at a constant height, or should it rise through the transition? Should the pavement be warped about the design path, the inner edge, or some other point? It is apparent that in either case the center of gravity can be kept level only for one particular speed or path; on all other paths the vehicle will either climb or fall through the transition. The design of Ford's Michigan track has the vehicle climb through the transition. The reason for this is shown in Figure 10.

The force which the driver perceives as a vertical acceleration is made up of two components: one is the difference between his true weight and the resultant of his weight with centrifugal force; the other component derives from the climb of the vehicle through the transition. The component from centrifugal force starts out slowly, increases rapidly near the midpoint, then levels off. The rate of increase is above the perception threshold for vertical jerk at the midpoint. To

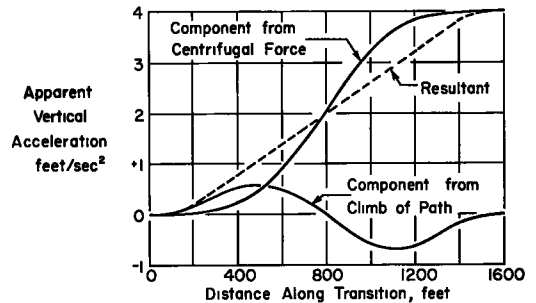


Figure 10. Apparent vertical acceleration (components perpendicular to pavement). Super-elevated spiral transition curve designed for minimum roll jerk.

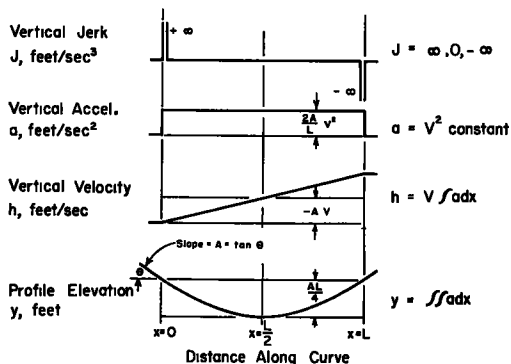


Figure 11. Characteristics of sag vertical curve designed for minimum acceleration.

As a final check on the transition design, all remaining motion characteristics were calculated. Longitudinal velocity was assumed constant, with no longitudinal acceleration or jerk. Pitch motions are extremely slight, and well below threshold values. In practice, they may be compensated by the slight longitudinal changes occasioned by the vehicle decelerating as it climbs through the transition. Yaw acceleration and jerk are likewise extremely small, and well below threshold.

VERTICAL CURVE DESIGN

Expressways which will steer the vehicles and keep them in the proper speed lane on curves, by requiring constant effort to drive in any other lane, are theoretically possible, but quite unlikely unless roads can be built for one-way traffic, and with nothing but right turns. An immediate application for sensitivity data does exist, however, in the design of vertical curves such as sags, crests, dips, and rises.

Present practice on vertical curves is to use a quadratic, or parabolic form, with characteristics as shown in Figure 11. With this form of curve, vertical acceleration is a constant minimum amount, but vertical jerk will be infinite at the beginning and end of the curve. While it may be possible to make such a curve long enough that the transition will be comfortable, a more efficient design would use a cubic form so that both vertical acceleration and jerk could be held to some desired finite value.

Figure 12 shows the vertical characteristics for a sag curve designed to minimize jerk. These equations can be solved to give the curve profile, the vertical velocity, the vertical acceleration, and the vertical jerk for any station along the curve in terms of the initial slope, the design speed, and the curve length.

The expression for the jerk can be equated to the jerk threshold value of 0.8 ft per sec per sec to find the minimum length of sag which will be imperceptible for any particular design speed and slope condition. With the length known, elevations of the curve profile can be found. The complete derivation and a sample calculation for a sag curve is given in the appendix.

By equating both jerk and acceleration to their respective threshold values of 0.8 ft per sec per sec per sec and 4 ft per sec per sec, and solving the equations simultaneously with the vertical velocity expression, it is found that acceleration will not become the governing value in establishing the minimum sag length until vertical velocities exceed 10 ft per sec. The

obtain a minimum jerk requires a uniform linear increase of apparent weight. This can be accomplished, and the vertical jerk brought within threshold limits, by climbing through the transition, giving the vehicle an additional vertical acceleration during the first half, decelerating it vertically during the last half of the spiral.

Integrating the required additional vertical component twice gives the required climb of the design path to give the desired minimum rate of change. It also is found that a cubic cross-section, warped about the inside edge, will give the correct rise to the design path, and for all other speeds as well.

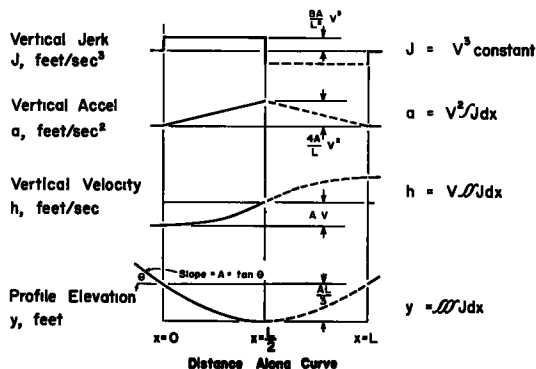


Figure 12. Characteristics of sag vertical curve designed for minimum jerk.

product of design speed and slope equals the vertical velocity. It is evident that design speeds much higher than are normally used on highways, or steeper slopes than are suggested in the AASHO minimum design standards, will be involved before acceleration rather than jerk governs the design; something like 70 mph on a 10 percent slope, for example.

The same shape curve as described in Figure 12 for a sag curve can be used, inverted, on crests, or modified for highway disturbances such as are common in the southwest for crossing dry washes. The same general dimensions of length and depth can be maintained as are now used, and the dip made imperceptible to the motorist. A sample calculation for shape which would convert an actual dip on a U. S. highway near Ford's Arizona proving ground from an especially severe jolt to an imperceptible wave is included in the appendix.

ACKNOWLEDGMENTS

The author wishes to thank personnel of General Motors, Chrysler, and Packard proving grounds for allowing him to visit and drive their automotive test tracks, and for supplying him with much useful design data on their facilities.

