The IQ Paradox: Still Resolved

William T. Dickens

The Brookings Institution

James R. Flynn*

University of Otago

February 5, 2002

Published in *Psychological Review*, Vol. 109 #4 (2002)

William T. Dickens
The Brookings Institution
1775 Massachusetts Avenue NW
Washington, DC 20036

wdickens@brookings.edu

Phone: (202) 797-6113
Fax: (202) 797-6181

ABSTRACT

In our original paper we formalized the consensus model that environment affects IQ and IQ affects environment and showed that it can resolve the apparent paradox between high heritability and large environmental effects. Our commentators suggest that that model has undesirable properties which call its usefulness into question. Loehlin argues that IQ is persistent and that incorporating persistence into the model causes problematic behavior. Rowe and Rodgers argue that an increasing correlation of IQ and environment should have caused growing variance of IQ. Empirical evidence suggests that IQ is not sufficiently persistent to cause the problems Loehlin finds and that the correlation of IQ and environment has not grown much over time so that the reciprocal effects model need not imply increasing variance.

The IQ Paradox: Still Resolved

Nearly everyone who studies IQ can agree on two things: environment can affect IQ and those with higher IQs will tend to be found in environments more conducive to developing and maintaining high IQ. These two propositions represent an informal model of IQ. Our contribution was to take this model and show that a formal version could resolve the paradox of the apparently small role played by environment in explaining cross-section variation in IQ and the large role environment must play in explaining IQ gains. We found that such a model can also provide simple explanations for a wide range of facts about measured intelligence – but only if the reciprocal effects of environment and IQ on each other are substantial. Although our commentators all subscribe to the consensus model they evidently do not fully accept that reciprocal effects are large enough to explain IQ gains over time.

Points at Issue

Loehlin's main concerns are that we need to specify the time period of the model, that we cannot model development, and that our models behave badly when persistence is added. We feel we were specific about the time period when needed, we disagree with the second point, and think the third irrelevant. Rowe and Rodgers argue that changes causing rising IQ must also cause an increase in the correlation of IQ and environment; and that in a model of reciprocal effects, a rise in that correlation implies that IQ variance is rising. They are right about the implication of an increase in the correlation, but the weight of evidence suggests that the correlation of environment and IQ has not risen.

Should Models of IQ Have Persistence?

Only Loehlin's simulations with high persistence suggest seriously counter intuitive behavior and then only if a period in the model is a year or more. But Loehlin provides no evidence that IQ is persistent. Instead he argues by analogy to learning to ride a bicycle that it probably is. This may be a misleading analogy. We would not model bicycle riding skill (or basketball skill) without persistence, but we probably would not include persistence in an equation for muscle conditioning measured annually. Just a few weeks in bed can cause considerable atrophy. Empirical evidence suggests that IQ is more like muscle conditioning than bicycle riding in its persistence.

The way Loehlin models persistence there is necessarily slow adjustment to environmental changes. This is evident in his simulations. In our article we noted that all the gains in IQ in enrichment programs seem to take place in the first year (Dickens & Flynn, p363). This is hard to reconcile with any substantial degree of persistence.

Fulker et. al. (1993) study children who are ages 1 to 4, 7 and 9 in three genetically informed longitudinal data sets. They find substantial year to year correlation in the effects of family environment and genes, but no evidence of correlation in the effects of non-shared environment. They fail to reject the hypothesis of no correlation in non-shared environment between years. Substantial persistence in IQ would cause a correlation in the effects of non-shared environment even if there were no persistence in the environment itself.

A similar study has been done for adult twins (Plomin et. al. 1994). From the results presented in figure 2 (p210) it is possible to compute that the correlation of non-shared environmental effects three years apart is .54. This result is inconsistent with Loehlin’s model with high persistence that would predict a correlation of about .75. It would be consistent with his model with moderate persistence, but this correlation could also be caused by persistence in environment only. We would expect adults’ environments to be correlated year to year.

Certainly Loehlin is right that if people took an IQ tests before they went to bed at night and then in the morning when they woke up the similarity of their scores would be best explained by persistence of the internal state of their minds rather than their environments. However, if we were to model measurements taken very close together we suspect that neither Loehlin's models nor ours would provide a satisfactory account of the dynamics. Such a model would have to include practice effects and, we suspect, non-linear persistence. However, for the purposes we used the models in our paper -- describing phenomena where IQ tests are taken a year apart or more – it seems that linear models with little or no persistence are adequate.

