Chapter 4

NUCLEAR PROCESSES, THE STRONG FORCE

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4.1 INTRODUCTION

To gain a practical understanding of nuclear, plasma and radiological phenomena one needs to understand the basic nuclear processes involving isotopes, nuclear reactions, radioactivity, as well as fission and fusion. The goal here is to deal with the strong force holding the nuclei together in the atomic nucleus and its possible release through the processes of fission and fusion. We first compare nuclear reactions to chemical reactions, then compare their energy releases. It is observed that nuclear reactions produce as much as 1 million times the energy per reaction as compared with chemical ones. We also consider the equivalence of mass to energy and the corresponding mass and momentum conservation relations governing nuclear reactions.

4.2 CHEMICAL REACTIONS

Chemical reactions as learned in the field of Chemistry involve the combination or separation of whole atoms. The interaction between the reactants and product nuclei is an atomic process involving the electronic cloud surrounding the nuclei. As examples, one can consider first the chemical reaction where carbon in fossil fuels, or in plant and animal metabolic processes, is burned or oxidized into carbon dioxide. Second, one can consider the reaction in which hydrogen is combined with oxygen chemically to produce water. Third, one can also consider the chemical reaction in which uranium dioxide is converted into uranium tetrafluoride UF_4 or the Green Salt through the interaction with hydrofluoric acid (HF) in the manufacturing of uranium fuel for nuclear reactors:

$$\text{Reactants} \rightarrow \text{Products}$$

$$C + O_2 \rightarrow CO_2 + 4\,eV \tag{1}$$

$$2H_2 + O_2 \rightarrow 2H_2O \tag{2}$$

$$UO_2 + 4HF \rightarrow 2H_2O + UF_4 \tag{3}$$

In the case of the first reaction one notices a release of energy in the reaction that can be measured through calorimetry as 4 eV. The electron Volt (eV) unit is the amount of kinetic energy acquired by a single electron if it is accelerated though a potential drop of 1 Volt. In this case that unit can be transformed into other units of energy as follows:

1 eV	=	1 electron Volt of energy
	=	1.5190×10^{-22} BTU (British Thermal Unit)
	=	$4.4400 \times 10\text{-}^{26}$ Kw.hr (Kilowatt.hour)
	=	1.6021×10^{-19} Joule (Where 1 Joule = 1 Watt.second)

Although the eV energy unit is adequate in chemical applications, in nuclear applications, a larger unit, the Million electron Volt is used where:

$$1 \text{ MeV} = 10^6 \text{ eV}.$$

In these chemical reactions, one can observe the following characteristics:

1. Each atom participates as a whole in the reaction and retains its identity after the reaction is completed,
2. The resulting product molecules are different from those entering the reactions as reactants,
3. A sharing or exchange of valence electrons occurs, and,
4. The nuclei of the participating atoms are unaffected.

4.3 NUCLEAR REACTIONS, TRANSMUTATIONS

Nuclear reactions differ substantially from chemical reactions in that the reactions involve the nuclei of the atoms rather than their electronic cloud as in chemical reactions. Consequently, the reactant nuclei do not necessarily show up as products of the reactions and transmutation as the dream of the Middle Age alchemists is achieved. We could find either isotopes of the reactants or completely different ones.

In balancing nuclear reactions we conserve nuclear particles rather than whole atoms like in chemical reactions. As examples the bombardment of aluminum or nitrogen nuclei with alpha particles, which are helium nuclei, can lead to the following nuclear reactions:

$$\text{Reactants} \rightarrow \text{Products}$$

$$_{13}Al^{27} + {}_2He^4 \rightarrow {}_{14}Si^{30} + {}_1H^1 + \Delta m \qquad (4)$$

$$_7N^{14} + {}_2He^4 \rightarrow {}_8O^{17} + {}_1H^1 + \Delta m \qquad (5)$$

In general, one can write:

$$_{Z1}X^{A1} + {}_{Z2}Y^{A2} \rightarrow {}_{Z3}U^{A3} + {}_{Z4}V^{A4} + \Delta m \qquad (6)$$

where:

X and Y represent the reactant nuclei,

U and V represent the product nuclei,

Z1 and Z2 represent the atomic numbers, or the charges, or the number of protons of the reactant nuclei, A1 and A2 represent the mass numbers, or the total number of nucleons in the reactant nuclei,

Z3 and Z4 represent the atomic numbers, or the charges, or the number of protons of the reactant nuclei,

A3 and A4 represent the mass numbers, or the total number of nucleons in the reactant nuclei.

In nuclear reactions the number of protons or the total charge is conserved on both sides of the nuclear reaction:

$$Z1 + Z2 \quad = \quad Z3 + Z4 \tag{7}$$

In addition, the total number of nucleons is conserved on both sides of the nuclear reaction:

$$A1 + A2 \quad = \quad A3 + A4 \tag{8}$$

As an extra feature of nuclear reactions, mass is also conserved. The conservation of mass process can be expressed through the statement that:

$$\Delta m \quad = \quad \text{mass (reactants)} - \text{mass (products)} \tag{9}$$

In addition, momentum and another nuclear property designated as "parity," are conserved.

4.4 MASS CONSERVATION IN NUCLEAR REACTIONS

Mass conservation in nuclear reactions allows us to calculate the energy releases from these nuclear reactions. To measure atomic and nuclear masses, a standardized unit designated as the atomic mass unit (amu) is used. Since carbon is abundant on earth in hydrocarbons and living matter, the amu unit of mass has been chosen as $1/12^{th}$ the mass of the most common isotope of carbon: ${}_6C^{12}$. Thus we can define the amu as:

$$1 \text{ amu} = \frac{m({}_6C^{12})}{12} \tag{10}$$

In older scientific papers and reports, the amu was defined as $1/16^{th}$ the mass of the O^{16} isotope.

If we now want to express the mass of the ${}_6C^{12}$ isotope, we can write that:

$$m({}_6C^{12}) = 12 \text{ amu}$$

We can also express the masses of the isotopes involved in the previous nuclear reactions as:

$$m({}_{13}Al^{27}) = 26.981541 \quad \text{amu}$$
$$m({}_2He^4) = 4.002603 \quad \text{amu}$$
$$m({}_{14}Si^{30}) = 29.973772 \quad \text{amu}$$
$$m({}_1H^1) = 1.007825 \quad \text{amu}$$

$m({}_7N^{14})$ = 14.003074 amu
$m({}_8O^{17})$ = 16.999131 amu

EXAMPLE

We can apply the conservation of mass law in terms of the nuclear masses in the previously listed nuclear reaction involving ${}_{13}Al^{27}$ as:

$$
\begin{aligned}
\Delta m &= m\,(\text{reactants}) - m\,(\text{products}) \\
&= [m({}_{13}Al^{27}) + m({}_2He^4)] - [m({}_{14}Si^{30}) + m({}_1H^1)] \\
&= [26.981541 + 4.002603] - [29.973772 + 1.007825] \\
&= 30.984143 - 30.981597 \\
&= +\,0.002540 \text{ amu}
\end{aligned}
$$

This difference in mass is positive in value implying an "exothermic" reaction.
If we apply the same mass conservation process to the reaction involving the ${}_7N^{14}$ isotope, we get:

$$
\begin{aligned}
\Delta m &= m(\text{reactants}) - m(\text{products}) \\
&= [m({}_7N^{14}) + m({}_2He^4)] - [m({}_8O^{17}) + m({}_1H^1)] \\
&= [14.003074 + 4.002603] - [16.999134 + 1.007825] \\
&= 18.005677 - 18.0069567 \\
&= -\,0.001279 \text{ amu}
\end{aligned}
$$

The difference in mass is this case is negative, implying an "endothermic" reaction. This suggests that we need to supply energy to the reactants in the form of kinetic energy to the reactants for the reaction to proceed.

Notice that this terminology is the opposite of the one used in chemical reactions.

4.5 AVOGADRO'S LAW

To convert the amu into units of grams, and later into units of energy, we invoke Avogadro's law from the field of Chemistry stating that:

"*The number of atoms or molecules in a mole of any substance is a constant: A_v = 0.602 x 10^{24} [atoms/mole]*"

where: 1 mole = 1 gram molecular weight
= Amount of a substance having a mass, in grams, equal to the atomic or molecular weight of the substance.

For instance:

1 mole of ${}_6C^{12}$ = 12 gm of ${}_6C^{12}$

and:

$$48 \text{ gm of } {}_6C^{12} \text{ contain} : 48/12 = 4 \text{ moles of } {}_6C^{12}$$

In general:

$$\text{Number of moles} = \frac{\text{Mass of Element}}{\text{Atomic or Molecular Weight}}$$

$$\simeq \frac{m({}_ZX^A)}{A}$$

Here we have approximated the atomic weight M, by the mass number A.

As an example, for 64 grams of oxygen O_2:

$$\text{Number of moles} \simeq \frac{64}{16+16} \simeq \frac{64}{32} \simeq 2$$

Since one gram mole contains Avogadro's number in atoms or molecules, and one gram mole of ${}_6C^{12}$ has a mass of 12 g, and contains N_A atoms, thus:

$$\text{Mass of one atom of } {}_6C^{12} = \frac{12}{A_v} = \frac{12}{0.602 \text{x} 10^{24}} = 1.99 \text{ x } 10^{-23} \text{ gm}$$

This can help us establish the value of the amu in grams:

$$1 \text{ amu} = \frac{m({}_6C^{12})}{12} = \frac{1.99 \text{ x } 10^{-23}}{12} = 1.66 \text{ x } 10^{-24} \text{ gm}$$

Avogadro's Law allows us to estimate the number of nuclei N, atoms, or molecules in a given mass g in grams of a substance:

$$N = \frac{g}{M} A_v \tag{11}$$

If we are interested instead in the number of nuclei, atoms, or molecules per unit volume, we use instead the number density form of the previous equation, by dividing by the volume V, or using the density of the substance instead of its mass resulting in a modified form of Avogadro's law as:

$$N' = \frac{N}{V} = \frac{(\frac{g}{V})}{M} A_v = \frac{\rho}{M} A_v \tag{12}$$

4.6 MASS AND ENERGY EQUIVALENCE

A result of Relativity Theory is that mass and energy are equivalent and convertible into each other. In particular the complete annihilation of a particle of rest mass m in grams releases an amount of energy E in units of ergs given by the formula:

$$E = mc^2 \text{ [ergs]} \tag{13}$$

where: c is the speed of light = 2.9979×10^{10} [cm / sec].

If Δm is the decrease or increase in mass when a number of particles combine to form the nucleus, then an amount of energy is absorbed or released in the process as:

$$\Delta E = \Delta m\, c^2 \text{ [ergs]} \tag{14}$$

This energy is the binding energy of the particles in the nucleus.

Thus the total annihilation of 1 gm of matter leads to the release of the following amount of energy:

$$\begin{aligned} E &= 1 \times (2.9979 \times 10^{10})^2 \\ &= 8.9874 \times 10^{20} && \text{[ergs]} \\ &= 8.9874 \times 10^{13} && \text{[Joules]} \end{aligned}$$

since: $1 \text{ erg} = 10^{-7}$ Joule.

This is a substantial amount of energy that is equal to about 25 million kilowatt.hours.