REFERENCES

1. H. M. Jacklin and G. J. Liddell, "Riding Comfort Analysis," Purdue University Research Bulletin No. 44, May, 1933.
2. H. M. Jacklin, "Human Reactions to Vibration," S. A. E. Journal, October, 1936.
3. R. N. Janeway, "Passenger Vibration Limits," S. A. E. Journal, August, 1948. See also publications of the S. A. E. Riding Comfort Research Committee.
4. W. E. Lay and L. C. Fisher, "Riding Comfort and Cushions," S. A. E. Transactions, Vol. 47, No. 5, November, 1940.
5. Tufts College Institute for Applied Experimental Psychology, "Handbook of Human Engineering Data for Design Engineers," Tech Report No. SDC 199-1-2, August, 1951.
6. W. E. Woodson, "Human Engineering Guide for Equipment Designers," Univ of Calif Press, 1954.
7. R. A. McFarland and Alfred L. Moseley, "Human Factors in Highway Transport Safety," Harvard School of Public Health, 1954.
8. R. A. McFarland, "Human Factors in Air Transportation," McGraw-Hill, 1953.
9. Joseph Barnett, "Transition Curves for Highways," Public Roads Administration, Washington, 1940.
10. F. G. Royal-Dawson, "Vertical Curves for Roads," E. and F. N. Spon, Limited, London, 1946.
11. K. A. Stonex, "Automotive Test Track Design," Technical Paper, American Society of Civil Engineers.
12. T. D. Kosier, "Transition Curves for Speedways and Highways," Technical Paper, Society of Automotive Engineers.
13. American Association of State Highway Officials, "Policies on Geometric Highway Design."

Appendix A

Transition Curve Design

Design of a Horizontal Transition Spiral

The problem is to develop a superelevated spiral transition from a straight to a curved path so that all angular and translational motions involved will be imperceptible.

Because of the extent of calculations involved, the problem will be reduced to defining the geometric shape of a particular transition used at Ford Motor Company's Romeo, Michigan, proving ground. The design speed and road width have been determined from projected vehicle performance, test requirements, traffic density, and economic considerations beyond the scope of this paper.

To avoid cumulative errors and because the business machines don't care, calculations have in many cases been carried out to a precision which is neither significant nor warranted, the rounding off being left to the man with the trowel.

Design conditions and constants are as follows:

Design speed: 140 mph (205.33 ft per sec) at center of outside lane.

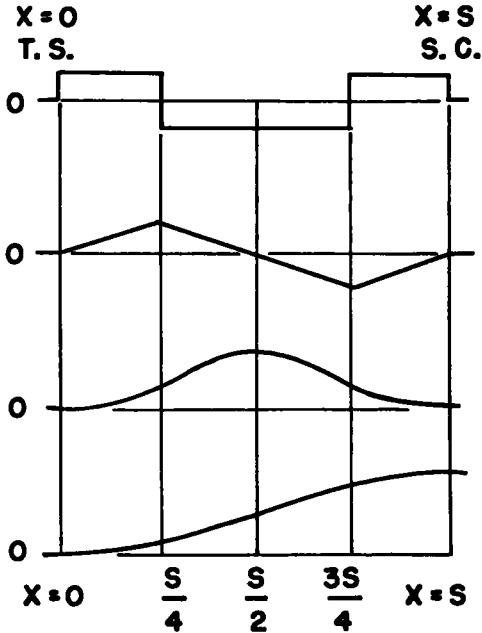
Final curve radius: 2,500 ft at design path, to keep below yaw and vertical (apparent weight increase) perception thresholds.

Track width: 60 ft.

Acceleration of gravity: 32.162 ft per sec per sec at Romeo.

Station interval: 25 ft, to avoid natural frequencies of tires and suspensions.

Step 1 — Diagram Roll Motion Characteristics



$$\text{Jerk} = J = \pm C \text{ deg/ft}^3 \\ = \pm CV^3 \text{ deg/sec}^3$$

$$\text{Acceleration} = A = \int J dx \text{ deg/ft}^2 \\ = V^2 \int J dx \text{ deg/sec}^2$$

$$\text{Roll velocity} = H = \iint J dx \text{ deg/ft} \\ = V \iint J dx \text{ deg/sec}$$

$$\text{Roll angle} = \phi = \iiint J dx \text{ degrees}$$

Where

V = design speed, ft/sec; and
S = spiral length, ft.

Step 2 — Develop Equations of Motion by Integration

From $X = 0$ to $X = \frac{S}{4}$:

$$\begin{aligned} J &= C \\ A &= CX \\ H &= \frac{CX^2}{2} \\ \phi &= \frac{CX^3}{6} \end{aligned} \quad (1)$$

From $X = \frac{S}{4}$ to $X = \frac{3S}{4}$:

$$\begin{aligned} J &= -C \\ A &= -CX + \frac{CS}{2} \\ H &= \frac{-CX^2}{2} + \frac{CSX}{2} - \frac{CS^2}{16} \\ \phi &= \frac{-CX^3}{6} + \frac{CSX^2}{4} - \frac{CS^2X}{16} + \frac{CS^3}{192} \end{aligned} \quad (2)$$

From $X = \frac{3S}{4}$ to $X = S$:

$$\begin{aligned} J &= +C \\ A &= CX - CS \\ H &= \frac{CX^2}{2} - CSX + \frac{CS^2}{2} \\ \phi &= \frac{CX^3}{6} - \frac{CSX^2}{2} + \frac{CS^2X}{2} + \frac{59CS^2}{96} \end{aligned} \quad (3)$$

Step 3 — Evaluate Constant C in Terms of Spiral Length, S

$$\text{When } X = \frac{S}{2}, \quad \phi = \frac{\Delta \theta}{2},$$

where $\Delta \theta$ is the total change in roll angle from straightaway to the curve. In the curve,

$$\tan \theta = \frac{V^2}{gR} = \frac{(205.333)^2}{(32.162)(2,500)} = 0.52433522$$

$$\theta = 27^\circ 40' 10.58''$$

In the straight,

$$\tan \theta = 0.01 \text{ (drainage slope)}$$

$$\theta = 0^\circ 34' 22.58''$$

$$\Delta \theta = 27^\circ 05' 48'' = 97,548''$$

$$\phi = 48,774'' = \frac{48,774}{3,600} \text{ degrees}$$

$$\text{at } X = \frac{S}{2}, \quad \phi = C \left(-\frac{S^3}{48} + \frac{S^3}{16} - \frac{S^3}{32} + \frac{S^3}{192} \right) = \frac{48,774}{3,600}$$

$$\therefore C = \frac{64}{S^3} \times \left(\frac{48,774}{3,600} \right) \text{ deg/ft}^3$$

$$= \frac{64}{S^3} \left(\frac{48,774}{3,600} \right) V^3 \text{ deg/sec}^3$$

Step 4 – Determine Spiral Length

$$\text{Maximum jerk} = C = \frac{64}{S^3} \left(\frac{48,774}{3,600} \right) \text{ sec/ft}^3$$

$$= \frac{64}{S^3} \left(\frac{48,774}{3,600} \right) V^3 \text{ deg/sec}^3$$

This value should be equal to or less than the 2-deg/sec³ threshold value. Therefore,

$$2 = \frac{64}{S^3} \left(\frac{48,774}{3,600} \right) (205.333)^3$$

$$S = 1,560 \text{ ft; use } 1,600 \text{ ft for spiral length.}$$

$$\text{Maximum acceleration, at } X = \frac{S}{4} = \frac{CS}{4} V^2 \text{ deg/sec}^2$$

$$= \frac{64}{(1,600)^3} \left(\frac{48,774}{3,600} \right) \frac{1,600}{4} (205.333)^2$$

$$= 3.57 \text{ deg/sec}^2$$

This is less than the 4.0-deg/sec² acceleration threshold, and is satisfactory.

$$\text{Maximum velocity, at } S = \frac{S}{2} = C \left(\frac{-S^2}{8} + \frac{S^2}{4} - \frac{S^2}{16} \right) V \frac{\text{deg}}{\text{sec}}$$

$$= \frac{64}{(1,600)^3} \left(\frac{48,774}{3,600} \right) \frac{(1,600)^2}{16} (205.333)$$

$$= 6.96 \text{ deg/sec}$$

This is less than the 8.0-deg/sec velocity threshold, and is satisfactory. Therefore, the spiral length of 1,600 ft will keep roll motions below the perception threshold.

