find applications to the construction of roof-shaped surfaces, of sun-dials, of regular polyhedrons and crystal-forms, of ellipses and of moment-ellipses in graphic statics. The second chapter, which deals with parallel-projection upon a single plane, includes sections on cavalier-perspective, shadows under parallel light-rays, affinity and its applications, on the foundations of general axonometry (including, of course, Pohlke's famous theorem), on the theory of involution and its applications to the polar properties of the circle and the ellipse. In the third chapter we find the customary orthographic projection upon two or more planes of projection, including a valuable section on conics. Whenever possible, full use is made of the advantages which the application of projective properties of conics and the principle of affinity afford in the graphic representation of geometric forms.

The second volume contains two chapters on perspective and various applications. Chapter IV, on central projection, treats of fundamental concepts, so-called restricted perspective, construction of shadows in perspective, invariance of cross ratio, involutory perspective, applications of perspective, perspective of circle and sphere, properties of conic sections and their applications, including Pascal's and Brianchon's theorems, and so-called free perspective.

Various applications and supplementary topics, such as plane curves, surface-ornaments, topographical surfaces, surfaces of revolution, helical and cycloidal curves and surfaces, ruled surfaces, interpenetrations and shadows, and finally a brief account of relief-perspective, form the contents of the concluding fifth chapter.

The level upon which Scheffers proceeds may be judged from the fact that even a discussion of Peano's surface is included, to show the student the danger of hasty generalizations. Peano's original surface has the form $z = (y^2 - 2px)(y^2 - 2qx)$ in which p and q are positive real integers. For the sake of convenient constructive treatment, Scheffers discusses the projectively equivalent surface

$$z = -\frac{1}{10}(x^2 - 5y)(x^2 - y).$$

Every plane through the z-axis cuts the surface in a quartic which has a maximum at the origin, so that one might expect a maximum for the surface at that point. Still it is possible to trace curves on the surface passing through O having a minimum at O.

The whole treatise is carefully written and is typographically faultless. It may be heartily recommended to teachers as well as to students of descriptive geometry.

ARNOLD EMCH.

Girolamo Saccheri's Euclides Vindicatus, edited and translated by George Bruce Halsted. Chicago, The Open Court Publishing Company, 1920. 30 + 246 pp.

The original title of Saccheri's now famous work is: Euclides ab omni naevo vindicatus, . . ., which appeared in Milan in 1733, and which Halsted translates as "Euclid freed of every fleck." In English *fleck* sounds rather Teutonic, and the reviewer suspects that *flaw*, or *blemish* would sound better to the American ear. It is highly commendable that the translator has arranged the Latin and English texts in parallel pages.

If Saccheri had drawn the last consequences of his keen logic, and if he had not unfortunately dragged in the then hazy notions of infinity and continuity, he would have established the first flawless non-euclidean geometry. On account of those concepts, foreign to euclidean geometry,* the latter portions of the book, according to Halsted, are comparatively unimportant. No less a mathematician than Segre says of this book: "Nevertheless the first seventy pages (apart from a few isolated phrases), up to Proposition 32 inclusive, constitute an ensemble of logic and of geometric acumen which may be called *perfect.*"

Considering the circumstance that the translation itself does add nothing new to well known facts, it must be said that the most valuable part of the book is the introduction written by Halsted, wherein he gives us interesting and valuable information on Saccheri's *Logica demonstrativa*. This remarkable work embodies a system of which the *Euclides Vindicatus*, which appeared later, may merely be called an application, and could obviously only originate in a mind of extraordinary ability. The first edition was published in Turin in 1697, the second in Turin in 1701, the third in Cologne in 1735. In an analysis of this logic Heath says: "Mill's account of the true distinction between real and nominal definitions was fully anticipated by Saccheri." According to Halsted, "In his *Logica demonstrativa* Saccheri lays down the clear distinction between what he calls *definitiones quid nominis* or *nominales*, and *definitiones quid rei* or *reales*, namely, that the former are only intended to explain the meaning that is to be attached to a given term, whereas the latter, besides declaring the meaning of a word, affirm at the same time the existence of the thing defined or, in geometry, the possibility of constructing it. The *definitio quid nominis* becomes a *definitio quid rei*" by means of a postulate, or when we come to the question whether the thing exists and it is answered affirmatively. "*Definitiones quid nominis* are in themselves quite arbitrary, and neither require nor are capable of proof; they are merely provisional, and are only intended to be turned as quickly as possible into *definitiones quid rei*," by means of certain postulates of existence or constructions and by means of demonstrations. Vailati, thus far the only protagonist of the *Logica demonstrativa*, says that the anticipation of Mill's distinction gives Saccheri the right to an eminent place in the history of modern logic.

On the whole, Halsted's edition of Saccheri's work is very well done and is a very praiseworthy scientific undertaking. Equally praiseworthy would be an English edition of the *Logica demonstrativa*, and we would suggest to Dr. Halsted to try to secure, for that purpose, a copy of the Cologne edition, or, for that matter, of any of the previous editions, and thus render a great service to the history of science.

ARNOLD EMCH.

* If, according to Weyl, we define as an *euclidean number* a number which is obtained from 1 by addition, subtraction, multiplication, and division, to which is adjoined the root of a positive number, then, within this system of definite content of euclidean numbers, all constructions of euclidean geometry may be carried out. It is not necessary to pour a continuous "space-broth" over it.