# Education and social mobility[1]

April 2009,
Revised January 2010

[1] We thank Marisa Hidalgo, Elke Lüderman and two anonymous reviewers for their comments and suggestions. Part of this paper has been written while the second author was visiting Yale University. He thanks Yale's Economics Department for its hospitality.

**Abstract**

This paper shows that the design of education policy involves a potential conflict between welfare and social mobility. We consider a setting in which social mobility is maximized under the least elitist public education system, whereas welfare maximization calls for the most elitist system. We show that when private education is available, the degree of elitism that maximizes social mobility increases, while the welfare-maximizing degree of elitism decreases. The ranking between the welfare- and mobility-maximizing degree of elitism may even be reversed. Utilitarian welfare is always higher when private supplementary education is available, but social mobility may be reduced.
**Keywords**: elitism , egalitarianism, private education.

**JEL classification**: H37.

# 1 Introduction

The literature on education often advocates "elitist" policies. The standard approach is to consider a population of individuals who differ in their ability to benefit from education. This heterogeneity typically implies a rather regressive distribution of public education: resources are concentrated on the most able individuals in order to get a "cake" as big as possible to share among individuals through income taxation; see *e.g.*, Brett and Weymark (2008), Bruno (1976), Hare and Ulph (1979) and Ulph (1977).[1] This recommendation relies on the assumption that education is not the only channel of policy intervention; there is also in a second stage an income tax that can alter social welfare.[2] If the exercise is restricted to the first stage, the solution is different and tends to be less regressive. This is shown for instance by Arrow (1971) who studies the optimal distribution of a given amount of public expenditure among individuals differing in their learning ability without accounting for the possibility of subsequent income redistribution. Note that this elitist distribution effect is mitigated when we introduce decreasing returns of educational spending.[3]

The literature that recommends elitism in education typically concentrates on a single generation. Consequently, the issue of social mobility does not arise. In reality, however, social mobility is often considered as an important issue for the assessment of education policies (Grossman and Kim (2003), Mejia and St-Pierre (2008), Speciale (2007), Iannelli and Paterson (2005) and the references therein). It is often valued for its own sake and independently of efficiency or (intragenerational) equity concerns. To understand the underlying problem suppose (just for the sake of the argument) that learning ability is transmitted by parents. We could end up with an educational policy that indeed maximizes social welfare at each period of time but at the expense of social mobility. Would such an outcome be acceptable? This is the issue dealt with in this paper. But first let us consider some basic facts.

[1] In a recent paper Cremer *et al.* (2009) put forward another reason to push for regressive education. It is not linked to heterogeneity in innate ability to benefit from education but to pervasive non-convexities that arise in the optimal income tax problem when individual productivities depend on education.

[2] See also De Fraja (2002) and Cremer and Pestieau (2006).

[3] Bovenberg and Jacobs (2005) and Maldonado (2007).

When looking at the educational system of relatively similar countries in terms of GDP, one is surprised by their wide heterogeneity. To characterize it, one can use the amount of expenditure devoted to education, the degree of elitism and the relative involvement of private market. We focus on the last two characteristics. The design of an educational system is an important, but also controversial and complex issue. Important because economic and human development are known to be crucially affected by the level and the structure of human capital. Controversial because nobody wants to admit that his educational system is elitist, even though this is often the case. Complex because it is not easy to measure the degree of elitism of an educational system. Measuring it by the way resources are allocated among students of different origins and learning capacities is not useful. What matters is the effective outcome, for example, the level of knowledge achieved by students of a given age.

Hanushek and Woesmann (2007) consider the share of students in each country that reach a certain threshold of basic literacy and the share of students that surpass a threshold of top performance. The first share can be used as a proxy for egalitarianism and the second as a proxy for elitism. Taking, in the sample of Hanushek and Woesmann, countries that have about the same GDP and the same relative level of educational spending, one observes that they differ quite a lot. For instance, the ratio of ninth decile to first decile in the prose literacy test performance varies from 1.4 for Denmark to 1.9 for the USA (p.18). Similar figures can be obtained from the OECD Programme for International Student Assessment (PISA 2006). As to the heterogeneity in public and private education expenditures, for an OECD average of 4.7% and 1.4% of GDP in 2004, we have 6.0% and 0.1% in Finland and 5.1% and 2.3% in the US.[4]

In this paper, we assume that society can control the degree of elitism of public education (for a given level of expenditures), and the availability of private supplements to it. We refer to a school system as elitist if it favors the uppertail of the distribution of learning capacities. An egalitarian system, on the other hand aims at equalizing opportunities of successful education. We assume that public authorities can determine the degree of elitism for instance through the design of school districts, the selection of

[4]See OECD (2007).

students leading to their assignment to different types of schools at a more or less early stage, differential investment in schools and students, selection of teachers, etc. Private supplementary education consists for instance of private tutoring which can range from informal arrangements with students or teachers to fully fledged professional tutoring.[5] Its availability can be affected by either making it tax deductible or by modifying the way public education programs are run (full day schedule or not). For instance, during the 2007 presidential elections in France, the socialist candidate has proposed to make it more difficult for public sector teachers to moonlight in the private sector and offer parents educational support after the regular public school hours. For simplicity, we assume that the availability of supplementary private education is a binary decision: it can either be allowed or forbidden. The degree of elitism of public education, on the other hand, is a continuous choice variable.

We consider two possible social objectives: utilitarian welfare and social mobility. These two objectives are often referred to in the assessment of education systems. For instance, Jesson (2007) studies both students performance and socioeconomic background in his recent assessment of England's grammar schools. He obtains that, although "one notable feature of the grammar schools is the high performance of their pupils in exams, [...they] do not offer a ladder of opportunity to any but a very small number of disadvantaged pupils."

Our objective is to develop a simple model that illustrates how stark the conflict between welfare maximization and social mobility can be in the determination of the optimal degree of elitism of public education, and how this degree of elitism is affected by the availability of private supplementary education.[6] The setting is the simplest one which can represent the main effects in a meaningful way. Within a two-skill setting

[5] Empirically, supplementary private education is important in many countries, and is on the rise: "Private supplementary tutoring has long been a major phenomenon in parts of East Asia, including Japan, Hong Kong, South Korea and Taiwan. In recent times it has grown dramatically in other parts of Asia and in Africa, Europe and North America" (Bray, 2005, p.1). Bray (2007) contains data that show the important scale of private supplementary tutoring in several countries, from Brazil to Zimbabwe, including Cambodia, Egypt, Japan and Malta.

[6] These two objectives appear to be conflicting in many countries and particularly in France and Germany. However, this is not necessarily the case. A good counterexample is provided by Finland where we simultaneously observe high social mobility, a high share of top performers and a high average performance; see e.g., PISA (2006), p. 184–5 and 189–90.

we assume that skilled parents are more likely to have skilled children than unskilled parents. When the educational policy becomes more elitist, the probability that a skilled parent has a child that is skilled as well increases and the probability that an unskilled parent has a skilled child decreases.

We first study the optimal degree of elitism when private educational supplements are not available. Utilitarian welfare increases with the steady state proportion of skilled agents which, in turn, increases with the degree of elitism of the public education system. On the other hand, elitism decreases the steady state proportion of heterogenous dynasties (those comprised of a skilled parent and an unskilled child, or vice versa) which is our measure of social mobility. Consequently, social mobility is maximized under the least elitist public education system, in stark contrast with the most elitist system implying maximum welfare.