A Model of Development

Loehlin implies that it is inappropriate to apply models such as ours to the study of child development because of our use of equilibrium analysis and constant parameters and genetic effects. While we often use the assumption of equilibrium to analyze our models they are meant to describe the behavior of individuals' IQs in or out of equilibrium. Also, stable genetic effects are not unreasonable approximation to the truth for children more than one year old. Fulker et. al.'s (1993) preferred model implies that genetic effects in years 2 to 7 are correlated .6 or higher. Except for year one, adjacent years are correlated .73 or higher.[1]

We suspect some of Loehlin's concern about our ability to model development follows from the ambiguity that he sees in the meaning of M in our models. We accept that we were not clear about this. To be precise, we imagine M to measure individual IQ as computed from raw scores using norms established for all age groups at one point in time. Thus if the mean of exogenous environment rises over time scores will rise and so will IQ computed using the norms from an earlier point in time. To see how a model of raw scores with changing parameters might translate into a model of age normed IQ with unchanging parameters consider the following equations for raw scores and IQ.

First, assume that the raw score for an individual i of age y at time t can be described by the equation

(1) $R_{iyt} = \sigma_{Ry} (a [G_i + z (y)] + v E'_{iyt} + w R_{i\,y-1\,t-1}/\sigma_{R\,y-1\,0})$.

Here raw scores grow as genetically determined ability $[G_i + z (y)]$ grows with age according to the function z. E' is a measure of environment. We include Loehlin's persistence term even though we think that w should equal zero if the model is to be applied to annual data. Now define M_{iyt} as $(R_{iyt} – E(R_{iy0}))/\sigma_{Ry0}$ where σ_{Ry0} was the standard deviation of R for people of age y at time 0 when the test was standardized and E() denotes the expectation of the value in parenthesis. If we normalize the expected value of G to zero, assume that the biological process yielding ability at each age does not change over time, that the standard deviations of R for each age do not change over time, and define E_{iyt} as $(E'_{iyt} – E(E'_{iy0}))$ this yields

(2) $M_{iyt} = a[G_i + z (y)]+v E'_{iyt}+wR_{i\,y-1\,t-1}/\sigma_{Ry-1}– az(y)–vE(E'_{iy0})–wE(R_{iy-1\,t–1})/ \sigma_{Ry-1}$

$= aG_i + v E_{iyt} + w M_{iy-1t}$.

With E interpreted as a deviation from an age normed environment, the parameters of the model are all stable. The model is very similar to those we presented in our paper.

We would not claim that this model is the right model of development for all purposes, and we would not deny that there is evidence that G changes as children age. As we stated in our paper we do believe that the equation describing environment must change as children age. What this does show is that an equation for IQ with constant G and unchanging parameters can provide a reasonable model of development so that we probably did not go far astray in using such a model to describe developmental phenomena in our original paper.

Loehlin's Other Points

Loehlin notes that the correlations of intelligence and environment in our models are high. The correlations for model 2 presented in the top panel of his table 1 are quite high and show how closely related IQ and total relevant environment must be if there is even a moderate effect of IQ on environment. Recall that in the absence of any effect of IQ on environment the correlation of environment and IQ will be .5 if h^2 is assumed to be .75. It must be higher if IQ affects environment and rises quickly as the direct effects of genes on IQ drop below .7. Correlations in some of Loehlin's other simulations are lower, but only because with lags and changing es, total IQ relevant environment includes not just current E, but all past Es as well.

Loehlin's correlations may seem impossibly high when compared to the correlations normally observed between IQ and particular aspects of environment such as education or occupational status. But Loehlin warns the reader not to draw inferences about correlations between observables based on his analysis of theoretical constructs. The theoretical E is a perfectly constructed index of *all* environmental effects on IQ which are also affected by IQ. An index of many different factors affecting and being affected by IQ will be much more highly correlated with IQ than any typical element of that index or even any partial imperfect index. To respond to Rowe and Rodgers' concerns we will construct a model in which there are many different aspects of environment that influence IQ. Each one will have a correlation with IQ in the range of .3 to .7, but the average of all of these influences have a correlation with IQ which is the same as in Loehlin's analysis.

