April 6, 2007

Physics 390: Homework set #7
Solutions

Reading: Tipler & Llewellyn, Chapter 13 (1-5), Chapter 14 (3-8)

Questions:

1. Discuss some ways by which we could distinguish antineutrons from neutrons. For example: how would an antineutron beta-decay? What effect would a magnetic field have on a beam of antineutrons? Could a nucleus capture an antineutron?

Solution: You could tell an antineutron from a neutron in several ways:

- It would beta decay into an antiproton and a positron: $\bar{n} \to \bar{p} + e^+ + \nu_e$.
- Its magnetic moment would have the opposite sign.
- It could not be captured by a nucleus since it would immediately annihilate with the neutrons and protons.

2. Some theorists have suggested that certain constants of nature may not really be constant—they may vary slightly with time. Suppose that the fine-structure constant $\alpha = e^2/4\pi\epsilon\hbar c$ changed by some fraction over a period of 10^{10} years. What kind of experiment can you think of to test this hypothesis?

Solution: You would like to test this by going back in time to see if α was different in the past. Actually, this is possible by looking at light reaching us from distant galaxies. This light was emitted millions or possibly billions of years ago. The position of emission and absorption lines, for example, depends not only on the redshift but on the value of the fine structure constant when the emission or absorption occurred. Indeed, some recent (controversial) measurements of absorption lines from very distant objects called quasars suggest that the fine structure constant may have been 0.001% smaller in the early universe than it is now. See `http://www.nature.com/nsu/010816/010816-8.html` for a discussion of these measurements.

3. Use the physical constants G, $\hbar$, and c, together with dimensional analysis, to construct a quantity with the dimensions of time. This is the *Planck time*. Construct corresponding quantities with the dimensions of mass and length—the *Planck mass* and *Planck length*, respectively. Evaluate these quantities numerically. In some sense these are the "natural" units for time, mass, and length.

Solution: The *Planck time* is

$$t_{\rm Pl} = \sqrt{\frac{\hbar G}{c^5}} = 5.39 \times 10^{-44} \text{ s.}$$

The other corresponding quantities are

$$M_{\rm Pl} = \frac{\hbar}{t_{\rm Pl} c^2} = \sqrt{\frac{\hbar c}{G}} = 2.18 \times 10^{-8} \text{ kg} = 1.2 \times 10^{19} \text{ GeV/c}^2$$

and

$$l_{\rm Pl} = c t_{\rm Pl} = \sqrt{\frac{\hbar G}{c^3}} = 1.61 \times 10^{-35} \text{ m.}$$

These are *very* short distances/times, and extremely large (on the scale of elementary particles) masses. At these scales, the quantum nature of gravity is manifest, and theory is currently unable to make predictions.

Problems:	Chapter 13:	6, 10, 14, 27, 28, 32, 49
	Chapter 14:	8, 21, 23

Problem 13-6: The reactions $p + p \to p + e^- + e^+ + \bar{p}$ and $p + p \to p + \bar{p}$ violate both charge and baryon number conservation.

Problem 13-10:

(a) The presence of a photon in the final state of the reaction ${}^{16}\text{O}^* \to {}^{16}\text{O} + \gamma$ is a sure sign that this is an electromagnetic interaction.

(b) Neutrinos participate in the weak interaction only. So the inelastic scattering process $\nu_e + e \to \nu_e + e$ must be a weak interaction.

(c) The presence of two photons in the final state indicates that the annihilation $p + \bar{p} \to \gamma + \gamma$ is an electromagnetic process.

(d) The inverse beta-decay process $p + \bar{\nu}_e \to n + e^+$ must be a weak interaction because it involves a neutrino.

(e) The inelastic scattering of a proton and a π^0 proceeds via the strong interaction.

(f) The beta decay of tritium, ${}^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e$ is mediated by the weak interaction, as you can tell from the neutrino in the final state.

Problem 13-14:

(a) The reaction $p \to n + e^+ + \bar{\nu}_e$ violates energy conservation, since $M_p < M_n + M_e$ (in fact, $M_p < M_n$). As if that weren't enough, this reaction also violates lepton number, since the electron number of the initial and final state is 0 and -2 respectively.

(b) The reaction $n \to p + \pi^-$ violates energy conservation, since $M_n < M_p + M_\pi$.

(c) $e^+ + e^- \to \gamma$ violates linear momentum conservation. To see this, consider boosting into the CM frame of the electron and positron. In this frame, $\mathbf{p} = 0$. But there is no frame in which the photon momentum is zero, since $p = h\lambda$. An electron and a positron can annihilate into two photons, when it's possible to conserve momentum.

(d) $p + \bar{p} \to \gamma + \gamma$ is allowed.

(e) $\nu_e + p \to n + e^+$ is not allowed because it violates lepton number. The electron neutrino has $L = 1$, while the positron has $L = -1$. So this reaction has $\Delta L = 2$. We could "fix" it by replacing the neutrino in the initial state with an electron antineutrino.

(f) $p \to \pi^+ + e^+ + e^-$ is forbidden by baryon number conservation.