Since the mass of the electron is 9.1095×10^{-28} g, the rest mass energy of the electron can be estimated as:

$$\begin{aligned} E &= 9.1095 \times 10^{-28} \times (2.9979 \times 10^{10})^2 \\ &= 8.1871 \times 10^{-7} \quad \text{[erg]} \\ &= 8.1871 \times 10^{-14} \quad \text{[Joule]} \\ &= 8.1871 \times 10^{-14} \text{ Joule} / (1.6022 \times 10^{-13} \text{ Joule/MeV}) \\ &= 0.511 \text{ MeV} \end{aligned}$$

where we used the definition of the electron Volt as the kinetic energy of an electron charge (1.6×10^{-19} Cb) accelerated through a one Volt potential drop:

$$1 \text{ eV} = 1 \text{ electron Volt} = 1.6 \times 10^{-19} \text{ Coulomb.Volt} = 1.6 \times 10^{-19} \text{ Joule.}$$

Since we have earlier established that the atomic mass unit (amu) is equal to 1.66×10^{-24} gm, hence the energy equivalent of 1 amu is:

$$1\ \text{amu} = \frac{1.66 \times 10^{-24}\ [\text{gm}]}{9.1095 \times 10^{-28}\ [\frac{\text{gm}}{\text{electron}}]} \times 0.511\ [\frac{\text{MeV}}{\text{electron}}],$$

or:

$$1\ \text{amu} = 931.481\ \text{MeV} \qquad (15)$$

4.7 THE CURVE OF BINDING ENERGY

The binding energy of a nucleus E_B is defined as the difference between the masses of the Z protons and the N neutrons in the free state and the mass of the nucleus containing the A = Z + N nucleons. We can use the masses of the neutral atoms instead of the nuclear masses, since the masses of the electrons will cancel out. Thus:

$$\begin{aligned} E_B &= Z.M_p + N.M_n - M_A \\ &= Z.(M_p + m_e) + N.M_n - (M_A + Z.m_e) \qquad (16) \\ &= Z.M_H + N.M_n - M \end{aligned}$$

where M_H is the atomic mass of the hydrogen atom of mass number 1, M_n is the mass of the neutron, and M is the mass of the isotope of atomic number Z and mass number A. This binding energy is expressed in amus or in the MeV units.

The binding energy per nucleon is defined as:

$$\frac{E_B}{A} = \frac{(Z M_H + N M_n - M)}{A} \qquad (17)$$

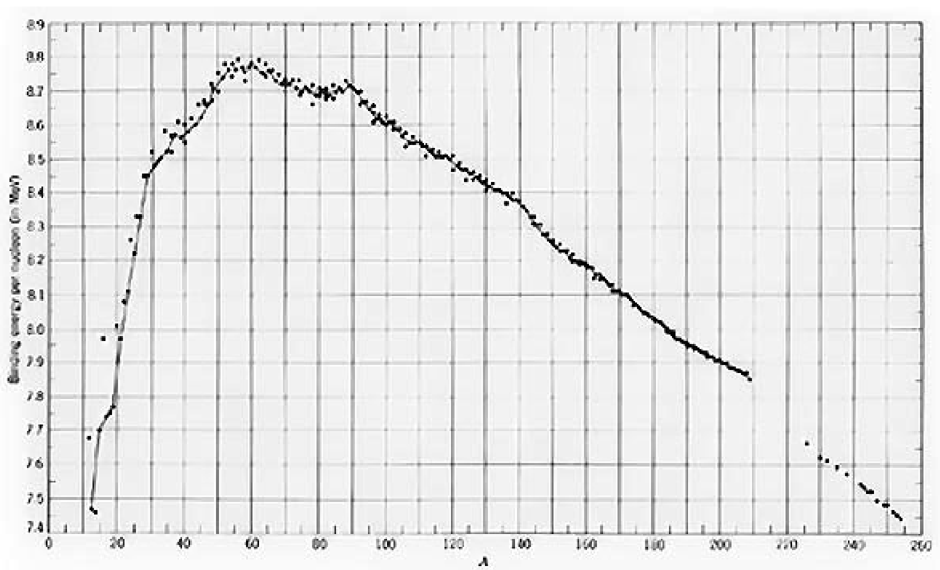


Figure 1. The curve of binding energy in MeV per nucleon as a function of the mass number A.

The graph that shows this quantity as a function of the mass number A, is designated as the "Curve of Binding Energy" and is shown in Fig. 1 for the naturally occurring isotopes.

For values of A larger than 20, the binding energy per nucleon increases slowly from 8 to 8.5 MeV/nucleon around the value of A = 60. It then decreases slowly to about 7.5 MeV/nucleon for the heaviest elements. For the values of A from 1 to 20, there are large variations in the values of the binding energy per nucleon.

The mass defect Δ is defined as the difference between the atomic mass M of an isotope and its mass number A:

$$\Delta = M - A \tag{18}$$

The values of the mass defect are tabulated in nuclear data libraries and are a useful quantity to calculate the atomic mass of any radioactive isotope. They are shown in Fig. 1 for the naturally occurring isotopes and the naturally occurring and artificial isotopes.

The packing fraction is defined as the mass defect of the whole atom per nucleon:

$$F = \frac{\Delta}{A} = \frac{(M - A)}{A} \tag{19}$$

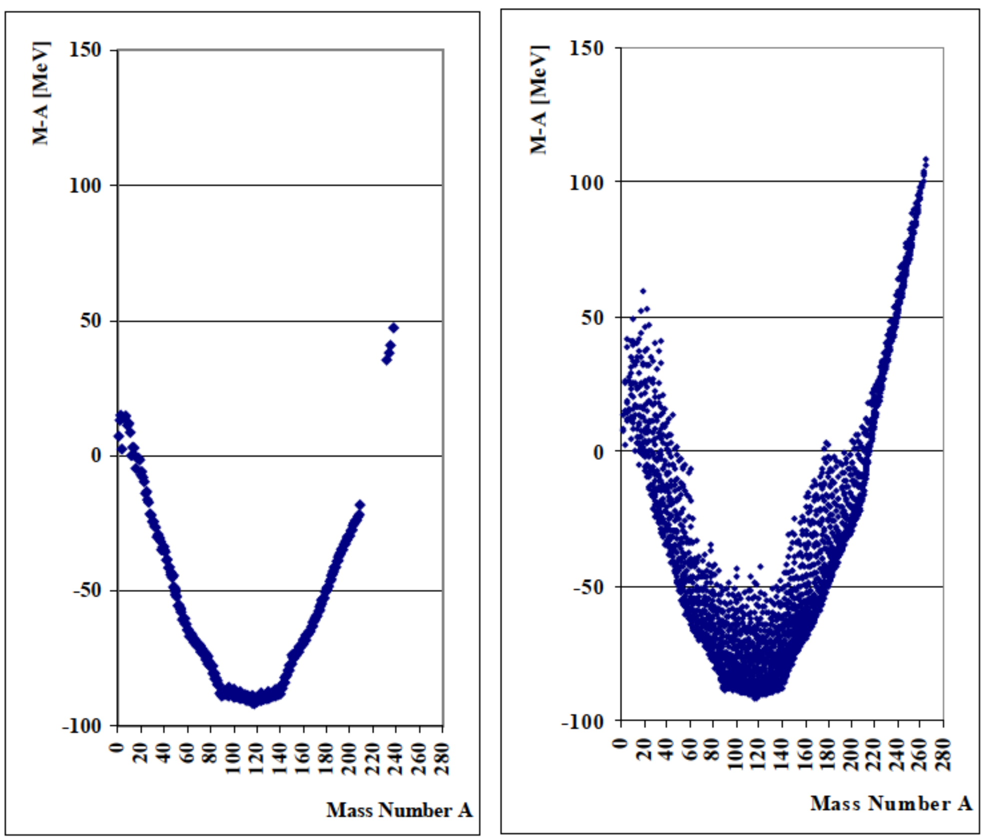


Figure 2. The mass defect diagram for the naturally occurring (left) and the artificially created and naturally occurring isotopes (right).

For mass numbers A between 20 and 180, the packing fraction is negative with a minimum at around 60. For the heavy and light elements, the packing fraction is positive.

The peak of the binding energy per nucleon corresponds to nuclei with mass numbers around 60, containing the iron, cobalt and nickel isotopes. Since the heavy elements like uranium, have a lower binding energy per nucleon, they can be split or fissioned into other nuclei near the peak of the curve, with a release of energy from the fission process.

Similarly, the light nuclei such as deuterium with a low binding energy per nucleon, can undergo a fusion process, leading to more stable nuclei, with an accompanying release of energy from the fusion process.

4.8 STABLE NUCLIDES OCCURRENCE

Nuclides found on Earth are either stable or are radioactive with long half lives in the range of billions of years, since they were produced in a distant past supernova that created the Earth about 4.6 billion years ago. If we plot the number of protons Z against the number of neutrons N, as shown in Fig. 3, we notice an average line of stability clustered around the straight line described by N = Z, among the light nuclei. For heavier nuclei, there is a deviation favoring a larger number of neutrons than protons in the nuclei. This can be attributed to the increased importance of the Coulomb force.

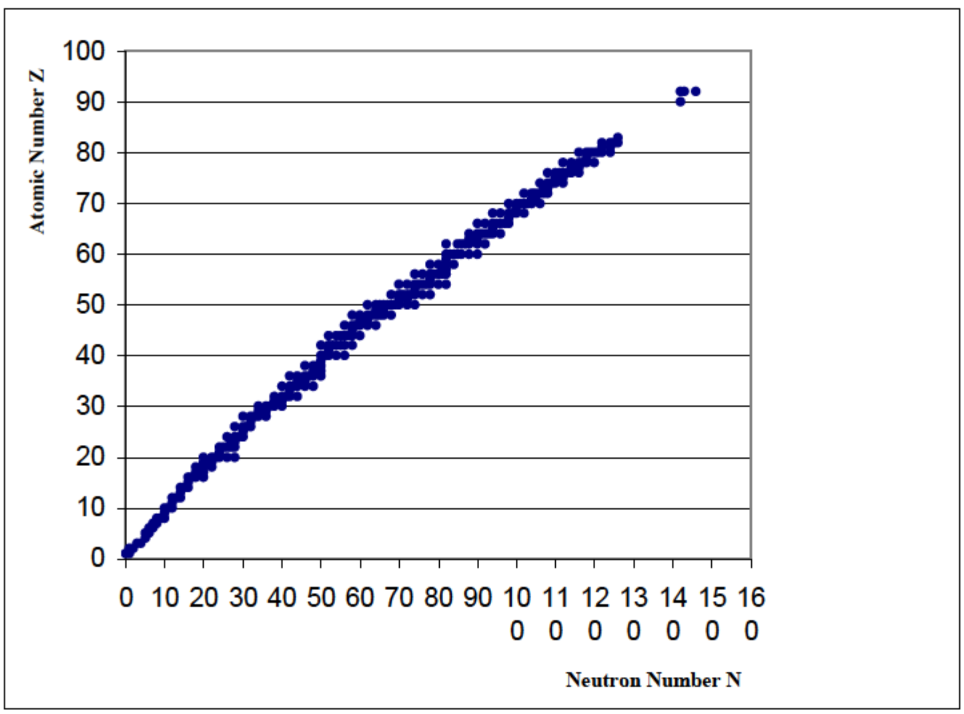


Figure 3. The proton Z neutron N diagram for the naturally occurring isotopes.