We then open up the possibility for skilled parents to invest in private supplementary education for their child. We assume that private education efficiency is larger when the public education system is more egalitarian (*i.e.*, less elitist). We study the impact of private education on the trade-off between social mobility and welfare. We show that the degree of elitism that maximizes social mobility increases, while the welfare-maximizing degree of elitism decreases, provided that the high skilled parents' productivity is large enough. We provide a numerical example where the ranking between the welfare- and mobility-maximizing degree of elitism is reversed when private education is allowed – *i.e.*, where the public education system that maximizes social mobility is more elitist than the one that maximizes welfare. Finally, we show that utilitarian welfare is always (weakly) higher when private supplementary education is available. On the other hand, to maximize social mobility it may be preferable to ban private supplements.

The rest of the paper is organized as follows. Section 2 presents the model and Section 3 gives the optimal choice of elitism in the absence of private education. Private education is introduced in Section 4 and a numerical example is given in Section 5.

## 2 The Model

Individuals care for their own consumption ($c$) and the educational attainment ($a$, as in altruism) of their (unique) child. All individuals have the same utility function

$$U(c,a) = u(c) + v(a),$$

with $u' > 0$, $u'' \leq 0$, $v' > 0$ and $v'' < 0$.

There are two types of individuals: high productivity/wage/income,$w_H$, and low productivity, $w_L$ with $w_L < w_H$. Educational attainment $a$ measures the child's probability to achieve a high productivity. This probability depends on the parent's productivity level and on a parameter $\alpha \in [0,1]$, which characterizes the degree of elitism of education policy. Observe that education policy is not represented by the level of expenditures which is implicitly assumed to be given. Instead, education policies are differentiated by their degree of elitism.

Formally, a child's probability of achieving a high productivity is given by $\phi_i(\alpha)$, $0 \leq \phi_i(\alpha) \leq 1$, where the index $i \in \{L, H\}$ refers to the parent's ability level. When $\alpha = 0$ the education system is egalitarian in the sense that $\phi_H(0) = \phi_L(0) = \bar{p}$. We assume *(i)* $\phi_H(\alpha) \geq \phi_L(\alpha)$ and *(ii)* $\phi'_H > 0$ and $\phi'_L < 0$. Assumption *(i)* states that a child with a high productivity parent never has a lower probability of achieving $w_H$ than a child with a low productivity parent. In other words, high productivity parents are more likely to have high productivity children than low productivity parents. This illustrates the importance of family background and of social, family-related skills which increase the productivity of formal education (see Introduction). The second assumption implies that the educational attainment function increases with $\alpha$ for the children of high productivity parents and decreases for those of low ability parents.

To complete the characterization of the attainment function define

$$p_H = \phi_H(1) \qquad \text{and} \qquad p_L = \phi_L(1) \tag{1}$$

with $p_H > \bar{p} \geq p_L$. Figure 1 depicts the relation $\phi_i(\alpha)$ starting at $\bar{p}$ for $\alpha = 0$ and ending at $p_H = \phi_H(1) > \bar{p} \geq p_L = \phi_L(1)$.[7]

[7] To draw this Figure, we use the functional forms presented in Section 5.

Insert Figure 1 around here

Fig.1: $\Phi_L$ and $\Phi_H$ (dashed) in the absence of private education

Our timeline spans several generations, so that the proportion of high type individuals in one generation is a function of the proportion in the previous generation and of $\phi_L(\alpha)$ and $\phi_H(\alpha)$. Let $p$ denote this proportion of high skilled individuals; its steady state level satisfies

$$p = (1-p)\phi_L(\alpha) + p\phi_H(\alpha),$$

so that the steady state level $p^*(\alpha)$ is given by

$$p^*(\alpha) = \frac{\phi_L(\alpha)}{1+\phi_L(\alpha)-\phi_H(\alpha)}. \tag{2}$$

We have that $p^*(0) = \bar{p}$ and, using (1)

$$p^*(1) = \frac{p_L}{1+p_L-p_H}.$$

Differentiating (2) shows that $p^*$ is not necessarily a monotonic function of $\alpha$, precisely because increasing $\alpha$ decreases $\phi_L$. If $p^*$ were to decrease with $\alpha$, there would be no conflict between welfare- and social-mobility maximization (see footnote 11 below). In order to introduce a conflict between these two objectives, we assume throughout the paper that

$$p^{*\prime}(\alpha) > 0 \qquad \forall \alpha \in [0,1]. \tag{3}$$

In words, we assume that a more elitist public education system increases the steady state proportion of skilled individuals. Intuitively, this requires that the benefit that high skilled parents derive from an increase in the degree of elitism of public education is large compared to the cost that it entails for low skilled parents (i.e., that $\phi_H'$ be large compared to $\phi_L'$, in absolute values).

## 3 The optimal level of $\alpha$

We use a transition matrix where rows denote the ability level of the parent while columns denote the child's ability. Each cell contains the corresponding proportion of the child population.

| | Child's ability | |
|---|---|---|
| Parent's ability | $w_L$ | $w_H$ |
| $w_L$ | $(1-p^*(\alpha))\,(1-\phi_L(\alpha))$ | $(1-p^*(\alpha))\,\phi_L(\alpha)$ |
| $w_H$ | $p^*(\alpha)\,(1-\phi_H(\alpha))$ | $p^*(\alpha)\phi_H(\alpha)$ |

Table 1: Proportion of families according to ability levels of parent and child

The cells $(w_L, w_L)$ and $(w_H, w_H)$ represent the homogenous dynasties (no social mobility). The remaining cells represent heterogenous dynasties (who experience mobility). The proportion of dynasties with upward mobility is given in cell $(w_L, w_H)$ while dynasties counted in cell $(w_H, w_L)$ experience downward mobility. Observe that the definition of $p^*(\alpha)$ as steady state level implies $(1-p^*(\alpha))\,\phi_L(\alpha) = p^*(\alpha)\,(1-\phi_H(\alpha))$. Given that $p^{*\prime}(\alpha) > 0$, $\phi_L'(\alpha) < 0$ and $\phi_H'(\alpha) > 0$, an increase in $\alpha$ increases the proportion of homogenous skilled dynasties and decreases the proportion of heterogeneous dynasties. The impact of $\alpha$ on the proportion of homogenous unskilled dynasties can go either way.

We consider two possible social objectives: social welfare maximization and social mobility (maximization of a mobility index). These two objectives are respectively denoted by $W(\alpha)$ and $M(\alpha)$. We now study the two objectives $W(\alpha)$ and $M(\alpha)$ in turn.

Social welfare $W(\alpha)$ is utilitarian and expressed as[8]

$$W(\alpha) = (1-p^*(\alpha))\,u(w_L) + p^*(\alpha)u(w_H). \tag{4}$$

Differentiating (4) yields

$$W'(\alpha) = p^{*\prime}(\alpha)\,(u(w_H) - u(w_L)) > 0.$$

Not surprisingly it thus appears that utilitarian welfare is maximized when the steady state proportion of high type individuals is at its maximum level. With $p^{*\prime}(\alpha) > 0$, the optimal value of $\alpha$ is then given by $\alpha^W = 1$.

