Ramifying Feedback Networks, Cross-Scale Interactions and Emergent Quasi-Individuals in Conway's Game of Life

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Abstract

Small patterns of state 1 cells on an infinite, otherwise empty array of Conway's Game of Life can produce sets of growing structures resembling in significant ways a population of spatially situated individuals in a non-uniform, highly-structured environment. Ramifying feedback networks and cross-scale interactions play a central role in the emergence and subsequent dynamics of the quasi-population. The implications are discussed: it is proposed that analogous networks and interactions may have been precursors to natural selection in the real world.

Keywords: feedback network, cross-scale interaction, cellular automata, emergence

1 Introduction

There are small patterns of state 1 cells on an infinite, otherwise empty array of Conway's Game of Life [5], which manifest a long-sustained increase in structural and dynamic complexity, and produce sets of growing structures resembling in significant ways a group of spatially situated individual organisms interacting with a non-uniform environment. These quasi-individuals' interaction with the environment and each other, but not their internal makeup, appears "lifelike": they grow – although having no metabolism, affect their neighbours' survival, and influence the appearance of new group members – although no genetic mechanism is involved. Ramifying (self-extending) feedback networks and cross-scale interactions are central to the emergence and subsequent dynamics of these quasi-individuals.

Conway's Game of Life (henceforth GoL) is a synchronous, deterministic, binary cellular automaton, run on a two-dimensional array of rectangular cells (infinite, or finite and toroidal). A cell's neighbors are the eight sharing an edge or vertex with it. A cell is in state 1 at step *t* if and only if at step *t-1*:

1) it, and either two or three neighbor cells were in state 1, or
2) it was in state 0, and exactly three neighbor cells were in state 1.

GoL is computationally universal [5], and designs exist for a Universal Turing Machine implemented as a GoL pattern [28]. In [5] there is a sketch of a proof that GoL can support *patterns*[1] which self-replicate by both copying a string of subpatterns and interpreting it as a set of instructions, as first shown to be possible for cellular automata by [32].

Others [6, 22] have shown that CA much simpler than von Neumann's 29-state automaton used can support this form of self-replication. However, these studies exhibit patterns that self-reproduce in an otherwise empty array, whereas real-world self-reproducers (cells and organisms) reproduce only within an environment containing other complex structures and processes, living and non-living. More recently, cellular automata which support emergent self-reproducing patterns have been studied [18, 27] – but in these cases, the facility for self-reproduction has been designed into the (moderately complex) CA, and the expected class of self-reproducing structures appear rapidly and reliably ([27] uses a CA which is an elaboration of GoL). The work reported here explores the hypothesis that quite simple patterns in some very simple CA (specifically, in GoL) may support the emergence of an "artificial biosphere": a pattern which would develop, given enough time, into an ecology of organisms, contained within and dependent on a structured environment, and evolving by natural selection ("enough time" might be more than any algorithm and practicably constructable computer could provide). No proof of this hypothesis is claimed, but the paper aims to establish its plausibility.

A "feedback network" is a collection of components (variables or structures) linked by a network of interactions including cyclical paths, so at least some components influence themselves indirectly. In what is called here a "ramifying feedback network", some interactions produce new components, and hence new interactions. The motivation for the term "cross-scale interaction" is that many complex systems comprise entities at several distinct spatial and temporal scales, each involving a characteristic set of processes that can to some extent be understood without considering larger or smaller scales; but this autonomy of scales is incomplete, and events that transgress it are dynamically important.

Complex structures involving ramifying feedback networks and extensive cross-scale interactions (although the terms are not used) can arise from a class of random GoL configurations with infinitely many state 1 cells [14, 15]; here, we focus on emergence from quite small and simple patterns. In the main example used,

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[1] Terms for GoL constructs are italicised on first use. *Configuration* refers to the complete state of any CA array; a pattern is the set of state-1 cells in an otherwise empty (state 0) configuration or more precisely, in any member of an equivalence class of such configurations under translation, since only the relative positions of the state 1 cells matter.

a set of long, thin, crystal-like structures appears, each growing at one end. These all begin in the same way, but due to ongoing cross-scale interactions, have unique and interacting “life-histories”: members of the set affect each others’ chances of continuing to grow, and may also change their mode of growth, producing more complex growing structures. Furthermore, these quasi-individuals can influence the formation of new members of the set. Although the primary influence is inhibitory, they could also initiate the formation of new set members that would not otherwise appear. Both the inhibitory and facilitative mechanisms will affect the probabilities of occurrence of specific spatial alignments between adjacent set members, raising the possibility of new levels of structure emerging, which could even support a form of natural selection.

2 Complex Systems, Ramifying Feedback Networks and Cross-Scale Interactions

Ramifying feedback networks may be important to the emergence of complexity in both biological and socio-technical domains. Chemical evolution involving increasingly numerous and complex chemical species and reactions appears a necessary precursor to the appearance of self-replicating entities. [20] argues that systems of polymer catalysts would pass a “critical complexity threshold” as the number of different polymers increases, past which subsystems would form where production of each member is catalysed by other subsystem members. In technological development, the identification of “components” is not straightforward, but [10] argues that artefact-activity pairs (e.g. a bicycle and the act of riding it) are the evolving entities. These components interact by coevolution (bicycles changed once roads were tarred) and transfer of materials and subcomponents; and although [10] does not mention the point, such interactions sometimes give rise to new artefact-activity pairs – as improvements in steam engines and rail tracks, both developed for mining, made possible steam locomotives. Institutional systems may also provide examples: interactions between evolving institutions may reveal conflict or ambiguity, prompting the development of new institutional “components” and hence, ramification of feedback networks.

Real-world feedback networks often contain multiple layers. Complex systems are often “lumpy” [17]: despite the association of complexity and self-organisation with scale-free structures and behavior [2], the most complex entities (organisms, ecosystems, societies) involve highly diverse kinds of entities and processes, at multiple scales. The biologically characteristic parts of an animal include organs, tissues, cells, organelles and macromolecules, each with its own characteristic range of scales and interactions with similar entities. In social systems, we can similarly distinguish individuals, households, settlements, economic and political units of various kinds, and civilisations. The different types of entity may not fit into a single hierarchy, particular types may cover wide and sometimes overlapping size ranges, and a scale-free size distribution may cover several magnitudes as claimed for cities [7] and firms [1]. Nonetheless, the kind of lumpiness described, and the relative autonomy of processes at different scales may be a necessary attribute of highly complex entities [29].

However, the different levels or scales of structure and process involved in an organism, ecosystem or society are not absolutely independent. Mutations in a macromolecule can turn a cell cancerous and kill the organism; an individual’s invention or decision can transform a society. In the downward direction, all chemical reactions between biomolecules are consistent with quantum mechanics, but which ones occur is partially determined by configurations of specialised structures (membranes and cytoskeletal elements) at the cellular or organelle level [4]. Similarly, how cells develop depends on their position in the organism; an animal’s behaviour (e.g. a reptile sunning itself) can alter processes at all finer structural levels; economic and political systems both constrain and facilitate individual action. Such cross-scale interactions appear universal in biological and social systems, and also occur in some purely physical ones.

Recent studies of cross-scale interactions range over plasma physics [8], oceanography [11], and the study of sepsis [31] and insect societies [30]. A model of wildfire spread is presented as an example the production of catastrophic events by cross-scale interactions and system feedbacks in [25], with additional examples from desertification, epidemics, and structural failures in engineering.

Extensive work concerns ecological and social-ecological systems. Peterson, Allen and Holling [26] propose a model of "cross-scale resilience" – ecological resilience being the amount of disturbance required to shift an ecosystem from one set of mutually reinforcing structures and processes to another. Different species with similar roles can increase resilience by operating at different spatial and temporal scales within an ecosystem. For example, insect outbreaks can devastate large forest areas, and their numbers are largely controlled by birds. Larger birds can only efficiently exploit larger agglomerations of insects but can travel further; hence, small birds prevent most insect outbreaks growing large, while large birds are drawn to and limit the size of large outbreaks. However, at least some ecosystems appear to have "keystone" species, removing which will transform the environment (while removing any other single species would make little difference). These are often megaherbivores such as elephants, which create spatial and hence species diversity by destroying or damaging large woody plants; but others prey on animals that would otherwise be present in very large numbers (e.g. sea otters of the North American Pacific coast, which control sea urchin numbers). This perhaps indicates a general feature of systems involving cross-scale interactions: some events at a particular scale will have no consequences beyond the immediate, others will transform the system, and doubtless others again will have intermediate effects.

Ramifying feedback networks and cross-scale interactions, then, are important in many systems involving nonlinear dynamics and spatial heterogeneity. How simple can a system be, in terms of local elements and interactions, and initial conditions, while generating such dynamics? Once in existence, do cross-scale interactions tend to generate further complexity, and in particular additional kinds of cross-scale interactions? This paper shows that ramifying feedback networks and cross-scale interactions can arise in remarkably simple systems, and that in some but not all of a range of cases of apparently equal initial complexity, they do produce additional kinds of cross-scale interaction, in a process that continues as far as it has been computationally possible to follow it. The example set of systems, which are necessarily explored in some detail (sections 3-6), have both features in common with the natural systems mentioned above, and marked differences from them.

3 Conway's Game of Life

3.1 Preliminaries

Random GoL configurations with 50% state 1 cells on toroidal arrays of thousands or millions of cells tend to resolve within some tens of thousands of steps into multiple small, isolated clumps of state 1 cells. Among these clumps, three successively more inclusive classes can be distinguished:

- *Still-lifes* are stable patterns: all state 1 cells remain in that state; no more are created. The commonest still-life is the four-cell *block* (figure 1).
- *Oscillators* are the same at step $t+n$ as at step t, for some n. Still-lifes are oscillators of period 1; the commonest higher period oscillator is the *blinker*, of period 2 (figure 1).
- *Repeaters* are patterns for which step $t+n$ is a translation of step t, for some n. Oscillators are repeaters for which the translation is the identity transformation. The commonest moving repeater is the *glider*, moving one cell diagonally in four steps (figure 1). The three next smallest also shown: the *light-weight spaceship* (LWSS), *middle-weight spaceship* (MWSS), and *heavy-weight spaceship* (HWSS), which move two cells orthogonally in four steps. *Spaceship* is used here as a general term for moving GoL repeaters.

If the state of one cell in a configuration is switched, the influence of the change cannot possibly spread faster than than one cell per step in any direction. This rate is referred to as the *speed of light*, or *c*. Gliders move at the maximum speed a complete GoL pattern can move diagonally: *c*/4; the LWSS, MWSS and HWSS at the maximum speed a complete GoL pattern can move orthogonally: *c*/2.

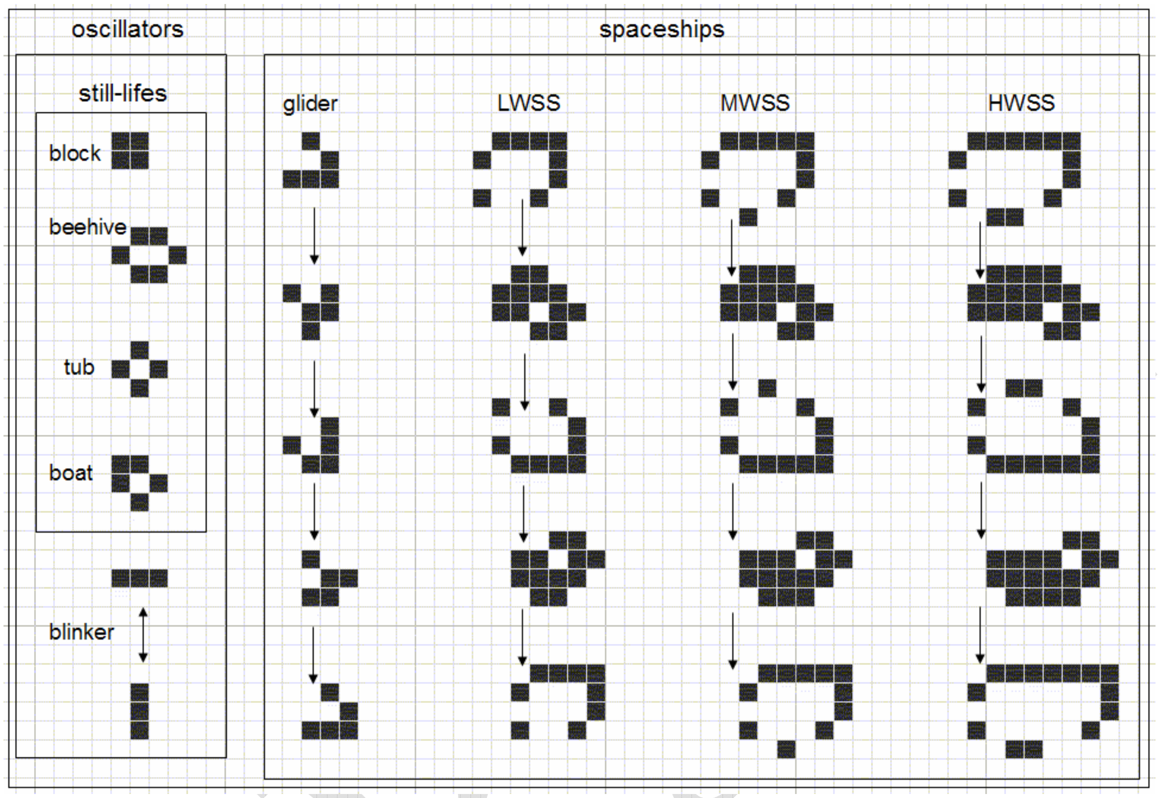

Figure 1: Some GoL repeaters.