Step 5 – Substitute values for S, V, and C in Equations 1, 2, and 3 to find roll angle, ϕ , as a function of distance, X.

$$\text{From } X = 0 \text{ to } X = 400 \text{ ft,}$$

$$\phi = 0.000127015625X^3 \text{ seconds}$$

$$\text{From } X = 400 \text{ ft to } X = 1,200 \text{ ft,}$$

$$\phi = 0.3048375 \left(\frac{160,000}{3} - 400X + X^2 - \frac{X^3}{2,400} \right) \text{ seconds}$$

$$\text{From } X = 1,200 \text{ ft to } X = 1,600 \text{ ft,}$$

$$= 0.3048375 \left(-\frac{4,160,000}{3} + 3,200X - 2X^2 + \frac{X^3}{2,400} \right) \text{ seconds.}$$

Transition Spiral Calculations

S = 1600'
R = 2500'

ΔS = 25'
V = 140 mph

Spiral Length to Station	Roll Angle ϕ	Angle Design Path θ	Slope Design Path $\tan \theta$	$\sin \frac{\beta}{2}$	Included Angle β	Offset Angle α	$\sin \alpha$	$\cos \alpha$	Δa	Δb	Coordinates		Tangent Angle
											a	b	
0	0° 00' 00"	0° 34' 23"	0.10000	.000095	0° 00' 00"	0° 00' 00"	.000000	1.000000	25 00.0	000000	0	0	0° 00' 00"
25	0° 00' 02"	0° 34' 25"	0.10009	.000095	0° 00' 39"	0° 00' 20"	.000095	1.000000	25 00.0	.002387	25 000	.002	0° 00' 39"
50	0° 00' 16"	0° 34' 38"	.010077	.000096	0° 00' 40"	0° 00' 59"	.000267	1.000000	25 0000	.007175	50 000	.010	0° 01' 19"
75	0° 00' 54"	0° 35' 16"	.010260	.000098	0° 00' 40"	0° 01' 39"	.000481	1.000000	25 0000	.012024	75 000	.022	0° 01' 59"
100	0° 02' 07"	0° 36' 30"	.010616	.000101	0° 00' 42"	0° 02' 20"	.000690	1.000000	25 0000	.017000	100 000	.039	0° 02' 41"
125	0° 04' 08"	0° 38' 31"	.011203	.000107	0° 00' 44"	0° 03' 03"	.000888	1.000000	25 0000	.022201	125 000	.061	0° 03' 25"
150	0° 07' 09"	0° 41' 31"	.012078	.000115	0° 00' 48"	0° 03' 49"	.001109	.999999	25 0000	.027750	150 000	.089	0° 04' 13"
175	0° 11' 21"	0° 45' 43"	.013300	.000127	0° 00' 52"	0° 04' 39"	.001352	.999999	25 0000	.033799	175 000	.122	0° 05' 05"
200	0° 16' 56"	0° 51' 19"	.014927	.000142	0° 00' 59"	0° 05' 34"	.001621	.999999	25 0000	.040528	200 000	.163	0° 06' 04"

1375	26° 41' 41"	27° 16' 04"	.515425	.004915	0° 33' 48"	13° 05' 53"	.226620	.973983	24 3496	5.665500	1369.004	86.064	13° 22' 47"
1400	26° 48' 52"	27° 23' 14"	.518071	.004940	0° 33' 58"	13° 39' 46"	.236207	.971703	24 2926	5.905182	1393.296	91.969	13° 56' 45"
1425	26° 54' 27"	27° 28' 50"	.520135	.004960	0° 34' 06"	14° 13' 48"	.245815	.969317	24 2329	6.145378	1417.529	98.114	14° 30' 51"
1450	26° 58' 39"	27° 33' 02"	.521688	.004975	0° 34' 12"	14° 47' 57"	.255432	.966827	24 1707	6.385808	1441.700	104.500	15° 05' 03"
1475	27° 01' 40"	27° 36' 03"	.522803	.004985	0° 34' 17"	15° 22' 11"	.265049	.964235	24 1059	6.626219	1465.806	111.126	15° 39' 20"
1500	27° 03' 41"	27° 38' 04"	.523550	.004992	0° 34' 19"	15° 56' 29"	.274656	.961543	24 0386	6.866400	1489.844	117.993	16° 13' 39"
1525	27° 04' 54"	27° 39' 17"	.524004	.004997	0° 34' 21"	16° 30' 50"	.284247	.958751	23 9688	7.106171	1513.813	125.099	16° 48' 00"
1550	27° 05' 32"	27° 39' 55"	.524237	.004999	0° 34' 22"	17° 05' 12"	.293816	.955862	23 8966	7.345390	1537.710	132.444	17° 22' 23"
1575	27° 05' 46"	27° 40' 09"	.524323	.005000	0° 34' 23"	17° 39' 34"	.303358	.952877	23 8219	7.583947	1561.532	140.028	17° 56' 45"
1600	27° 05' 48"	27° 40' 11"	.524335	.005000	0° 34' 23"	18° 13' 56"	.312871	.949796	23 7449	7.821770	1585.277	147.850	18° 31' 08"

Notes: (1) (2) (3) (4) (5) (6) (7) (8) (9)

- (1) ϕ obtained from roll equations, page A-4
- (2) $\theta = \phi$ plus drainage angle of $1\frac{1}{2} = \phi + 0^\circ 34' 23''$
- (3) $\sin \frac{\beta}{2} = \frac{1}{2} \text{ chord} \div \text{radius} = \frac{12.5}{r}$; $\text{radius} = \frac{v^2}{g \tan \theta} \therefore \sin \frac{\beta}{2} = \frac{12.5}{v^2} \tan \theta = .0095354 \tan \theta$
- (4) α determined from progression; $\alpha_1 = \frac{\beta}{2}$; $\alpha_2 = \beta_1 + \frac{\beta^2}{2}$; $\alpha_n = \beta_1 + \beta_2 + \dots + \beta_{n-1} + \frac{\beta_n}{2}$
- (5) $\Delta a = 25 \cos \alpha$
- (6) $\Delta b = 25 \sin \alpha$
- (7) $a = \sum \Delta a$
- (8) $b = \sum \Delta b$
- (9) Tangent Angle = $\sum \beta$

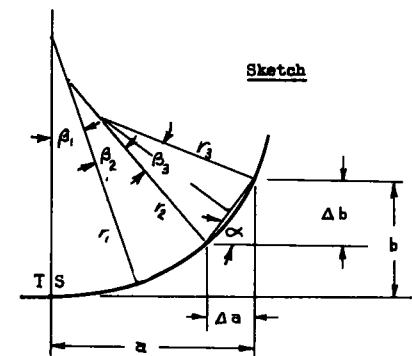


Figure A-1.

