Proposal for the redefinition of the astronomical unit of length (ua) through a fixed relation to the SI metre

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Presentation outline

- The astronomical units
- The role of the astronomical unit (of length)
- Recent evolution and new situation
- The proposal

The astronomical units

The current System of astronomical constants includes 3 astronomical units
(defined in the *IAU 1976 System of astronomical constants*)

– *The astronomical unit of time: a time interval of one day (D) of 86 400 s*

- provides a unit of time of "convenient" size for astronomy,
- is related to the SI second by a defining number.

– *The astronomical unit of length: ua*

- is a specific astronomical unit for expressing distances in the solar system,
- its definition is based on the Gaussian gravitational constant *k,*
- its value in SI has to be determined experimentally.

– *The astronomical unit of mass: the mass of the Sun,* M_{Sun}

- is a specific astronomical unit for expressing masses in the solar system,
- its value in SI has to be derived from the SI values of (1) the heliocentric gravitational constant, GM_{Sun}, and (2) the gravitational constant G (current relative uncertainty ~ 1×10^{-4}),
- the value of GM_{Sun} in SI is obtained from the SI values for the ua and the day,
- the value of the astronomical unit of mass in SI depends on the ua.

Definition and role of the astronomical unit of length (ua)

The ua is defined by the value of the Gaussian gravitational constant **k** (with $k^2 = G$), called a "defining constant"

Definition of the ua in the IAU-1976 System of astronomical constants

The astronomical unit of length is that length (*A*) for which the Gaussian gravitational constant (*k*) takes the value of 0.017 202 098 95 when the units of measurements are the astronomical unit of length, mass and time. The dimensions of k^2 are those of the constant of gravitation (*G*), i.e., $L^3M^{-1}T^{-2}$. The term "unit distance" is also for the length *A*.

Definition of the of the ua in the SI brochure (intended to non-astronomers)

The astronomical unit is approximately equal to the mean Earth-Sun distance. It is the radius of an unperturbed circular Newtonian orbit about the Sun of a particle having infinitesimal mass, moving with a mean motion of 0.017 202 098 95 radians per day (known as the Gaussian constant).

(Table 7 of the SI Brochure: Non-SI units whose values in SI units must be obtained experimentally)

For any planet, n is measured in SI (i.e. rad/s), its semi major axis can be expressed as $a^3 = k^2/n^2$ (perturbations are neglected), hence the distances to other planets are in ua.

→ the accuracy of the time measurements is transferred into the relative distances

This definition of the ua was explained by the lack of precise measures of distances in the solar system, while it provides accurate relative distances.

It let open the problem of precise scaling of the solar system in the SI.

The IAU 1976 System of astronomical constants

		ϵ_{76}	Diff. (76)–(64)		ϵ_{64}
I. Defining Constants		(in units of the last decimal)			
1. Gaussian gravitational constant	$k = 0.017\ 202\ 098\ 95$	0		0	0
II. Primary constants					
2. Speed of light	$c = 299\ 792\ 458$ m s^{-1}	1.2	-	42	500
3. Light-time for unit distance	$\tau_A = 499.004\ 782$ s	µs: 6	-	7 218	11 000
4. Equatorial radius for Earth	$a_e = 6\ 378\ 140$ m	5	-	20	80
5. Dynamical form-factor for Earth	$J_2 = 0.001\ 082\ 63$	1	-	7	30
6. Geocentric gravitational constant	$GE = 3.986\ 005 \times 10^{14}$ m^3 s^{-2}	3	-	25	30
7. Constant of gravitation	$G = 6.672 \times 10^{-11}$ m^3 kg^{-1} s^{-2}	4	+	2	5
8. Ratio of mass of Moon to that of Earth	$\mu = 0.012\ 300\ 02$	5	-	10	152
9. General precession in longitude, per Julian century, at standard epoch 2000	$p = 5\ 029''.096\ 6$				
10. Obliquity of the ecliptic, at standard epoch 2000	$\epsilon = 23°26'21''.448$				
11. Constant of nutation, at standard epoch 2000	$N = 9''.205\ 5$				
III. Derived constants					
12. Unit Distance	$c\tau_A = A = 1.495\ 978\ 70 \times 10^{11}$ m	km: 2	-	2 130	3 000
13. Solar parallax	$\arcsin(a_e/A) = \pi_\odot = 8''.794\ 148$				
14. Constant of aberration, for standard epoch 2000	$\kappa = 20''.495\ 52$				
15. Flattening factor for the Earth	$f = 0.003\ 352\ 81$				
16. Heliocentric gravitational constant $= k^2A^3/D^2$	$GS = 1.327\ 124\ 38 \times 10^{20}$ m^3 s^{-2}				
17. Ratio of mass of Sun to that of Earth	$S/E = 332\ 946.0$				
18. Ratio of mass of Sun to that of Earth + Moon	$(S/E)/(1+\mu) = 328\ 900.5$	3	-	115	215
19. Mass of the Sun	$S = 1.989\ 1 \times 10^{30}$ kg	12	-	7	22

IERS Conventions 2003
uncertainty in the ua: 6 m
(JPL ephemerides DE403, Standish 1995)

The scale distance in the solar system is provided by the value A of the ua in m as fitted to a planetary ephemeris.

Recent evolution and new situation

1. The context of the recent IAU Resolutions on reference systems (1991, 2000, 2006)

Definition of the celestial reference systems in a GR framework; improvements in the concepts, models,

$\longrightarrow$ the definition of the ua should be best compliant with this modern IAU framework.