While IQ and measurable aspects of environment never have correlations as high as .9, it is not uncommon to encounter correlations that high when variables have substantial reciprocal effects. For example gross domestic product is thought to be both a cause of consumption demand and to be caused by it. Between 1959 and 1999 those two variables had a correlation of .99. When the time trend is removed from the two variables they are still correlated .94.[2]

Loehlin makes several comments about the relative magnitudes of the parameters of the model and how they change when persistence is added. One should keep in mind that the meaning of the parameters changes completely when persistence is added in the way Loehlin prefers. For example, in Loehlin's models with high persistence even very low values of the coefficient of environmental effects produce huge multipliers. This is because in Loehlin's model, effects accumulate not only through the multiplier process but also through the effect of past values of environment and IQ on current values.

Loehlin seems to suggest that our explanation for the Dutch gains is inadequate because we can't square developmental changes for individuals with IQ gains over time (p11). Simulations of our model 3 produce the results he cites, but the models we presented in our paper were meant to provide simple illustrations of our substantive points and are not the models we would use to fit real data. As we noted in footnote 19 and in the text on page 365 of our paper there are several things we would want to add to a model meant to describe year-to-year development. An important one would be simultaneous determination of IQ and environment. In a model with simultaneous effects and realistic persistence most multiplier effects would work themselves out within a year and nearly all would be present in 5 or 6 years. As Loehlin notes "the multiplier of 4.5 which they use would be appropriate for a given u continuously input until equilibrium is reached which is not happening in this simulation." In the model we would use to capture both development and IQ gains over time this is what would be happening and the multiplier analysis we did in our original paper appropriate.

Expanding Variance and Rising IQ

Both comments suggest that the variance of IQ should rise, at least initially, when IQ rises if our explanation for IQ gains is correct. To be clear, when we talk about the variance of IQ rising we mean the variance of IQ computed based on norms established at one point in time.

Loehlin simulates increasing IQ as resulting from a change in exogenous environment that slowly diffuses through the population. As such the effective variance of exogenous environmental effects on IQ rises and then falls as the change takes place and consequently the variance of IQ rises and falls by a multiple of that change. Rowe and Rodgers argue that increases in the quality of environment over the last century have been accompanied by increases in the correlation of IQ and environment; and that therefore, substantial reciprocal effects of IQ and environment on each other imply growing variance of IQ - something that has not been observed.

What We Believe

We agree with both commentators that if we were able to measure the IQs of individuals living in traditional societies where work and social roles were not highly differentiated, we would expect that the variance of IQ would be lower than in modern society. We have been unable to find data on the variance of IQ for people who both have little or no contact with modern society and whose score variance has not been artificially reduced by floor effects.

As for industrial societies, the days of homogenous social and work roles were long gone when the IQ test was invented in the early part of the last century. Such societies are the only ones for which we have multiple measurements on the same IQ tests over time. IQ gains seem to have been going on for the entire time for which we have IQ data, and gains in the skills IQ tests measure may have been going on before that. Further, it is not clear whether the dispersion of the cognitive demands of work and social roles has grown or shrunk over the last century and it is certainly not clear whether the correlation of IQ with IQ relevant environment has risen or fallen. We thus see no reason to expect that the variance of IQ has risen over the last century.

Could We Tell if the Standard Deviation of IQ Has Risen?

Rowe and Rodgers are correct that increasing the effect of IQ on environment in a model with reciprocal effects will cause a rise in the correlation of IQ and environment and an increase in the variance of IQ, but the analysis they do is inadequate to judge the magnitude of this effect. They examine the effect on the variance of IQ of increasing the correlation of environment and genotype from 0 to .8 in our model 1 and imply that this is the change in variance that should have been expected historically if reciprocal effects were substantial. From Loehlin's simulations we know that a value of .8 for the GxE correlation is not unreasonable, but certainly the correlation of genes and environment was greater than 0 in Rowe and Rodgers starting year of 1932. If we assume that the increase in the correlation of G and E from 1932 to 1972 was less than .4 then Rowe and Rodgers' table 1 implies increases in the standard deviation of IQ of less than 2.75 IQ points (assuming an initial standard deviation of 15 points). Even if the correlation increased from zero to .8 the increase in the standard deviation of IQ would be less than 5.1 IQ points in all three columns.