Transition Cross-Section

<u>Station 32 - 800' Along Trans.</u>					<u>Station 64 - 1600' Along Trans.</u>				
x	(x-12)	$K(x-12)^3$	+ .01x	= y	x	(x-12)	$K(x-12)^3$	+ .01x	= y
0	-	-	-	0.0000	0	-	-	-	0.0000
12	0	0.0000	0.1200	0.1200	12	0	0.0000	0.1200	0.1200
14	8	.0004	.1400	.1404	14	8	.0008	.1400	.1408
16	64	.0029	.1600	.1629	16	64	.0062	.1600	.1662
18	216	.0099	.1800	.1899	18	216	.0210	.1800	.2010
20	512	.0234	.2000	.2234	20	512	.0498	.2000	.2498
22	1000	.0456	.2200	.2656	22	1000	.0972	.2200	.3172
24	1728	.0789	.2400	.3189	24	1728	.1679	.2400	.4079
26	2744	.1253	.2600	.3853	26	2744	.2667	.2600	.5267
28	4096	.1870	.2800	.4670	28	4096	.3981	.2800	.6781
30	5832	.2662	.3000	.5662	30	5832	.5668	.3000	.8668
32	8000	.3652	.3200	.6852	32	8000	.7775	.3200	1.0975
34	10648	.4861	.3400	.8261	34	10648	1.0349	.3400	1.3749
36	13824	.6311	.3600	.9911	36	13824	1.3436	.3600	1.7036
38	17576	.8023	.3800	1.1823	38	17576	1.7082	.3800	2.0882
40	21952	1.0021	.4000	1.4021	40	21952	2.1335	.4000	2.5335
42	27000	1.2325	.4200	1.6525	42	27000	2.6242	.4200	3.0442
44	32768	1.4958	.4400	1.9358	44	32768	3.1848	.4400	3.6248
46	39304	1.7942	.4600	2.2542	46	39304	3.8200	.4600	4.2800
48	46656	2.1298	.4800	2.6098	48	46656	4.5345	.4800	5.0145
50	54872	2.5049	.5000	3.0049	50	54872	5.3331	.5000	5.8331
52	64000	2.9216	.5200	3.4416	52	64000	6.2202	.5200	6.7402
54	74088	3.3821	.5400	3.9221	54	74088	7.2007	.5400	7.7407
56	85184	3.8886	.5600	4.4486	56	85184	8.2791	.5600	8.8391
58	97336	4.4433	.5800	5.0233	58	97336	9.4602	.5800	10.0402
60	110592	5.0485	0.6000	5.6485	60	110592	10.7496	0.6000	11.3486

$\tan \theta_T = .25157737$ $\tan \theta_T - .01 = .24157737$ $\frac{\tan \theta_T - .01}{.5292} = K = .0000456495$	$\tan \theta_T = .52433522$ $\tan \theta_T - .01 = .51433522$ $\frac{\tan \theta_T - .01}{.5292} = K = .0000971911$
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Figure A-2.

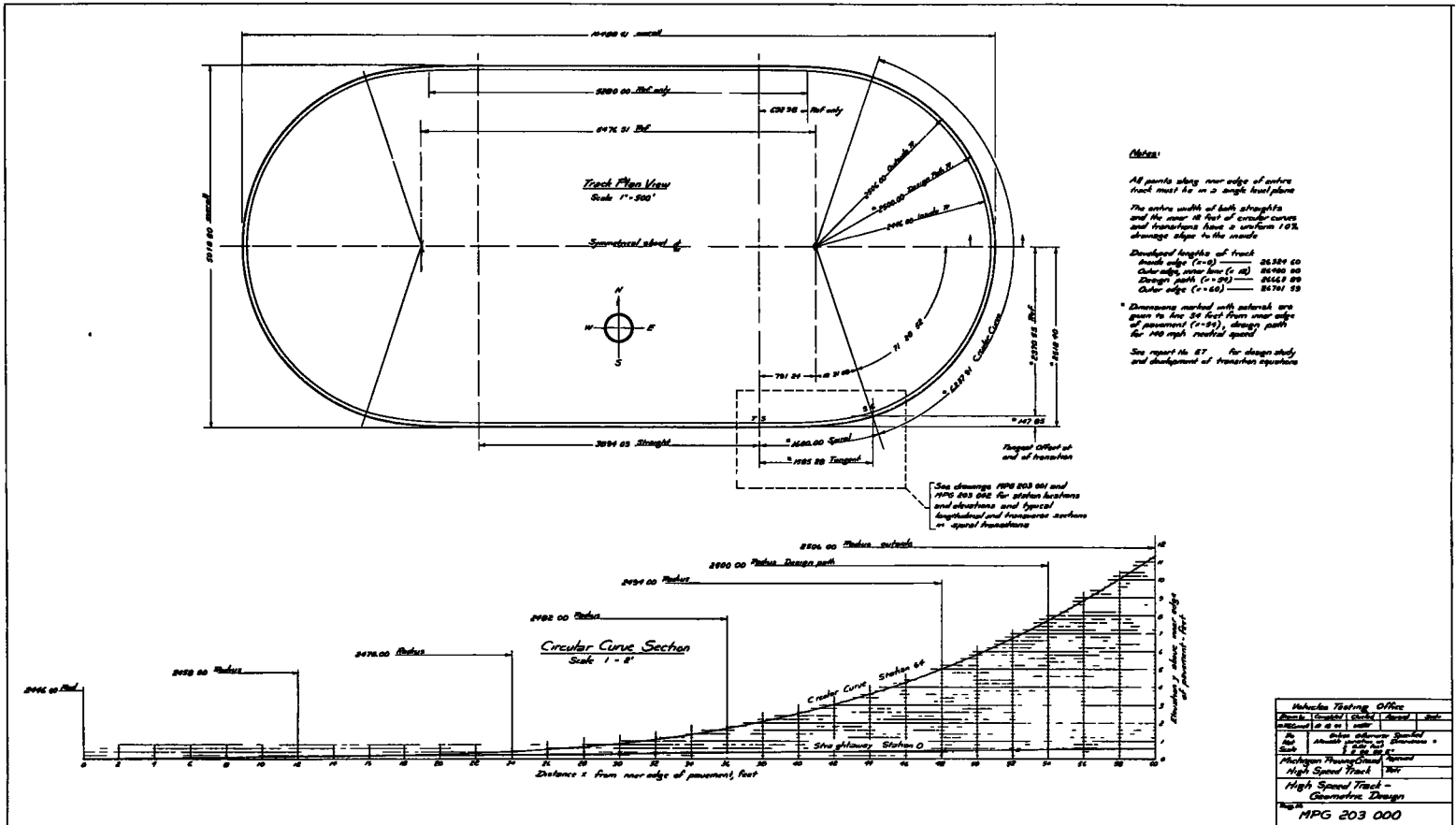


Figure A-3.