[8]Observe that we launder the individual preferences in the sense that we do not take into account the impact of $\alpha$ on $v(.)$. See on this Hammond (1987) and Andreoni (2006). This is a standard, albeit sometimes debated assumption. It is made to avoid "double-counting" of altruistic considerations in social welfare. We discuss in Section 5 how our results would be affected if we did not launder preferences.

Let us now turn to the mobility index $M(\alpha)$ which is defined as the proportion of heterogenous dynasties in the steady state[9]

$$M(\alpha) = (1 - p^*(\alpha))\, \phi_L(\alpha) + p^*(\alpha)\, (1 - \phi_H(\alpha)) = 2\, (1 - p^*(\alpha))\, \phi_L(\alpha). \tag{5}$$

Differentiating (5) shows that

$$M'(\alpha) < 0,$$

given that $\phi'_L < 0$ and $p^{*\prime}(\alpha) > 0$. Social mobility as measured by the index $M$ is thus maximized when $\alpha = \alpha^M = 0$.[10]

These results are summarized in the following proposition.[11]

**Proposition 1** *In the absence of supplementary private education, and under assumption (3), the maximization of utilitarian social welfare yields the most elitist public education system ($\alpha^W = 1$) while social mobility is maximum when the least elitist public education system is adopted ($\alpha^M = 0$).*

# 4 Introducing private education

We now open up the possibility for high ability parents to invest in (supplementary) private education in order to increase the probability that their child be highly productive. We assume that low ability parents never invest in private education.[12] Formally

[9] Alternatively, we could have used the more sophisticated approach advocated by Atkinson and Bourguignon (1982) or Gottschalk and Spolaore (2002) which model the concern for mobility as aversion to multi-period inequality. Our more pedestrian approach is much easier to deal with analytically, especially since we want to contrast the results obtained with a purely welfarist planner and with a planner only interested in social mobility. It is true that the objective of maximum social mobility, reached when all skilled parents have unskilled children, and vice versa, may not look very appealing to everyone. Observe that we never attain this situation in our model, since the probability to have a high skilled child is always weakly higher for high skilled than for low skilled parents.

[10] When $\alpha = 0$, the probability of being highly skilled is the same for both types of parents. This corresponds to a perfect equality of opportunities (where your situation in life is independent of your parent's). In this sense, Proposition 1 continues to holds when we replace the objective of social mobility maximization by one of equalization of opportunities.

[11] As mentioned earlier, if $p^{*\prime}(\alpha) < 0$ there would be no trade-off between efficiency and social mobility. Adopting a more egalitarian system would enhance both welfare and mobility.

[12] This assumption is made to simplify the exposition. In reality, as the Director of UNESCO's International Institute for Educationnal Planning writes: "Families with the necessary resources are able to secure not only greater quantities but also better qualities of private tutoring. Children receiving such tutoring are then able to perform better in school, and in the long run to improve their lifetime earnings. By contrast, children of low-income families who do not receive such benefits may not be able

$\phi_H$ is now (redefined as) a function of expenditure on private education, $e$, and $\alpha$, while $\phi_L$ continues to be a function of the sole variable $\alpha$.

In the remainder of the paper we will also adopt specific functional forms for these expressions. Let

$$\phi_H(\alpha, e) = (1-\alpha)(\bar{p}+e) + \alpha p_H, \tag{6}$$

so that private education is especially efficient when $\alpha$ is low — *i.e.*, when the public school system is very egalitarian. The rationale for this assumption is that an an elitist public system already invests more in brighter kids, so that the marginal benefits they could obtain from additional private education is low. An egalitarian public system, on the other hand, does not devote extra resources to brighter students, who may then benefit a lot from additional private education. The function $\phi_L$ continues to be specified by

$$\phi_L(\alpha) = (1-\alpha)\bar{p} + \alpha p_L. \tag{7}$$

We further assume from now on that $u(c) = \ln(c)$ and that $v(d) = \ln(d)$.[13]

We start by solving for the individual decision of how much to invest in private education.

### 4.1 The private education choice

We assume a constant price, normalized to one, for private education services. High productivity individuals solve the following problem:

$$\max_e u(w_H - e) + v(\phi_H(\alpha, e)),$$

where $w_H - e$ denotes the consumption level of a type $H$ once the private education cost $e$ is subtracted from income $w_H$. The first-order condition with respect to $e$ is

$$u'(c) = v'(d)(1-\alpha),$$

to keep up with their peers and may drop out of school at an earlier age." (Bray, 2005, p.1) We discuss the robustness of our results to this assumption in the concluding section.

[13] We need to introduce a functional form in order to solve explicitly for the individually optimal supplementary education level.

from which we obtain that

$$\begin{aligned} e^o(\alpha) &= \frac{1}{2}(w_H - \bar{p} - \frac{\alpha}{1-\alpha}p_H) \text{ if } \alpha \leq \tilde{\alpha} = \frac{w_H - \bar{p}}{w_H + p_H - \bar{p}} < 1, \\ &= 0 \text{ if } \alpha > \tilde{\alpha}, \end{aligned}$$

where $e^o$ denotes the most preferred value of $e$ of a high ability individual. We assume that $w_H > \bar{p}$ so that $e^o(0) > 0$.[14] It is clear that $e^{o\prime}(\alpha) < 0$ when $\alpha < \tilde{\alpha}$. Intuitively, since the efficiency of private education decreases with $\alpha$, so does the optimal amount of private education bought by skilled parents. The individually optimal private education amount increases with the individual's income $w_H$ (since a larger $w_H$ decreases the marginal utility from consumption, and thus the marginal utility cost of investing in supplementary education) and decreases with both $p_H$ and $\bar{p}$ (because increasing either $p_H$ or $\bar{p}$ increases the probability of raising a high-skilled child for any given private education level, making private education less attractive at the margin).[15] We then have that $\tilde{\alpha}$ increases with $w_H$ and decreases with both $p_H$ and $\bar{p}$.

Substituting $e^o(\alpha)$ into $\phi_H(\alpha, e)$ yields

$$\begin{aligned} \phi_H^o(\alpha) &= \phi_H(\alpha, e^o(\alpha)) \qquad (8) \\ &= \frac{1}{2}(\bar{p}(1-\alpha) + w_H(1-\alpha) + p_H\alpha) \text{ if } \alpha \leq \tilde{\alpha}, \\ &= \phi_H(\alpha, 0) = (1-\alpha)\bar{p} + \alpha p_H \text{ if } \alpha > \tilde{\alpha}. \end{aligned}$$

This function is linear in two parts. Skilled parents buy private education provided that its marginal productivity is large enough — i.e., that the public education system is not too elitist ($\alpha < \tilde{\alpha}$). As long as $\alpha < \tilde{\alpha}$, increasing $\alpha$ has two effects of opposite signs on $\phi_H^o$. On the one hand, the public education system becomes more elitist, which increases $\phi_H$ for a given value of $e$. On the other hand, skilled parents buy less private education, which decreases $\phi_H$, for a given $\alpha$. With our formulation, the net effect cannot be signed without additional assumptions on $\bar{p}$, $p_H$ and $w_H$: $\phi_H^o$ decreases with $\alpha$ if $w_H$ is large enough ($w_H > p_H - \bar{p}$), because a large value of $w_H$ amplifies the second effect. As $\alpha$ increases above $\tilde{\alpha}$, the second effect disappears (since skilled agents do not

[14] If it were not the case, nobody would ever buy private education and the analysis contained in the previous section would carry through.