There are 279 stable isotopes which can be divided into 4 categories depending on the parities of their atomic number Z, their neutron number N, and their mass number A, as shown in Table 1. It is obvious that the even A, N and Z nuclides predominate as stable nuclides. There are about as many nuclides with odd A with an even number of protons as there are with an odd number of protons. This suggests that the nuclear strong force tying the nucleons in the nucleus does not depend on whether the nucleons are protons or neutrons. Odd Z and odd N stable nuclei are rather rare. Their scarcity can be attributed to the pairing energy between nuclei in the same nuclear shell.

Table 1. Stable nuclides distribution as even and odd numbers nuclei.

Number of nuclide	168	57	50	4	Total 279
Neutron number N	Even	Odd	Even	Odd	
Atomic number Z	Even	Even	Odd	Odd	
Mass number A	Even	Odd	Odd	Even	

In Fig. 3, elements along the diagonals perpendicular to the N = Z line possess the same mass number A and these nuclides are designated as *isobars*.

Nuclides with the same number of neutrons N, are designated as *isotones*.

For odd A, only one stable isobar exists except for A = 113 and 123. For even A, only even N and even Z stable nuclides exist, except for A = 2, 6, 10, and 14, which have odd Z and odd N isobars.

4.9 RADIOACTIVE NUCLIDES

The use of nuclear reactors and particle accelerators has led to the generation of more isotopes of the elements than occur in nature. The terms *isomer* or *metastable states*, refers to isotopes that have different radioactive properties in different long-lived energy states. At present there are 2004 nuclides and 423 isomers known. Of these only 279 are stable or naturally occurring isotopes.

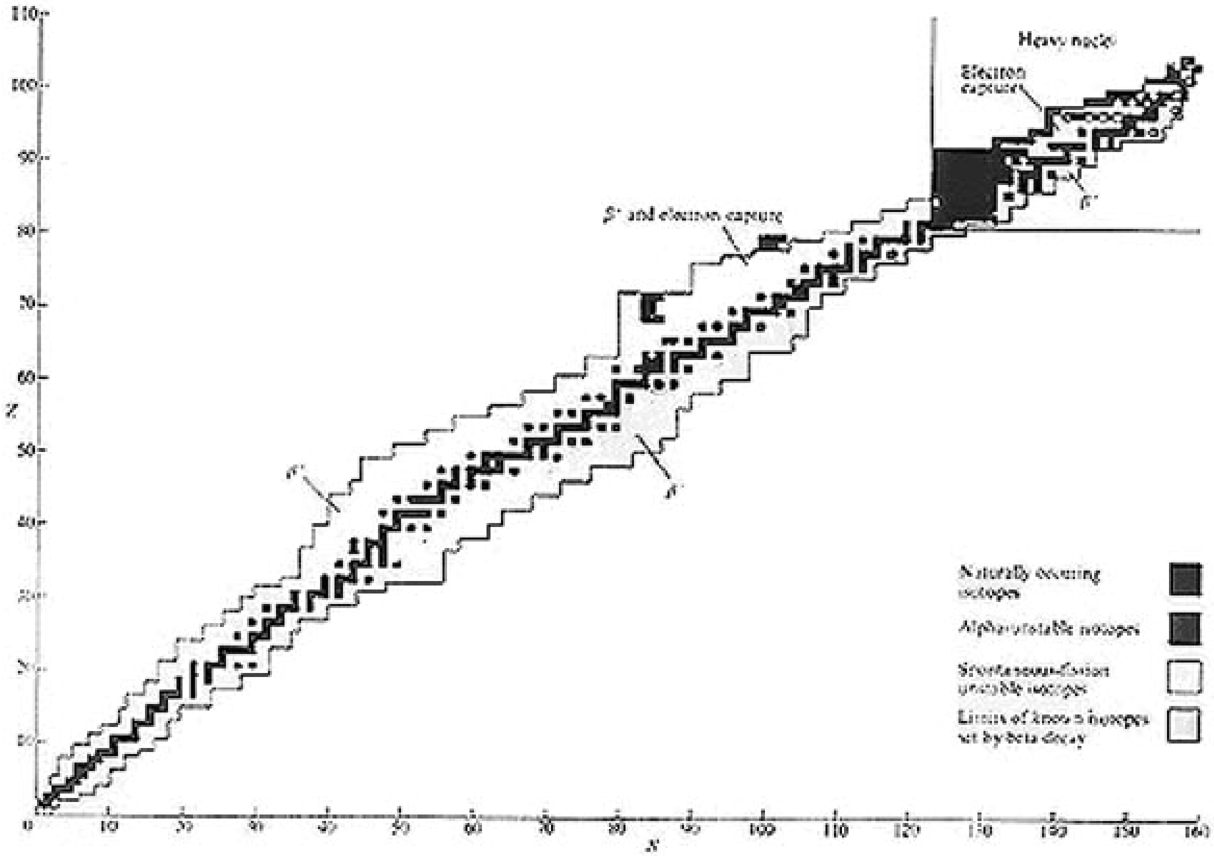
The known nuclides are classified in the Chart of the Nuclides shown in Fig. 4 in the same way that the chemical elements are classified in the periodic table of the elements or the Mendeleev table. The black dots correspond to the naturally occurring isotopes which constitute a band below the Z = N straight line.

Nucleons above that band are proton rich, and the nuclei try to reach the region of stability by emitting a positive electron or positron, or by the process of electron capture, where an inner shell electron is captured by the nucleus.

Nucleons below the stability range are neutron rich, and try to reach the stability region by emitting a negative electron or beta particle. Some of the heavy elements emit an alpha particle or a helium nucleus. The radioactive decays can leave the nucleus in an excited state, which is accompanied by gamma rays emission to reach the ground states.

EXAMPLE

Cesium^{137} is a product of the fission process and has a half-live of 30.17 years. It decays by negative beta emission to a short-lived daughter barium^{137} with a half live of 2.552 minutes. The latter emits a gamma ray photon with energy of 0.66164 MeV, to the stable barium^{137} isotope. The ground state is a stable element, which can be considered as having an infinite half-life.



Figue 4 The proton neutron diagram for the artificially created and the naturally occurring isotopes.

4.10 REST MASS ENERGY, KINETIC ENERGY AND TOTAL ENERGY

From the Special Theory of Relativity, for a relativistic particle, the total energy is expressed as:

$$\text{Total Energy} = \text{Kinetic Energy} + \text{Rest Mass Energy} \tag{20}$$

Expressed in terms of the mass of the particle m and the square of the speed of light, this equation can be written as;

$$mc^2 = E_k + m_0 c^2 \tag{21}$$

From this equation the kinetic energy of a relativistic particle is:

$$E_k = mc^2 - m_0c^2 = (m - m_0)c^2 \tag{22}$$

The relativistic mass of the particle depends on the ratio of its velocity v to the speed of light c as

$$\beta = \frac{v}{c} \tag{23}$$

It can be written as:

$$m = \frac{m_0}{\sqrt{1-\beta^2}} \tag{24}$$

Substituting for the relativistic mass Eqn. 24 into the expression for the kinetic energy Eqn. 22, we get:

$$\begin{aligned} E_k &= \left(\frac{m_0}{\sqrt{1-\beta^2}} - m_0 \right) c^2 \\ &= \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) m_0 c^2 \end{aligned} \tag{25}$$

Using the binomial expansion:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n!}{(n-r)!r!}x^r + \ldots \tag{26}$$

For small values of the ratio of the particle's speed to the speed of light, we can expand the square root in Eqn. 25 as:

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{1}{(1-\beta^2)^{\frac{1}{2}}} \cong 1 + \frac{1}{2}\beta^2 + \ldots \tag{27}$$

Substituting from Eqns. 23 and 27 into Eqn. 25 we obtain for the kinetic energy for a particle with a velocity that is small compared to the speed of light:

$$E_k = \left(1 + \frac{1}{2}\beta^2 + \ldots - 1\right) m_0 c^2$$

$$= \frac{1}{2}\left(\frac{v}{c}\right)^2 m_0 c^2 = \frac{1}{2} m_0 v^2 \qquad (28)$$

This shows that the kinetic energy of classical mechanics is a special case of the relativistic kinetic energy when the particle's energy is much less than the speed of light.

4.11 NUCLEAR REACTIONS Q VALUES

Let us consider a nuclear reaction between two reactants a and b, leading to two products c and d:

a + b → c + d

reactants → products.

We can write a mass-energy balance involving the kinetic energies, KE of the particles in the reaction as well as their rest mass energies as:

$$(\text{KE} + \text{rest mass energies})_{\text{reactants}} = (\text{KE} + \text{rest mass energies})_{\text{products}} \qquad (29)$$

Explicitly, we denote the kinetic energies as E, the masses as M, and the rest mass energies as mc^2. The mass energy balance can thus be expressed as:

$$(E_a + m_a c^2) + (E_b + m_b c^2) \rightarrow (E_c + m_c c^2) + (E_d + m_d c^2) \qquad (30)$$

Gathering the kinetic energies on the left hand side (LHS) of the equation and the masses at the right hand side (RHS) of the equation we can define the Q value of the reaction as the difference between the kinetic energies of the products minus the kinetic energies of the reactants, which is also equal to the difference between the masses of the reactants minus the masses of the products:

$$Q = (E_c + E_d) - (E_a + E_b) = (m_a c^2 + m_b c^2) - (m_c c^2 + m_d c^2) \qquad (31)$$

Since it is easier to access the data about the masses of the nuclei undergoing the reaction rather than measuring their velocities to calculate their kinetic energies, the Q values for nuclear reactions are calculated by using the masses of the reactants and products:

$$Q = [(m_a + m_b) - (m_c + m_d)]\, c^2 \qquad (32)$$

The masse in Eqn, 32 are the masses of the nuclei, whereas the masses that are measured experimentally using mass spectrometers are the masses of the neutral atoms, or the masses of the nuclei and the surrounding electrons. To account for the electron masses, we use the conservation of charge equation for the nuclear reaction considered as:

$$Z_a + Z_b = Z_c + Z_d \tag{33}$$

By considering the mass of the electron as m_e we can rewrite the expression for the Q value in Eqn. 22 in terms of the neutral atoms masses, not just the nuclei masses as:

$$Q = \{[(m_a + Z_a m_e) + (m_b + Z_b m_e)] - [(m_c + Z_c m_e) + (m_d + Z_d m_e)]\}\, c^2 \tag{34}$$

When the value of Q is positive, the reaction is called exothermic, and would lead to a positive energy release. When the value Q is negative, the reaction needs to be fed energy to proceed, normally in the form of kinetic energy of the reactants.

EXAMPLE

For the DT fusion reaction between the two isotopes of hydrogen deuterium (D) and tritium (T):

$$_1D^2 + {}_1T^3 \rightarrow {}_0n^1 + {}_2He^4 \tag{35}$$

Data mining the appropriate data warehouses such as the Chart of the Nuclides for the neutral atom masses of the reactants and products yields:

$m({}_1D^2) = 2.014102$ amu,
$m({}_1T^3) = 3.016049$ amu,
$m({}_0n^1) = 1.008665$ amu,
$m({}_2He^4) = 4.002604$ amu.