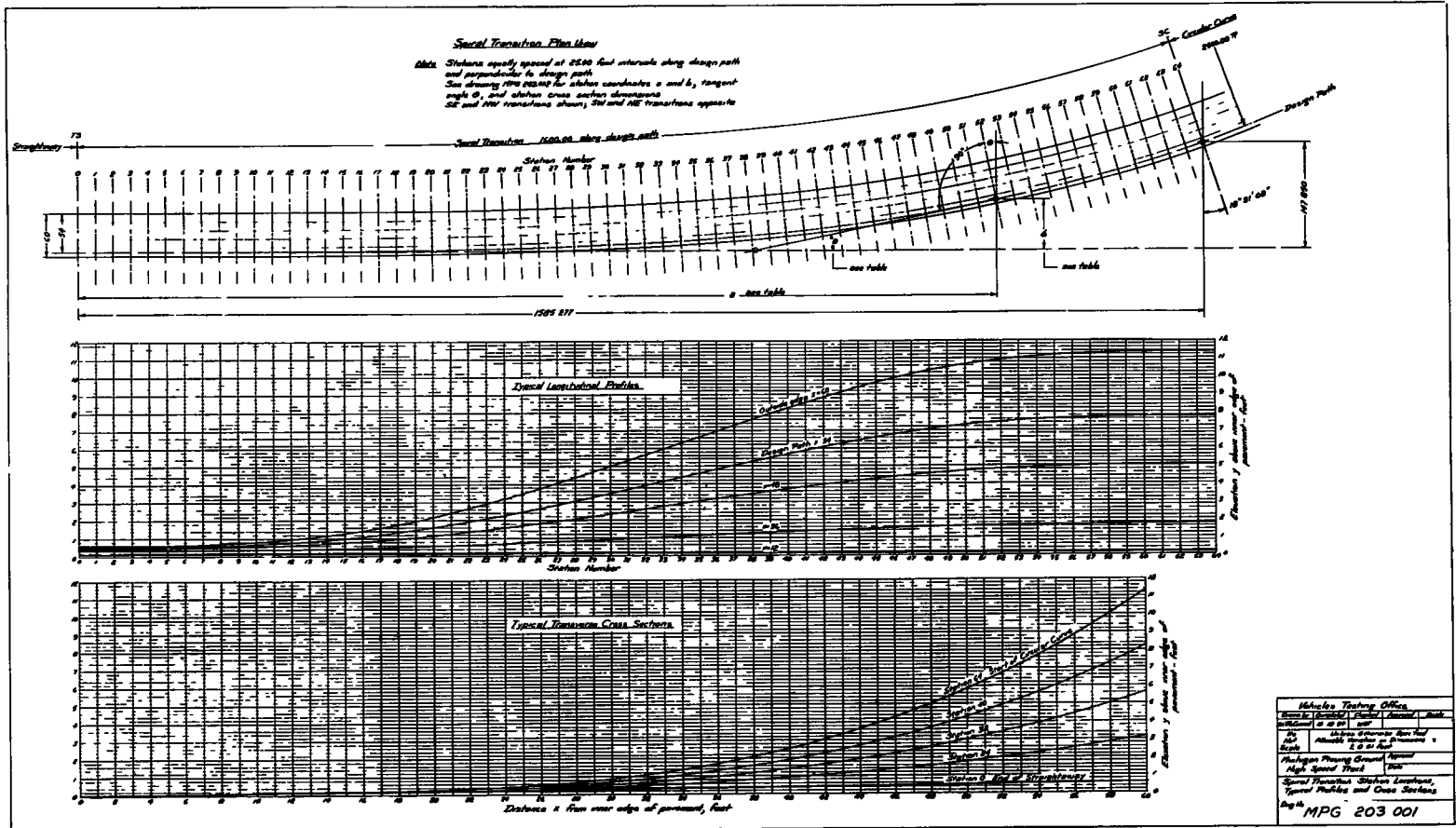


Figure A-4.