Problem 13-22: The approximate range of a weak interaction mediated by a W^+, according to Eqn. 11-50 and Table 13-2, is

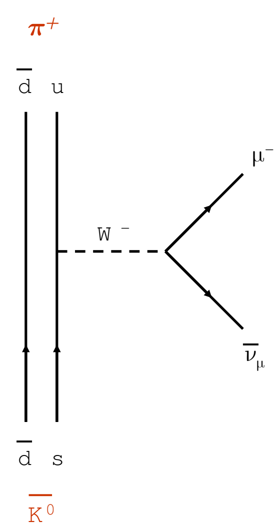
$$R = \frac{\hbar\, c}{mc^2} = \frac{0.1973\ \text{GeV} \cdot \text{fm}}{81.91 GeV} = \underline{2.41 \times 10^{-3}\ \text{fm}}.$$

Problem 13-28:

The Feynman diagram for the decay

$$\overline{K^0} \quad \to \quad \pi^+ + \mu^- + \overline{\nu}_\mu$$

is like



Problem 13-32:

(a) From Table 13-4, we see that the K$^+$ has charge $Q = +1$, $B = 0$, and $S = +1$. Being a meson, the K$^+$ is constructed of a quark-antiquark pair. The only combination with charge $Q = +1$, and strangeness $S = +1$ is the $u\,\bar{s}$, as you can find in Table 13-7.

(b) The K^0 meson has $Q = 0$, $B = 0$, and $S = +1$. The quark-antiquark structure to produce these quantum numbers is $d\,\bar{s}$.

Problem 13-49:

(a)

$$\Delta t = t_2 - t_1 = \frac{x}{u_2} - \frac{x}{u_1} = x\left(\frac{1}{u_2} - \frac{1}{u_1}\right) = x\frac{u_1 - u_2}{u_1 u_2} = \frac{x\Delta u}{u_1 u_2} \approx \frac{x\Delta u}{c^2},$$

since $u_1, u_2 \approx c$.

(b) From Eqn. 2-10, $E = \frac{mc^2}{\sqrt{1-u^2/c^2}}$. Solving for u/c, we have

$$\frac{u}{c} = \sqrt{1 - \frac{mc^2}{E}} \approx 1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2,$$

where in the last step we used the fact that $mc^2/E \ll 1$.

(c)

$$\begin{aligned}
u_1 - u_2 &= c\left[1 - \frac{1}{2}\left(\frac{mc^2}{E_1}\right)^2 - 1 + \frac{1}{2}\left(\frac{mc^2}{E_2}\right)^2\right] \\
&= \frac{c}{2}\left(\frac{mc^2}{E_2}\right)^2 - \frac{c}{2}\left(\frac{mc^2}{E_1}\right)^2 = \frac{c(mc^2)^2}{2}\left[\frac{E_1^2 - E_2^2}{E_1^2 E_2^2}\right] \\
&= \frac{c(2.4\text{ eV})^2}{2}\left[\frac{(20 \times 10^6\text{ eV}) - (5 \times 10^6\text{ eV})}{(20 \times 10^6\text{ eV})(5 \times 10^6\text{ eV})}\right] \\
&= \frac{c(2.4\text{ eV})^2}{2}\left[\frac{(20)^2 - (5)^2}{(20)^2(5)^2(10^6\text{ eV})^2}\right] = \underline{1.08 \times 10^{-13}c}.
\end{aligned}$$

Then

$$\Delta t \approx \frac{x\Delta u}{c^2} = \frac{(170,000\ c \cdot \text{yr})(1.08 \times 10^{-13}c)}{c^2} = \underline{1.84 \times 10^{-8}\text{ yr}} = \underline{0.58\text{ s}}.$$

(d) If the neutrino rest mass energy is 40 eV, then $\Delta u = 3.00 \times 10^{-11}c$ and $\Delta t \approx 161$ s. The difference in arrival times can thus be used to set an upper limit on the mass of the neutrino.

Problem 14-8: A star's luminosity is $L = 4\pi r^2 f$ and the difference in the apparent magnitudes of two stars m_1 and m_2 is defined as $m_1 - m_2 = -2.5\log(f_1/f_2)$. Thus,

$$L_p = 4\pi r_p^2 f_p \quad \text{and} \quad L_B = 4\pi r_B^2 f_B.$$

If the two stars have the same luminosity $L_p = L_B$, then

$$r_p^2 f_p = r_B^2 f_B,$$

or

$$r_B^2 = r_p^2\left(\frac{f_p}{f_B}\right).$$

Furthermore,

$$\log\left(\frac{f_p}{f_B}\right) = \frac{m_p - m_B}{-2.5} = \frac{1.16 - 0.41}{-2.5} = -0.30$$

so $f_p/f_B = 0.50$. Because $r_p = 12$ pc,

$$r_B = r_p\left(\frac{f_p}{f_B}\right)^{1/2} = 12\sqrt{0.5} = \underline{8.5\text{ pc}}.$$

Problem 14-21: According to Wien's law (Eqn. 3-11)

$$\lambda_{max} = \frac{2.898 \text{ mm} \cdot \text{K}}{T} = \frac{2.898 \text{ mm} \cdot \text{K}}{2.728 \text{ K}} = 1.062 \text{ mm}.$$

This wavelength is in the microwave region of the electro-magnetic spectrum.

Problem 14-23:

If Hubble's law applies for an observer in the Milky Way, say A, then the relative velocity between observer B and A, and between observer C and A is

$$v_{BA} = H\, r_{BA}, \quad \text{and} \quad v_{CA} = H\, r_{CA}.$$

From mechanics, we know that

$$v_{BC} = v_{BA} - v_{CA} = H(r_{BA} - r_{CA}) = H\, r_{BC}.$$

Thus, Hubble's law applies in C as well, and by extension in all other galaxies.