Indefinite growth patterns increase their count of state 1 cells without bound. The smallest, and the only ones ever seen to emerge from random configurations, incorporate the pattern shown in figure 2, top left, the *switch engine*. After 48 steps (top center) this produces a glide-reflected copy of itself, plus additional debris; after 96 steps (top right) the switch engine has moved 8 cells diagonally, for a speed of *c*/12, and produced a larger debris clump. The debris eventually overtakes and destroys a lone switch engine, but there are numerous ways of stabilising the pattern: for example, a block placed as shown at bottom left in figure 3 interacts with the debris to produce an endless trail of blocks as the switch engine progresses (bottom right). (This pattern contains 12 state 1 cells; [16] reports 10-cell patterns that produce a stabilised switch engine, and that 10 is the minimum number of state 1 cells needed for indefinite growth.)

The current study shows that certain small and apparently quite simple patterns grow in structural and dynamic complexity over large numbers of steps. Clearly, the complexity must be inherent in the combination of the GoL transition rule, the initial pattern, and the otherwise empty infinite array; it is cross-scale interaction that makes it manifest. The type of complexity generated is markedly different from that produced from simple initial patterns by “type 3” cellular automata [33] and even from any previously demonstrated for “type 4” cellular automata [33], such as GoL and the one-dimensional CA 110 [9]. In the GoL patterns of figures 1-2, we can already distinguish levels of structure. The lowest consists of individual cells. The next level, or set of levels, consists of clumps of state 1 cells. A *d-cluster* is a maximal set of state

1 cells such that each pair of cells in the set is linked by a neighbor-to-neighbor path that never goes through more than *d* successive state 0 cells. The term *cluster* on its own refers to a set of cells constituting a *d*-cluster for some *d*. *1-clusters* are of particular significance, as the state 1 cells in a 1-cluster cannot share any neighbors with state 1 cells outside it, so the states of the 1-cluster's cells, and their state 0 neighbors, can be determined for one step without considering any other cells. Any configuration can be assigned a hierarchical cluster structure, with 0-clusters grouped into 1-clusters, 2-clusters… *n*-clusters. Dynamically, however, what is most important is how the 1-clusters composing a pattern merge, divide and disappear over time: the *1-cluster dynamics* of the pattern. The 1-cluster dynamics of the patterns considered in detail here have intricate patterns of nested components, isolated for longer or shorted periods.

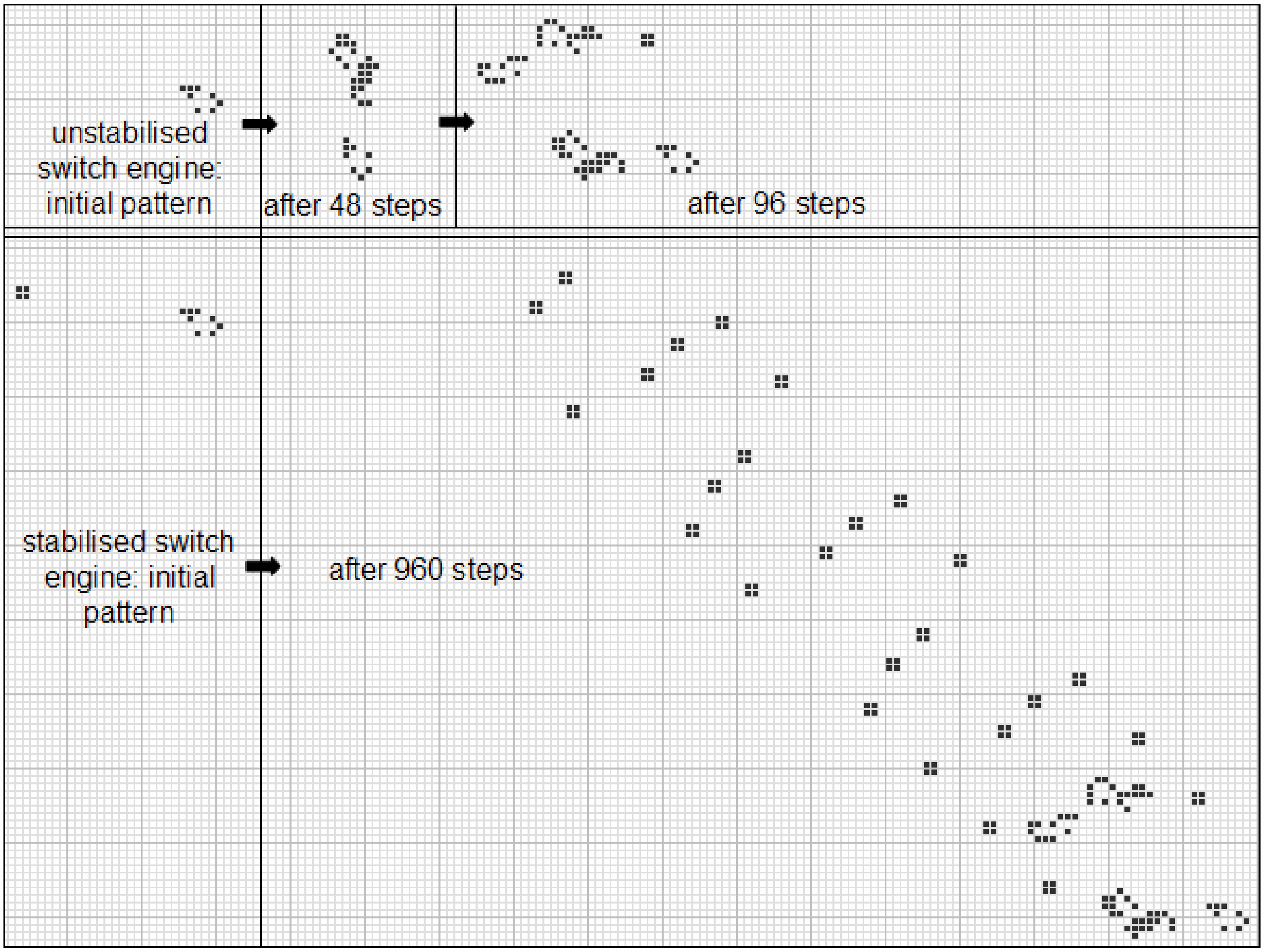

Figure 2: The switch engine.

3.2 Puffers

The stabilised switch engine of figure 2 is a *puffer* (puffers form a subcategory of indefinite growth patterns). A puffer has a *head* which moves indefinitely across the array, leaving a trail of oscillators. It may also produce streams and/or waves of mobile repeaters: its *limbs*. (In a stream, all the mobile repeaters follow the same path, while in a wave, successive mobile repeaters follow different but parallel paths; streams produced by a puffer travel parallel or antiparallel to the head's movement, while waves do not.)

The trail left by the stabilised switch engine of figure 2 is typical in having an initial irregular part (the *tail*), followed by an indefinitely lengthening regular *body*. The switch engine's head, however, is unusual among GoL puffers in consisting of a single 1-cluster through most of its cycle, and being "indivisible": no proper subpart of it can traverse the array alone. Other such puffers have considerably larger heads, while all other small puffers have heads with multiple separate but interacting parts and a three-level structure: individual cells, 1-clusters, and the head as a whole.

The first puffer discovered in the exploration of GoL (by R. William Gosper) was the *puffer train* [12], which can begin with a minimum of 22 state 1 cells. The head includes two LWSSs and an additional cluster, moves orthogonally at *c*/2 and in its original form, goes through a 140-step cycle producing oscillators but no waves or streams. Many puffers have alternative stabilisations, or *orbits*: that is, adding a small cluster to the original pattern, either behind or ahead of the head can cause a different body and in some cases limbs to form. If the additional cluster is located ahead of the head, the puffer can start in one orbit and switch when the additional cluster is encountered – and such switches may be repeated. These effects are reminiscent of the way some macromolecules can be shifted between alternative configurations, by interaction with small signalling molecules.

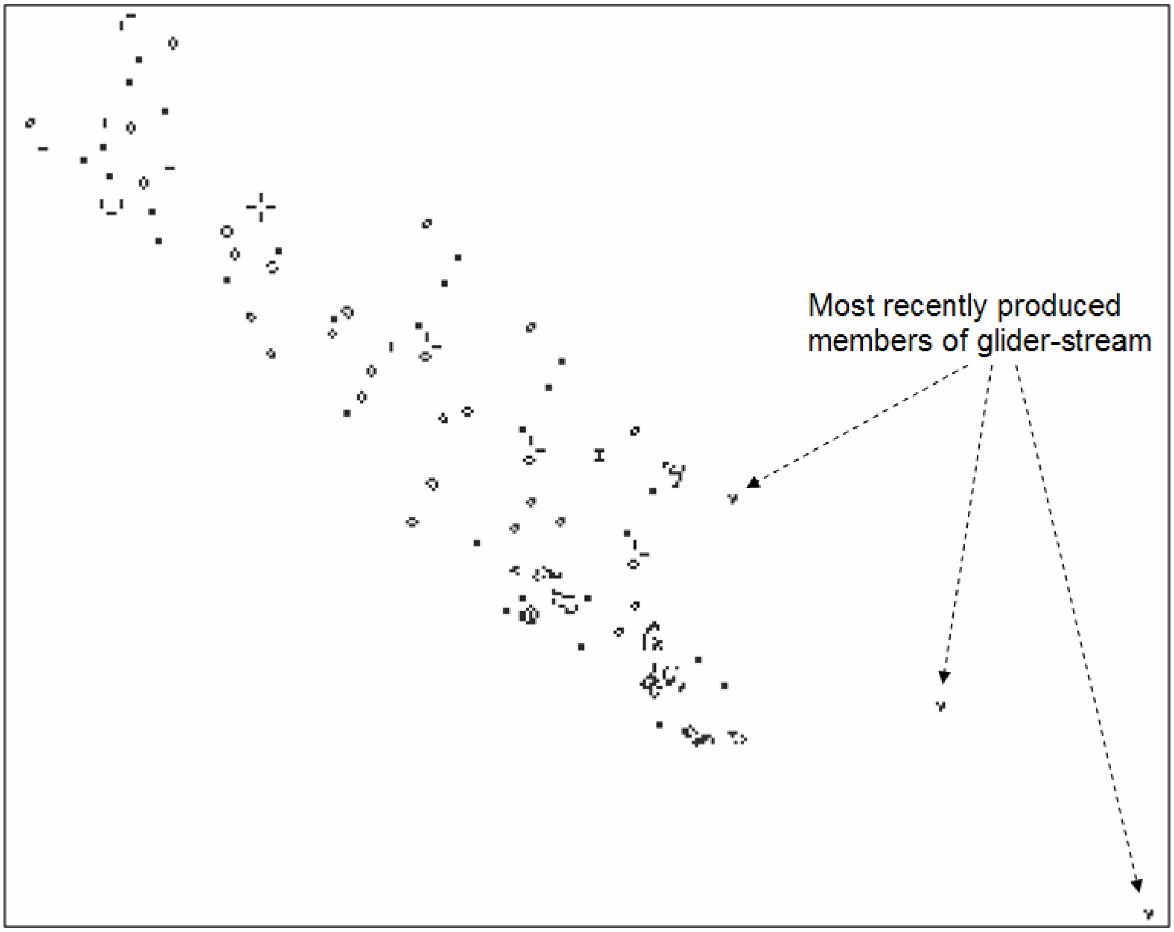

Figure 3: Glider-stream switch engine.

The switch engine has just two known orbits. The one shown in figure 2 is the *block-laying switch engine*. The other is the *glider-stream switch engine* (figure 3), which produces a stream of gliders ahead of itself (the head moves at 1/3 of a glider's speed). A block placed at various points relative to the 8-cell switch engine head can bring about this stabilisation, but it can also arise from crashing a glider into the front of a block-laying switch engine. The puffer train has several orbits. The one of importance here (figure 4) requires at minimum an initial 25 state 1 cells. It produces a wave of gliders, moving at 135º degrees to the puffer's head. This pattern will be referred to as *gosper2*. It settles into a 100-step cycle, producing a body with a spatial periodicity of 50 cells. There are many points during the 100-step cycle at which, if all the state 1 cells in a column perpendicular to the direction of growth are removed, it reverts to the 140-step cycle of the original puffer train, which appears more stable.