Gains of these magnitudes could not be rejected given the large fluctuations in the measured standard deviation in Rowe and Rodgers table 2. In fact, a regression of the standard deviations on the year in table 2 yields a coefficient of .06 with a standard error of .05. This means that the best fitting line suggests an increase in the standard deviation of IQ of 2.4 IQ points over the 40 years covered by the table. The confidence interval on this predicted increase includes zero, and thus one cannot reject the hypothesis of no change in the standard deviation over time. However, one would also be unable to reject the hypothesis that the standard deviation of IQ had increased as much as 5.5 points at the .05 level in a one-tailed test given Rowe and Rodgers' data. Thus in the only example Rowe and Rodgers present (their table 1), even implausibly large changes in the correlation of genes and environment create changes in the standard deviation of IQ too small to be ruled out given Rowe and Rodgers own data.

An Alternative Model of Changing Variance

However, Rowe and Rodgers example may understate the true effect of increasing gene x environment or IQ x environment correlation. Their analysis assumes that variance of environment remains constant while the variance of IQ increases. While arguments could be made that increasing the precision of the match of IQ and environment would not necessarily increase the variance of environment, we want to consider the worst case for reciprocal effects.

A problem with doing this, as Loehlin's simulations show, is that there isn't much room for correlations between IQ and a full measure of IQ relevant environment to increase. Further, it is correlations between IQ and observable aspects of the environment that inform our intuition about what is or isn't a big change in a correlation. Any data that might be brought to bear will be correlations of IQ with observables. Thus we present a model of IQ where environment is represented not by one variable, but a large number of variables, at least some of which we imagine to be observable.

We specify

(3) $$M_i = aG_i + v\sum_{j=1}^{N} E_{ij} / N$$

(4) $$E_{ij} = b\, M_i + e_{ij}$$

so that the IQ of person i is equal to the direct effect of genetic endowment on IQ (a) times that person's genetic endowment plus v times the average value of N environmental influences. Each of those environmental influences is affected by IQ. The model is fully simultaneous (there are no lags) which is equivalent to assuming that the time period of analysis is long enough to allow the full multiplier effect to take place. For analytical convenience we assume that all environmental influences affect and are affected by IQ to the same extent. We further assume that M, G and each E are initially standardized to have mean zero and variance 1 and that $cor(e_{ij},e_{kl}) = 0$ for all $i \neq k$ or $j \neq l$ and that none of the es are correlated with G. We will then ask what an increase in b will do to the variance of M.

Table 1 illustrates the range of results the model is capable of producing. In all cases we have assumed that $h^2 = .75$. Once heritability, the number of environmental variables (N), and the direct effect of genes on IQ (a) are specified the remaining parameter values can be deduced from equations 3 and 4 and the normalizations.

Several things are notable about this table. First, the correlations between IQ and total environment are the same high values as in Loehlin's simulations (except for his random simulation errors), but the correlations between IQ and each environmental variable are much smaller and of the size we typically observe for variables such as education, income, or occupational status. Second, moderate increases in the size of the effect of IQ on environmental variables (b) cause moderate increases in the correlation of IQ and each element of environment. We have chosen the increases in b to produce changes in the correlations that keep them in the range of correlations that have generally been observed. Finally, increasing the effect of IQ on environment (b) enough to increase the correlation of IQ and environmental variables about .1 is not enough to produce a change in the standard deviation of IQ that would be noticeable in Rowe and Rodgers data. However, if the change in the correlation is much larger than this the change in the standard deviation of IQ becomes substantial and should have been noticeable. Thus it becomes an empirical question. If the typical increase in the correlation between IQ and and IQ relevant variables has risen .15 or more we should have noticed an increase in the variance of IQ.

Are IQ and Environment More Correlated Now Than Early Last Century?

Despite the essential role played by increasing correlations in their argument, Rowe and Rodgers say very little about why they expect that correlations have increased. They devote only two paragraphs (p6 and p10-11) to the question. They speculate that jobs today are more diverse in their intellectual demands than in the past based mainly on the disappearance of farm labor. But considerably less than half of all jobs were on farms in 1932 (Council of Economic Advisors 2001, p316) so the disappearance of farm labor since then could have either decreased or increased the heterogeneity of the cognitive demands of work depending on how heterogenous farm farm work was and how different its cognitive demands were from the mean demands of the non-farm sector.