Using the mass energy equivalence that we derived earlier where 1 amu is equivalent to 931.481 MeV of energy, we get for the energy release or Q value from this reaction as:

$$\begin{aligned} Q &= \{[(2.014102) + (3.016049)] - [(4.002604) + (1.008665)]\}\ 931.481 \\ &= 0.018882 \text{ x } 931.481 \text{ (amu). (MeV/amu)} \\ &= 17.6 \text{ MeV.} \end{aligned}$$

4.12 REACTIONS INVOLVING POSITRON EMISSION

Positron decay leaves the product nucleus with an atomic number diminished by one unit, leaving the mass number A unchanged, and is accompanied by a neutrino emission. In general, the calculation of the Q values is equal to the difference between the masses of the reactants neutral atom masses, and the products masses. An exception exists in the case of positron emission. Let us consider a positron emission reaction:

$$_{6}C^{11} \rightarrow {}_{5}B^{11} + {}_{+1}e^{0} + \nu \qquad (36)$$

Ignoring the mass of the neutrino, but not the electron masses, the Q value is written in terms of the nuclear masses as:

$$Q = [m(C) - 6m_e] – [m(B) – 5m_e] - m_e$$

This can be rewritten as:

$$Q = m(C) - m(B) – 2m_e \qquad (37)$$

Thus two electron masses must be subtracted from the difference in the neutral atom masses.

For the reaction to occur spontaneously:

$$Q > 0,$$

implying that:

$$m(C) - m(B) > 2m_e > 2 \times 0.51 \text{ MeV} > 1.02 \text{ MeV} \qquad (38)$$

Thus the difference of masses between parent and daughter should be larger than 2 electron masses for positron decay to occur.

4.13 REACTIONS INVOLVING ORBITAL ELECTRON CAPTURE

Orbital electron capture decreases the atomic number of the nucleus by one unit while leaving the mass number unchanged, just like positron decay. Let us consider the K electronic shell electron capture reaction:

$$_{29}Cu^{64} + {}_{-1}e^{0} \rightarrow {}_{28}Ni^{64} + \nu \qquad (39)$$

The neutrino acquires the entire energy Q released by the reaction leaving little for the recoil nucleus. We must notice that the captured electron releases its entire mass when it is absorbed in the nucleus minus the mass equivalent of the binding energy of the electron in the atomic shell, E_B. Therefore the Q value can be calculated as:

$$Q = [m(Cu) – 29m_e] – [m(Ni) – 28m_e] + (m_e – E) \qquad (40)$$

Thus:

$$Q = m(Cu) – m(Ni) - E_B \qquad (41)$$

Since E_B is small, for instance 0.024 MeV for the K shell of palladium^{103} it can be ignored and the Q value can be calculated on the basis of the neutral atomic masses.

Electron capture competes with positron decay. However positron decay needs at least the energy equivalent of two electron masses to occur, whereas electron capture needs the energy equivalent of the binding energy of the electron in the K shell.

Since electron capture leaves an empty inner shell, characteristic x rays of the product nucleus is always emitted, and is used in addition to Auger electrons and the recoil of the product nucleus to detect the electron capture process.

Auger electrons are emitted as a result of a radiationless transition, for instance, instead of a K x-ray being emitted when an L electron goes to the K shell, the energy is used to eject an electron from the L level.

4.14 INTERNAL CONVERSION

Internal conversion occurs when the energy of an excited nuclear state is transferred to an atomic electron from the K or L shell, ejecting it from the atom. Auger electrons are internal conversion electrons. Internal conversion occurs as an alternative to gamma ray emission from the nucleus.

The kinetic energy of the ejected electron E_e is equal to the excitation energy of the nucleus minus the binding energy of the electron in its shell:

$$E_e = E_{excitation} - E_B \tag{42}$$

Internal conversion is prevalent in heavy nuclei and increases as Z^3 but decreases with $E_{excitation}$. Gamma decay predominates in light nuclei. Examples of nuclei with internal conversion are Cs^{137} and Tc^{99m}.

4.15 REACTIONS INVOLVING NEGATIVE BETA DECAY

These reactions occur in the neutron rich nuclei. The result is a nucleus with an atomic number increased by one unit, an unchanged mass number. An example is the reaction involving the decay of the tritium isotope of hydrogen:

$${}_1T^3 \rightarrow {}_{-1}e^0 + {}_2He^3 + \nu^* \tag{43}$$

The Q value of the reaction is:

$$Q = [m(T) - m_e] - [m(He) - 2m_e] - m_e \tag{44}$$

from which:

$$Q = m(T) - m(He) \tag{45}$$

Thus the neutral atom masses of the reactants and the product can be used ignoring the mass of the emitted electron.

4.16 REACTIONS INVOLVING ALPHA DECAY

Alpha decay occurs among the heavy nuclei. An example is the reaction that transforms the man-made Pu^{239} into the normally naturally occurring U^{235}:

$$ {}_{94}Pu^{239} \rightarrow {}_{2}He^{4} + {}_{92}U^{235} \tag{46}$$

$$Q = [m(Pu) - 94m_e] - [m(U) - 92m_e] - [m(He) - 2m_e] \tag{47}$$

from which:

$$Q = m(Pu) - m(U) - m(He) \tag{48}$$

In this case the mass of the emitted alpha particle must be subtracted as a product nucleus, in contrast to negative beta decay where the mass of the emitted electron is not.

4.17 ENERGY PARTITIONING IN NUCLEAR REACTIONS, CONSERVATION OF MOMENTUM

It is of interest to be able to calculate how the energy release apportions itself among the products of a nuclear reaction such as the one described by Eqn. 32. We need to supplement the conservation of mass energy equation by the equation of conservation of momentum. Considering the particles as initially at rest, forming a compound nucleus then splitting apart in two opposite directions, conservation of momentum requires that:

$$m_c v_c = m_d v_d \tag{49}$$

Squaring both sides yields:

$$m_c^{\ 2} v_c^{\ 2} = m_d^{\ 2} v_d^{\ 2}$$

or:

$$m_c\, m_c\, v_c^{\ 2} = m_d\, m_d\, v_d^{\ 2}$$

Multiplying both sides by ½ yields:

$$m_c .\ \tfrac{1}{2}\ m_c v_c^{\ 2} = m_d\, \tfrac{1}{2}\ m_d v_d^{\ 2}$$

Noting that the kinetic energies of the product particles are:

$$E_c = \tfrac{1}{2}\ m_c v_c^{\ 2}$$

$$E_d = \frac{1}{2} m_d v_d^{\;2}$$

We can write:

$$m_c E_c = m_d E_d \tag{50}$$

Now, conservation of energy for particles initially at rest implies that:

$$Q = E_c + E_d \tag{51}$$

or:

$$E_c = Q - E_d \tag{52}$$

Combining Eqns. 50 and 52 yields:

$$m_c (Q - E_d) = m_d E_d$$

$$m_c Q = (m_c + m_d) E_d$$

Which implies that:

$$E_d = \frac{m_c}{m_c + m_d} Q \tag{53}$$

and similarly:

$$E_c = \frac{m_d}{m_c + m_d} Q \tag{54}$$

This leads to the interesting result that the energy release apportions itself in *inverse* proportion to the masses of the product nuclei.

EXAMPLE

We consider the fusion reaction in Eqn. 35 whose Q value is 17.6 MeV. If we want to calculate the kinetic energies of the neutron and the helium nucleus, we can use as an approximation the mass numbers instead of the actual atomic masses to yield from Eqns. 53 and 54:

$$E_{neutron} = \frac{4}{4+1} 17.6 = \frac{4}{5} 17.6 = \frac{80}{100} 17.6 = 14.08 \ MeV$$

$$E_{helium} = \frac{1}{4+1}17.6 = \frac{1}{5}17.6 = \frac{20}{100}17.6 = 3.52\ MeV$$

These are approximation to the more exact values of 14.06 and 3.54 MeV. It must be noted that the lighter particle, the neutron, carries 80 percent of the released nuclear energy, whereas, the heavier particle, the helium nucleus, carries only 20 percent of the energy release from the reaction.

4.18 THE FISSION PROCESS

The absorption of a neutron in some of the fissile nuclei with even Z and odd A, even with zero energy can cause them to fission into two fission product and a few neutrons. The two fission products primarily share the energy release. The neutrons released can leak from the system, get absorbed in the structural materials, or be available to fission other fissile nuclides and generate a fission chain reaction as shown in Fig. 5.

A typical reaction is the fission of the U^{235} isotope, which could yield:

$$_{0}n^{1} + {}_{92}U^{235} \rightarrow {}_{36}Kr^{97} + {}_{56}Ba^{137} + 2{}_{0}n^{1} + Q \qquad (55)$$

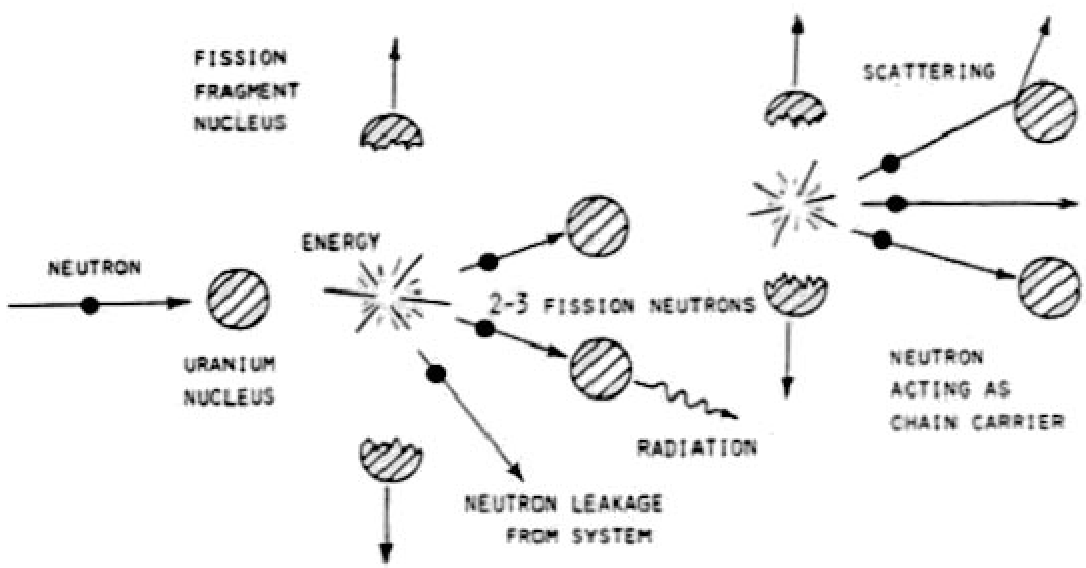


Figure 5. Schematic of a fission chain reaction showing the fission products and the prompt neutrons.

The fission products yields appear with asymmetric values of the mass number as shown in the fission yield graph in Fig. 6 for fission with thermal neutrons at a speed of 2,200 m/sec. As the energy of the neutron increases say to 14.08 MeV as the neutrons originating from the DT fusion reaction considered earlier, the two humps flatten out, with the yield curve rising up in its central section.