[15] A higher value of $\bar{p}$ could be associated, for instance, with a higher quality/level of public education.

buy private education for large values of $\alpha$) and $\phi_H^o$ increases with $\alpha$, as in the previous section.

The steady state proportion of high ability individuals is now a function of $\alpha$ and $e$. However, using $e^o(\alpha)$ we return to a function of the single variable $\alpha$ and can redefine $p^*$ as

$$p^*(\alpha) = \frac{\phi_L(\alpha)}{1 + \phi_L(\alpha) - \phi_H^o(\alpha)}. \tag{9}$$

When $\alpha > \tilde{\alpha}$ (so that $e^o(\alpha) = 0$) and for the functional forms defined by (6) and (7) this steady state proportion is then given by

$$p^*(\alpha) = \frac{\bar{p} - \alpha(\bar{p} - p_L)}{1 - \alpha(p_H - p_L)}. \tag{10}$$

We make the same assumption as in the previous section, namely that the steady state proportion of high type individuals is increasing in $\alpha$ *when the spending on private education is zero.* Differentiating expression (10) shows that this is the case when

$$p_L(1 - \bar{p}) - \bar{p}(1 - p_H) > 0, \tag{11}$$

a condition which is satisfied when $p_H$ or $p_L$ are high enough.

When $\alpha \leq \tilde{\alpha}$, on the other hand, private education spending is positive and the expression for $p^*$ is more complicated so that $p^*$ may well be a decreasing function of $\alpha$. Figure 2 illustrates this possibility. It depicts $\phi_H^o(\alpha)$ (dashed curve), $p^*(\alpha)$ (thick curve) and $\phi_L(\alpha)$ (thin curve) for $p_L = 0.3$, $p_H = 0.8$, $\bar{p} = 0.5$, $w_L = 0.33$ and $w_H = 1$ (the values upon which the simulations of section 5 are based). These values satisfy condition (11) so that $p$ increases when private education spending is zero (when $\alpha > \tilde{\alpha}$). For low values of $\alpha$ on the other hand, when high type individuals buy private education, the steady state proportion of high type individuals is decreasing in $\alpha$: we have $\phi_H^o$ decreasing in a for $\alpha < \tilde{\alpha}$ (since $w_H > p_H - \bar{p}$), which is a sufficient (although not necessary) condition to have $p^{*\prime} < 0$ over this range of values of $\alpha$.

Insert Figure 2 around here

Probabilities of a high productivity child when private education is available.

## 4.2 The level of elitism

We first consider a utilitarian welfare function given by

$$W^{PE}(\alpha) = (1 - p^*(\alpha))\, u(w_L) + p^*(\alpha) u(w_H - e^o(\alpha)). \tag{12}$$

This expression differs from (4) in two aspects that reflect the impact of private education spending. First, the utility level of the high productivity individuals now depends on $\alpha$ and, second, $p^*(\alpha)$ is now defined by (9). Consequently, maximizing social welfare is no longer equivalent to maximizing $p^*$. Observe that as in the previous section, we launder utilities and do not take into account the utility parents obtain from the probability that their kid is of a high type. The (laundered) utility of the high productivity individual increases with $\alpha$ as long as $\alpha < \tilde{\alpha}$, and is constant for higher values of $\alpha$:

$$\begin{aligned} \frac{\partial u(w_H - e^o(\alpha))}{\partial \alpha} &= \frac{p_H}{(1-\alpha)((1-\alpha)(\bar{p} + w_H) + p_H \alpha)} > 0 \text{ if } \alpha < \tilde{\alpha}, \\ &= 0 \text{ if } \alpha > \tilde{\alpha}. \end{aligned}$$

As the public education system becomes more elitist (up to $\tilde{\alpha}$), high type individuals invest less in private education and rely more exclusively on public education. Consequently, their (laundered) utility increases. The utility of low type individuals, on the other hand, remains independent of $\alpha$. Finally, recall that by assumption (3) the steady state proportion of high type individuals increases with $\alpha$ when $\alpha > \tilde{\alpha}$.

Putting these observations together, we obtain

**Proposition 2** *With private education and $p^{*\prime}(\alpha) > 0$ for*

$$\alpha > \tilde{\alpha} = \frac{w_H - \bar{p}}{w_H + p_H - \bar{p}},$$

*the level of $\alpha$ (denoted $\alpha^W$) that maximizes utilitarian welfare, is either equal to one or belongs to $[0, \tilde{\alpha}]$. Consequently, we can exclude $\alpha^W \in ]\tilde{\alpha}, 1[$. Moreover, a necessary condition to obtain $\alpha^W \leq \tilde{\alpha}$ is that*

$$w_H > \tilde{w}_H = 2 - \frac{\bar{p}(2(1 - p_H) + p_L)}{p_L}.$$

**Proof:** See Appendix

Proposition 2 states that a large value of $w_H$ is necessary for a somewhat egalitarian system with effective private education ($\alpha < \tilde{\alpha}$) to yield a higher level of welfare than an elitist system without any private education at equilibrium ($\alpha = 1$). This property can easily be understood. The consumption level of a high ability type is lower when he invests in private education. A low level of $\alpha$ (inducing positive private education spending) can thus only yield a higher level of welfare than $\alpha = 1$ when it implies a larger steady state proportion of high productivity individuals. This proportion increases with $w_H$ when $\alpha$ is low, because richer parents buy more private education. Proposition 2 provides a lower bound $\tilde{w}_H$ on $w_H$ that guarantees that the steady state proportion of high skilled agents is larger with effective private education. This bound depends upon the three determinants of the functions $\phi_L$ and $\phi_H$, namely $\bar{p}$, $p_L$ and $p_H$. More precisely, $\tilde{w}_H$ decreases with $\bar{p}$ because a larger value of $\bar{p}$ increases the steady state proportion of high-skilled people with a very egalitarian public system (taking into account the amount of supplementary education chosen by high-skilled parents) but not with a very elitist system, making the egalitarian public education scheme more attractive. Similarly, $\tilde{w}_H$ increases with $p_L$ and $p_H$ because they both increase the steady state proportion of high-skilled type with a purely elitist public system but not with an egalitarian one, making the public education scheme with $\alpha = 1$ more attractive. The next section provides a numerical example where social welfare is larger with an elitist system without effective private education if $w_H$ is low while an egalitarian system with private education is preferred if $w_H$ is large enough.

We now turn to the maximization of the social mobility index which continues to be defined by (5) but with $p^*(\alpha)$ redefined by (9). Recall that this index simply measures the proportion of heterogeneous dynasties which at the steady state is given by $2\,(1-p^*)\,\phi_L(\alpha)$ (see Table 1). Social mobility is decreasing on $[\tilde{\alpha}, 1]$ because $p^*$ increases with $\alpha$ over that range, while $\phi_L$ decreases. When $\alpha < \tilde{\alpha}$, social mobility may increase or decrease with $\alpha$, because the steady state proportion of unskilled individuals may increase with $\alpha$, while $\phi_L$ decreases with $\alpha$. We then obtain

| | Welfare | Social Mobility |
|---|---|---|
| No PE | $\alpha^W = 1$ | $\alpha^M = 0$ |
| PE | $\alpha^W = 1$ when $w_H$ is not too large<br>$\alpha^W \in [0, \tilde{\alpha}[$ only when $w_H$ large enough | $\alpha^M \in [0, \tilde{\alpha}]$ and weakly increases with $w_H$ when $w_H$ is large enough |

Table 2: Main results, without and with private education (PE), according to the social objective (welfare or mobility).