Is there any evidence that the correlation of IQ and the cognitive demands of work has incrased? To our knowledge no one has addressed this question directly, but three studies suggest that the answer is probably no. Flynn (2000) shows that the correlation of children's IQ with their parents' occupational status remained the same or declined from 1932 to 1989 with all reliable correlations falling in the range of .26 to .38 . Weakliem et. al. compare birth cohorts in the General Social Survey and find smaller differences in vocabulary scores between occupations for younger as opposed to older cohorts.

Rowe and Rodgers cite two sources in support of their view. Rowe et. al. (1999) present no evidence of an increasing correlation. Their other citation is Herrntein and Murray's *The Bell Curve*. This book contains a wide range of arguments as to why we might expect that IQ and environment would be more closely matched today than in the past, but, as far as we can tell, no direct evidence that they are. We do not find the arguments or the indirect evidence persuasive in the face of direct evidence to the contrary. For example, Herrnstein and Murray document the increasing use of tests to determine admissions to elite private schools, but only a tiny fraction of the population attends elite colleges so the allocation of those slots will have almost no effect on population correlations. It is possible that the quality of education is more highly correlated with IQ today than in the early part of the 20th century, but we know of no studies. However, Jencks (1972 pp324-326) concludes that the correlation of IQ and educational attainment was about .55 in some restricted samples in the 1920s and about .57 in a comparable sample in 1968. Correlations between armed forces induction tests and educational attainment appear to have declined over roughly the same period from values of about .75 to .63 or less.

Herrnstein and Murray suggest that the earnings of those with more education have risen relative to those with less. In fact the ratio has fluctuated considerably over the last century with most of the biggest increase coming since 1980 (Goldin and Katz 1999). While these changes have been large they have come about mainly as a result of changes in the variance of income rather than changes in the correlation of education and job status or the cognitive demands of work as the evidence cited previously shows. Herrnstein and Murray speculate that the correlation of husbands' and wives' IQs have increased, but that correlation was already high in the 1920s and 1930s with correlations in more representative groups ranging from .42 to .74. (Smith, 1941, p698). The evidence that Herrnstein and Murray discuss, that college educated men are more likely to marry college educated women today than in the past, could result simply from a greater number of college educated women even if the correlation of couples IQs have remained constant.

Rowe and Rodgers also argue that an improvement in environment is typically accomplished by making the environment more diverse. They provide an example in which the introduction of books and libraries increases the mean quality of environment, but since people may or may not read the books it also increases the variance environment and its correlation with IQ. But environmental change need not take this form. It could be that existing books just become more demanding. For example books teaching computer programming could be replacing books on how to fix things around the house with no increase in the variance of the cognitive demands of reading material and no change in the correlation of the cognitive demands of reading material chosen and IQ.

Overall we see no evidence to support the view that the typical correlation between IQ and environment has risen by more than .1 over the last century. In fact, where we have several data the trend is flat or very slightly down. Under these conditions, the reciprocal effects model does not predict increasing variance of IQ, so no such 'prediction' is grounds for rejecting it.

Rowe and Rodgers Other Points

Rowe and Rodgers cite the dissimilarity of siblings, and the lack of large effects of the elimination of tracking, as indictments of our social multiplier effect. They note that "…sharing a family environment hardly forces IQ-similarity on siblings," but our social multiplier doesn't force IQ-similarity on the population sharing social effects. People in our model are just as different from one another as are people in the real world despite the social multiplier effect. However, Rowe and Rodgers point suggests a test of the reasonableness of the magnitude of the social multipliers we used in our paper. If we assume that some fraction of the social multiplier effect comes from contact within ones family and combine that with assumptions about the variance of average family IQs we can compute the magnitude of such effects and compare them to the effect sizes implicit in estimates of the share of variance attributable to shared environment. We have done this for a wide range of assumptions and, in general, values of the social multiplier equal to or less than what we termed “moderate” produce effect sizes much less than those implied by estimates of the share of variance attributable to family environment. However, the larger value of the social multiplier we used in our paper is only consistent with typical estimates of shared environmental effects when individual multipliers are less than 1.5.

