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APPENDIX A

REPRODUCTION OF A MEMORANDUM
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THE USE OF HIGH-SPEED VACUUM TUBE DEVICES FOR CALCULATING

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There are many sorts of mathematical problems which require calculation by formulas which can readily be put in the form of iterative equations. Purely mechanical calculating devices can be devised to expedite the work. However, a great gain in the speed of calculation can be obtained if the devices which are used employ electronic means for the performance of the calculation, because the speed of such devices can be made very much higher than that of any mechanical device. It is the purpose of this discussion to consider the speed of calculation and the advantages which may be obtained by the use of electronic circuits which are interconnected in such a way as to perform a number of multiplications, additions, subtractions or divisions in sequence, and which can therefore be used for the solution of difference equations. Since a sufficiently approximate solution of many differential equations can be had simply by solving an associated difference equation, it is to be expected that one of the chief fields of usefulness for an electronic computer would be found in the solution of differential equations.

As will be brought out in the following discussion, the electronic computer may have certain advantages other than the single one of high speed when compared with the differential analyzer of the usual mechanical construction. For instance, the mechanical analyzer has an accuracy limited by the way in which the slip and back-lash enter into its operation, whereas the electronic device, operating solely on the principle of counting, can without great difficulty, be made as accurate as is required for any practical purpose. Secondly, whatever errors are introduced by the electronic device (when it is properly operating) are mathematically determined errors which arise through the use of a difference equation in place of a differential equation; and since these errors are mathematically determined, they are reproduceable. Hence, a check calculation may be run at any time; and whatever the result may be for the first run, that result should be obtained exactly on every succeeding test run. Third, the ease with which the various components of such a computing device can be interconnected by cables and switching units makes it possible to set up a new problem without much difficulty. It is also to be appreciated that additional component parts may be built and connected in whenever more complicated problems require them, and that additional spare components may be kept in reserve and quickly interchanged with any components which fail in operation. Fourth, a convenient way of diagnosing faults and isolating defective units is available through the use of standard electric testing procedures. Fifth, not only can the regular problems which have to be solved be run through quickly on the electronic computer, but also the test problems, and hence less time is lost in discovering any failure which may develop during operation. Sixth, both the mechanical analyzer and the electronic computer require maintenance by skilled labor, but the number of persons required to turn out the same amount of finished work should be appreciably less in the case of the electronic computer.

It is now necessary to justify the foregoing comparison. Let us first consider the mode of operation. As already stated, the electronic computer utilizes the principle of counting to achieve its results. It is then, in every sense, the electric analogue of the mechanical adding, multiplying and dividing machines which are now manufacutred for ordinary arithmetic purposes. However, the design of the electronic computer allows easy interconnection of a number of simple component devices and provides for a cycle of operations which will yield a step by step solution of any difference equation within its scope. The result of one calculation, such as a single multiplication, is immediately available for further operation in any way which is dictated by the equations governing the problem, and these numbers can be transferred from one component to another as required, without the necessity of copying them manually onto paper or from one component to another, as is the case when step by step solutions are performed with ordinary calculating machines. If one desires to visualize the mechanical analogy, he must conceive of a large number, say twenty or thirty, calculating machines, each capable of handling at least ten-digit numbers, and all interconnected by mechanical devices which see to it that the numerical result from an operation in one machine is properly transferred to some other machine, which is selected by a suitable program device; and one must further imagine that this program device is capable of arranging a cycle of different transfers and operations of this nature, with perhaps fifteen or twenty operations in each cycle. It may be said that even though such a mechanical device were constructed, and even though its speed of operation were considered satisfactory, the number of problems which it would solve within the lifetime which is determined by the war of its parts would undoubtedly be very small.

In stating that the electronic computer consists of components which are exactly analogous to the ordinary mechanical computing machine, it is intended that this analogy shall be interpreted rather completely. In particular, just as the ordinary computing machine utilizes the decimal system in performing its calculation, so may the electronic device. When a number, such as 1216, is to be injected into a particular register, it is not necessary for 1216 counts to be used. Instead, a total of 10 counts would be sufficient, one in the thousands register, 2 in the hundreds register, one in the tens register, and 6 in the units register. It is only in this way that almost unlimited accuracy can be obtained without unduly prolonging the time of operation. Electronic devices have been proposed for which this is not true, but such do not seem to merit consideration here.