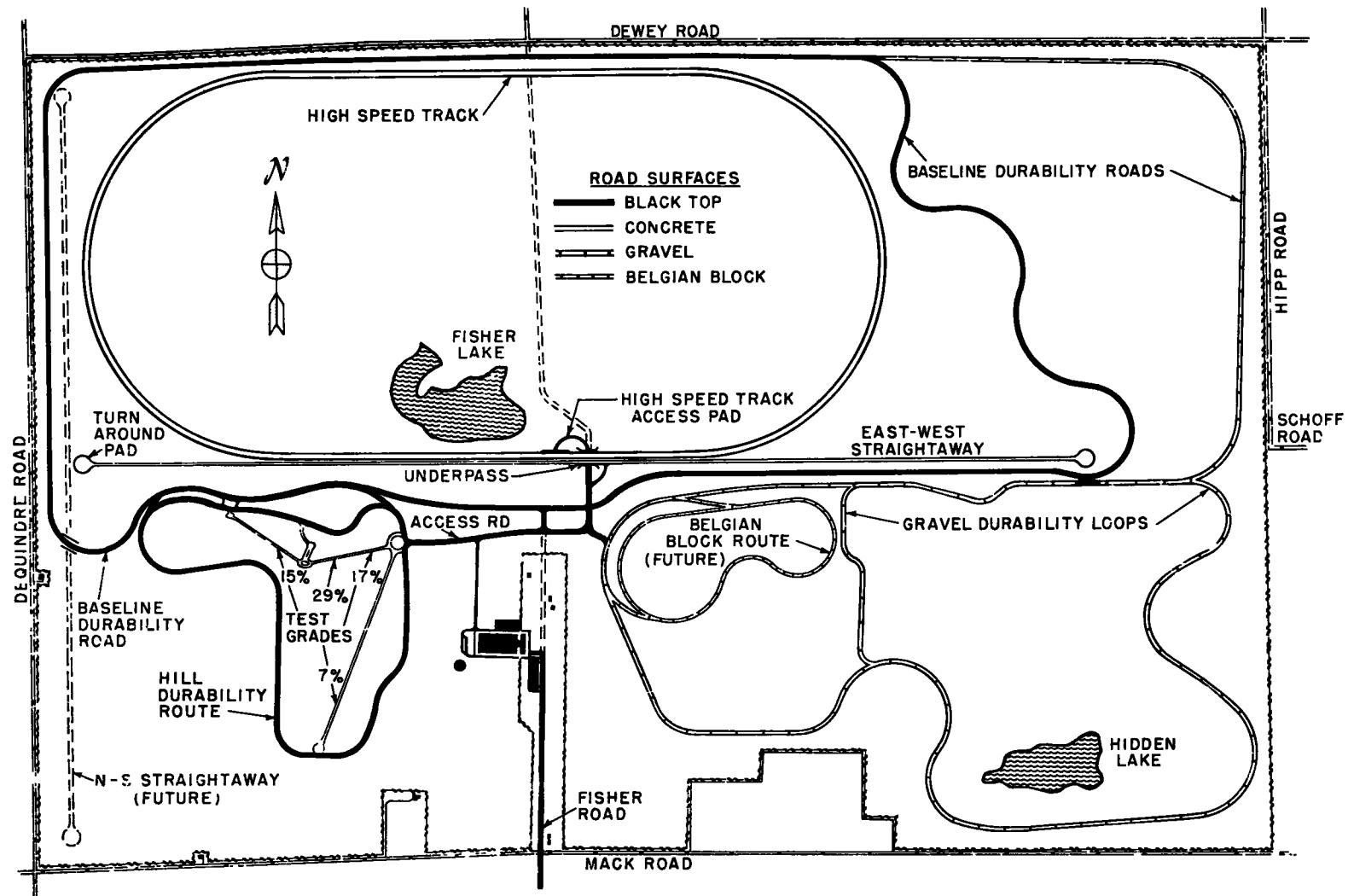


Figure A-6. Ford Michigan proving ground.

Step 6 — Calculate roll angles and slopes for each 25-ft station through transition

Determine by trigonometric relations the horizontal coordinates of the design patch at each station, and the angle of the tangent at that point, for field layout use.

These calculations are detailed in Figure A-1 for several stations.

Step 7 — Develop cross-section equation

The inner lane is to be flat, with a 1 percent drainage slope to the inside.

The remaining cross-section is to be a cubic parabola, with the origin at the inside edge. The cubic must be adjusted for a slope and elevation match where it joins the inside lane, the first portion being discarded.

$$\begin{aligned} \text{From } x = 0 \text{ to } x = 12, \\ y = 0.01x \end{aligned} \quad (4)$$

$$\begin{aligned} \text{From } x = 12 \text{ to } x = 60, \\ y = 0.01x + K(x - 12)^3 \end{aligned} \quad (5)$$

$$\frac{dy}{dx} = 0.01 + 3K(x - 12)^2$$

at $x = 12$, $y = 0.12$ ft for elevation match with inner lane.

$$\frac{dy}{dx} = 0.01 \text{ for slope match.}$$

at $x = 54$ (design path),

$$\begin{aligned} \frac{dy}{dx} &= \tan \theta \text{ for proper roll motion} \\ &= 0.01 + 3K(42)^2 \end{aligned}$$

$$\therefore K = \frac{\tan \theta - 0.01}{5,292} \quad (6)$$

Where

x = horizontal distance from inner edge, in feet;

y = elevation above inner edge, in feet; and

$\tan \theta$ = slope of cross-section at design path.

Figure A-2 shows cross-section calculations for two typical stations. These calculations can be performed rapidly by machine; previous calculations could not, as trigonometric functions were not available on IBM cards to the required accuracy.

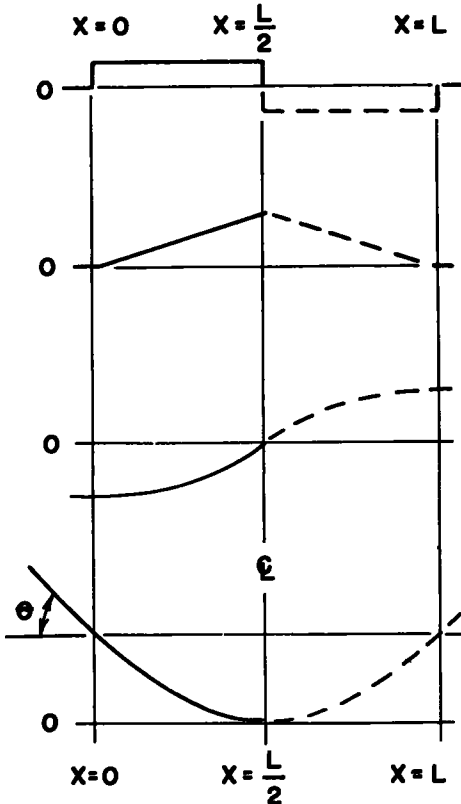
Figures A-3, A-4, and A-6 show plan, profile and general layout drawings. Spiral transition dimensions are given in Figure A-5.

Appendix B

Sag Vertical Curve Design

The problem is to develop an equation for a sag vertical curve with imperceptible vertical acceleration and jerk.

Step 1 – Diagram motion characteristics and write their equations



$$\begin{aligned} \text{Jerk} &= J = \text{constant} \\ J &= C \text{ ft}/\text{ft}^3 \\ &= CV^3 \text{ ft}/\text{sec}^3 \end{aligned}$$

$$\begin{aligned} \text{Acceleration} &= a = \int J dx \\ a &= CX \text{ ft}/\text{ft}^2 \\ &= CXV^2 \text{ ft}/\text{sec}^2 \end{aligned}$$

$$\begin{aligned} \text{Vertical velocity} &= H = \iint J dx \\ H &= \left(\frac{CX^2}{2} - \frac{CL^2}{8} \right) \text{ ft}/\text{ft} \\ &= \left(\frac{CX^2}{2} - \frac{CL^2}{8} \right) V \text{ ft}/\text{sec} \end{aligned}$$

$$\begin{aligned} \text{Elevation} &= y = \iiint J dx \\ y &= \left(\frac{CX^3}{6} - \frac{CL^2}{8}x + \frac{CL^3}{24} \right) \text{ feet} \end{aligned}$$

where

L = sag length, in feet; and
 V = horizontal velocity or design speed, in feet per second.

Step 2 – Evaluate constant C

When $x = 0$, slope = $\tan \theta = A$

$$\frac{dy}{dx} = H = A = \frac{CX^2}{2} - \frac{CL^2}{8} \quad \therefore C = \frac{8A}{L^2}$$

Step 3 – Substitute constant in equations

$$\text{Jerk} = \left(\frac{8A}{L^2} \right) V^3 \text{ ft}/\text{sec}^3 \quad (7)$$

This value should be equal to or less than the 0.8-ft/sec³ perception threshold. Equate jerk to threshold value, substitute desired values for slope, A , and design speed, V , to determine minimum sag length.

$$\text{Acceleration} = \left(\frac{8A}{L^2} \right) x V^2 \text{ ft}/\text{sec}^2 \quad (8)$$

This value should be equal to or less than the 4.0-ft/sec² perception threshold.

$$\text{Elevation} = \left(\frac{8A}{L^2} \right) \left(\frac{X^3}{6} - \frac{L^2 X}{8} + \frac{L^3}{24} \right) \text{ feet} \quad (9)$$

Substitute value of L from Equation 7 and desired slope A in this equation to determine elevation at any station, x .