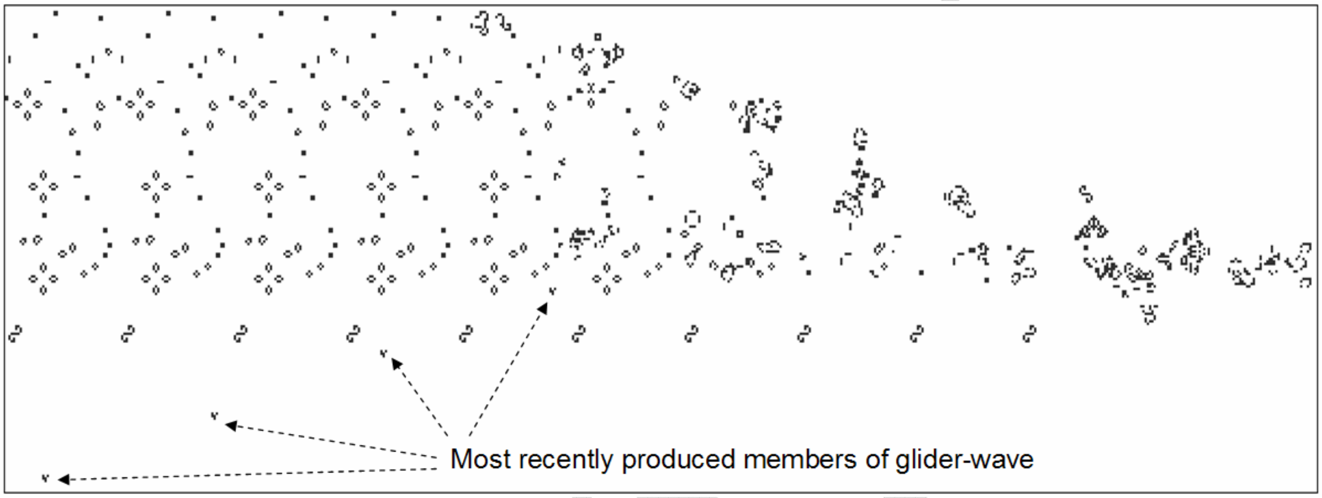

Figure 4: Gosper2.

4 Pairs of patterns

Combining any pair of patterns in a two-dimensional cellular automaton produces a Z^2 collection of larger patterns, each member being defined by the pair's relative placement. Interesting periodicities arise within such sets of patterns in GoL. We identify a particular member of the Z^2 collection by the relationship between the two subpatterns' *northwest cells* – those at the northwest corners of their minimal containing rectangles. Figure 6 shows two (mirror-image) gosper2s, with northwest cells marked. Placements causing the two components to overlap require special treatment: here, we ignore such cases. Clearly, if each of the two subpatterns would on its own remain forever within a finite block of cells, placing them far enough apart will prevent interaction. Even where this is not so a complete analysis may be possible: although a glider can be placed at any distance from a block and still interact with it eventually, all members of the Z^2 collection of possibilities divide into a few classes, depending on how (if at all) the glider eventually hits the block.

More complex but still fully analysable classes of examples involve a glider-stream switch engine and a small cluster such as a block or glider. The forward glider-stream can interact with such a cluster to construct a trail of six-cell still-lifes called *beehives* ahead of the switch engine. This trail is *glider-crystal* (figure 6), and once the interaction between glider-stream and glider-crystal enters a regular cycle, it takes 22 gliders to add two pairs of beehives, extending the crystal toward the switch engine's head. When the latter overtakes the crystal, interactions between head and crystal send glider-waves to either side.

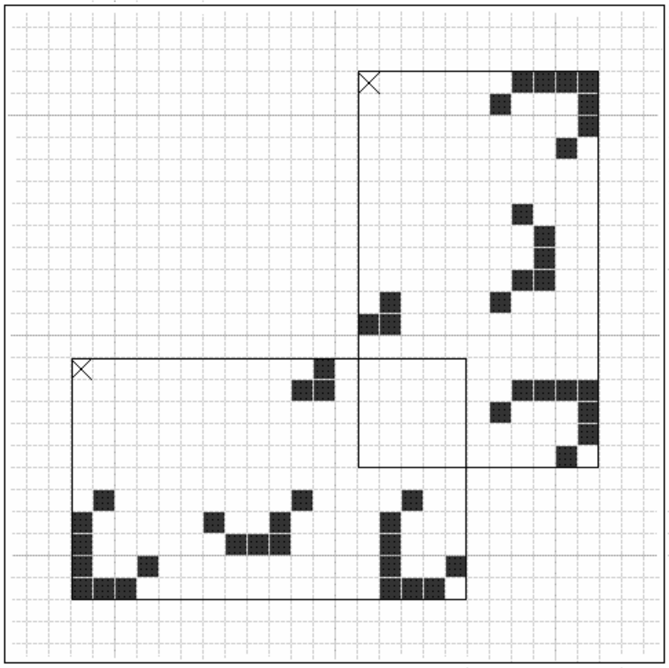

Figure 5: Initial pattern of g2-g2-s0sw13 (see section 5.1 for an explanation of the nomenclature used for this class of patterns). Crosses mark northwest cells of the two gosper2s constituting the pattern.

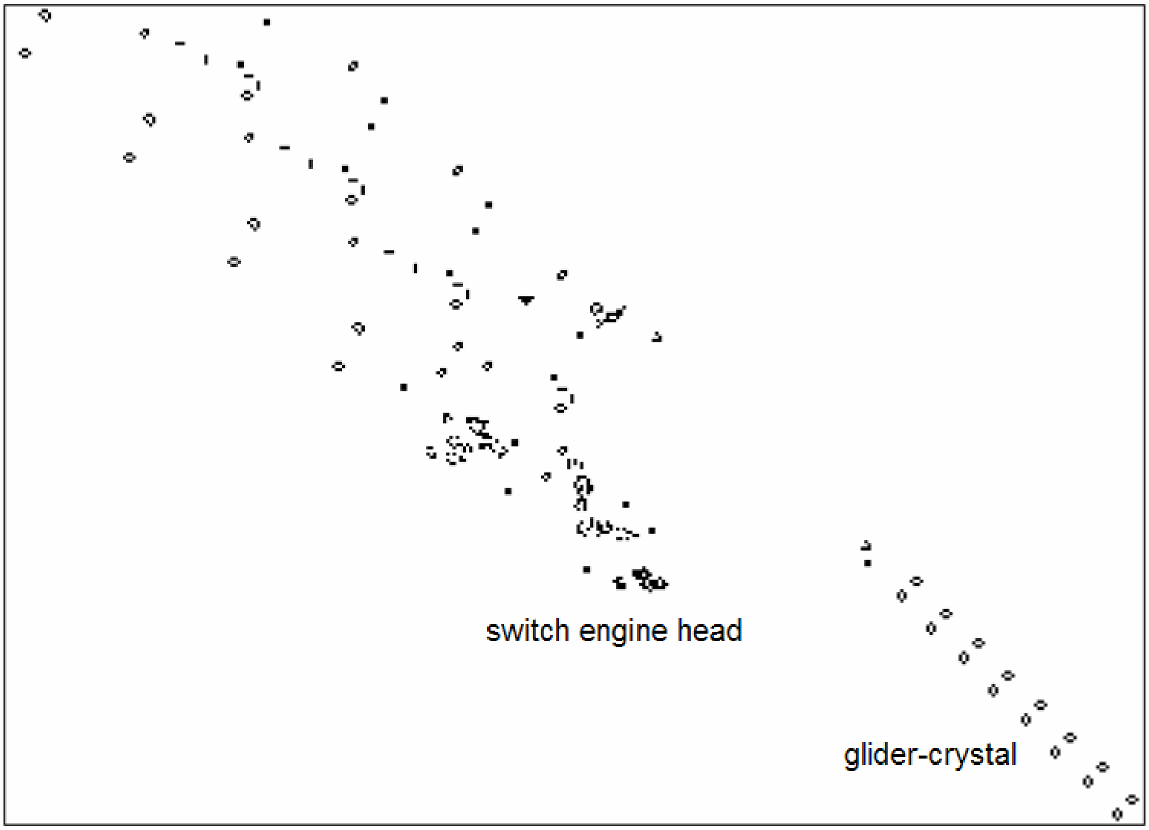

Figure 6: glider-stream switch engine constructing glider crystal.

Depending on the phase of crystal building and of the head when overtaking begins, additional spaceships, or even a block-laying switch engine heading at right angles to the glider-stream switch engine, may also be produced. This last possibility allows a pair of mutually-stabilising switch engines (an *ark*) and a ten-cell glider-stream switch engine to be combined in a 26-cell pattern that is the smallest known to show quadratic cell-count growth.

In other cases again, a complete analysis may be impossible. The next two sections explore one such class of patterns, formed by a pair of gosper2s diverging at right angles.

5 Seeking a Biosphere

5.1 Diverging puffer pairs, expanding feedback loops, and ramifying feedback networks

A gosper2 can be grown from 25 cells, by adding a blinker or a *preblock* (three state 1 cells in an orthogonal "L", which becomes a block in one step) in various positions to the 22 required for the original puffer train. Here, all the gosper2 pairs used start with two mirror-image copies of the same 25-cell pattern, as shown in figure 5. The two puffers travel *east* and *south* (left-to-right and top-to-bottom of the array), and are oriented so members of the backward wave of gliders from each travel toward the other's body. The ability of gliders (and other spaceships) to have effects arbitrarily distant from their point of origin in these patterns is itself a simple form of cross-scale interaction.

Successive members of the glider-wave from a gosper2 follow paths 50 *half-diagonals* apart (moving 1 cell southeast is a shift of two half-diagonals southeast, while moving 1 cell east or south is a one half-diagonal shift southeast). For the gosper2 headed east, the *n+1*th member of the wave is 75 cells east and 25 cells north of the *n*th, and follows a path 50 half-diagonals southeast of it. Shifting one puffer relative to the other along a northeast-southwest line leaves the relationship between the diagonals along which members of the glider-waves travel unchanged, while shifting along a north-south or east-west line, unless by a multiple of 50 cells, does not. This makes it convenient to use non-orthogonal coordinates, specifying the spatial relationship of the two gosper2s in terms of how far north or south, and how far southwest or northeast, the northwest cell of one is from that of the other. For the pattern g2-g2-s0sw13 (figure 5), the northwest cell of the south-headed gosper2 is 13 cells south and 13 cells west of that of the east-headed gosper2, but in the coordinate system used here, that is 0 south and 13 southwest.

Consider the set of patterns g2-g2-s*j*sw*k*, where *j* and *k* can take any integer values. For any *k*, sufficiently low values of *j* mean that either the two patterns will overlap, or the head of one gosper2 will crash into some part of the other; our interest is in the patterns where *j* is high enough to avoid these situations. Holding *k* constant, as the value of *j* modulo 50 is varied from 0 to 25, the spatial relationship between members of the two glider-waves as they either collide or pass through each other changes; as *j* modulo 50 is varied from 26 to 49, however, the spatial relationships that occurred as it ranged from 1 to 24 are repeated, but in reverse order and with the roles of the two waves reversed. Indeed, any g2-g2 pattern where $j > 25$ mod 50 is an exact enantiomorph of one where $j < 24$ mod 50. Hence we need consider only the patterns where $0 \leq j \leq 25$ mod 50, which we divide into *families* 0 through 25. For families 0 through 6, members of the two waves collide; for families 7 through 25 the waves pass through each other, and the component gliders hit the other gosper2's body. (In both cases, the first gliders may do something different, depending on the gosper2s' exact initial placement, but this seems to make no fundamental difference to the type of outcome that ensues.) In some of the wave-collision families the collisions generate further gliders that then collide with the body of one of the gosper2s, but none of these collisions generate any further gliders (or other spaceships). For most of the remaining families, members of the wave from each gosper2 hit the other's body without generating any secondary spaceships; the exceptions are families 7, 15, 17, 18, 19 and 24, where secondary waves of gliders are generated. In family 19, each collision with the body of the south-headed gosper2 generates a single secondary northeast-headed glider which hits the body of the east-headed gosper2 but generates nothing more. The other four families are more complicated, and divide into *subfamilies* according to whether secondary glider-waves interact with the northeast primary wave produced by the south-headed gosper2. This is determined by the value mod 25 of *k* in g2-g2-s*j*sw*k* (g2-g2-s*j*+50sw*k* is in the same subfamily as g2-g2-s*j*sw*k*). We briefly examine the simplest of the families, family 15, before turning to what appears to be the most complex, family 24.