EXAMPLE

To calculate the Q value of the fission reaction in Eqn. 55 we use the neutral atom masses:

$$
\begin{aligned}
m\,({}_0n^1) &= 1.00867 \text{ amu}\\
m\,({}_{92}U^{235}) &= 235.04390 \text{ amu}\\
m\,({}_{36}Kr^{97}) &= 96.92120 \text{ amu}\\
m\,({}_{56}Ba^{137}) &= 136.90610 \text{ amu}
\end{aligned}
$$

from which the Q value of the reaction is:

$$
\begin{aligned}
Q &= [m\,({}_0n^1) + m\,({}_{92}U^{235}) - \{m\,({}_{36}Kr^{97}) + m\,({}_{56}Ba^{137})\} - 2m({}_0n^1)]\, c^2\\
&= [(1.00867 + 235.04390) - (96.92120 + 136.90582) - 2 \text{ x } 1.00867] \text{ x } 931.481\\
&= (236.05257 - 233.82702 - 2.01734) \text{ x } 931.481\\
&= 0.20821 \text{ x } 931.481 \text{ [amu] [MeV/amu]}\\
&= 193.94 \text{ MeV}\\
&= 193.94 \text{ x } 1.52 \text{ x } 10^{-16} \text{ [MeV] [BTU/MeV]}\\
&= 2.95 \text{ x } 10^{-14} \text{ BTU} \qquad (56)
\end{aligned}
$$

Thus it can be concluded that a single fission event leads to the release of 194 MeV or about 200 MeV of energy. This is about 10 times the energy release from the DT fusion reaction at 17.6 MeV. Thus fission can be construed to be energy rich per reaction compared with fusion as typified by the DT fusion reaction.

It is of great interest from the perspective of fission power generation from fission to estimate the energy release from a given mass g of fissile material. Applying Avogadro's law from Eqn. 11 yields the number of nuclei of U^{235} in grams as:

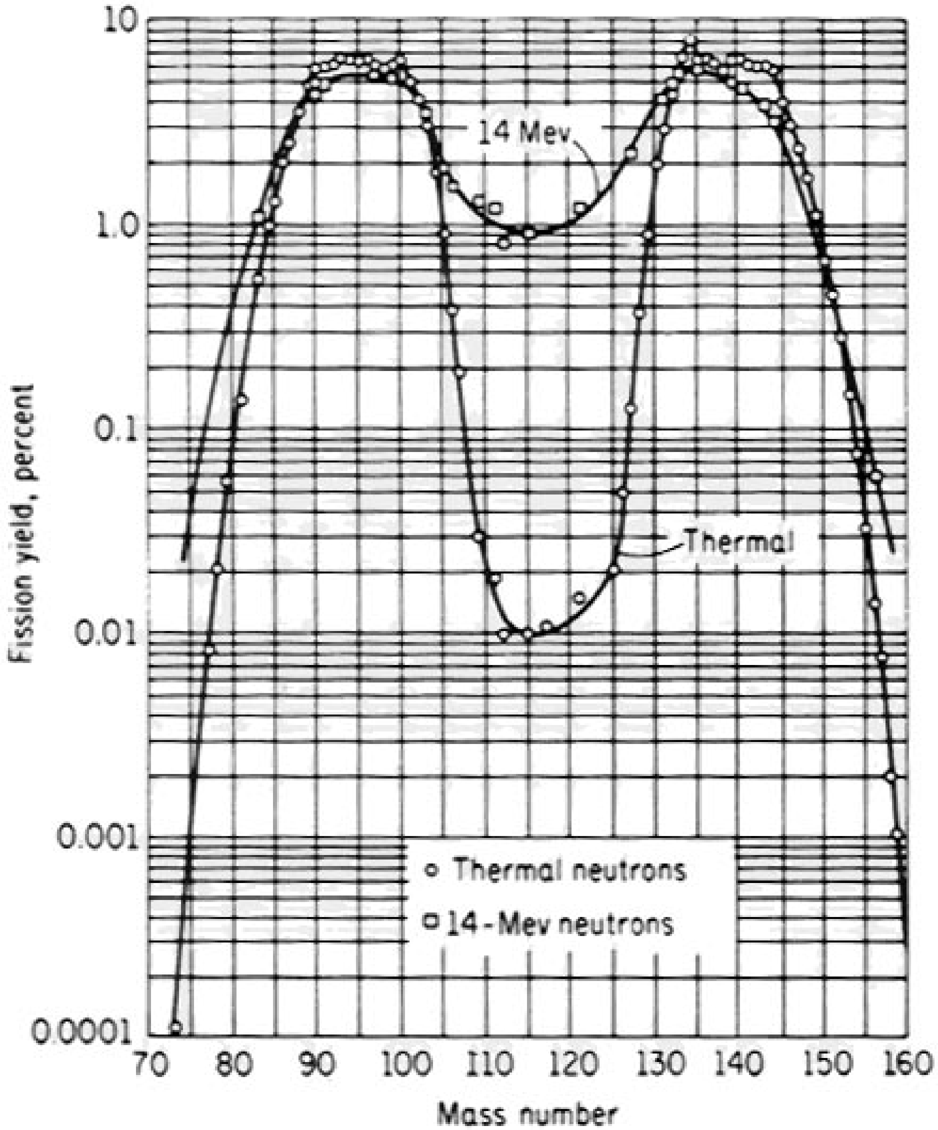


Figure 6. The fission yield curve from thermal and fast fissions of U^{235}.

$$N(U^{235}) = \frac{g}{M} A_v = \frac{g}{235.0439} 0.60225 \times 10^{24} [nuclei]$$

If we consider that the fission event yields about:

200 [MeV/nucleus]

of which 10 MeV are carried out by the antineutrinos and are not extractable, then we consider that the extractable energy per fission event is:

$$200 - 10 = 190 \text{ [MeV/nucleus]}$$

The complete fission of g grams of U^{235} will release the energy E:

$$\begin{aligned}
E &= N(U^{235}) \times 190[nuclei][MeV / nucleus] \\
&= \frac{g}{235.0439} \times 0.60225 \times 10^{24} \times 190[MeV] \\
&= 0.487 \times 10^{24} g[MeV] \\
&= 2.162 \times 10^{4} g[kW.hr] \\
&= 900.6 g[kW.day] \\
&= 0.9006 g[MW.day]
\end{aligned} \tag{57}$$

An expression for the power release can be deduced for the complete fission of g' [gm/day] as:

$$P = \frac{E}{t} = 0.9 g'[MWth] \tag{58}$$

This implies that in a fission reactor the complete fission of 0.9 gm of U^{235} per day corresponds to about 1 MWth of thermal power production.

EXAMPLE

A typical fission power reactor produces 3,000 megawatts (MWth) of thermal power. If it were completely burning its fuel inventory in U^{235}, it would fission:

3,000 [MWth] x 0.9 [gm/(day.MWth)] = 2,700 [gm/day] = 2.7 [kgs/day]

The amount that would be fissionned in a year would be:

2.7 x 365 [kgs/day][days/year] = 9855 [kgs/year] = 0.9885 [metric tonne/year]

Thus a minimum amount of about 1 metric ton of U^{235} fuel is needed per year for a typical 3,000 MWth fission power reactor.

4.19 SPONTANEOUS FISSION

Some heavy nuclei decay in a process where the nucleus breaks up into two intermediate mass fragments and several neutrons. It occurs in with nuclei with mass number A > 230.

Since the maximum binding energy per nucleon occurs at A = 60, nuclides above A > 100 are unstable with respect to spontaneous fission, sine a condition for spontaneous fission is:

$$m(A,Z) > m(A',Z') + m(A-A',Z-Z')$$

in the spontaneous fission reaction:

$$_{Z}X^{A} \rightarrow {}_{Z'}X^{A'} + {}_{Z-Z'}X^{A-A'}$$

Because of the high Coulomb barrier for the emission of the fission fragments, spontaneous fission is only observed in the heaviest nuclei.

4.20 ENERGY DISTRIBUTION OF NEUTRONS FROM FISSION

The average number of neutrons emitted per fission of a U^{235} nucleus by slow or thermal neutrons is:

$$\nu = 2.47 \pm 0.03[\frac{neutrons}{fission}] \qquad (59)$$

These fission neutrons possess a distribution in energy shown in Fig. 7.

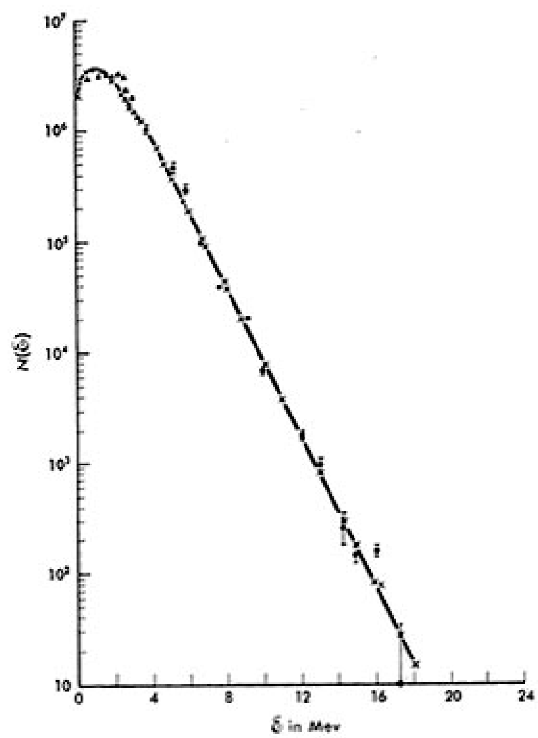


Figure 7. Energy distribution of fission neutrons from U^{235}.

The curve has a maximum or most probable energy at 0.75 MeV and an average energy of 1.98 MeV. It is represented by the semi empirical equation, known as the Watt's curve as:

$$N(E) = C\sinh(2E)^{\frac{1}{2}} e^{-E} \qquad (60)$$

4.21 DELAYED NEUTRONS FROM FISSION

Neutron emission from the fission products can occur on a delayed basis, in addition to those prompt neutrons emitted at the time of fission. Delayed neutron emission occurs if the parent nucleus had already decayed through beta decay, but is still left in an excited state with an energy that exceeds the binding energy of a neutron of this nucleus. Since the neutron emitting nucleus was formed in a beta decay process, the neutron activity will have the same half-life of the parent nuclide. Examples of delayed neutrons emitting fission products are Br^{87} and I^{137}, whose decay diagrams are shown in Fig. 8.

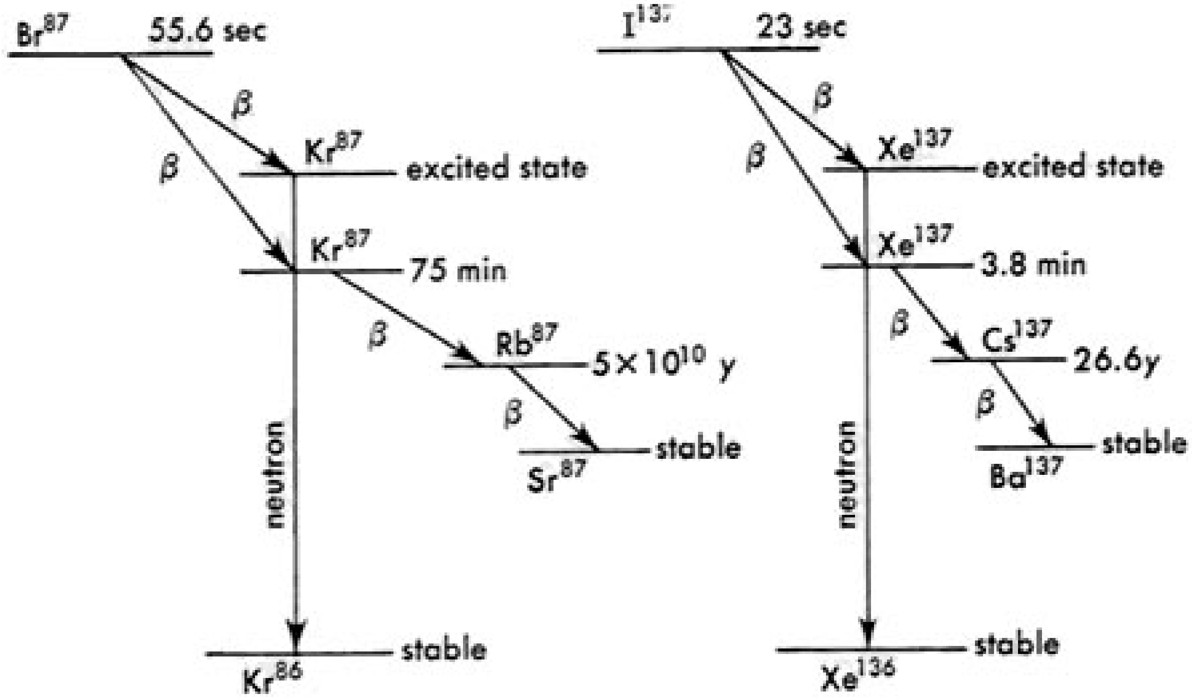


Figure 8. Delayed neutrons emission from the two precursors Br^{87} and I^{137}.