After what has just been said it is easy to see that the total time required for the completion of any cycle of operations can be rather closely estimated if one knows only the time required for injecting a single count. Existing electronic circuits are capable of counting electric pulses at rates in excess of 100,000 per second. (In much of the literature on counters, often referred to as scaling circuits and not to be confused with the Geiger-Mueller counters also discussed in the literature, the counting rate is stated for pulses which are randomly spaced. This counting rate is of necessity a great deal lower than that at which the same circuit will count when the pulses are uniformly spaced. Although the pulses which would be used in a computing device must be non-uniformly spaced, because they must be interrupted at various times, there is nevertheless a minimum time interval between pulses which is never violated, a situation quite different from that obtaining when the pulses are randomly spaced.) It is therefore quite possible that counting rates in excess of 1,000,000 per second might be used in an electronic calculator, but a conservative estimate, based on art already disclosed, might better put the counting rate at some figure in the neighborhood of 100,000 pulses per second. In what follows, the counting rate will be designated by the latter R, and whenever desired, some representative figure, such as 10^5 per second, may be adopted for the sake of illustration.

Let us analyze the time required for the addition of an N digit number to some register. In order properly to time operations which are taking place simultaneously in various parts of the device, it is necessary to allow for any addition whatever a time equal to the maximum time required for any addition. The largest number of pulses which must be injected into any signle digit register is ten-9 for the maximum digit which may be added and 1 more for the carrying operation. Just as in a mechanical computing machine the digits which represent various powers of 10 are added simultaneously, so in the electronic device. Therefore, no greater time is needed for adding a number having 10 digits than is needed for one having 2 digits. An estimate for the time of addition is therefore 10/R, no matter how large the number added. However, because the programming device, which causes such additions to be made, requires a certain time for its operation, we will set our estimate as 11/R.

Next let us consider the time necessary for multiplication of an N digit number by some other number of any number of digits. This multiplication is carried out by successive addition, as it is in mechanical calculators. Hence, it is immaterial how many digits one of the numbers has. This number has to be added a total number of times equal to the sum of the digits in the multiplier. Again we must allow for the maximum time which such a multiplication may require. A multiplier of N digits can have at most a total digit sum of 9 N, so that we must allow for 9 N additions. Furthermore, we must allow a time for the operation analogous to shifting the carriage in a mechanical computer and so must estimate a total operation time equivalent to 10 N additions; since each addition requires a time of 11/R, we estimate that the use of an N digit multiplier requires a total time of operation of 110 N/R. Any additional time required for the programming device in initiating the operation of multiplication will be negligible in comparison with this and so need not be considered. Let us now suppose that R equals 10^5 seconds and that N is 9. The use of a 9 digit multiplier at this pulse speed then results in an estimated rate of 1,000 multiplications per second.

The rate at which divisions may be performed will, of course, be of the same order of magnitude.

Analysis of the time required for each cycle or step in the solution of a numerical difference equation of the sort encountered in ballistics problems indicates that the total time required for each step will be about that needed for 8 or 10 multiplications. Assuming the higher figure, we arrive at the estimate that these steps may be taken at the rate of 100 per second. When similar equations are numerically integrated with mechanical machines manually operated no more than 100 steps are usually necessary. We are not justified in concluding that the electronic computer will be able to run through a complete trajectory in 1 second. This is because in the manual computation sufficiently exact results can be obtained from 100 steps only if the methods of proceeding from one step to the next are somewhat increased in refinement. On the other hand, simplicity of construction and operation in the electronic computer is achieved by utilizing only the simplest step by step methods, for which many more than 100 steps are needed to obtain the same final accuracy. It is obvious, however, that by the use of even 10,000 steps at the speed of the electronic device, the time for the overall integration is still extremely short--100 seconds in comparison to the time required for an equivalent manual integration--at least several hours--and also quite short in comparison to the time required for solution with less accuracy on a mechanical differential analyzer.

Note added April 2, 1943: The figures given in the last paragraph above have been superseded by other data given in the report of which this is an appendix