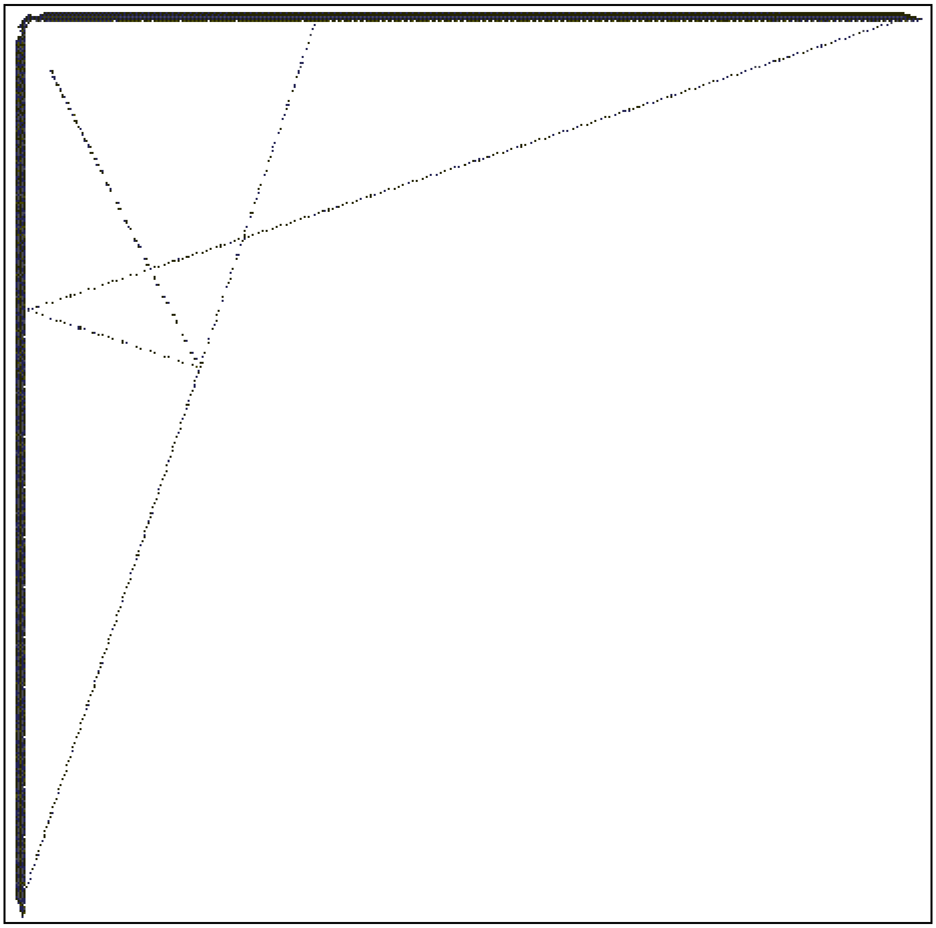

Figure 7: g2-g2-s15sw27.

In family 15, the southwest primary wave generates a southeast secondary wave when it hits the south-headed gosper2. The members of this wave may pass between members of the northeast primary wave (as in g2-g2-s15sw13), or every member may collide with a member of that primary wave. In this case the gliders may mutually annihilate; or oscillators and/or a new glider or gliders may be created (in g2-g2-s15sw27 four blocks are produced). As should just be visible in figure 7, the remnant oscillators may then remove some members of the southwest primary wave. When a southwest primary is removed in g2-g2-s15sw27, the southeast secondary it would have created, and the set of four blocks *that* would have created, do not form; and the southeast primary those blocks would have removed survives to produce a later set of blocks, which remove a later southeast primary. Hence, an *expanding feedback loop* is created. The line of oscillators produced by collisions between northeast primary and southeast secondary gliders in g2-g2-s15sw27, presents a binary sequence of present and absent remnants, closely related to the Thue-Morse sequence [19]. Other members of the same subfamily produce similar sequences.

In family 24 as in family 15, the southwest primary wave creates a secondary southeast wave when colliding with the south-headed gosper2, and in some subfamilies, this interacts with the northeast primary wave. However, the interaction with the gosper2 is more complicated than in family 15: the collisions of successive gliders with the gosper2 interact, they create a northeast secondary wave as well as the southeast secondary, and both secondary waves are *compound waves*. A compound wave consists of two or more *simple waves*; all the gliders in a simple wave (like the secondary southeast wave in the 15 family) are produced in exactly the same way, and will (in g2-g2 patterns) follow paths that are multiples of 50 half-diagonals apart, and be separated by multiples of 25 cells in both orthogonal directions. In subfamilies where the southeast secondary wave does not interact with the northeast primary, such as that represented by g2-g2-s24sw13 (figure 8), the northeast secondary wave consists of three simple waves (which collide with the east-headed gosper2 without creating any tertiary waves), the southeast secondary of five: for every six members of the southwest primary wave, four northeast gliders (two belonging to the same simple wave), and five southeast gliders are created. The members of one of the five simple southeast waves collide and mutually annihilate with every sixth member of the southwest primary wave, so only five out of six actually reach the south-headed gosper2, and only four of the five simple waves survive.

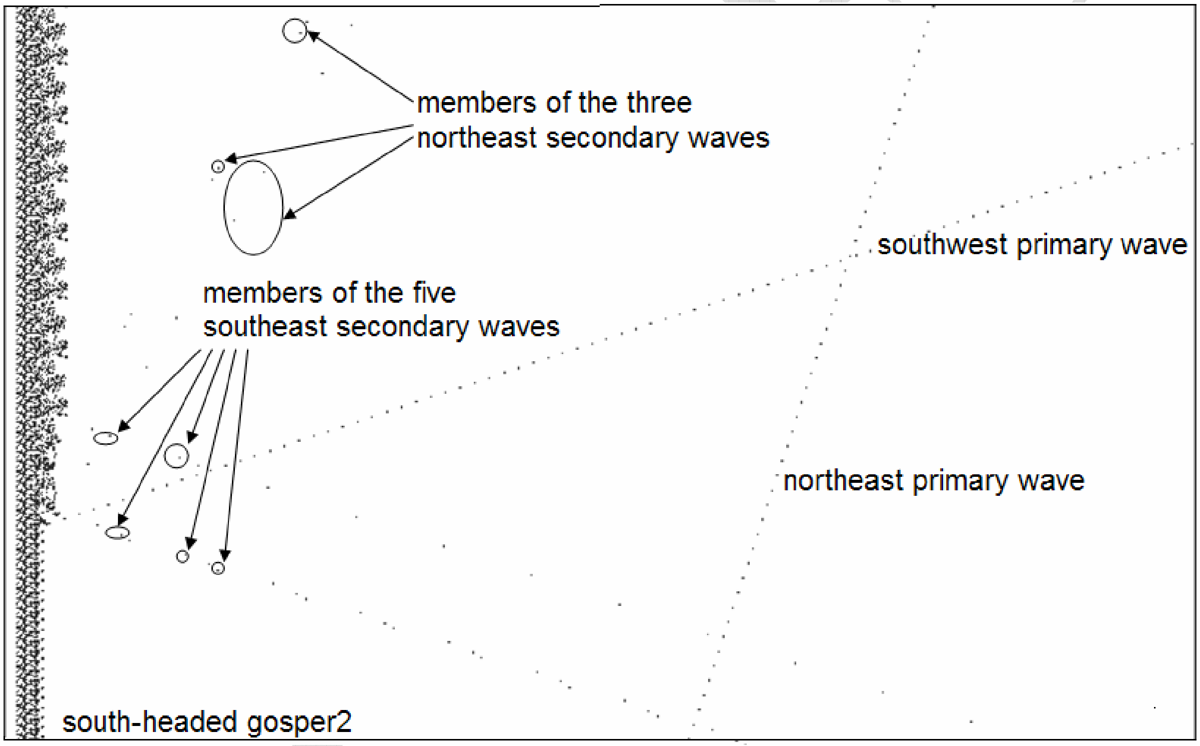

Figure 8: g2-g2-s24sw13: closeup of interaction between southwest primary wave and south-headed gosper2.

For many subfamilies of family 24, all members of the southeast waves either pass through the northeast primary wave without collision, or mutually annihilate with members of that wave. For some, collisions with northeast primary gliders create small remnants that the southeast primary avoids, while for three (subfamilies 4, 7 and 22), remnants are created with which members of the southeast primary wave collide. Of these, only subfamily 22, which produces patterns that increase in manifest complexity fastest, is described in detail here, but figure 9 shows a member of subfamily 4, g2-g2-s24sw29, as it appears after 2^{24} steps[2] (at the right scale, it would appear the same after any greater number).

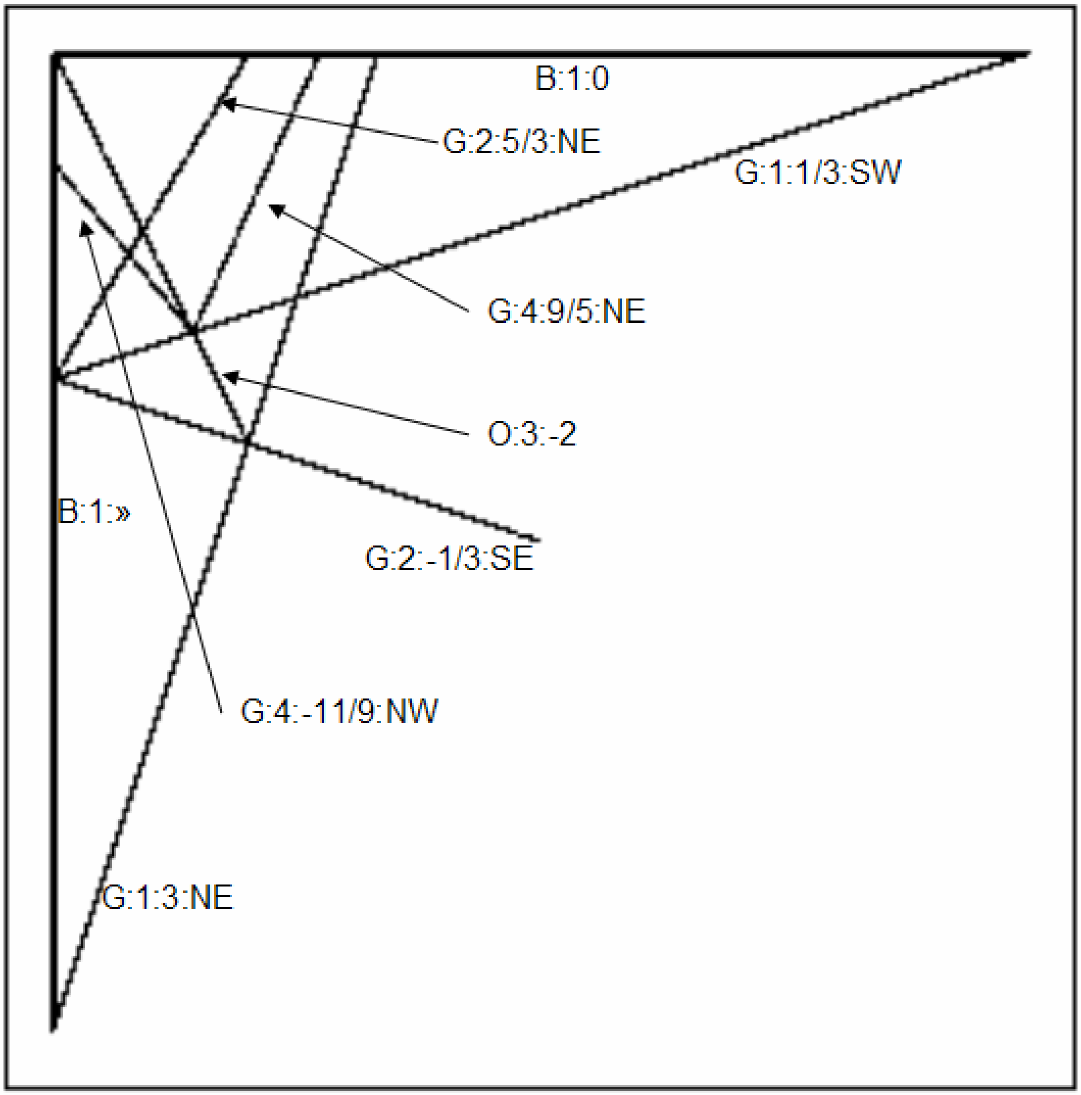

Figure 9: g-g-s24sw29, after 2^{24} steps.

Each line is labelled with a type (B, G or O – for gosper2 body, glider-wave and oscillator ray respectively), a *generation number*, a slope (distance travelled from south to north (positive) or north to south (negative) when moving a unit distance west to east along the line – for vertical lines, the symbol "»" is used), and in the case of glider-waves, the gliders' direction of travel. The gosper2 bodies and primary glider-waves are given generation number 1, indicating production directly by the gosper2 heads; other waves, and the oscillator rays, are all products of interaction between pairs of lines with lower generation numbers, and are given a number one greater than the maximum of those of these lines. Thus the two lines labelled "G:2:5/3:NE" and "G:2:-1/3:SE" are the secondary waves created when the primary southwest wave

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[2] All the investigation of these patterns was done using free software: Johan Bontes' Life32 (http://www.xs4all.nl/~jbontes/), Tom Rokicki's hlife (http://tomas.rokicki.com/hlife/), and Golly (http://sourceforge.net/projects/golly/), developed by Rokicki and Andrew Trevorrow.

(G:1:1/3:SW”) hits the south-headed gosper2 body (B:1:»); the line labelled “O:3:-2” is the ray of oscillators created by collisions between members of G:1:3:NE and G:2:-1/3:SE (which are both modified by the collisions); G:4:9/5:NE and G:4:-11/9:NW are glider-waves created when G:1:1/3:SW interacts with O:3:-2 (again, both interacting lines are themselves modified). The *moving intersection* between any two lines itself follows a straight-line path away from where the gosper2s started as the entire pattern grows; at each such moving intersection, one or more specific types of interaction may occur repeatedly – or the two lines may simply pass through each other without interacting, as in the case of G:2:5/3:NE and G:4:-11/9:NW here. If any of the types of interaction between the waves produce oscillators, the path of the moving intersection becomes visible as an oscillator ray. In g2-g2-s24sw29, the range of interactions occurring at every moving intersection soon (i.e. within some tens of thousands of steps) becomes fixed.