Delayed neutrons are important in the process of building control systems for controlling fission chain reactions, and the operation of fission reactors.

4.22 THE LIQUID DROP MODEL FOR NUCLEI

The radius of a nucleus is proportional to its mass number to the 1/3 power and is given by:

$$R = 1.25 \times 10^{-13} A^{1/3} \text{ [cm]} \quad (61)$$

The volume of a nucleus can be estimated from the equation of its radius as:

$$V = 4/3\ \pi R^3 = 4/3\ \pi\ (1.25 \times 10^{-13} A^{1/3})^3$$

$$= 4/3\ \pi\ (1.25 \times 10^{-13})^3 \times A \text{ [cm}^3\text{]}. \quad (62)$$

This implies that the volume of the nucleus is proportional to A. The density of nuclear matter, or the number of nucleons per unit volume of the nucleus is given by:

$$A/V = 1/\{4/3\ \pi\ (1.25 \times 10^{-13})^3\} = \text{constant}, \quad (63)$$

for all nuclei, whether they are large or small. This uniform density suggests that nuclei are similar to a drop of liquid (Fig. 9). This analogy accounts for many of the physical properties of nuclei.

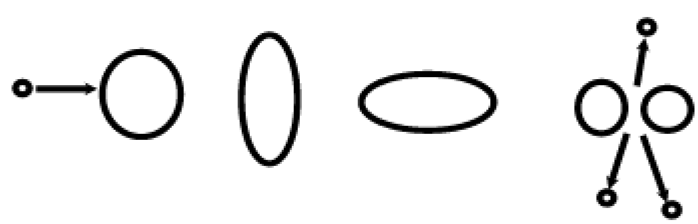


Figure 9. Liquid drop model of the neutron fission of a fissile nucleus.

Both the volume and the total binding energy of nuclei are nearly proportional to the total number of nucleons present A. This implies that nuclear matter is incompressible. It also implies that nuclear forces must have a saturation character, that is, a nucleon in a nucleus can interact with only a small number of neighboring nuclei, just as an atom in a liquid is strongly bound to its neighbor atoms.

The nuclear binding energy of a nucleus is the difference between its mass M expressed in energy units, and the masses of its constituent nuclei as protons, electrons and neutrons. Expressed mathematically this is:

$$E = M - \{ Z M_p + Z M_e + (A-Z) M_n \} \text{ [MeV]}$$

$$= M – \{ Z M_H +(A-Z)M_n\} \text{ [MeV]} \quad (64)$$

where: M_H is the mass of the hydrogen atom, in energy units,
and: M_n is the mass of the neutron, in energy units.

Using a semi-empirical approach, an equation can relate the binding energy, or total mass, of any nucleus to its nuclear composition in terms of the atomic number (or number of protons) Z, and its mass number (or total number of nucleons) A.

For values of A > 40, the energy required to dissociate a nucleus into its constitutive nucleons is given by:

$$E = 14.0A – 13.1 A^{2/3} – 0.585 Z(Z-1) A^{-1/3} – 18.1 (A-2Z)^2A^{-1} + \delta A^{-1} \text{ [MeV]} \quad (65)$$

This equation can generate a three dimensional plot of the binding energies of all nuclei as a function of A and Z.

The term {14.0 A}, is the dominant term, and expresses the fact that the binding energy is proportional to the total number of nucleons A, and is a consequence of the short range and the saturation character of the nuclear forces. When four nucleons, 2 protons and 2 neutrons are joined together, these forces become saturated as in the cases of the nuclei of ${}_2He^4$, ${}_6C^{12}$, and ${}_8O^{16}$.

The term $\{– 13.1 A^{2/3}\}$ expresses the fact that the surface of a nucleus has unsaturated forces, and consequently, a reduction in the binding energy proportional to that surface is to be expected. Since the radius of the nucleus is proportional to $A^{1/3}$, the surface of the nucleus is proportional to $A^{2/3}$. When the size, or radius, of the nucleus increases the surface to volume ratio of the nucleus:

$$S / V = (4 \pi R^2) / (4/3 \pi R^3) = 3 / R \quad (65)$$

decreases. Thus this term becomes less dominant as the size of the nucleus increases.

The term $\{– 0.585 Z(Z-1) / A^{1/3}\}$ expresses the Coulomb repulsive forces between the protons. Each of the Z protons interacts with the other (Z-1) protons over a distance of a nucleus radius, which is in turn proportional to $A^{1/3}$. This term is mostly dominant for Z > 20, and accounts for the fact that stable nuclei contain more neutrons than protons in the Chart of the Nuclides.

The term $\{– 18.1 (A-2Z)^2 / A\}$ is a symmetry term accounting for the fact that most stable nuclei tend to have an equal number of neutrons and protons. Thus a term containing the absolute value of the difference between the number of neutrons and protons:

$$| N - Z | = | A – Z – Z | = | A – 2Z| \quad (66)$$

is added. The binding energies of unstable light nuclei show that this effect is symmetric around the value N = Z. The 1/A dependence comes about from the binding energy contribution per neutron-proton pair. This is proportional to the probability of having such a pair within a volume, which is determined by the range of the nuclear forces. This probability is in turn inversely proportional to the nuclear volume that is proportional to A.

The term $\{ \delta / A\}$ expresses the fact that the binding energies depend upon whether N and Z are even or odd.

For the most stable nuclei,

Z is even, N is even, then: $\delta = 132.0$ (67)

when:

Z is even, N is odd, then: $\delta = 0.0$
Z is odd, N is even then: $\delta = 0.0$
Z is odd, N is odd then: $\delta = -132.0$

The values of δ given above are suitable for the range A > 80. For A < 60, a lower value in the range of $|\delta| = 65$ should be used. This term is called the pairing term since it is related to the tendency of two like particles to complete an energy level in the nucleus by pairing of opposite nuclear spins.

The difference in the stabilities of these four types of nuclei is expressed in the stable nuclides. There exists as shown in Table 1:

	168	stable even Z, even N nuclei,
	57	stable even Z, odd N nuclei,
	50	stable odd Z, even N nuclei,
and only:	4	stable odd Z, odd N nuclei.

The latter category includes the following nuclei:

$${}_1D^2,\ {}_3Li^6,\ {}_5B^{10},\text{ and }{}_7N^{14}.$$

The greater stability of even-even nuclei with filled energy states is clear as apparent from their greater abundance relative to other nuclei. Elements of even Z are 10 times more abundant than elements of odd Z. For these elements with even Z, the isotopes of even N, and consequently also even A, account for 70 to 100 percent of each element, with beryllium, xenon, and dysprosium being exceptions.

The binding energy curve which has a maximum at A = 60, displays an increasing trend before the maximum, followed by a decreasing trend beyond the maximum. This is caused by the relative contributions of surface energy, decreasing with A, and the Coulomb and symmetry energies, which are increasing with A.

It is possible to express the masses of nuclei in terms of the binding energy equation in the form:

$$M = \{ Z M_H + (A-Z)M_n\} - E \quad [\text{MeV}] \qquad (68)$$

By substituting the expression for the binding energy E, we can obtain the semi-empirical mass equation as:

$$M = 925.55A - 0.78Z + 13.1\,A^{2/3} + 0.585\,Z(Z-1)\,A^{-1/3} + 18.1\,(A-2Z)^2 A^{-1} - \delta A^{-1}\ [\mathrm{MeV}] \quad (69)$$

This equation is a quadratic in Z and can be rewritten as:

$$M = aZ^2 + bZ + c - \delta A^{-1} \quad (70)$$

where

$$a = 0.585\,A^{-1/3} + 72.4\,A^{-1},$$
$$b = -0.585\,A^{-1/3} - 73.18,$$
$$c = 943.65A + 13.1\,A^{2/3}.$$

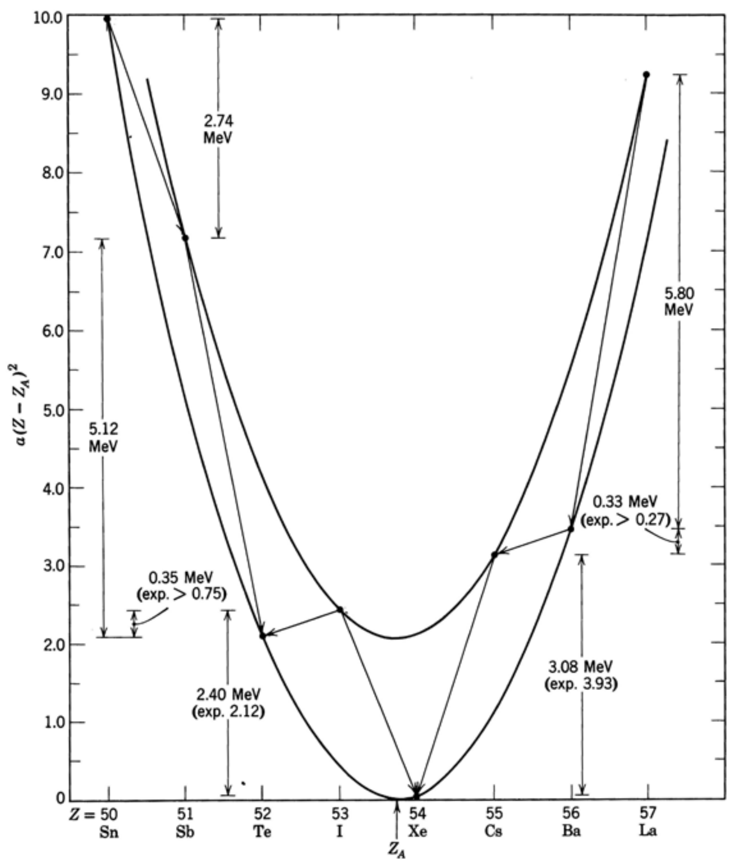


Figure 10. Double parabolas for the mass number A=128. The beta decay energies for the neighboring isobars are shown compared to the experimental values in parentheses.

For a given constant A, the coefficients a, b and c are constant, and we can generate a parabola for a given value of δ. For odd value of A, $\delta = 0$, and we obtain a single parabola. For an even value of A, $\delta = +/-132$, and we obtain two parabolas displaced from each other by a value of $(2\delta/A)$ along the energy axis (Fig. 10).