Tracking is a different story. Our reading of the literature suggests that normal tracking produces little or no decline in achievement test scores for less able children and perhaps some gains for the more able (Ferguson 1998 pp330-335). This is exactly what we would expect if social multiplier effects and gains from an appropriate curriculum were both at work. Less able children would gain from the more appropriate curriculum but lose from lack of contact with the more able children while the more able children would gain from both effects. However, the effects would be tiny. Take the following extreme example. Suppose that IQ in a class before tracking was normally distributed with a standard deviation of 1 and children chose their friends within the class at random. Suppose that half of all social multiplier effects came through classroom contacts, assume a moderate sized social multiplier coefficient of .4 and an individual multiplier of 1.5. Now if tracking is instituted and perfectly sorts the children into equal size groups on the basis of their IQs (above or below the class mean) and students now only socialize with randomly chosen members of their new tracked class, the IQ of their typical social contact will rise or decline by .8. Multiplying that by .4 for the social effect and .5 for this being only half of that effect, and then by 2 for the individual multiplier produces a change in IQ of .32sd or 4.8 IQ points. A gain of this magnitude would only happen if tracking was perfect and children didn't already segregate themselves to some degree by IQ. More likely, gains should be 2 points or less – roughly the size of the gains from tracking Ferguson reports for the more able.

Recently there has been an explosion of good studies of peer effects on a wide range of behaviors including academic achievement. These studies use things like random assignment of roommates in college to produce random variation in peer groups and examine their effects. None that we are aware of have looked at IQ, but several suggest at least some important effects on achievement test scores (Hoxby 2000, Hanushek et. al. 2001). Boozer and Cacciola (2001) attribute most of the gains in the Tennessee STAR experiment to peer group effects.

At worst the collection of all these results suggest that values for the social multiplier should be somewhat smaller than what we described as moderate social effects in our original paper. Such effects could still contribute substantially to explaining IQ gains over time. However, someone who believed that social effects were as large as the larger value we used in our paper could easily accommodate these results by making less extreme assumptions than we did in deducing the social multiplier effects.

Rowe and Rodgers ask whether the formal model of reciprocal effects we specify can be falsified. We suggested several ways in our original article to test our explanation for IQ gains and these comments and our reply have suggested others. By specifying formal models we have made the implications of the theory of reciprocal effects clearer and even quantifiable. This should make it easier rather than harder to falsify.

Finally, Rowe and Rodgers suggest several verbal explanations for IQ gains that they argue are more parsimonious than our models, but the parsimony is specious. Verbal arguments may seem simple, but were they to formalize the arguments they are suggesting their parsimony would disappear. As we explained earlier, they would also be subject to the full force of Jensen's argument for why environmental effects cannot be large – unless they adopted the reciprocal effects model. Formalizing the arguments makes precise what can and cannot be claimed.

Conclusion

Any reader who is still convinced that the reciprocal effects model has serious flaws should take note of what must then be given up. There is ample evidence that IQ is matched to environment and that at least some elements of environment affect IQ. Therefore, if the reciprocal effects model is wrong, it must be because those aspects of environment affected by IQ are ones that do not substantially affect it in return. This would put us back in the world of the simple linear model with little or no GxE correlation and leave us with no coherent explanation for IQ gains or the other phenomena we discussed in our original paper.

Neither of the comments calls into question the logic by which we reconcile high heritability with large environmental effects so in that sense we feel the IQ paradox is still resolved. But we agree with Loehlin's concluding comment that "…we have some way to go yet" to entirely resolve the IQ paradox. Solving the logical paradox is only the first step to finding a complete explanation for IQ gains over time and coherent accounts of the development of IQ and its relation to the environment.

Footnotes

* This reply has benefited from extended helpful correspondences with both sets of commentators. In particular, John Loehlin provided us with extensive comments on our original paper before it went to press. Consequently we believe we were able to anticipate most of his concerns in that paper. If we were successful it was only because of Loehlin's generosity. We should also mention that the developmental extension to our model presented in this reply was constructed to respond to concerns Loehlin raised and that that effort would have been more difficult were it not for what we learned from thinking about Loehlin's modeling efforts. We felt his contribution was sufficiently important that we offered him co-authorship of a paper presenting those results. He declined our offer. Finally, as we write this, David Rowe is very ill. He has our best wishes for recovery, but this still seems an appropriate place to acknowledge a major intellectual debt to him. Reading his paper with Cleveland (1996) on the black-white IQ gap, and thinking about how it might apply to IQ gains over time provided the first glimpse of the insights that became our paper.