Example – Determine profile of sag curve between two 3 percent slopes for minimum length with no perceptible change. Design speed, 140 mph (205 ft/sec).

$$\text{Jerk} = \left(\frac{8A}{L^2} \right) V^3 = 0.8 \text{ ft}/\text{sec}^3 \text{ (threshold)}$$

$$L^2 = \frac{(8)(0.03)(205)^3}{0.8} = 2,560,000$$

$$L = 1,600 \text{ ft (sag length).}$$

$$\text{Acceleration} = \left(\frac{8A}{L^3}\right) V^2 X;$$

$$\text{max value at } X = \frac{L}{2}:$$

$$A_{\text{max}} = \frac{4A}{L} V^2 = 3.15 \text{ ft/sec}^2$$

Maximum value is below the 4.0-ft/sec² threshold.

$$\text{Elevation} = y = \left(\frac{8A}{L^3}\right) \left(\frac{X^3}{6} - \frac{L^2 X}{8} + \frac{L^3}{24}\right) \text{ feet}$$

$$= \frac{X^3}{64,000,000} - 0.03X + 16$$

	x feet	y feet
Start of sag	0	16.0
	100	13.0
	200	10.1
	300	7.4
	400	5.0
	500	3.0
	600	1.4
	700	0.4
Bottom of sag	800	0

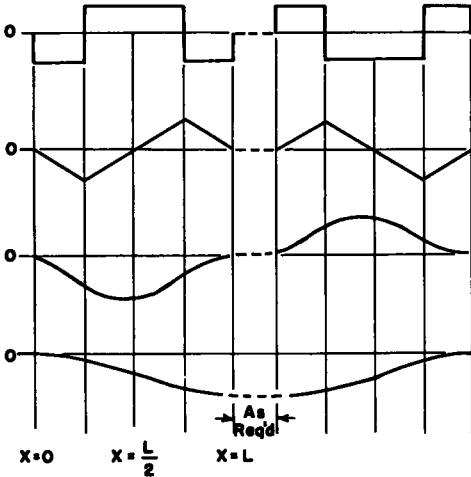
Note: Sag is symmetrical about bottom point, so calculations are necessary for first half only.

Appendix C

Highway Dip and Rise Design

The problem is to develop an equation for a highway dip with imperceptible vertical motions. Rises, such as a grade separation, use the same equations, have the same form, but with all characteristics inverted.

Step 1 – Diagram motion characteristics



$$\text{Jerk} = J = \pm C \text{ ft/ft}^3$$

$$= \pm CV^3 \text{ ft/sec}^3$$

$$\text{Acceleration} = a$$

$$= \int J dx \text{ ft/ft}^2$$

$$= V^3 \int J dx \text{ ft/sec}^2$$

$$\text{Velocity} = H$$

$$= \iint J dx \text{ ft/ft}$$

$$= V \iint J dx \text{ ft/sec}$$

$$\text{Depth} = y = \iiint J dx \text{ ft}$$

Dip is symmetrical, so derivation is confined to one side only.

L = length of vertical curve, in feet;
and,

V = horizontal velocity, or design speed, in feet/second.

Step 2 – Develop equations of motion by integration

$$\text{From } x = 0 \text{ to } x = \frac{L}{4}$$

$$J = -C$$

$$a = -CX$$

$$H = \frac{-CX^2}{2}$$

$$y = \frac{-CX^3}{6}$$

$$\text{From } x = \frac{L}{4} \text{ to } x = \frac{3L}{4}$$

$$J = +C$$

$$a = CX - \frac{CL}{2}$$

$$H = \frac{CX^2}{2} - \frac{CLX}{2} + \frac{CL^2}{16}$$

$$y = \frac{CX^3}{6} - \frac{CLX^2}{4} + \frac{CL^2 X}{16} - \frac{CL^3}{192}$$

Step 3 – Evaluate constant C

When $x = \frac{L}{2}$, $y = \frac{d}{2}$, where d is the desired depth of the dip.

$$\therefore y = \frac{d}{2} = \frac{CL^3}{48} - \frac{CL^3}{16} + \frac{CL^3}{32} - \frac{CL^3}{192}$$

$$C = -\frac{32d}{L^3}$$

Step 4 – Substitute constants in equations

$$\text{Jerk} = J = \pm \frac{32d}{L^3} V^3 \text{ ft/sec}^3 \quad (10)$$

J should be equal to or less than the 0.8-ft/sec³ threshold limit. Substitute the 0.8 value for J , and desired values for depth, d , and design speed, V , and solve for minimum length, L .

$$\text{Acceleration} = a = \frac{32dx}{L^3} V^2 \text{ ft/sec}^2$$

a is maximum at $x = \frac{L}{4}$, or

$$a_{\text{max}} = \frac{8d}{L^2} V^2 \text{ ft/sec}^2 \quad (11)$$

Check to make sure maximum acceleration is below the 4.0-ft/sec² threshold limit.

$$\text{Depth} = y = \frac{16dx^3}{3L^3} \text{ feet, from } x = 0 \text{ to } x = \frac{L}{4} \quad (12)$$

$$y = \frac{16dx^3}{3L^3} + \frac{8dx^2}{L^2} - \frac{2dx}{L} + \frac{d}{6} \text{ feet} \quad (13)$$

from $x = \frac{L}{4}$ to $x = \frac{3L}{4}$, substitute value of L from Equation 10 and desired value for d . Solve for y at various stations, x . It is not necessary to use equations beyond $x = \frac{L}{2}$, as elevations between $x = \frac{L}{2}$ and $x = L$ can be found from the symmetry of the curve.

