SHORTER NOTICES.

Lezioni sulle Teoria delle Superficie. By GREGORIO RICCI. Padova, 1898.

THIS lithographed volume of about 400 pages gives the formal development and application to the theory of surfaces of the instrument of analysis termed Calcolo differenziale assoluto by the author. The method leads to formulæ and equations always presenting themselves under the same form for any system of independent variables, and the difficulties which are incidental and formal rather than intrinsic are thus to some extent done away with, and the research assumes a uniformity absent in other methods. The entire discussion is based on the properties of differential quadratic forms, and the Introduction, of about 130 pages, contains a rather complete exposition of the methods of the calculus with a more or less full treatment of differential quadratic forms in general and binary forms in particular. The novelty and generality of the method and notation require of the reader not a little effort; but, these difficulties of form once overcome, the various geometrical applications follow easily.

Chapter I is devoted to generalities on linear homogeneous partial differential equations of the first order and to complete systems. Chapters II and III treat differential quadratic forms and develop the absolute calculus for n variables. In chapters IV and V these forms are classified and differential invariants are formed, while chapter VI applies the calculus to two variables.

Part I, comprising the second third of the book, treats surfaces as flexible and inextensible, while Part II is taken up with the theory of surfaces considered as having a rigid form in space. Chapter IV of this part treats special classes of surfaces, while chapter VI is devoted to surfaces of the second degree.

The treatise is by no means a complete exposition of the whole theory of surfaces, such subjects as infinitesmal deformations, pseudospherical geometry, and triply orthogonal systems being barely mentioned; but the idea of the volume seems to be to give some notion of the power and elegance of the absolute calculus. This has a wider application than that indicated here, many questions of pure mathematics and of mathematical physics being advantageously treated by it; and those accustomed to the classic method and notation, as found in Darboux and Bianchi for instance, will find the book well worth reading if for no other reason than to look at the theory from another and entirely different aspect.

G. O. James.

Leçons sur la Théorie des Formes et la Géométrie analytique supérieure. Par H. Andoyer. Volume I. Paris, Gauthier-Villars, 1900. 8vo, pp. vi + 508. Price, 15 francs.

It would be asserting too much to say that M. Andoyer's first volume fulfills all reasonable expectations. In this volume only binary and ternary forms are treated, the quaternary field being reserved for a second, which is announced as already in press. All who read the author's introduction (lithographed) a few years ago must have expected to find in the present work a treatise not only compendious but also elementary. Compendious it certainly is, covering a surprisingly wide range, but the student beginning in this subject and reading it alone will find it impenetrable. It is lucid, it is concise, but it is extremely condensed. Therein lies, however, its great merit. As a work of reference, or as a syllabus to accompany a lecture course, it will supersede anything hitherto published in the same field. Its field is geometry—invariantive geometry, algebra taking the second place. Accordingly one cannot yet dispense with Salmon, Faa di Bruno, and Elliott. Nor is it intended to precede the study of projective geometry of curves; rather it presupposes a large amount of geometrical knowledge, and aims to recast and systematize it. To quote from the preface: "Je me suis proposé, en l'écrivant, d'exposer d'une façon didactique la théorie des formes et son interprétation géométrique générale."

Binary forms are treated in ten chapters, occupying 145 pages. After two excellent but too compact preliminary chapters, linear and quadratic forms and systems of forms are fully discussed, together with formal treatment of resultants and discriminants. Cubics, quartics, and quintics with their full covariant systems are given, the last only in list without any detail. All systematic discussion of *complete* form systems is excluded, the reader being referred to Gordan and Hilbert. Finally forms in two sets of variables are taken up, otherwise correspondences, and the metric geometry on a line. The chapter most novel in this binary division is that on the doubly quadratic form, or the (2, 2) correspondence on a line. Of course the problem of derived correspondences (2, 2) and the conditions for the occurrence