1. Genetic effects in year 9 are correlated .89 with those in year 7, but only .3 to .4 with the genetic effect in years 2-4. The factor structure for year 9 seems strange and this may be because IQ was assessed in that year with a test given over the telephone.
2. Authors' computations from data from Council of Economic Advisors (2001) pp 276 & 311.

References

Boozer, M.A. & Cacciola, S.E., Inside the ‘black box’ of project STAR: estimation of peer effects using experimental data, New Haven CT: Yale University Economic Growth Center, discussion paper #832.

Council of Economic Advisors (2001) Economic Report of the President. Washington, DC: United States Government Printing Office.

Ferguson, R. (1998) Can schools narrow the black-white test score gap? In C. Jencks & M. Phillips The Black-White Test Score Gap, Washington DC:Brookings.

Flynn, J.R, (2000) IQ trends over time: intelligence, race, and meritocracy. In K. Arrow, S. Bowles & S. Durlauf Meritocracy and Economic Inequality Princeton NJ: Princeton University Press.

Fulker, D.W., Cherny, S.S. & Cardon, L.R. (1993) Continuity and change in cognitive development. In R. Plomin and G.E. McClearn Nature, Nurture, and Psychology (pp. 77-97) Washington DC: American Psychological Association.

Goldin, C. & Katz, L. (1999) The returns to skill in the United States across the twentieth century, Cambridge MA:National Bureau of Economic Research, working paper #7126.

Hanushek, E.A., Kain, J. F., Markman, J.M. & Rivkin, S.G. (2001) Does peer ability affect student achievement? Cambridge MA:NBER, working paper #8502.

Hoxby, C. (2000) Peer effects in the classroom: learning from gender and race variaiton, Cambridge MA:NBER, working paper #7867.

Plomin, R. Pedersen, N.L, Lichtenstein, P., & McClearn, G.E. (1994) Variability and stability in cognitive abilities are largely genetic later in life. Behavior Genetics, 24(3), 207-215.

Rowe D.C. & Cleveland H.H., (1996) Academic achievement in Blacks and Whites: Are the developmental processes similar? Intelligence, 23, 205-228.

Smith, M. (1941) Similarities of marriage partners in intelligence, American Sociological Review, 6 (5, October), 697-701.

Weakliem, D. McQuillan, J. & Schauer, T. (1995) Toward meritocracy? Changing social-class differences

Table 1
Effects of Increasing Correlation of IQ and Environment

Direct Effect of Genes on IQ (a)	.2	.3	.4	.4	.5	.5	.6	.7
Number of Environmental Variables (N)	100	30	20	40	10	50	15	5
Multiplier [1] (v/[1-bv])	6.1	3.5	2.6	3.5	1.8	3.7	2.0	1.2
Correlation of IQ and Total Environment	.99	.98	.96	.96	.92	.92	.85	.75
Correlation of IQ with Each E_j	.58	.62	.54	.41	.53	.27	.34	.41
Percent change in IQ's Effect on Environment (b)	5%	10%	30%	15%	20%	50%	20%	100%
Correlation of IQ with Each E_j after change in b	.66	.72	.76	.51	.63	.49	.40	.62
Change in SD of IQ in IQ points	3.0	3.5	8.1	3.2	2.6	8.7	1.5	4.7

Derivations for the formulas used to compute the values in this table can be found in the appendix.

[1] The multiplier shows the effect on IQ in standard deviations of increasing the mean of e for each environmental variable by one standard deviation of E.

Appendix

To obtain the values in table 1 we first obtain the reduced form of the system (equations with each endogenous variable as a function of exogenous variables and parameters only) of N+1 linear simultaneous equations with N+1 unknowns represented by equations 3 and 4. We can then use the restrictions implied by normalizations and the assumption that measured heritability should be .75 to reduce the number of parameters so that by specifying the number of environmental variables and the direct effect of genes on IQ we can compute any other parameter.