Example – Design a highway dip with a maximum depth of 12 ft, for a design speed of 50 mph (75 ft/sec) with no perceptible vertical motions.

$$J = \frac{32d}{L^3} V^3 = 0.8 \text{ ft/sec}^3 \text{ (threshold)}$$

$$L^3 = \frac{32 \cdot 12 \cdot (75)^3}{0.8}$$

$$L = 600 \text{ ft (approx)}$$

$$a_{\text{max}} = \frac{8d}{L^2} V^2 = 1.5 \text{ ft/sec}^2 = < 4 \text{ (threshold)}$$

From $x = 0$ to $x = 150$,

$$y = \frac{16dx^3}{3L^3} = \frac{x^3}{3,375,000}$$

From $x = 150$ to $x = 450$,

$$y = -\frac{x^3}{3,375,000} + \frac{x^2}{3,750} - \frac{x}{25} + 2.$$

	Station, x, ft	Depth, y, ft
Level	0	0
	50	0.0
	100	0.3
	150	1.0
	200	2.3
	250	4.0
L/2	300	6.0
	350	8.0
	400	9.7
	450	11.0
	500	11.7
Bottom	550	12.0
	600	12.0

Appendix D

Photographs

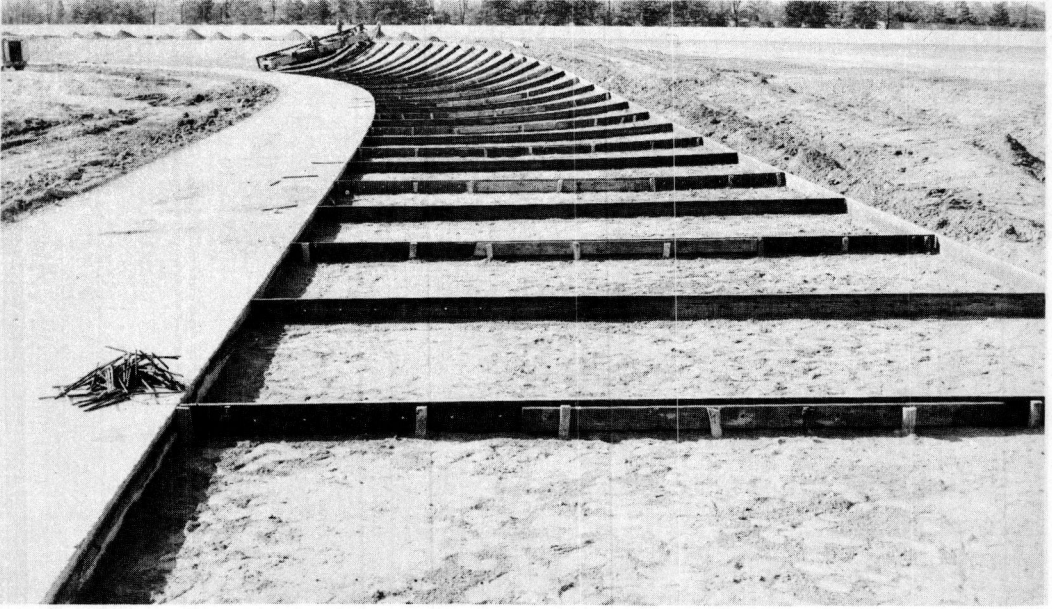


Figure D-1. Ford Dearborn test area, construction of transition curve.



Figure D-2. Ford Dearborn test area transition curve.



Figure D-3. Paving superelevation - Ford Arizona track.

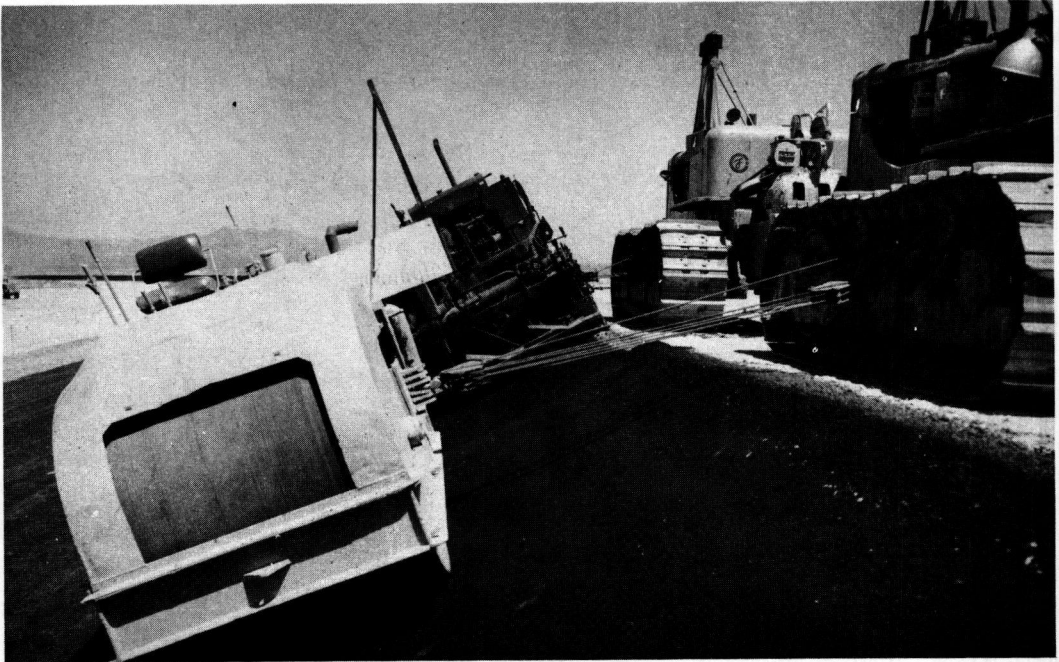


Figure D-4. Rolling superelevation - Ford Arizona track.

THE NATIONAL ACADEMY OF SCIENCES—NATIONAL RESEARCH COUNCIL is a private, nonprofit organization of scientists, dedicated to the furtherance of science and to its use for the general welfare. The ACADEMY itself was established in 1863 under a congressional charter signed by President Lincoln. Empowered to provide for all activities appropriate to academies of science, it was also required by its charter to act as an adviser to the federal government in scientific matters. This provision accounts for the close ties that have always existed between the ACADEMY and the government, although the ACADEMY is not a governmental agency.

The NATIONAL RESEARCH COUNCIL was established by the ACADEMY in 1916, at the request of President Wilson, to enable scientists generally to associate their efforts with those of the limited membership of the ACADEMY in service to the nation, to society, and to science at home and abroad. Members of the NATIONAL RESEARCH COUNCIL receive their appointments from the president of the ACADEMY. They include representatives nominated by the major scientific and technical societies, representatives of the federal government, and a number of members at large. In addition, several thousand scientists and engineers take part in the activities of the research council through membership on its various boards and committees.

Receiving funds from both public and private sources, by contribution, grant, or contract, the ACADEMY and its RESEARCH COUNCIL thus work to stimulate research and its applications, to survey the broad possibilities of science, to promote effective utilization of the scientific and technical resources of the country, to serve the government, and to further the general interests of science.

The HIGHWAY RESEARCH BOARD was organized November 11, 1920, as an agency of the Division of Engineering and Industrial Research, one of the eight functional divisions of the NATIONAL RESEARCH COUNCIL. The BOARD is a cooperative organization of the highway technologists of America operating under the auspices of the ACADEMY-COUNCIL and with the support of the several highway departments, the Bureau of Public Roads, and many other organizations interested in the development of highway transportation. The purposes of the BOARD are to encourage research and to provide a national clearinghouse and correlation service for research activities and information on highway administration and technology.