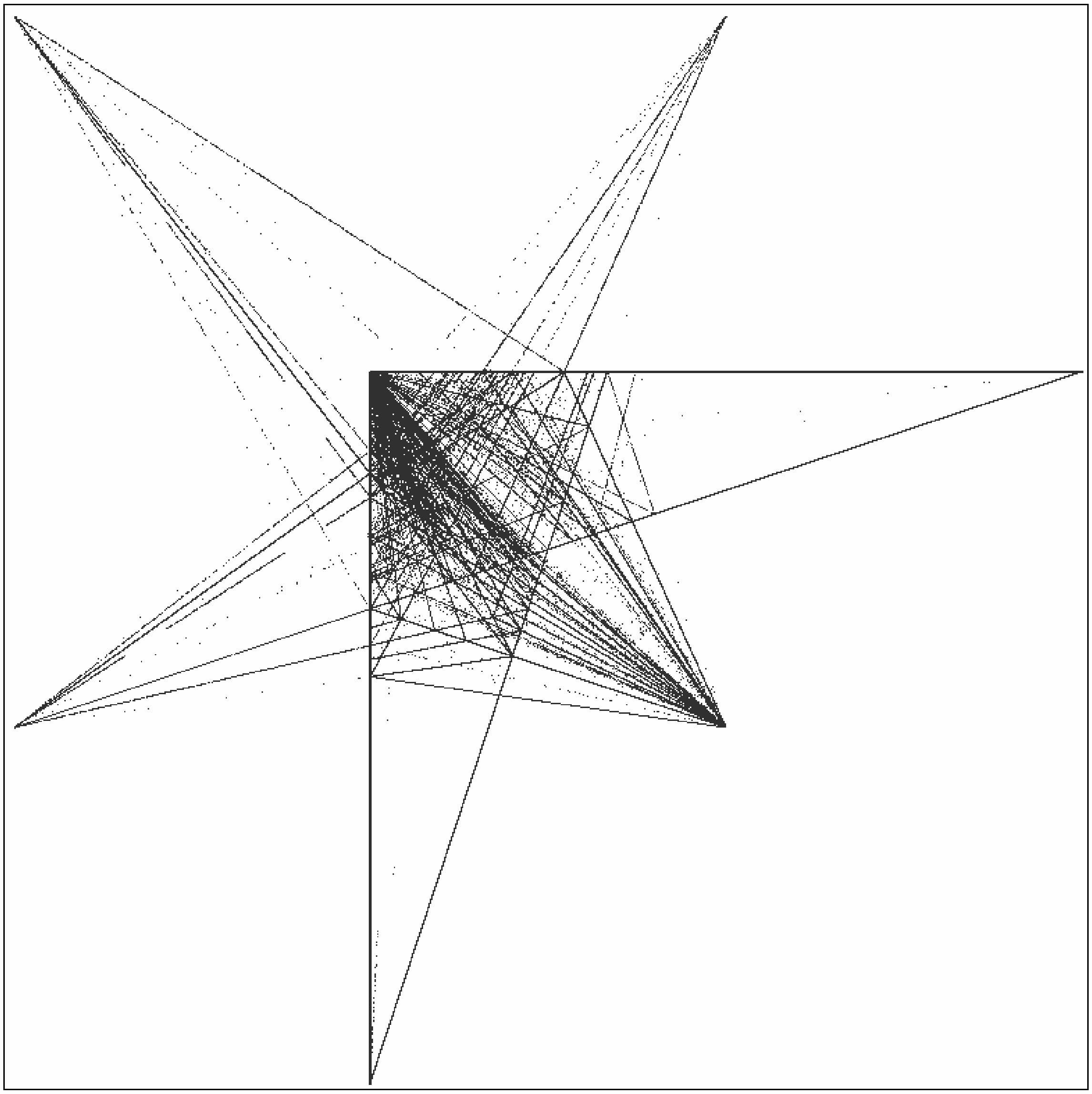

Figure 10: g2-g2-s24sw72, after 2^{32} steps (entire pattern shown).

Pictures of members of subfamily 22 are less easy to interpret because the number of lines increases continually (at least, as far as the patterns’ development has been followed), and later lines are sparser than earlier ones, so it may be unclear what lines there are. Figure 10 shows a member of this family, g2-g2-

s24sw72, after 2^{32} steps. The dots in the outer parts of the eastern and southern points of the irregular six-pointed star are orthogonally travelling spaceships, mostly LWSSs. All the lines leading to the other points of the star are glider-waves, with the gliders moving outwards. Other members of the subfamily appear similar in general, but different in detail. Figure 11 shows part of the same pattern after 2^{27} steps, in which multiple oscillator rays and glider-waves of different densities, some with long gaps, are clearer. The patterns in this subfamily, and some other g2-g2 patterns, can surely be characterised as ramifying feedback networks: multiple, interacting (and in this case expanding) feedback loops in which new types of local interactions (collisions between shapeships, or between spaceships and oscillators) are continually generated. Some of the early stages in the development of the ramifying feedback network in g2-g2-s24sw72 (and other members of its subfamily) can be understood from figure 11:

- The lines labelled O:3:-2, and G:3:-9/7:NW are created by the interaction between members of G:1:3:NE and G:2:1/3:SE. These interactions can create oscillators, and send gliders in all four directions.
- Without the modifications these collisions cause, G:1:3:NE would pass through G:1:1/3:SW, but the modifications cause interactions, leaving O:2:-1 as evidence, and removing gliders from both waves.
- Both O:3:-2, and G:3:-9/7:NW also interact with and modify G:1:1/3:SW. Each remnant in O:3:-2 affects a part of G:1:1/3:SW that was northeast of the collision creating that remnant when it occurred, while the gliders in G:3:-9/7:NW created in that same collision affect members of G:1:1/3:SW that were created earlier, and were due north of the collision when it occurred. Thus the collision between G:1:3:NE and G:2:1/3:SE indirectly modifies G:1:-1/3:SW at two different points, which become increasingly distant with time.
- The interaction between G:1:-1/3:SW and O:3:-2 also creates G:4:11/5:NE, which collides with B:1:0 creating G:5:-11/5:SE, which in turn collides with G:1:1/3:SW, modifying a part of it that has not yet been through any of the other interactions mentioned.

Where a linear macrostructure such as a wave or ray undergoes interactions with several other such macrostructures, all these interactions occur simultaneously, involving different parts of the macrostructure. The relationships between these interactions can become confusing because of the different scales involved: any given part of G:1:1/3:SW encounters G:5:-9/5:SW before it encounters O:3:-2; but any particular member of G:5:-9/5:SW results from a collision between an earlier member of G:1:1/3:SW, and a member of O:3:-2.

Figure 11 shows macrostructures made both of oscillators (the gosper2 bodies and the oscillator rays), and of spaceships (the glider-waves): the former simply grow at one end, while the latter both grow and move. Cutting across the oscillator/spaceship distinction is one between structures produced by a single growth point (the gosper2 bodies and primary waves), and those produced by successive collisions between clusters belonging to pre-existing macrostructures and taking place at a moving intersection (the oscillator rays and the remaining glider-waves). As already noted (in relation to G:1:1/3:SW and B:1:»), there are cases where successive collisions of this kind interact with each other: where chains of local events are linked into a single, spatio-temporally extended, connected subgraph of the pattern's 1-cluster dynamics. Such a sequence may persist until a specific event disrupts it, establishing an alternative pattern, which itself persists until disrupted. Removing a single glider from G:1:1/3:SW in g2-g2-s24sw13 can produce such a switch into an alternative sequence, in which every six gliders in G:1:1/3:SW produce just two gliders in G:2:-1/3:SE and two in G:2:5/3:NE, instead of four in G:2:-1/3:SE and four in G:2:5/3:NE (the glider removed must be the third after one of those that mutually annihilates with a southeast glider shortly before it reaches B:1:», as described above). Removing another glider reverses the switch, as do a variety of changes – some quite minor – to the gosper2 body. Such switching is a form of cross-scale interaction: a local change that can – if the conditions are right – have an effect on a scale which is non-locally determined. It creates *meso-structures*, distinct from both fixed-size micro-structures and ever-growing macro-structures.

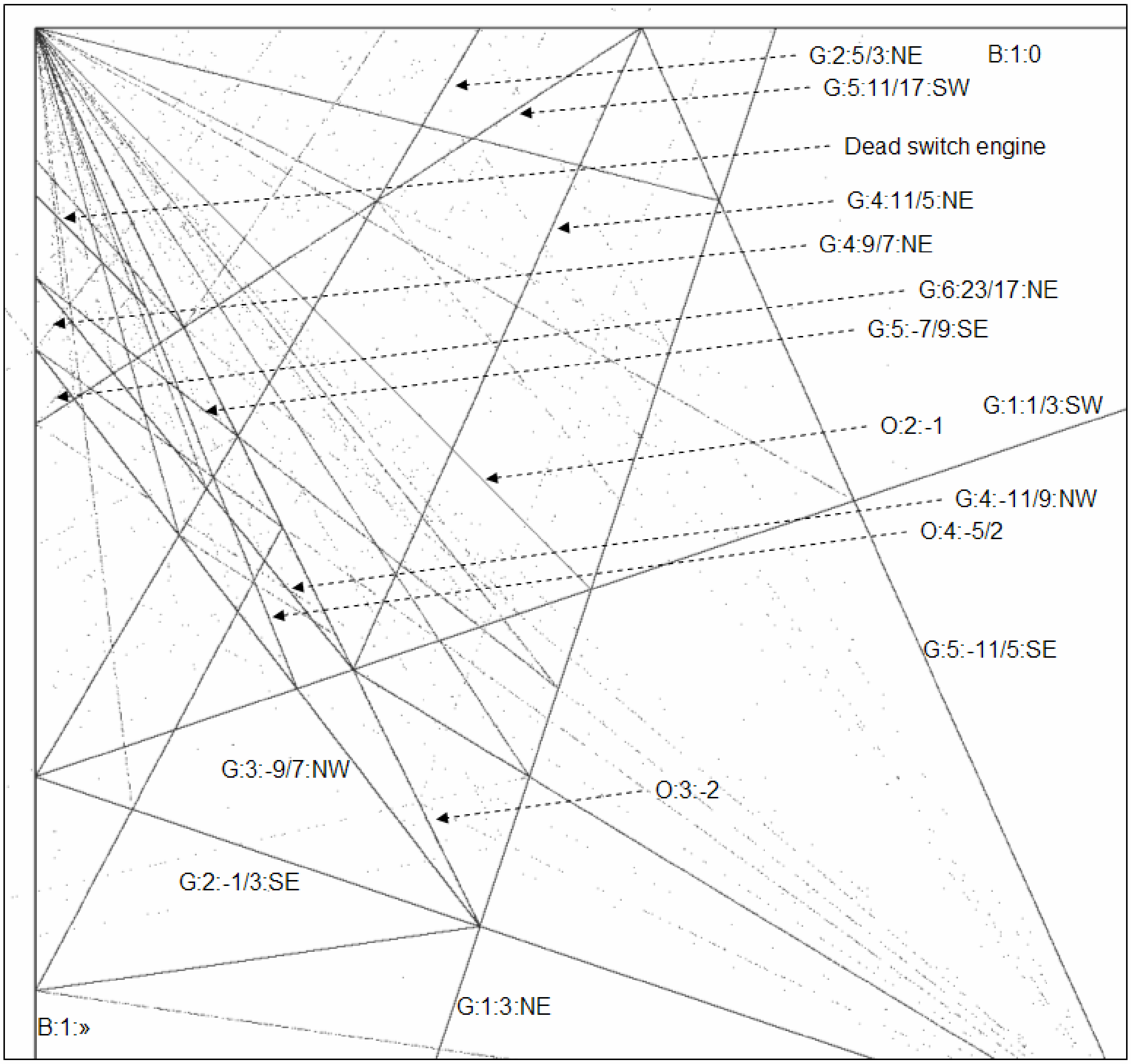

Figure 11: g2-g2-s24sw72, after 2^{27} steps (main area of activity shown).

Why does the feedback network continue ramifying in g2-g2-s24sw72, but not in g2-g2-s24sw29? In g2-g2-s24-sw72 the interaction between G:2:-1/3:SE and G:1:3:NE produces larger and more varied oscillator clusters in O:3:-2 than in g2-g2-s24sw29, and new gliders going in all four directions (no new gliders are produced in g2-g2-s24sw29); and further interactions involving these products and G:1:1/3:SW lead to a considerable variety of glider-clusters in this wave, and oscillator clusters in O:3:-2 and B:1:», in turn leading to a considerable number of additional types of collision. Given sufficient variety of collisions, at least some will produce novel oscillator and glider-clusters and hence the opportunity for further novel collisions; there may thus be quite a sharp division between diverging puffer pair patterns that produce enough initial variety to initiate this positive feedback loop, and those that do not.