The reduced form of the system can be derived by substituting the N equations represented by equation 4 into equation 3 to get

$$(A1) \qquad M_i = aG_i + v\sum_{j=1}^{N}\frac{bM_i + e_{ij}}{N} = aG_i + vbM_i + v\sum_{j=1}^{N}\frac{e_{ij}}{N} = \frac{aG_i + v\sum_{j=1}^{N}\frac{e_{ij}}{N}}{1-vb},$$

and then substituting A1 into the N equations represented by 4 to get

$$(A2) \qquad E_{ij} = \frac{abG_i + vb\sum_{k=1}^{N}\frac{e_{ik}}{N}}{1-vb} + e_{ij} = \left[abG_i + vb\sum_{\substack{k=1 \\ k\neq j}}^{N}\frac{e_{ik}}{N} + (1 - bv(1-\frac{1}{N}))e_{ij}\right] \Big/ (1-vb).$$

Assuming that the es are all uncorrelated (as stated in the text) and have a common standard deviation σ_e, this system of equations has four parameters: a, b, v and σ_e. Assuming that M and each E are normalized to have variance 1 before any change in b, and assuming that the total genetic variance before any changes equals h^2 gives three restrictions so that any three of the parameters of the model can be written as functions of N, h^2, and any one other parameter. We chose *a*.

Assuming that the variance of G is standardized to 1 and that G is uncorrelated with all es we observe first (from A1) that

(A3) $h^2 = a^2/(1-vb)^2$ => $vb=(1-a/h)$ => $v=(1-a/h)/b$.

Next we compute the variance of M_i as

$$(A4) \qquad Var(M) = \frac{a^2 + v^2\sigma_e^2 / N}{(1-vb)^2}.$$

Using the normalization that the variance of M is initially 1 and A3 allows us to rewrite A4 to get the equation for the starting value for b as

$$(A5) \qquad b_0^2 = \frac{\sigma_e^2(h-a)^2}{Na^2(1-h^2)}.$$

These same assumptions allow us to write the variance of any E_j as

$$(A6) \qquad \begin{aligned} Var(E) &= \frac{b^2a^2 + \sigma_e^2\left[b^2v^2\,\frac{(N-1)}{N^2} + (1-bv(1-1/N))^2\right]}{(1-vb)^2} \\ &= \frac{b^2a^2N + \sigma_e^2\left[(N-1)(1-bv)^2+1\right]}{N(1-vb)^2}. \end{aligned}$$

Setting the variance of E equal to 1 and substituting A5 for b^2 in A6, A3 for vb in A6 and solving for σ_e yields

$$(A7) \qquad \sigma_e = \sqrt{\frac{N(1-h^2)a^2}{(1-h^2)a^2(N-1) + h^2(h-a)^2 + (1-h^2)h^2}}.$$

This gives σ_e as a function of N, a and h only. By substituting A7 into A5 one can get b_0 as a function of those same values, and then substituting A5 into A3 yields v as a function of these three values. We are now in position to compute the values in table 1.

Increasing the mean of each e_{ij} by one in A1 will increase the expected value of M by v/(1-bv). Substituting for b and v from above yields the values in the third row of the table.

Defining total environment as the average of the E_{ij}s the correlation of total environment and M in the population will be

(A8) Cor(M,E*) = Cov(M,E*)/(Var(M)Var(E*))$^{.5}$.

From 3 e can write $M_i=aG_i+vE^*_i$, so that setting the right-hand-side of this equation equal to the right-hand-side of A1, and solving for E* yields

$$(A9) \qquad E_i^* = \frac{baG_i + \sum_{j=1}^{N} e_{ij} / N}{(1-bv)}.$$

The Variance of E* will be

$$(A10) \qquad Var(E_i^*) = \frac{b^2a^2 + \sigma_e^2 / N}{(1-bv)^2}$$

and

$$(A11) \qquad Cov(M, E^*) = \frac{ba^2 + v\sigma_e^2 / N}{(1-bv)^2}.$$

The correlation of M and E* can be constructed by substituting A4, A10, and A11 into A8 and then substituting A7, A5 and A3 into the result to compute the correlation from N, h and a.

The correlation of each E_{ij} can be constructed similarly with the covariance of E and M which is

$$(A12) \qquad Cov(M, E_j) = \frac{ba^2 + bv^2 \dfrac{\sigma_e^2(N-1)}{N^2} + \dfrac{v(1-bv+bv/N)\sigma_e^2}{N}}{(1-bv)^2} = \frac{ba^2 + v\sigma_e^2 / N}{(1-bv)^2}.$$

Again, the appropriate substitutions yield the correlation as a function of h, a and N. Similar substitutions in A4 yield the variance from which the standard deviation of M can be computed.