**Proposition 3** *With private education and $p^{*\prime}(\alpha) > 0$ for $\alpha > \tilde{\alpha}$, the level of $\alpha$ (denoted $\alpha^M$) that maximizes social mobility belongs to $[0, \tilde{\alpha}]$ . Consequently, social mobility is maximized when private education is effective (high type individuals invest in private education). Moreover, the value of $\alpha^M$ is weakly increasing with $w_H$ if $w_H$ is large enough ($w_H > p_H - \bar{p}$).*

**Proof:** See Appendix

Recall that, without private education, both the steady state proportion of low ability types $(1-p^*)$ and the proportion of agents with a low ability parent who achieve a high productivity ($\phi_L$) decrease with $\alpha$. This explains why the steady state proportion of heterogeneous dynasties decreases with $\alpha$, and why social mobility is maximized with a purely egalitarian public education ($\alpha = 0$). When private education is introduced, an increase in $\alpha$ induces high ability agents to reduce their investment in private education (because its return is lower with a more elitist public system) and this may increase the steady state proportion of low type individuals, resulting in a larger number of heterogenous dynasties. Furthermore, a larger value of $w_H$ reinforces this impact of $\alpha$ on the steady state proportion of low type individuals, so that $\alpha^M$ weakly increases with $w_H$. Table 2 summarizes our results.

It is straightforward that utilitarian welfare may only increase when private education becomes available. The welfare level for $\alpha^W = 1$ is the same under both systems (high productivity individuals do not buy private education when $\alpha = 1 > \tilde{\alpha}$). Consequently, when $\alpha^W < 1$ is chosen when private education is available it must yield a larger welfare level. As for social mobility, the allocation chosen when private education is not available is no longer feasible when private education is introduced. Consequently, we cannot be certain that social mobility[16] increases when private education becomes available.

These results are summarized in the following proposition.

**Proposition 4** *(i) If the income of the high skilled is small enough, the availability of private education does not affect* $\alpha^W$ *nor* $\alpha^M$*. (ii) If the high-skilled income is large enough, the availability of private education may decrease* $\alpha^W$ *while increasing* $\alpha^M$*. (iii) The maximum welfare level when private education is available is the same as without private education if the high skilled income is small enough, and may be larger if the high skilled income is large enough. Consequently, with a welfarist social objective it is never desirable to forbid private educational supplements. (iv) The maximum level of social mobility may be lower with private education, whatever the income of the high skilled. Consequently, when the objective is to maximize social mobility it may be desirable to forbid private educational supplements.*

# 5 Numerical example

We now resort to numerical simulations, with several objectives in mind. First, we would like to show that the introduction of private education may effectively strictly decrease $\alpha^W$ (provided that the high skilled income is large enough) and strictly increase $\alpha^M$. Second, the analytical results do not show whether the ranking of the degrees of elitism achieved under the two social objectives may be reversed when private education is introduced. We do know that $\alpha^M = 0 < \alpha^W = 1$ in the absence of private education. But can we have $\alpha^M > \alpha^W$ once private education is introduced? Third, numerical examples allow us to compare absolute levels and to determine, for instance, whether social mobility of welfare is improved when we allow for supplementary private education. They also enable us to perform a comparative static analysis of the optimum welfare or social mobility levels with respect to high skill productivity, accounting for the induced changes in the degree of elitism. Finally, the numerical examples show how robust our results are to the laundering of utilities when computing social welfare.

[16] Or equalization of opportunities, for that matter.

| $w_H$ | $\alpha^M$ | $M(\alpha^M)$ | $p^*(\alpha^M)$ | $\alpha^W$ | $W(\alpha^W)$ | $p^*(\alpha^W)$ |
|---|---|---|---|---|---|---|
| 0.6 | 0 | 0.5 | 0.5 | 1 | -0.750 | 0.6 |
| 0.65 | 0 | 0.5 | 0.5 | 1 | -0.702 | 0.6 |
| 0.8 | 0 | 0.5 | 0.5 | 1 | -0.577 | 0.6 |
| 1 | 0 | 0.5 | 0.5 | 1 | -0.443 | 0.6 |
| 1.1 | 0 | 0.5 | 0.5 | 1 | -0.386 | 0.6 |
| 1.15 | 0 | 0.5 | 0.5 | 1 | -0.360 | 0.6 |
| 1.3 | 0 | 0.5 | 0.5 | 1 | -0.286 | 0.6 |
| 1.5 | 0 | 0.5 | 0.5 | 1 | -0.200 | 0.6 |

Table 3: No private education

We adopt the logarithmic utility functions and the parameters of the linear probability functions (6) and (7) are given by $p_L = 0.3$, $p_H = 0.8$, $\bar{p} = 0.5$ . Finally we set $w_L = 0.33$, while $w_H$ varies from 0.6 to 1.5.

Table 3 studies the case without private education and reports, for several values of $w_H$, the values of $\alpha^M$ and the corresponding social mobility level and steady state proportion $p^*$ attained, and the value of $\alpha^W$ with the welfare level and value of $p^*$ reached in that case. The social mobility maximizing policy does not depend upon $w^H$, and is such that $\alpha^M = 0$ (see Proposition 1) and that one half of families are heterogenous at the steady state equilibrium.[17] This also corresponds to the allocation equalizing opportunities, since all children have a 50% probability of being highly skilled, independently of the status of their parent. The steady state proportion of skilled agents is also 50% in that case. The welfare-maximizing value of $\alpha$ is equal to one, whatever the value of $w_H$ (see Proposition 1). We have reported the maximum value of welfare (as given by equation (4)) attained, which is of course increasing in $w_H$. The underlying steady state proportion of high skilled type remains constant at 60% for all values of $w_H$.