The minimum mass, or maximum binding energy for a given value of A is obtained at a given value of Z_{min}, by taking a partial derivative of M with respect to Z, with A as a constant, and equating the derivative to zero:

$$\delta M/\delta Z = 2a\, Z_{min} + b = 0,$$

from which:

$$Z_{min} = -b/2a.$$

We can use the expression for Z_{min} to rewrite the mass equation for M as:

$$M = a(Z-Z_{min})^2 - \delta A^{-1} + \{c - (b^2/4a)\} \text{ [MeV]} \qquad (71)$$

This equation is found in the literature to express the total atomic mass of a nucleus according to the liquid drop model.

4.23 THE SHELL MODEL OF THE NUCLEUS

High stability and high abundance with respect to the neighboring nuclides are characteristics for nuclei for which the *magic numbers* for N or Z are:

2, 8, 20, 28, 50, 82, 126.

The magic nuclei are more tightly bound and require more energy to be excited than the non magic nuclei. These correspond to closed shell in the structure of the nucleus in the same way that we encounter closed electronic shells in the structure of the atom. Figure 11 shows the frequency of the stable isotones as a function of the neutron number N. Higher frequencies of occurrences correspond to the magic numbers.

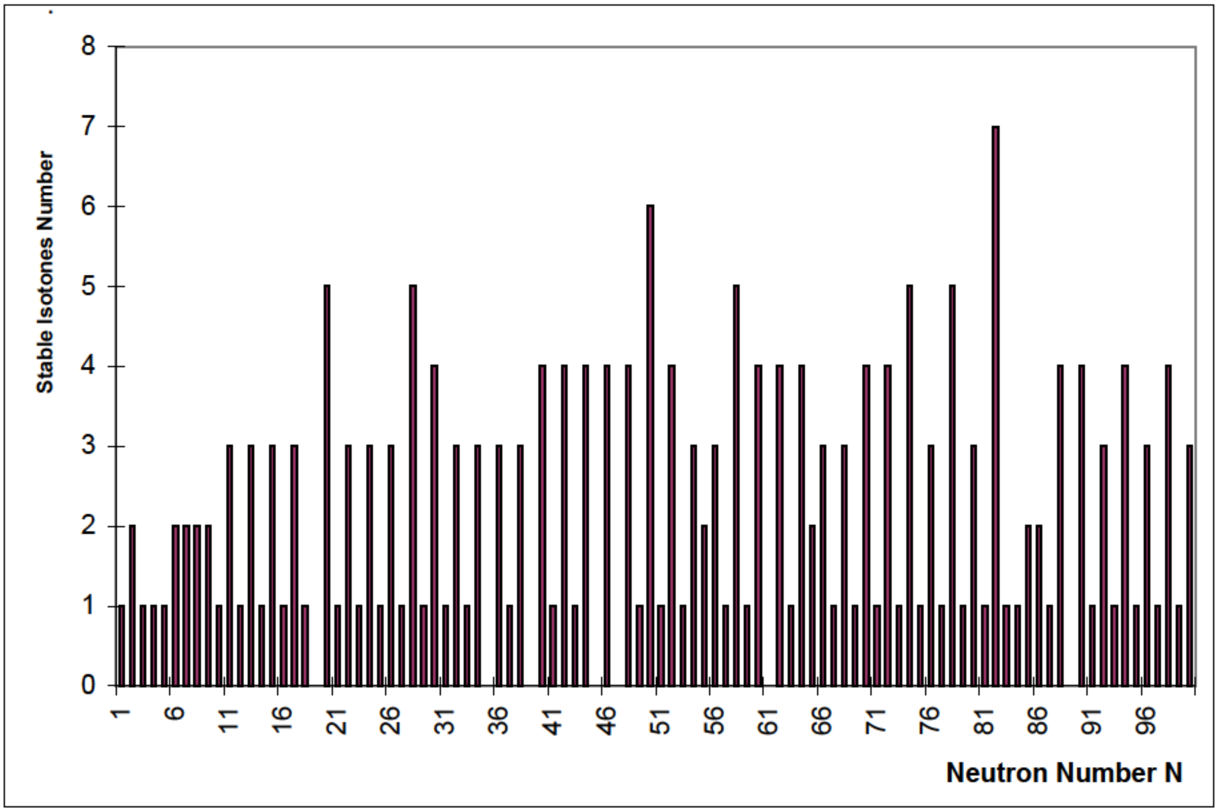


Figure 11. The frequency of the stable isotones as a function of the neutron number N. Higher frequencies correspond to the magic numbers.

A refinement to the shell model of the nucleus involves the spin-orbit coupling model. This model suggests that a strong interaction should exist between the orbital angular momentum and the intrinsic spin angular momentum of each nucleon. A natural consequence of the spin-orbit model is the existence of the *isomeric states* near the shell breaks. If the magnitude of the spin-orbit splitting is properly adjusted, the major shell breaks occur at the experimentally determined magic numbers, which the original shell model does not succeed in predicting.

A limitation of the spin-orbit shell model is that it cannot explain why even Z - even N nuclei do not always possess a zero ground state spin, or why any even number of identical nucleons couples to zero ground state spin. As the nuclear numbers deviate from the magic numbers, cooperative effects appear among the nucleons, and these are taken into accounts by the *collective models* of the nucleus. In this case rotational motion and vibrational motion are built into the models. The *meson theory* of nuclear forces explains more of the nuclear properties.

4.24 ISLANDS OF STABILITY AND THE SUPERHEAVIES

From A = 200 to 220 an empty area in the Chart of the Nuclides occurs, followed by an "island" containing the isotopes of the naturally occurring heavy elements which re members of the actinides series in the periodic table of the elements (Fig. 12). As the atomic number is increased for the heavier nuclides, shorter and shorter half-lives are expected. This is caused by the higher Coulomb repulsion between protons, which overcomes the attractive strong nuclear force between the nucleons. Spontaneous fission among the heavy nuclides becomes more likely as their atomic number increases beyond 100. The process of alpha particle decay also increases as the atomic number increases. The shell model provides a way to suggest that when the protons and neutron shells are closed, a higher level of stability can be attained. The magic numbers for which such an extra stability would be attained is Z=114 and N=184. Other suggested numbers are Z = 126, 116, 124 and 127.

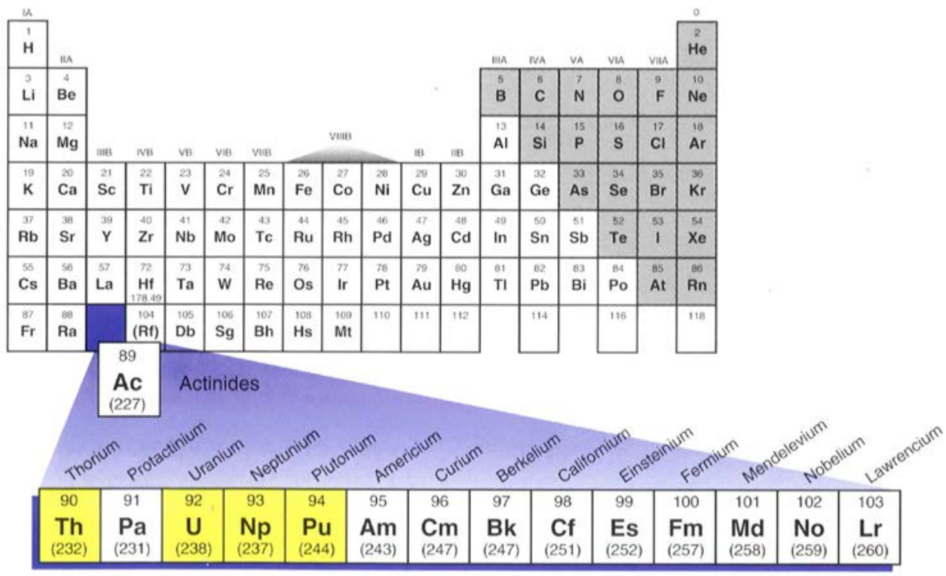


Figure 12. The Actinides series in the periodic table of the elements.

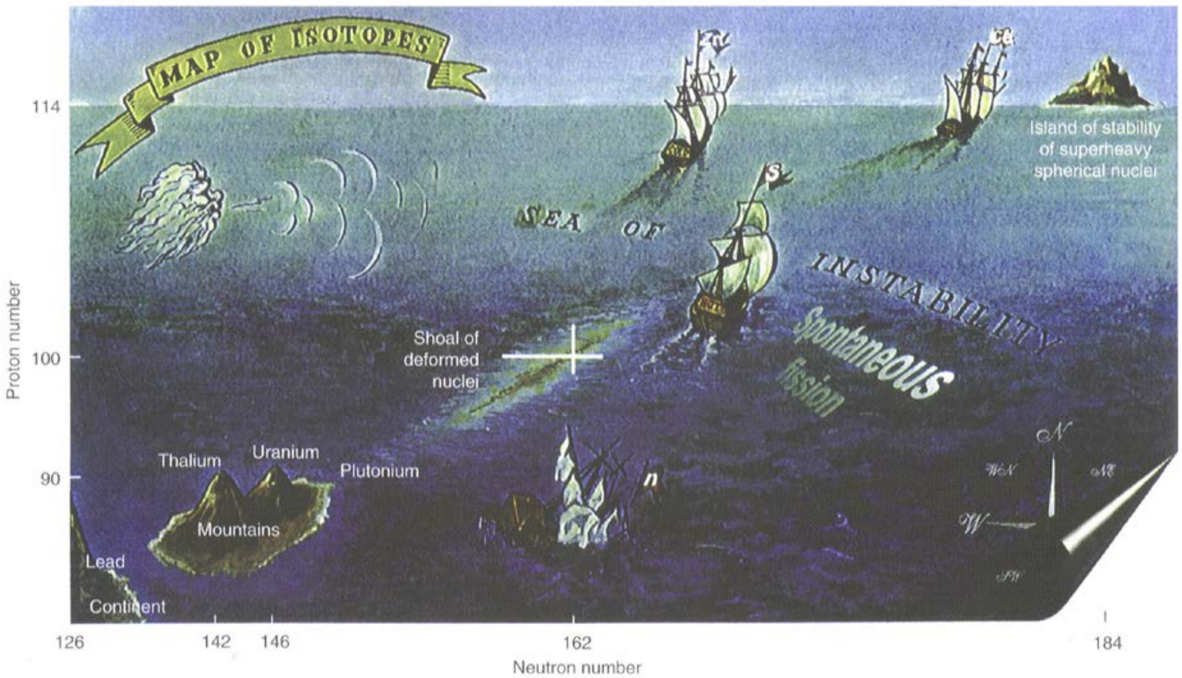


Figure 13. Map of the Isotopes expresses the elusive search for an island of stability of superheavy spherical nuclei beyond the sea of instability.

There has been a prediction started in the sixties that another island of stability could exist with super heavy elements reaching another region of stability in the shell structure. A suggestion was made that they could have been created before the earth itself was created, and that they could still be present if they half lives are long enough. Researchers looked at all kinds of ores minerals, rocks including manganese nodules at the bottom of the oceans, moon rocks, gold and platinum ores, meteorites, hot springs water, volcanic lava, and medieval stained glass, without much success.

Indirect evidence for the one time existence of elements 113 to 115 in the Allende meteorite has been reported by a group at the University of Chicago. Synthesis of the super heavy elements has been attempted by bombarding curium^{248} with a beam of accelerated calcium^{48} ions, but met with negative results. The search for the super heavies remains as a quest for the Holy Grail in the field of nuclear science (Fig. 13).