5.2 Emergence of additional puffers

As noted, linear features of the g2-g2-s24 family of patterns are produced by micro-level mechanisms interacting in two different ways: through the action of growth points (gosper2 heads) existing in the starting pattern, and by the repeated interaction of components of other macrofeatures. Here, the focus is on linear features arising in a third way: by the production of new puffers – specifically, block-laying switch engines. In this kind of cross-scale interaction a sequence of local interactions widely distributed across two-dimensional space gives rise to a new (meso-scale) linear structure.

Novel microprocesses are continually generated by the ramifying feedback network in a g2-g2-s24sw22 subfamily pattern. Unpublished work by Paul Callahan suggests that if 8x8 patches of an otherwise empty array are randomly (at 50% probability) filled with state 1 cells, around 1 in 10^6 will produce a switch engine. How rapidly the count of distinct local interactions rises in g2-g2-s24-sw22 subfamily patterns has not been calculated; the fact that all the new puffers yet discovered arise in the same way may indicate that – even after 2^{32} steps – the number has not reached millions. At any rate, the infrequent but repeated generation of switch engines from pairs of small puffers is not unique to these patterns: in a class of patterns involving two perpendicularly diverging arks, additional switch engines are produced in diverse ways.

The block-laying switch engines produced in g2-g2-s24sw22 subfamily patterns appear from the moving intersection between B:1:» (the south-headed gosper2's body) and G:3:-9/7:NW, the glider-wave produced by collisions between G:2:-1/3:SE and G:1:3:NE. This moving intersection travels down B:1:» at a speed of *c*/14. For a block-laying switch engine to be produced, a section of B:1:» must have been "prepared" in the right way. Full details of the process are too lengthy for this paper, but figures 11 and 12 illustrate an outline account. Gliders beginning as parts of primary glider-waves at four separate points on B:1:0, and three on B:1:», contribute to the final result.

Gliders setting off southwest from around point D on B:1:0 in figure 12 reach B:1:» at D'. To contribute to production of a switch engine the wave must, when it reaches D', include the sequence 01101(111)*0111 (the *key sequence*), where 1 indicates a glider, 0 a gap where a glider has been removed, and (<subsequence>)* a subsequence that can be repeated any non-zero number of times. The third 0 must represent a glider removed in a collision at point Y (leaving a cluster of oscillators there), with a glider headed northwest from B": the origin of this glider is described below. The other two gliders may have been removed in any of several ways. The start of the key sequence institutes a shift from the most stable form of interaction between B:1:» and an unaltered G:1:1/3:SW to a second, less stable interaction. The final missing glider in the key sequence reverses this shift, and the gliders sent southeast as part of G:2:-1/3:SE around this time will interact with gliders from E' to send gliders back northwest (as members of G:3:-9/7:NW). These can interact again with B:1:» around D' to produce a southeast-headed switch engine.
For this to occur, however, a small change (one small still-life – a *tub* – being turned into another – a *boat*) must be made to B:1:» by a southeast headed glider arriving on exactly the right diagonal from a second collision at point Y. Producing this *adjuster* involves interactions between stretches of B:1:0 around points A, B and C in figure 12, and stretches of B:1:» around C' and E':

- The interaction between B:1:» and G:1:1/3:SW around A' produces gliders in G:2:-1/3:SE that interact with G:1:3:NE gliders from C' to produce a collection of oscillators at A" (belonging to O:3:-2).
- Gliders in G:1:1/3:SW generated around C hit these oscillators, sending gliders back as part of G:4:11/5:NE.
- These in turn collide with B:1:0 to produce gliders belonging to G:5:11/17:SW, which in a further encounter with the oscillators at A" produce a glider (the *preadjuster*) that proceeds to point X.
- Here it may meet a glider produced in a similar journey from B, involving collisions at B' and B". The latter sends five gliders northwest as part of G:3:-9/7:NW. It is one of these that must cause the third gap in the key sequence, leaving a remnant at Y; another must meet the preadjuster at X, sending another glider back to Y to collide with the remnant there, producing the southwest headed adjuster.

This sequence of collisions was neither designed nor foreseen; the ramifying feedback network produces many such sequences, and it "just happens" that this one produces a switch engine – although of course the fact that it does so must be a logical consequence of the rules of GoL and the initial pattern used.

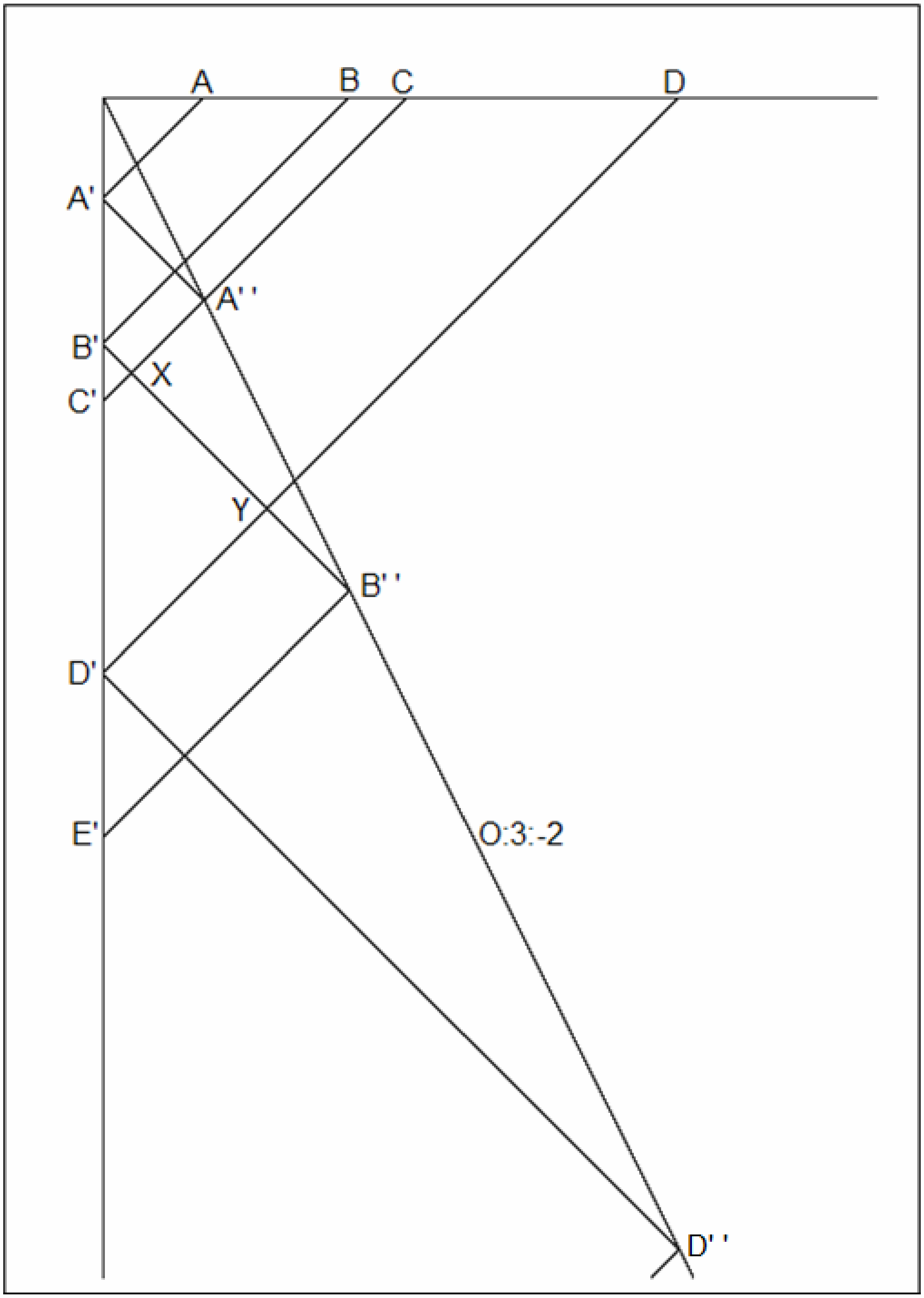

Figure 12: Construction of block-laying switch engines in g2-g2-s24sw22 subfamily of patterns.

If successive positive integers are assigned to the members of G:1:1/3:SW (call the integer assigned to a member its *index*), those contributing to the construction of a switch engine as described must have an index with certain residues in relation to specific moduli. In particular, a glider producing the remnant at Y and hence leaving the final gap in the key sequence is constrained to have a specific index residue modulo 119 (= 7*17) which is fixed for a particular member of the subfamily, and varies in a systematic way across subfamily members. Since members of every simple wave of gliders are necessarily separated from each other by multiples of fifty half-diagonals (because of the regularities in the gosper2 body and primary glider-wave), the possible tracks of the members of G:1:1/3:SW that take part in the collision leaving the final 0 in a key sequence are separated by multiples of 5950 half-diagonals, and hence the positions down B:1:» from which switch engines can grow are separated by multiples of 5950 cells.

Empirically, there is also a bias in the values modulo 3 of the starting positions of switch engines. Any two members of the g2-g2-s24sw22 subfamily of patterns can be mapped onto each other by shifting the relative position of the two gosper2s in the initial pattern by a multiple of 50 cells in the north-south direction, and then by a multiple of 25 cells in the northeast-southwest direction. Three groups within the subfamily can be defined by the residue modulo 3 of the north-south distance between the northwest cells of the two gosper2s in the initial 50-cell pattern, minus the east-west distance between the same two cells (which is the value of *j* in g2-g2-s*j*sw*k*). For each group, the north-south distance between B:1:0 and the starting positions of switch engines shows a strong bias toward one residue modulo 3: 108 of the 114 switch engines found have one of the three residues (*standard 3-residue* switch engines), 6 have the residue reached by adding 1 to this standard residue (*standard-plus-1 3-residue*), while none have yet been found in the *standard-plus-2 3-residue* class. This bias has not been fully accounted for, but appears to stem from the interaction of G:1:3:NE and G:2:-1/3:SE to create O:3:-2, which consists of several types of remnant, each a cluster of oscillators. For any particular type of remnant, only every third member of G:1:3:NE can take part in the collision that creates it. Each remnant type has a particular effect on the G:1:1/3:SW wave, generally removing gliders from it, and the same G:1:3:NE glider index modulo 3 is involved in the two most common collisions with G:2:-1/3:SE. Both of these leave a large remnant in O:3:-2 that will remove at least two gliders from G:1:1/3:SW. These interactions will give a modulo 3 bias to the index of the gliders, and particular sequences of gliders and gaps, remaining in G:1:1/3:SW after its encounter with O:3:-2, and hence a modulo 3 bias to the north-south positions at which particular sequences reach B:1:». There are also apparent differences in the ease with which switch engines form in the three groups of patterns defined above, according to a systematic study of 72 patterns followed for at least 2^{30} steps, and 144 more followed for at least 2^{28}. Again, this may be due to the way O:3:-2 removes gliders from G:1:1/3:SW.

6 Interacting secondary puffers: emergence of a quasi-population

Once a switch engine is formed, it grows southeast, the head moving both south and east at *c*/12. As long as its growth continues, the head is thus south of the moving intersection between B:1:» and G:3:-9/7:NW (which moves at c/14), and south of any subsequently generated switch engine (heads of all growing switch engines form a straight line with a slope of -1/7). The switch engine head would eventually crash into the O:3:-2 remnant that produced the gliders that created it – hence, a switch engine's maximum lifespan is determined by its time of origin – but it is often destroyed before this can happen. Figure 13 shows, for all the switch engines followed from "birth" to "death" in all members of the g2-g2-s24sw22 subfamily investigated, how far they grew before a fatal collision. The horizontal scale (logarithmic for clarity) shows the distance south of the tail of B:1:» at which a switch engine was formed; the vertical scale shows the proportion of the distance to O:3:-2 the switch engine grew before dying. Several points may be noted:

- The proportion of switch engines alive at the last point at which the pattern was observed grows toward the right of the plot, so the apparent decline in the average proportion of the distance to O:3:-

2 covered before death may be partly an artefact. However, this would not explain the larger proportion of later-born switch engines killed before reaching 2/3 of the way to O:3:-2.

- A number of switch engines reached O:3:-2.
- At least two switch engines died extremely close to 0.2 (1/5) of the way to O:3:-2, several extremely close to 0.4 (2/5) of the way, or extremely close to 0.667 (2/3) of the way. These are hypothesized to have been killed by collision with members of pre-existing waves of gliders or lines of oscillators.
- Most of the remaining switch engines are believed to have been killed by an indirect process beginning when a glider hits the switch engine body. Several glider-waves hit a growing switch engine from the side (figure 14): some gliders pass through without hitting any blocks, but others do hit them; these may generate further gliders going in any direction. Any going onward form part of the original wave (which is generally thinned); those going back form a new wave; those going to left and right form glider-streams rather than waves (not simple streams such as a glider-stream switch engine produces: gliders appear at irregular intervals and may follow slightly different paths). Gliders in the southeast stream will overtake the head, and may then collide with something and either form an obstacle to the switch engine's progress, or produce gliders travelling northwest, which may collide with the switch engine head or with a later member of the stream. Hence, local interactions between the switch engine's body and its environment can lead indirectly to its "premature death".