2. The context of the modern observations in the solar system

High accuracy observations in the solar system: ranging to planets, spacecraft observations, VLBI, etc.,

$\longrightarrow$ ranging observations are so accurate that there is no longer a reason to have a scale-invariant description of the solar system as provided by the current definition of the ua.

3. The context of the recent ephemerides

- The primary determination of the GM_i of the planets are obtained in km^3/s^2 (TDB-compatible values) and then converted into values in astronomical units (Folkner et al. 2008).

- The direct estimation of GM_{Sun} has been tested in the INPOP08 ephemerides (Fienga et al. 2009),

$\longrightarrow$ the ua appears as an intermediate unit only used for historical purposes.

4. The possibility of a direct detection of a variation of the solar mass in the near future

The decrease of the solar mass is expected to be detectable when the accuracy has been improved by a factor 10,

$\longrightarrow$ using time-dependent units of length and mass to measure such a variation will be a non-sense.

The status of the ua and GM_S should be reformed to be more in agreement with the modern context (Klioner 2008, Capitaine & Guinot 2009)

The definition of the ua in the relativistic framework

Several options can be considered for the astronomical units in the GR context
(Guinot 1995, Capitaine & Guinot 1995, Brumberg & Simon 2004, Standish 2005, Pitjeva 2005, Klioner 2008).

1) Necessity to extend the Gaussian gravitational constant to the GR framework: several possible options but the geometrical interpretation through the motion around the Sun would be more delicate.

2) Necessity to take into account the use of TCB or TDB, i.e. the scaling factor $F=(1 - L_B)$:
$\text{quantity}_{TDB} = F_* \text{quantity}_{TCB}$:several options: *scale only units, scale only values, scale both, etc.*
out of which the two most logical are:

- **Option 1:** $M_S=1$ and k has the same value with both TCB and TDB. This requires:
  unit of distance: $AU_{TDB} = F^{1/3} {}_* AU_{TCB}$,
  distance: TDB-compatible value = $F^{2/3}$ $_*$ TCB-compatible value,

very confusing because of the unusual scaling.

- **Option 2:** $AU_{TDB} = AU_{TCB}$ and scale the value. This requires:
  distance: TDB-compatible value = F $_*$ TCB-compatible value,

but GM_S is no longer k^2, either with TDB or with TCB, which kills the uniqueness of the Gaussian constant k.

Relativistic scaling from TCB to TDB makes the definition of the ua even trickier.

The IAU 2009 System of astronomical constants

Defining Constants

Constant	Description	Value	Reference
		Natural Defining Constant	
c	Speed of light	$2.997\,924\,58 \times 10^{8}$ ms^{-1}	
		Auxiliary Defining Constants	
k[1]	Gaussian gravitational constant	$1.720\,209\,895 \times 10^{-2}$	[14, 11]
L_G	1-d(TT)/d(TCG)	$6.969\,290\,134 \times 10^{-10}$	[15, 25]
L_B	1-d(TDB)/d(TCB)	$1.550\,519\,768 \times 10^{-8}$	[16]
TDB_0[2]	Bias between TDB and TCB	-6.55×10^{-5} s	[16]
θ_0[3]	Earth rotation angle at J2000.0	0.779 057 273 264 0 revolutions	[15, 4]
$d\theta/dt$[3]	Rate of advance of Earth rotation angle	1.002 737 811 911 354 48 revolutions UT1-day^{-1}	[15, 4]

CODATA, 2006

Current Best Estimates

Constant	Description	Value	Uncertainty	Reference
		Natural Measurable Constants		
G	Constant of gravitation	$6.674\,28 \times 10^{-11}$ $m^3kg^{-1}s^{-2}$	6.7×10^{-15} $m^3kg^{-1}s^{-2}$	
		Derived Constants		
au[4]	Astronomical unit	$1.495\,978\,707\,00 \times 10^{11}$ m	3 m	
GM_S	Heliocentric gravitational constant	$1.327\,124\,420\,99 \times 10^{20}$ m^3s^{-2} (TCB-compatible) $1.327\,124\,400\,41 \times 10^{20}$ m^3s^{-2} (TDB-compatible)	1.0×10^{10} m^3s^{-2} (TCB-compatible) 1.0×10^{10} m^3s^{-2} (TDB-compatible)	

CODATA, 2006

Pitjeva & Standish 2009, *CMDA* 103, 365

Folkner et al. 2008, from the ua fitted to DE 421: Memorandum IOM 343R-08-003

Proposal

- As already suggested before the 2009 IAU General assembly, it is proposed to re-define the ua as a fixed number of SI metres through a defining constant.
- This would mean:
  - dropping the k constant and abandoning the experimental determination of the ua in SI unit,
  - determining experimentally GM_{Sun}.
- Such a change of status for the ua would limit its role to that of a unit of length of "convenient" size for some applications.
- The defining number to be adopted for the conventional definition of the ua should be, for continuity reason, the value for the current best estimate of the ua in m as adopted by IAU 2009 Resolution B2 (i.e. ua = 1.495 978 707 00 x 10^{11} m exactly).

The CCU declared its support to move to a fixed relationship to the SI metre through a defining number

Conclusion

- A re-definition of the ua is necessary in the modern context in order to make the system of astronomical constants best compliant with modern dynamical astronomy.

- From the point of view of the principles, the important point is the change of status for the astronomical unit of length (and not the value of its defining number).

- Such a change of status of the ua:
  - would be a great simplification for the users of the astronomical constants,
  - will let possible variations of the mass of the Sun (and/or *G*) to appear directly (which is the option that has the most physical meaning), and
  - would avoid an unnecessary deviation from the SI.

- This should be largely discussed within the astronomical community in order to be proposed at the next IAU GA (2012).

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