4.25 COSMOLOGICAL INVENTORY

The rotational speed of the galaxies and other lines of evidence indicate that the universe contains more mass than the sum of the mass of the luminous stars. Some of the missing mass designated as dark matter could be ordinary atoms and molecules that are not hot enough to emit light. If all the mass of dark matter was ordinary matter as neutrons and protons by weight, it would be inconsistent with the density of ordinary matter inferred from the Big Bang model and from the relative abundances of light nuclides. A search is ongoing for ordinary matter which could reside in the outer reaches of galaxies in the Massive Astrophysical Compact Halo Objects

(MACHOs) and for more exotic forms of matter such as the Weakly Interacting Massive Particles (WIMPs) or the axion.

Luminous matter in the universe including the stars and galaxies, accounts for just 0.4 percent of the known universe. Radiation accounts for a mere 0.005 percent. Nonluminous matter such as intergalactic gas (3.6 percent), neutrinos (0.1 percent), supermassive black holes (0.004 percent), account together for only 3.7 percent of the cosmological inventory of the universe. The remaining mass and energy of the universe is divided as about 23 percent dark matter and about 73 percent dark energy.

What makes these components is subject to many theories suggesting that dark matter is predominantly a remnant of unknown and new elementary particles, perhaps axions, from the beginning of the universe at the time of the Big Bang (Table 2).

Table 2. Cosmological inventory of matter and energy.

Component	Percent	Properties
Dark energy	73.000	Possible parallel universe
Dark matter	23.000	Possible parallel universe
Luminous matter: Stars and luminous gas Radiation	 0.400 0.005	Known
Other nonluminous components: Intergalactic gas Neutrinos Supermassive black holes	 3.600 0.100 0.004	Known

Cosmologists are divided in support of two explanations: cold dark matter or extremely massive neutrinos other than the three known types of neutrinos. Cold matter would be some particle that did not acquire much velocity in the early universe. These particles are considered at the same speed as the stars in the galaxy, which is a much slower speed than the speed of light.

Massive neutrinos would have been moving fast at the birth of the universe and they would still have velocities close to the speed of light. This is considered unlikely since the density needed to make up a significant portion of dark matter would have smoothed out irregularities and prevented the formation of the structures that evolved into the galaxies in the first billion years of the universe.

If the predominant form of dark matter is cold dark matter, then two elementary particles are suggested as candidates:

1. A stable Weakly Interacting Massive Particle or WIMP that is about 10 to 100 billion eV in mass equivalent.

2. An axion, which is theoretically considered to be a very light particle of a mass equivalent of just 10^{-6} to 10^{-3} eV with neither electric charge nor spin, thus it interacts hardly at

all. These axions are supposedly so minute that their density is assumed to be 10 [trillions/cm^3] of space in our galaxy. Those occupying a sugar cube volume of space would still weigh less than does half of a proton. Their extreme lightness and nearly nonexistent coupling to radiation make these particles extremely long lived to maybe 10^{50} seconds.

The age of the universe itself is estimated at 10^{18} seconds or 32 billion years old. The axions could be considered as a stable particle. If they exist, they would constitute the bulk of dark matter. What they lack in mass each, they make up for in sheer numbers.

The axions would resolve a difficult issue in the Standard Model theory attempting at explaining fundamental particles and how they interact. It was first suggested as a solution to a thorny problem of particle physics concerning the absence of Charge Parity (CP) violations in nuclear strong force interactions. This has to do with the principle in physics that if two systems are mirror image of each other but are otherwise identical, and if parity is conserved, all subsequent evolution of these two systems should remain identical except for the mirror difference. Nature prefers symmetry to asymmetry. It has no preference for right handed versus left handed behavior as far as particle physics are concerned.

In 1956, Chen Ning Yan and Tsung Dao Lee realized that although many experiments had been conducted to show that minor symmetry was true for the strong interaction force and for the electromagnetic interaction force, no experiments had been conducted for the weak interaction force. Experiments by Chieng-Shiung Wu showed that contrary to what was expected, weak interactions did show a preference or handedness in Co^{60}. When the decay of Co^{60} is observed in a strong magnetic field, there was a preferred direction for emitting beta particles in a decay proving that the weak interaction violated the conservation of parity. This ran counter to all expectations.

In the 1970s, Quantum Electro Dynamics (QED) was developed to describe the strong force interaction, and it held that large amounts of CP violations. These violations had not been observed in experiments. In an effect to explain the phenomenon, Roberto Peccei and and Helen Quinn proposed a new symmetry of nature that resulted in the particle dubbed the axion. Their predicted mass for the axion was about 50 keV.

As a result of observations done in 1987 on the neutrinos of a supernova, a conclusion was made that the mass of the axion is about 10^{-3} eV, suggesting that the supernova core would have been cooled not just by the emitted neutrinos, but by the axions as well. If that was the case, the length of the neutrino burst associated with the supernova would have been much shorter than was observed.

Axions lighter than a microelectron volt could have been produced in the Big Bang resulting in the universe being more massive than it is, which would be at variance with current observations where dark matter is at most ¼ of the closure or the critical density of the universe. Refined calculations suggest that the axion mass is between 10^{-6} to 10^{-3} eV.

Several experiments world wide are dedicated to the WIMP search. Two experiments are tracking down the axion. One experiment in Japan designated as CARRACK2 at Kyoto University's Institute for Chemical Research, and the other at the Lawrence Livermore National Laboratory (LLNL) in the USA. The LLNL experiment is based on the theory that an axion when it interacts decays into two photons with frequencies in the microwave region of the electromagnetic spectrum. There is an idea suggesting that an axion could be stimulated to decay into a single photon in the presence of a large magnetic field threading a microwave cavity. The setup uses an 8 Tesla 6 tons superconducting magnet coil that is wound around the outside of a copper plated stainless steel cylinder, the size of an oil drum. A set of tuning rods inserted into the cylinder's cavity is moved by stepper motors to tune the cavity's frequency. Liquid helium cools the cavity and reduces the background noise and amplifiers attempt at boosting the faint axion signal.

If the axion is not found then something else, maybe another particle, a symmetry, or path or another new theory must be developed to explain what makes up dark matter.

4.26 OBSERVATION OF DARK MATTER

The Bullet Cluster is thought to have formed after two large clusters of galaxies collided, and passed trough each other in the most energetic event since the postulated Big Bang. Until present, the existence of dark matter was inferred by the fact that galaxies had only 1/5 of the visible matter needed to create the gravity that keeps them intact; implying that the balance must be invisible to light telescopes.

Figure 14. Bullet Cluster known as galaxy cluster 1E0657-56. Hot stellar gas is shown in red and dark matter in blue. NASA photograph.

The observations were conducted using a combination of observations from the orbiting Hubble Space Telescope and the Chandra x ray Observatory, along with the ground-based European Southern Observatory Very Large Telescope and the Magellan Telescope. A paper in the Astrophysical Journal Letters, a journal of the American Astronomical Society, describes the observations.

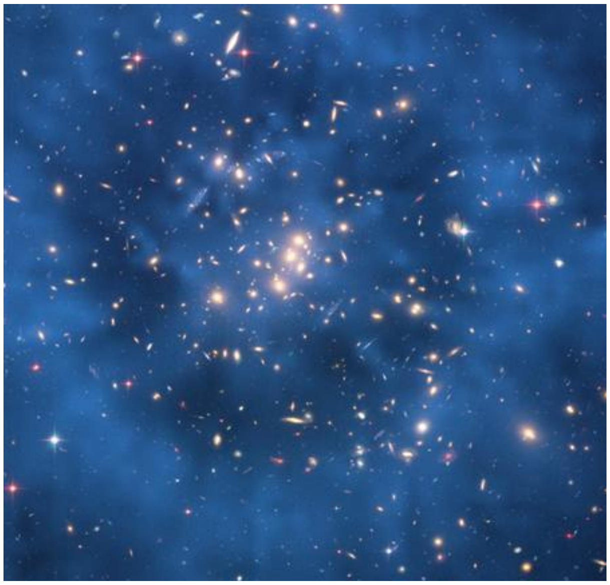


Figure 15. Hubble Space Telescope image of a ring of dark matter in the galaxy cluster Cl 0024+17. Source: NASA.

Images taken by NASA's orbiting Hubble Space Telescope in 2007 allowed astronomers to detect this ring of dark matter created by the collision of two galaxy clusters 5 billion light-years from Earth. Scientists came across the new evidence while studying the distribution of dark matter within a galaxy cluster designated as Cl 0024+17. Wondering about the genesis of this ring, the researchers came across earlier work showing that the galaxy cluster had run into another cluster 1 billion to 2 billion years ago.

The collision between the two galaxy clusters created a ripple of dark matter that left distinct footprints in the shapes of the background galaxies. This is like looking at the pebbles on the bottom of a pond with ripples on the surface. The pebbles' shapes appear to change as the ripples pass over them. Dark matter previously has been detected in other galaxy clusters, but has never been previously found to be so distinctly separated from the hot gas and the galaxies that make up these clusters.

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EXERCISES

1. In a possibly future matter/antimatter reactor, use the mass to energy equivalence relationship to calculate the energy release in ergs, Joules and MeV from the complete annihilation of:

   a. An electron/positron pair.

   b. An antiproton/proton pair.

Consider the following masses:

$m_{electron} = m_{positron} = 9.10956 \times 10^{-28}$ gram

$m_{proton} = m_{antiproton} = 1.67261 \times 10^{-24}$ gram.

2. Balance the following nuclear reactions by applying conservation of charge and of nucleons. Next, apply mass/energy conservation and calculate the energy releases in units of MeV for the following fusion reactions:

   a. ${}_1D^2 + {}_1T^3 \rightarrow {}_0n^1 + ? + Q1$ (DT fusion reaction)

   b. ${}_1D^2 + {}_1D^2 \rightarrow {}_1H^1 + ? + Q2$ (Proton branch of the DD fusion reaction)

   c. ${}_1D^2 + {}_1D^2 \rightarrow {}_0n^1 + ? + Q3$ (Neutron branch of the DD fusion reaction)

   d. ${}_1D^2 + {}_2He^3 \rightarrow {}_2He^4 + ? + Q4$ (Aneutronic or Neutronless DHe^3 reaction)

3. For the Fusion reactions listed in the last problem, using conservation of momentum, calculate how the energy release (Q) apportions itself among the reaction products.

4. Calculate the energy release or the Q-values of the following fission reactions:

${}_0n^1 + {}_{92}U^{235} \rightarrow 3\,{}_0n^1 + {}_{53}I^{137} + {}_{39}Y^{96}$

${}_0n^1 + {}_{92}U^{235} \rightarrow 3\,{}_0n^1 + {}_{54}Xe^{136} + {}_{38}Sr^{97}$

${}_0n^1 + {}_{92}U^{235} \rightarrow 2\,{}_0n^1 + {}_{56}Ba^{137} + {}_{36}Kr^{97}$

5. Calculate the Q values of the following nuclear reactions:

${}_6C^{11} \rightarrow {}_5B^{11} + {}_{+1}e^0 + \nu$ (Positron decay reaction)

${}_{29}Cu^{64} + {}_{-1}e^0 \rightarrow {}_{28}Ni^{64} + \nu$ (Orbital electron capture reaction)

${}_1T^3 \rightarrow {}_{-1}e^0 + {}_2He^3 + \nu^*$ (Negative beta decay reaction)

${}_{94}Pu^{239} \rightarrow {}_2He^4 + {}_{92}U^{235}$ (Alpha particle decay reaction)