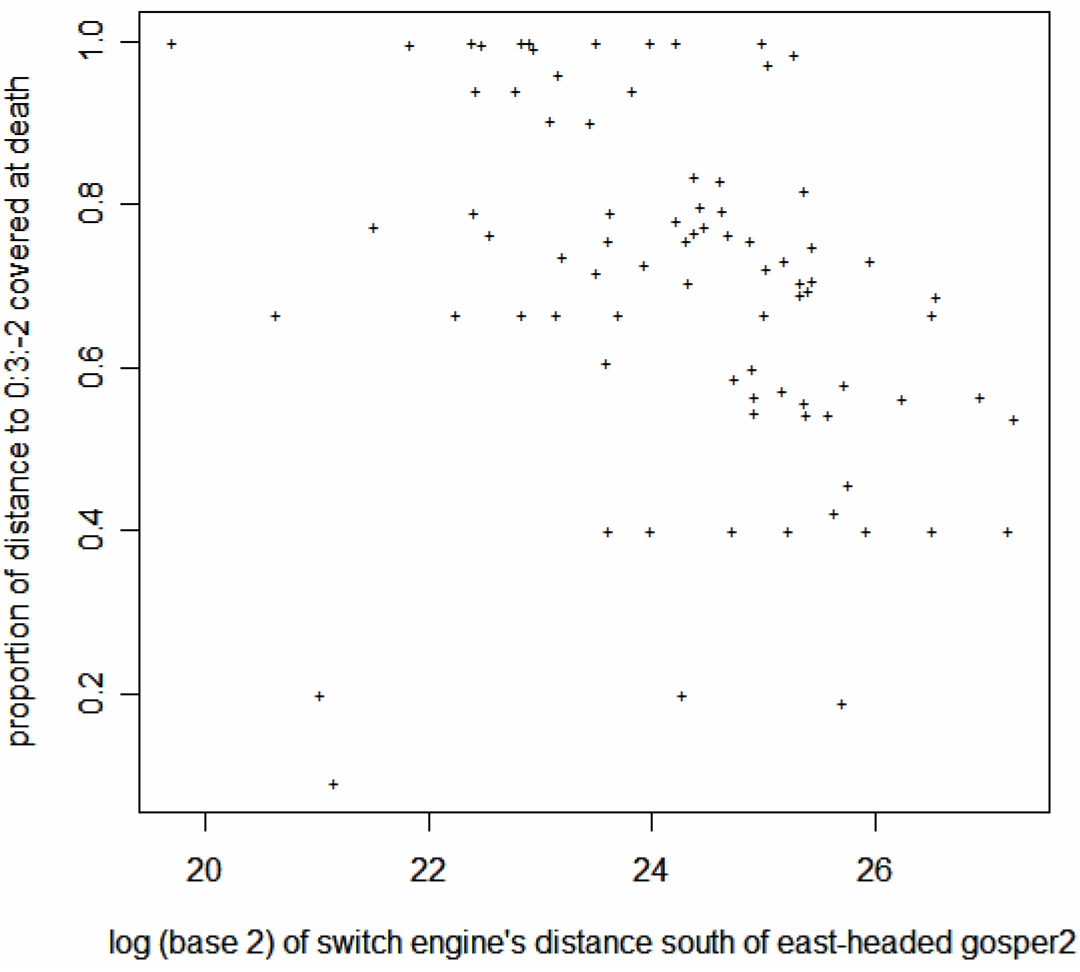

Figure 13: Switch engine survival in g2-g2-s24sw22 subfamily of patterns.

Even if the deaths at 1/5, 2/5 and 2/3 of the way to O:3:-2 are excluded, figure 13 shows apparent clusters of deaths: one to the right and between 0.5 and 0.6, a second spread more evenly across and between 0.7 and 0.85, and possibly a third above 0.9. The last two groupings might merge when further data is collected, but there are likely explanations for there to be two groupings at least. Taking the second and third groupings together, the densest wave a switch engine crosses is G:5:11/17:SW; it does so when it has gone 0.4 (2/5) of the way to O:3:-2. From then on, interactions between the two will generate a glider-stream ahead of the switch engine. The leaders of this will reach O:3:-2 when the switch engine has completed 0.6 (3/5) of its journey, and northwest-headed gliders created by the reaction between the stream and the O:3:-2 remnants it hits cannot meet the head before it has completed 0.7 (7/10) of its journey. Hence, the number of switch engines dying should rise sharply at this point, then tail off (since if an earlier interaction kills the switch engine, a later one cannot). The peak in numbers dying between 0.5 and 0.6 probably results from the switch engine's interaction with two less dense waves. G:5:9/7:NE first hits the switch engine right near its root in B:1:», and resulting northwest-headed gliders can start meeting the switch engine halfway to O:3:-2. However, later interaction with G:6:23/17:NE can produce lumps on the switch engine body that block most of the southeast gliders produced by G:5:9/7:NE, while producing fewer southeast gliders itself.

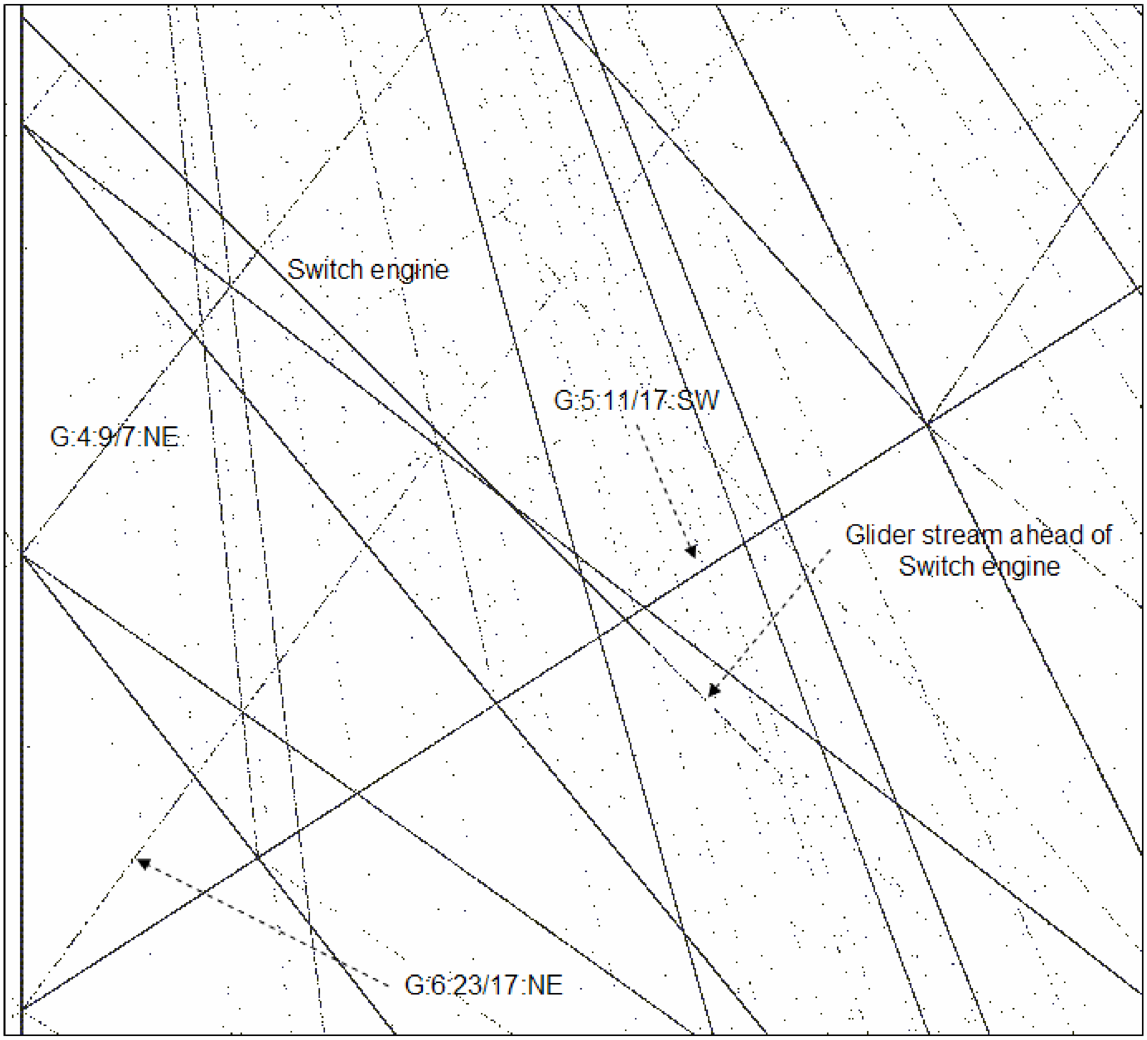

Figure 14: g2-g2-n126sw297 after 2^{30}, showing growing switch engine's interaction with its environment.

How long a switch engine lives will, in general, depend on a large number of local interactions, most of them taking place along its body, which acts as a kind of "catalytic surface" in the two-dimensional array, linking local interactions in a complex causal network: another form of cross-scale interaction. Thus the exact sequence of gliders hitting a switch engine determines how long it lasts, and hence its effect on the wider pattern – but omitting some gliders will make no non-local difference, while others will be crucial, as has been suggested is characteristic of cross-scale interaction.

Additionally, switch engine bodies act as a kind of "semi-permeable membrane", thinning the glider-waves that cross them, and altering their composition – sometimes producing altogether novel glider-clusters. This is another form of cross-scale interaction, as the switch engine alters the relative frequency of different types of glider-cluster in a wave (eventually, the switch engine will die and the wave will, later, move past its southeast end, so their interaction can be regarded as a meso-scale event – non-local but limited in size and duration). Block-laying switch engines produce a body with a spatial periodicity of 48 half-diagonals. Of the 48 ways a single glider can approach the body from the side, only eight allow the glider to pass through, one produces an onward glider shifted by two half-diagonals toward the head, and one produces gliders going in all three other directions, but none going onward. For glider-clusters the situation is more complicated, but usually fewer travel onward than were originally in the cluster. The changes to the wave will affect lines it subsequently intersects, so the switch engine's influence will extend through the pattern's network of lines. This effect on its environment will continue even after its death, although any individual switch engine should become less influential as the main areas of activity move south and east.

Switch engines with starting positions on B:1:» which are not a multiple of 48 cells apart can interact differently with glider-waves, if the latter also show certain types of spatial periodicity. Specifically, G:5:11/17:SW, has a modulo 300 periodicity in the northwest-southeast direction: any two clusters of gliders of the same type (i.e., produced by homologous processes) will be separated by a multiple of 300 half-diagonals in that direction. Thus for any such type of cluster, and a given switch engine, there are eight possible interactions, and switch engines belonging to the three different 3-residue classes will have non-overlapping sets. Hence the effects of the interaction with this wave will be different for the three classes.

If a block-laying switch engine's head hits an oscillator or oncoming glider, the outcome depends on the obstacle's exact position and the switch engine's phase. The switch engine may be killed, as discussed above; the disturbance will be transitory; or the switch engine can change phase, so that once production of a regular trail of blocks resumes, it will have shifted in the northwest-southeast direction (for a southeast headed switch engine) by 16 or 32 half-diagonals modulo 48. Finally, conversion into a glider-stream switch engine has been observed in one pattern. The glider-stream switch engine itself was quite soon killed, but another outcome is possible, although not yet observed: if the glider-stream hits an obstacle in the right way, it may build up glider crystal which the head may subsequently overtake.

Switch engines will influence each other, particularly if more than one is or has recently been growing, as should become more common over time (across the 142 patterns followed as far as 2^{30} steps, the total number of switch engines produced is 0 after 2^{23} steps, 1 after 2^{24}, 1 after 2^{25}, 2 after 2^{26}, 13 after 2^{27}, 23 after 2^{28}, 44 after 2^{29}, and 85 after 2^{30}, indicating that the rate of switch engine production is roughly constant at least up to 2^{30} steps). Consider two switch engines starting their growth within a relatively short time. As soon as the second begins to grow, it will interpose itself between G:4:9/7:NE and the first. Conversely, once the second switch engine has completed 2/5 of its journey, the first switch engine will be interposed between it and G:5:11/17:SW, the densest wave that interacts with switch engines. Such interposition is likely to be protective on average, since interactions with the interposing switch engine will tend to reduce the number of gliders hitting the other, and hence the number in the stream of gliders overtaking the head. However, interaction with the first switch engine may produce gliders that would not otherwise have existed, and in some cases these may generate southeast gliders when hitting the second.