Table 4 reports the same information as Table 3, but in the case where private supplementary education is available to high skilled parents. We start by looking at the objective of social mobility maximization. When the highly skilled people are less

[17] This means that 25% of families have a skilled parent and an unskilled child, while another 25% have a low skill parent and a high skill child.

than roughly twice as rich as low-skilled agents ($w_H < 0.65$), we have $\alpha^M = 0$ as in the case without private education. The value of $\alpha^M$ then increases with $w_H$, in accordance with Proposition 3. Comparing the third columns of Tables 4 and 3, shows that the maximum level of social mobility is always strictly lower when private education is available than when it is not. As explained in Proposition 4, the original optimal allocation without private education is not available anymore when private education is introduced. We can distinguish three cases in our example. When $w_H$ is low ($w_H = 0.6$), $\alpha^M$ remains at zero, as in Table 3. Social mobility is lower in Table 4 because high skilled parents do buy private education, which increases both the probability that their children are highly skilled and the steady state proportion of high skilled agents (compare the fourth columns of Tables 3 and 4). When $w_H$ is intermediate ($w_H = 0.65$ in Tables 3 and 4), the social-mobility maximizing value of $\alpha$ is higher with than without supplementary education, and private education is bought at the optimum (*i.e.*, $0 < \alpha^M < \tilde{\alpha}$). Compared to Table 3, the planner increases the value of $\alpha$ to mitigate the negative impact of high skilled parents buying private education on social mobility. With higher values of $w_H$ ($w_H \geq 0.8$), the mobility-maximizing value of $\alpha$ is equal to $\tilde{\alpha}$, meaning that no private education is bought at equilibrium. In that case, the social planner increases the degree of elitism of the public education system in order to prevent high skilled parents from buying private education. We then obtain that social mobility is decreasing in $w_H$, because the public education system has to be made more elitist to prevent high skilled families from buying private education as they get richer. Interestingly, it is the mere availability of supplementary private education that drives the public education system to be more elitist and that decreases the maximum social mobility level, and not whether supplementary education is actually bought at optimum or not.

Also interestingly, even though the maximum social mobility level is monotonically decreasing in $w_H$, the steady state proportion of high skilled agents is not, since it first decreases and then increases with $w_H$. Observe that it increases with $w_H$ when $\alpha = \tilde{\alpha}$ at the optimum. This is intuitive, since $\alpha^M = \tilde{\alpha}$ increases with $w_H$, and since $p^{*\prime}(\alpha) > 0$ when no private education is bought (irrespective of the value of $w_H$). For low values of

| $w_H$ | $\alpha^M$ | $M(\alpha^M)$ | $p^*(\alpha^M)$ | $\alpha^W$ | $W(\alpha^W)$ | $p^*(\alpha^W)$ |
|---|---|---|---|---|---|---|
| 0.6 | 0 | 0.474 | 0.526 | 1 | -0.750 | 0.6 |
| 0.65 | 0.118 | 0.461 | 0.517 | 1 | -0.702 | 0.6 |
| 0.8 | 0.273 | 0.431 | 0.516 | 1 | -0.577 | 0.6 |
| 1 | 0.385 | 0.402 | 0.524 | 1 | -0.443 | 0.6 |
| 1.1 | 0.429 | 0.391 | 0.527 | 1 | -0.386 | 0.6 |
| 1.15 | 0.448 | 0.385 | 0.529 | 1 | -0.360 | 0.6 |
| 1.3 | 0.500 | 0.373 | 0.533 | 0 | -0.273 | 0.833 |
| 1.5 | 0.556 | 0.358 | 0.538 | 0 | 0 | 1 |

Table 4: Private education allowed

$w_H$, we have that the individually optimal amount of private education decreases when $\alpha$ increases, and the impact of this effect is larger than the direct impact of increasing $w_H$, leading to a lower value of $p^*$. Finally, observe that the steady state proportion of high skilled individuals is always strictly larger in Table 4 than in Table 3, whatever the value of $w_H$.

We now move to the welfare maximizing objective (last three columns of Tables 4 and 3). As predicted by Proposition 2, the welfare-maximizing value of $\alpha$ remains at one when $w_H$ is low enough ($w_H \leq 1.15$ in our example, while $\tilde{w}_H = 5/6$) but jumps from one to zero when $w_H$ is large enough ($w_H \geq 1.3$ here). The maximum welfare level remains the same as in Table 3 when $w_H$ is low enough so that $\alpha^W$ is unchanged. It is strictly larger when $\alpha^W$ is zero in Table 4: as explained in Proposition 4, the welfare maximizing allocation without private education remains attainable when supplementary education is introduced. Consequently, if the optimal value of $\alpha$ is changed (here, decreased from one to zero) when private education is introduced, the new allocation must correspond to a strictly larger welfare level. Similarly, the steady state proportion of high skilled agents is the same in Tables 3 and 4 when $w_H$ is low enough. It is larger in Table 4 when $\alpha^W$ equals zero, since Proposition 2 has shown that $p^*(0) > p^*(1)$ is a necessary condition for a low value of $\alpha$ to be preferred to one when private education is introduced. Finally, $p^*$ is increasing with $w_H$ when $w_H$ is large enough for $\alpha^W$ to be equal to zero. In our simulation, it reaches the maximum value of one when $w_H = 1.5$.

To summarize the main results from Table 3 and 4, allowing private education is, in our example, always detrimental for social mobility. This is true whatever the value of $w_H$ and irrespective of whether supplementary education is effectively bought when available. Allowing private education is at worst innocuous and at best beneficial for a welfare maximizing planner, provided that high skilled parents are rich enough. The optimal degree of elitism of the public education system is effectively larger in our example for a mobility maximizing than for a welfare maximizing planner, provided once more than high skilled parents are rich enough.

Numerical examples give us the opportunity to discuss how our results would be affected if we did not launder preferences in the definition of social welfare. Observe first that several of our theoretical results would, in any event, not be affected. Obviously, the analysis of social mobility maximization would not be affected: $\alpha^M$ remains equal to zero in the absence of private education, while Proposition 3 continues to hold when supplementary education is available. As for welfare maximization, unlaundered welfare, given by

$$W^{NL}(\alpha) = p^*(\alpha)(u(w_H) + v(\phi_H(\alpha))) + (1 - p^*(\alpha))(u(w_L) + v(\phi_L(\alpha))),$$

remains increasing in $\alpha$ in the absence of private education (*i.e.*, Proposition 1 holds) provided that the sub-utility function $v(a)$ (pertaining to the educational achievement of the child) is not too concave.[18] This is confirmed by Table 5, which reports welfare-maximizing results for the exact same specification as for Tables 3 and 4, except that social welfare is not laundered. The first columns of Table 5 show that $\alpha^W = 1$ when preferences are not laundered. The only difference between Tables 3 and 5 is the actual welfare level reached when $\alpha = 1$.[19]

Proposition 2 also remains valid: the intuition as to why a large value of $w_H$ is necessary for the welfare-maximizing value of $\alpha$ to be lower than $\tilde{\alpha}$ remains essentially the same (since the laundered utility of high skilled agents is increasing in $\alpha$ for $\alpha < \tilde{\alpha}$),[20]

[18] The impact of $\alpha$ on unlaundered welfare is threefold: it decreases $v(a)$ for low skilled agents but increases both the steady state proportion of high skilled (and thus high utility) agents and $v(a)$ for high skilled parents. If the function $v(a)$ is not too concave, the first (negative) impact of a higher $\alpha$ on welfare is less than the sum of the other two (positive) impacts.

[19] Welfare levels are lower in Table 5 since we add $v(a)$ to those reported in Table 3, with $a < 1$ (since it is a probability) and $v(a) = \log(a)$.