The effect will also depend on the exact distance between the starting positions of the two switch engines. The north-south distance between the starting positions of two switch engines produced within the same pattern may have any even residue modulo 48; although because of the modulo 3 bias in starting positions noted earlier, the residues 0, 6, 12, 18, 24, 30, 36 and 42 are (over the cases known) most common. Any particular residue modulo 48 in starting position corresponds to a distinct way in which the trails of blocks making up the bodies of the switch engines line up with each other in a southwest-northeast direction. If (and only if) the residue modulo 48 is 0 or 24, any glider passing through one switch engine body will pass through the other. Only for residues 0, 2, 22, 24, 26 and 46 will there be *any* path a glider can take through both switch engine bodies without a collision. Hence, the difference in residue modulo 48 may make a big difference in the effect each switch engine has in altering the interactions the other has with glider-waves. Pairs with distances of residue 0 or 24 modulo 48 are likely to offer each other most protection, as any glider that passes through one without interaction will also pass through the other in the same way, whereas such gliders may cause the production of southeast gliders in the case of other residues.

The fact that switch engines can cross G:5:11/17:SW means they can inhibit production of new switch engines, because the preadjuster glider belongs to G:5:11/17:SW. Standard residue and standard-plus-1 3-residue switch engines will block 3/4 of preadjusters, while standard-plus-2 residue switch engines (if any exist), and most pairs of switch engines that are not aligned in the northwest / southeast direction modulo 24 will block all preadjusters. If the starting position of a switch engine is r cells south of B:1:0, some or all switch engines may be prevented from forming that would have had starting positions between $17r/5$ and $17r/3$ cells south of B:1:0. Restricting attention to relationships between standard 3-residue switch engines, a switch engine with starting position m mod 24 cells south of B:1:0 that grows all the way to O:3:-2, will only permit the formation of possible switch engines between $17r/5$ and $17r/3$ cells if they have a starting position m-6 mod 24 cells south of B:1:0 (for patterns where j in g2-g2-s$j$sw$k$ is 0 mod 4); and only if they have a starting position m+6 mod 24 cells south of B:1:0 (for patterns where j in g2-g2-s$j$sw$k$ is 2 mod 4).

Consider two sequences of standard 3-residue switch engines arising in similar patterns and growing across G:5:11/17:SW, sequence A having north-south gaps in their starting positions of residue 0 modulo 24, sequence B including a mix of gaps of 0, 6, 12 and 18 modulo 24. Sequence A will increase the chance of several successive residue-3 switch engines being separated by residue 0 gaps modulo 24 at a later point, by blocking most of the preadjusters that could give rise to switch engines in different residue classes modulo 24. Sequence B, on the other hand, will block *all* preadjusters so long as any two with different residues modulo 24 are across G:5:11/17:SW. This will produce a stretch of B:1:» with no switch engines at all – but that in turn will increase the likelihood that there will be a period when all preadjusters get through and create another sequence of switch engines including many gaps that are non-zero modulo 24. Moreover, if the effect in at least some patterns is to lead to sequences of switch engines in which most adjacent pairs have a gap of 0 modulo 24 (the interactions discussed above between switch engines that simultaneously interact with the same glider-waves make this plausible), a slightly stronger form of quasi-hereditary effect could arise. The sequence of gaps between the starting positions of successive standard 3-residue switch engines would consist mostly of 0s, with an occasional 6, 12 or 18. Ignoring the 0s, consider the sequence of non-zero residues that remains. Each member of the sequence now represents a switch from one batch of standard 3-residue switch engines with gaps of residue 0 modulo 24 to another, with three different types of switch possible. Any switch or subsequence of switches might then have a tendency to produce a copy of itself further down the sequence. For example, suppose a batch of switch engines with starting position residues of 0 modulo 24 is followed by a batch with residues of 12 modulo 24. If the pattern is of the $j = 0$ mod 4 type, then while only members of the first batch lie across G:5:11/17:SW, preadjusters giving rise to switch engines with starting position residue 0, 6 or 12 mod 24 will be blocked; among standard 3-residue switch engines, only those with residue 18 can be created on a stretch of B:1:» further down (non-standard 3-residue switch engines with residues 2 or 34 could be created, but as noted, these are much rarer). There is then likely to be a period when residue 0 and residue 12 switch engines both lie across G:5:11/17:SW, blocking all preadjusters. Once the last residue 0 engines no longer lie across G:5:11/17:SW, preadjusters able to create standard 3-residue switch engines with starting positions of residue 6 can get through. Hence, once these preadjusters have done their work, a batch of residue 18 engines will be followed by a residue 6 batch, replicating the 12 in the non-zero gap-residue sequence. Parallel arguments can be constructed for patterns of the $j = 2 \bmod 4$ type, for a non-zero gap-residue of 6 or 18, and for sequences of two or more non-zero gap-residues.

This line of reasoning is tenuous, but raises intriguing possibilities. Given the ways switch engines can influence each other's survival by modulating the effect of glider-waves discussed above, different non-zero gap-residues are likely to give the batches of engines they separate different expected lifetimes – and hence, differing abilities to influence the production of future engines, and non-zero gap residues. Even the possibility that partially heritable differences in ability to self-replicate might emerge from the ramifying feedback networks and cross-scale interactions in g2-g2 patterns is surely of considerable interest. There are also ways in which switch engines could promote the formation of additional switch engines by manufacturing additional adjusters. Those so far discovered (but not yet observed in growing patterns) involve at least two transformations of block-laying switch engines into the glider-stream variety, and the interaction of at least one of these with a length of glider-crystal. They could, like the inhibitory mechanism described above, affect the relative frequency of occurrence of gaps of different sizes between the starting positions of adjacent switch engines.

As far as available computational resources have allowed g2-g2-s24sw22 patterns to be followed, their apparent complexity continues to grow. There is no proof this will continue, but conversely, new complexity-generating mechanisms could arise – for example, additional switch-engines, or other forms of puffer. The long-term fate of these patterns, therefore, remains unknown.

7 Discussion

Patterns in any binary CA with the same neighbourhood as GoL have a 1-cluster dynamics analogous to GoL patterns, but usually far simpler. For example, if at least 4 neighbours are needed to shift a cell into state 1, 1-cluster mergers soon cease; for "3-4 Life", in which a cell is in state 1 at $t+1$ if and only if either 3 or 4 of its neighbours (not including itself) were in state 1 at t, any random blob of sufficient size quickly consolidates into a single large 1-cluster and expands apparently forever, possibly throwing off small spaceships which move faster than blob expansion and thus escape. These CA thus do not support the temporally extended merging, splitting and vanishing dynamics of 1-clusters that GoL does. Even within GoL, almost all moderate-sized and dense random blobs (i.e. where the initial density of state 1 cells is anywhere near 50%) support this only for thousands or tens of thousands of steps. Prolonged interactions between large and varied collections of physically distinct "objects" of different sizes, may be a precondition for the appearance of complexity; certainly their existence in the physical world, at both astronomical and molecular levels, appears to have supported life's emergence. Note that GoL puffers already have at least three distinct structural levels: individual cells, 1-clusters, and repeated sequences of splitting and merging among 1-clusters. The extent to which small patterns with distinctive dynamic properties, such as LWSSs and switch engines, can combine with each other and with other small patterns to produce larger patterns with more sophisticated dynamic properties is one root of GoL's ability to generate complexity – rather as the ability of small molecules with distinctive chemical properties to combine into macromolecules underlies the complexity of organic chemistry and ultimately, life.

Initially, patterns in the g2-g2 class consist of two small subpatterns, each of which turns into a head, body and glider-wave. The heads thereafter remain the same size, while the bodies and waves grow linearly with time; we can thus distinguish micro or local objects and processes, of fixed size, from macro or global objects and processes, with apparently unending growth. Within the micro-scale there are at least three functionally important subscales: individual cells, 1-clusters, and groups of 1-clusters which interact in a regular cycle, or else are produced close together in space and time, like the clusters of oscillators or gliders constituting most of the oscillator rays and compound glider-waves discussed. Within the macro-scale we can distinguish subscales for the pattern as a whole (the area covered expands quadratically, but the number of state 1 cells linearly, as far as any pattern has been followed), and the ever-growing linear features. All these grow at a constant rate, but the rate constants differ between lines, as do the average density of the lines in terms of state 1 cells per unit length. We have noted distinctions between rays and waves, and between lines produced from a single growth point and those produced by interactions between other lines. These interactions produce secondary micro-features, the points of intersection, each of which moves at a constant velocity. The lines also divide a pattern into subareas: the puffer bodies separate off the south-east quadrant; rays divide the inner part of this quadrant into segments, and waves subdivide each segment into parts which shift and grow as the waves move south and east, but also get subdivided as additional waves form. In those patterns which produce ramifying feedback networks, later-formed rays and lines are generally less dense, so the areal division of the south-east quadrant has a continually developing hierarchical aspect: the edges of an area act as barriers to and modulators of the spaceship-clusters moving through them, and the denser barriers generally make most difference. Lines running in semi-cardinal directions, and to a much lesser extent orthogonal lines, can also act as quasi-catalytic sites, as collisions with them can generate spaceships which travel along the line until they meet an obstruction or spaceship from another collision.

While both micro and macro scales have complex structure including subscales, there are phenomena in g2-g2-s24-sw22 subfamily patterns that grow over time but not forever, and can be described as meso-scale. Most of the kinds of cross-scale interaction noted in these patterns involve meso-scale phenomena. The exception is the existence of the primary expanding feedback loops themselves. Here, the global structure of the pattern (the pair of diverging puffers sending glider-waves toward each other), combined with the exact spatial relationship between the two that define the subfamily, permit early members of G:1:1/3:SW to initiate chains of local interactions that remove later wave members, and add new types of glider-cluster to it. Once this type of cross-scale interaction initiates the formation of a ramifying feedback network, however, further types of cross-scale interaction either produce meso-scale phenomena, or involve these phenomena in further interaction with micro- or macro-scale phenomena, or both. These include:

- The reversible switches in the form of interaction occurring between G:1:1/3:SW and B:1:» that can occur when a member of the former is removed. It is these that give the secondary southeast wave sufficient variety to trigger the ramification of the feedback network.
- The production of block-laying switch-engines through a specific sequence of local interactions.
- The subsequent death of such switch engines, or their transformation into glider-stream switch engines, by collision with a glider or oscillator.
- The influence of such switch engines, once produced, on the "population dynamics" of particular cluster types, as they act as semi-permeable and cluster-transforming membranes; and as catalytic sites, bringing about the interaction of glider-clusters that would otherwise remain functionally unconnected.
- The way specific sequences of glider-clusters hitting a switch engine, along with any that have interacted with the O:3:-2 remnant, can determine the switch engine's fate, via the production of a compound stream of gliders that overtakes the head of the switch engine – and hence the fate, or indeed existence, of other switch engines.

We have already noted the involvement of both ramifying feedback networks and cross-scale interactions in real-world complex systems. Clearly, the specific entitites and processes involved in the apparent growth in complexity in the GoL patterns described here are unlike those in any real-world domain, and specifically any involved in the origin of life. However, there is at least one current proposal for the origin of life where pre-existing groups of meso-scale structures, themselves formed through what can be seen as cross-scale interactions and influencing each others' formation and persistence, play a crucial role. A major problem in understanding life's beginnings lies in the apparent need for self-replicating molecules or sets of molecules, and semi-permeable cell walls or membranes, to arise simultaneously. This is, specifically, a problem for the idea that life arose in a "primordial soup" of interacting prebiotic chemicals, within which molecules could move freely and so encounter each other: if reaction products are free to diffuse away, it is unclear how (in the absence of highly evolved cell membranes), prebiotic chemicals could be sufficiently concentrated to support the emergence of self-replicating polymers, or how selection could get started. (The "soup" metaphor has influenced artificial life research into the origins of life [13, 24] although [24] confines the soup within a "continuously stirred flow reactor".) Among alternatives to the primordial soup idea is that proposed in [21, 23] : that life arose around hydrothermal vents on the ocean floor, where the geothermally-driven flow of "exudate" rich in metal sulphides and other reactive molecules, into a cooler and more acid early ocean, could both create a continuously growing network of metal-sulphide lined compartments (which would themselves influence the flow of exudate, although these authors do not say so) and provide chemical building blocks for the formation of self-replicating sets of organic polymers. The formation of compartments requires the "cooperation" of processes on the molecular and geological scales, and would in turn permit the concentration of particular chemical species, while their surfaces could catalyse the production of more complex from simpler molecules. Free-living cells could have arisen by the later construction of lipid membranes, independently for archaebacteria and eubacteria. In this connection, the role of switch engine bodies as both semi-permeable barriers and sites of "catalysis" is of particular interest.

More generally, the way in which ramifying feedback networks give rise to an ever-growing variety of novel local interactions in the patterns examined here, and eventually to some which are capable of producing non-local dynamic effects and of further expanding the range of processes occurring, is surely relevant to the study of a broad range of systems showing increasing complexity: in GoL we have a system with a completely transparent "basic physics", remote from real physics, in which deterministic processes operating on quite simple initial conditions can generate structures and processes which show cross-scale but scale-dependent complexity reminiscent of that found in many real-world domains. Perhaps, contrary to proponents of various "anthropic principles" [3], complexity sufficient to serve as the basis for natural selection, and so for much greater increases in complexity culminating in the evolution of intelligent observers, would arise in a very wide variety of logically possible worlds.

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