[20] As long as $\partial\phi_H(\alpha, e)/\partial\alpha > 0$, *i.e.*, when $p_H > \bar{p} + e$.

| | No private education | | | Private education allowed | | |
|---|---|---|---|---|---|---|
| $w_H$ | $\alpha^W$ | $W^{NL}(\alpha^W)$ | $p^*(\alpha^W)$ | $\alpha^W$ | $W^{NL}(\alpha^W)$ | $p^*(\alpha^W)$ |
| 0.6 | 1 | -1.365 | 0.6 | 1 | -1.365 | 0.600 |
| 0.65 | 1 | -1.317 | 0.6 | 1 | -1.317 | 0.600 |
| 0.8 | 1 | -1.193 | 0.6 | 1 | -1.193 | 0.600 |
| 1 | 1 | -1.059 | 0.6 | 0 | -0.984 | 0.667 |
| 1.1 | 1 | -1.002 | 0.6 | 0 | -0.834 | 0.714 |
| 1.15 | 1 | -0.975 | 0.6 | 0 | -0.752 | 0.741 |
| 1.3 | 1 | -0.902 | 0.6 | 0 | -0.476 | 0.833 |
| 1.5 | 1 | -0.816 | 0.6 | 0 | 0 | 1 |

Table 5: Table 5: Without laundering

although the analytical determination of the lower-bound on $w_H$ is made very complex. This is confirmed by Table 5, where the optimal value of $\alpha^W$ jumps from 1 when $w_H$ is low (up to 0.8) to 0 for a large enough $w_H$ (larger than 1). Tables 5 and 4 also show that the value of $w_H$ above which $\alpha^W = 0$ is lower without laundering of preferences than with laundering. The intuition is that private education increases the probability that one's child is highly productive, which gives additional utility to the parent through the sub-utility function $v(a)$. Under laundering this positive impact of private education is neglected, which biases the value of $\alpha^W$ towards one. Finally, as with laundering, the maximum welfare level attained is higher with private education when $\alpha^W < 1$, as can be seen from a comparison of the third and penultimate columns in Table 5.

In conclusion, our analysis is robust to whether we launder utilities or not in the definition of social welfare. However, as our numerical example illustrates, the specific value of $\alpha^W$ that is chosen for a given value of $w_H$ may of course differ.

# 6 Conclusion

We have considered two alternative objectives of education policy: utilitarian welfare and social mobility. These two issues are often referred to in the assessment of education systems. We have developed a simple model that studies the determination of the degree of elitism of public education. It has shown that there may be a stark conflict between welfare maximization and social mobility.

We have first studied the optimal degree of elitism when private educational supplements are not available. Utilitarian welfare increases with the degree of elitism of the public education system. On the other hand, elitism decreases the steady state proportion of heterogenous dynasties (those comprised of a skilled parent and an unskilled child, or vice versa) which is our measure of social mobility. Consequently, social mobility is maximized under the least elitist public education system.

Next, we have introduced the possibility that skilled parents invest in private supplementary education. We have shown that the appropriate degree of elitism now depends on induced adjustment in private education. In particular, an egalitarian system may be welfare maximizing while some degree of elitism may be good for mobility. Both results occur because an elitist system discourages private education expenditures. More precisely, we have shown that the degree of elitism that maximizes social mobility increases, while the welfare-maximizing degree of elitism decreases, provided that high skilled productivity is large enough. We have provided a numerical example where the ranking between the welfare- and mobility-maximizing degree of elitism is reversed when private education is allowed, *i.e.*, where the public education system that maximizes social mobility is more elitist than the one that maximizes welfare. Finally, we have shown that utilitarian welfare never decreases when private supplementary education becomes available. However, to maximize social mobility it may be preferable to ban private supplements.

Our analysis is based on a number of simplifying assumptions. We now comment on three of them and look at the robustness of our results to changes in these assumptions. First, the assumption that low skilled parents do not buy supplementary private education is admittedly strong; it is made to simplify the algebra. Our results would carry through when both low- and high- ability parents are allowed to buy supplementary private education, as we now explain. The simplest way to extend our model to this case would be to modify the function $\phi_L(\alpha)$ in the same way as we have modified the function $\phi_H(\alpha)$ at the beginning of Section 4, to obtain $\phi_L(\alpha, e) = (1-\alpha)(\bar{p}+e)+\alpha p_L$. We would then need to introduce two threshold levels of $\tilde{\alpha}$, denoted by $\tilde{\alpha}_L$ and $\tilde{\alpha}_H$, above which, respectively, low- and high-skilled parents do not buy private education. Our results that a large value of $w_H$ is necessary to have *i)* a low value of the welfare-maximizing public education elitism degree and *ii)* a value of the social mobility maximizing degree of elitism that increases with $w_H$ would both remain correct.

Second, our objective of maximum social mobility, reached when all skilled parents have unskilled children, and vice versa, may not look very appealing to everyone. Observe that we never attain this situation in our model, since the probability to have a high skilled child is never lower for high skilled than for low skilled parents. The largest degree of social mobility that we attain at equilibrium is one where the probability of being highly skilled is the same for both types of parents. This corresponds to an objective of perfect equality of opportunity (where your situation in life is independent of your parent's), which is probably easier to defend on normative grounds than the one of maximizing social mobility. Social mobility maximization and equalization of opportunities yields the same results when private education is not available. With the availability of private education, Figure 2 shows that perfect equalization of opportunities is not within reach anymore.[21] In that sense, the result that a planner bent on maximizing social mobility may wish to forbid supplementary private education (unlike a welfarist planner) remains correct when the planner's objective is changed to equalizing opportunities.

Finally, we have also shown in section 5 that most of our results are robust to the assumption that we launder individual utilities when computing social welfare.

In the literature, maximum social welfare has lead many authors to recommend an elitist and regressive educational policy. In this paper, we have shown that this recommendation can be challenged when introducing considerations of social mobility. This is not the only way an elitist policy can be questioned. Here are two examples. First, one can deem that education not only brings more productivity but has a value *per se*. If education were introduced as an argument in the individual utility function, the case for a regressive educational policy would be weakened. Second, there is also the effect of earnings distribution on growth. There exists a rich literature comparing

[21] It would require an egalitarian public education scheme. This, however, would drive high skilled parents to invest in private education, which would prevent complete equalization of opportunities.

the growth incidence of two polar systems of education that can be labeled for short egalitarian and elitist. The former one tries to induce equalization of human capital while the latter tends to perpetuate or even exacerbate its initial inequality. Benabou (1996) addresses the question of which system promotes faster growth. He shows that their short run effects are ambiguous, but that in the long run the egalitarian system is clearly desirable. This suggests that, when growth is accounted for, the conflict between welfare and mobility may be less drastic than in our setting.

# APPENDIX

## A Proof of Proposition 2

First, the optimal value of $\alpha$ cannot belong to $]\tilde{\alpha}, 1[$ because the utility of both types of individuals is constant over this interval while the proportion of high type (and thus high utility) individuals increases with $\alpha$ over this interval.

Second, the necessity of a large $w_H$ to obtain $\alpha^W < 1$ is established as follows. A necessary condition to have $\alpha^W < 1$ is

$$p^*(\alpha^W) > p^*(1) = \frac{p_L}{1 - p_H + p_L},$$

because $\alpha^W < \tilde{\alpha} < 1$. Using equations (2), (7) and (8), we obtain that

$$p^*(\alpha^W) = \frac{2\bar{p}(1-\alpha^W) + 2p_L\alpha^W}{2 + \bar{p} - w_H(1-\alpha^W) - \alpha^W(\bar{p} + p_H - 2p_L)},$$

which increases with $w_H$. Combining these two expressions shows that a necessary condition to obtain $\alpha^W < 1$ is that

$$w_H > 2 - \frac{\bar{p}(2(1-p_H) + p_L)}{p_L} + \frac{\alpha^W}{1-\alpha^W}p_H.$$

Observe that this lower bound on the value of $w_H$ increases with $\alpha^W$. If this condition is satisfied for some $\alpha^W < \tilde{\alpha}$, it thus must also hold for $\alpha^W = 0$, so that a necessary condition to obtain $\alpha^W < 1$ is

$$w_H > 2 - \frac{\bar{p}(2(1-p_H) + p_L)}{p_L}.$$

## B Proof of Proposition 3

The problem is given by

$$\max_{\alpha} \; M(\alpha) = [1 - p^*(\alpha)]\phi_L(\alpha),$$

where $p^*(\alpha)$ is given by (9) and $\phi_L(\alpha)$ by (7). Consequently, when $\alpha^M$ is an interior solution it is determined by

$$M'(\alpha) = -p^{*\prime}(\alpha)\phi_L + (1-p^*)\phi_L'(\alpha) = 0.$$

We have that[22]

$$\begin{aligned}
\operatorname{sign}\left(\frac{\partial \alpha^M}{\partial w_H}\right) &= \operatorname{sign}\left(\frac{\partial^2 M}{\partial \alpha \partial w_H}\right) \\
&= \operatorname{sign}\left(-\frac{\partial^2 p^*}{\partial \alpha \partial w_H}\underset{+}{\phi_L} - \underset{+}{\frac{\partial p^*}{\partial w_H}}\underset{-}{\frac{\partial \phi_L}{\partial \alpha}}\right) \qquad (13)
\end{aligned}$$

so that $\partial^2 p^*/\partial\alpha\partial w_H < 0$ is sufficient to yield $\partial\alpha^M/\partial w_H > 0$.

Differentiating (9), the expression for $p^*$ yields

$$\frac{\partial p^*}{\partial \alpha} = \frac{\underset{-}{\frac{\partial \phi_L}{\partial \alpha}}(1-\phi_H^o) + \phi_L \underset{-}{\frac{\partial \phi_H^o}{\partial \alpha}}}{(1-\phi_H^o+\phi_L)^2} < 0 \text{ if } w_H > p_H - \bar{p}$$

and

$$\begin{aligned}
\frac{\partial^2 p^*}{\partial \alpha \partial w_H} &= \frac{\left(\phi_L \frac{\partial^2 \phi_H^o}{\partial \alpha \partial w_H} - \frac{\partial \phi_L}{\partial \alpha}\frac{\partial \phi_H^o}{\partial w_H}\right)(1-\underset{+}{\phi_H^o}+\phi_L)^2}{(1-\phi_H^o+\phi_L)^4} \\
&\quad + \frac{2\underset{-}{\left(\frac{\partial \phi_L}{\partial \alpha}(1-\phi_H^o)+\phi_L\frac{\partial \phi_H^o}{\partial \alpha}\right)}(1-\underset{+}{\phi_H^o}+\phi_L)\underset{+}{\frac{\partial \phi_H^o}{\partial w_H}}}{(1-\phi_H^o+\phi_L)^4} \\
&< 0 \text{ if } \phi_L \frac{\partial^2 \phi_H^o}{\partial \alpha \partial w_H} - \frac{\partial \phi_L}{\partial \alpha}\frac{\partial \phi_H^o}{\partial w_H} < 0 \text{ and } w_H > p_H - \bar{p}.
\end{aligned}$$

Using the definitions of $\phi_L$ and $\phi_H^o$, (7) and (8) we obtain that

$$\phi_L \frac{\partial^2 \phi_H^o}{\partial \alpha \partial w_H} - \frac{\partial \phi_L}{\partial \alpha}\frac{\partial \phi_H^o}{\partial w_H} = -\frac{p_L}{2} < 0,$$

which completes the proof because from (13) $\partial^2 p^*/\partial\alpha\partial w_H < 0$ implies $\partial\alpha^M/\partial w_H > 0$.

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**Fig .1 : $\Phi_L$ and $\Phi_H$ (dashed) in the absence of private education**

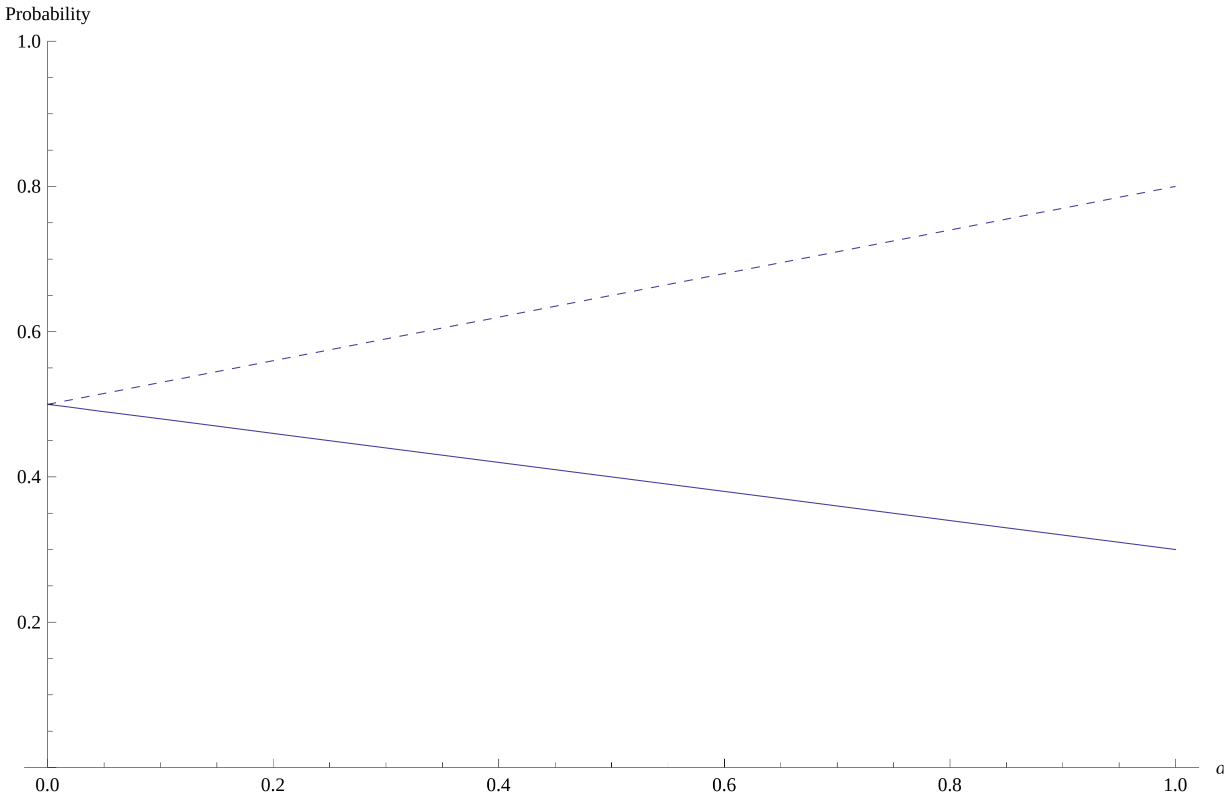

**Fig2. : Probabilities of a high productivity child when private education is available.**